Does the CAPM Predict Returns?*

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ABSTRACT

We provide strong empirical evidence that asset returns can be predicted using the dynamic CAPM. Indeed, the predictive power of the market return predictor transmits to the product of the asset's conditional beta and the market expected return. The dynamic CAPM yields a monthly out-of-sample R^2 of about 4% across all test assets, which is of the same order of magnitude as the out-of-sample R^2 obtained by predicting market returns using the market return predictor. Strategies exploiting the predictive power of the dynamic CAPM have Sharpe ratios up to 100% larger than those of the corresponding buy-and-hold strategies.

JEL Classification: D53, G11, G12

Keywords: capital asset pricing model, predictability, cross-section of stock returns

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1. Introduction

The static capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965a,b), and Mossin (1966), a Nobel Prize winning theory, states that an asset expected excess return should be equal to the product of the asset's exposure to market risk (beta) and the market expected excess return. Graham and Harvey (2001) show that the CAPM is widely used in the financial industry to compute the cost of capital, while Berk and van Binsbergen (2016) provide evidence that investors use the CAPM to time the market. Although the CAPM has been and will continue to be taught in business schools for a long time, Black, Jensen, and Scholes (1972) and Fama and French (1992, 2004) document that it is unequivocally rejected by the data.

With the hope of providing more realistic theoretical predictions, Merton (1973) extends the original CAPM framework by considering a dynamic version of it. A particularly interesting prediction of this dynamic model is that if investors' hedging motives are nil, then the CAPM holds but in a dynamic fashion. That is, the time-t expected future excess return of an asset is equal to the time-t beta of the asset times the time-t expected future excess return of the market. Using realized excess returns to perform their tests, Hasler and Martineau (2019) show that the dynamic CAPM cannot be rejected by the data for a large panel of test assets at both the daily and monthly frequencies.

Building on these findings, this paper provides evidence that the dynamic CAPM has particularly strong predictive power for the next period return of a large cross-section of test assets both in- and out-of-sample. In particular, we show that the monthly out-of-sample R^2 obtained using the dynamic CAPM is about 4% over the past 25 years, which is substantial. Indeed, such out-of-sample predictive power implies that the Sharpe ratio of a strategy exploiting return predictability can be up to 100% larger than the Sharpe ratio of the corresponding buy-and-hold strategy.

To use the dynamic CAPM as a predictive model, we need to estimate at each point in time using past data only the expected next period market excess return and the conditional beta of each test asset. As a predictor of the next month market excess return, we choose the variance risk premium (VRP) because of its particularly high predictive power (Bollerslev, Tauchen, and Zhou, 2009). Indeed, we show that regressing the next month market excess return on the VRP yields an in-sample R^2 of about 5% over the period 1990-2017, while the out-of-sample R^2 is about 4% over the period 1995-2017.[1] In what follows, we call "expected market return" the fitted value obtained from the aforementioned regression. To estimate the conditional monthly betas of the test assets, we follow Hasler and Martineau (2019) and use a 24-month rolling window of past monthly excess returns. Our test assets consists of ten beta-sorted portfolios constructed using monthly data from CRSP (see, e.g., Black et al., 1972, Savor and Wilson, 2014, Ben-Rephael, Carlin, Da, and Israelsen, 2018, Hendershott, Livdan, and Rosch, 2018, Jylha, 2018) as well as 24 (4×6) sorted portfolios formed on size-and-book-to-market, size-and-investment, size-and-profitability, and size-and-momentum obtained from Kenneth French's website.

We first show that the dynamic CAPM has strong in-sample predictive power. Indeed, regressing the next month asset return on the product of the current expected market excess return and current beta of the asset yields an R^2 of about 5%. This result shows that the in-sample predictive power of the VRP for the market excess return transmits to the entire cross-section of test assets via the dynamic CAPM. Furthermore, the regression features an intercept that is statistically insignificant and a slope that is not statistically different from one, as predicted by the dynamic

[1] The VRP is available at the monthly frequency from 1990 to 2017 on Hao Zhou's website. To perform the out-of-sample analysis, we consider a 5-year burn-in (estimation) period from 01/1990 to 12/1994.

CAPM. Overall, the in-sample analysis suggests that the dynamic CAPM is also likely to have strong out-of-sample predictive power.

To gauge the out-of-sample predictive power of the dynamic CAPM, we compute the out-of-sample (OOS) R^2 statistics (Welch and Goyal, 2007, Campbell and Thompson, 2008) for each test asset separately and by pooling assets together. The OOS R^2 measures the model's ability to outperform the benchmark historical mean at predicting returns out-of-sample. A positive (resp., negative) OOS R^2 means that the predictive model performs better (resp., worse) than the historical mean. Predicting the next month asset return using the product of the current expected market return and current beta of the asset yields OOS R^2 of about 4%, which is substantial. In comparison, over roughly the same period of time and using the same test assets, Gu, Kelly, and Xiu (2019) obtain a monthly OOS R^2 of at most 1.9% across both their different portfolios and predictive machine learning methods.

We then show that the out-of-sample predictive power of the dynamic CAPM translates into large investment gains. Indeed, the Sharpe ratios of trading strategies exploiting return predictability are up to 100% larger than those of the corresponding buy-and-hold strategies. Furthermore, most trading strategies' Sharpe ratios are substantially larger than the Sharpe ratio of the market portfolio. In particular, strategies investing in portfolios for which the dynamic CAPM has particularly strong out-of-sample predictive power have Sharpe ratios up to 50% larger that of the market portfolio.

Following Black et al. (1972), Savor and Wilson (2014), Ben-Rephael et al. (2018), Hendershott et al. (2018), and Jylha (2018) among others, we finally estimate the security market line (SML) for the ten beta-sorted portfolios, but also for the ten trading strategies exploiting the predictive power of the dynamic CAPM and investing in the respective ten beta-sorted portfolios. Consistent with existing findings, regressing the average excess returns of the ten beta-sorted portfolios on their respective unconditional betas yields an economically large and statistically significant intercept and a statistically insignificant slope. This confirms that the static CAPM of Sharpe (1964), Lintner (1965a,b), and Mossin (1966) does not hold when tested on the ten beta-sorted portfolios. However, a particularly interesting result is that the security market line features a statistically insignificant intercept and a slope that is not statistically different from the average excess return of the market when tested on strategies exploiting the predictive power of the dynamic CAPM. In other words, there is a strong, positive relationship between the risk (unconditional beta) and the return of trading strategies exploiting return predictability, as predicted by the static CAPM.

We believe that this finding could explain why mutual fund investors use the static CAPM to make their capital allocation decisions (Berk and van Binsbergen, 2016). Indeed, since actively managed funds are likely to exploit return predictability, the relationship between the risk (unconditional beta) and the return of these funds should be positive, as documented in our paper. Therefore, the static CAPM should indeed be the benchmark model used to properly allocate capital into actively managed funds.

This paper is closely related to the literature focusing on return predictability. Welch and Goyal (2007) show that the market return predictors documented in the literature have actually no or only negligible out-of-sample predictive power. By imposing theoretically motivated restrictions on the regressions performed in Welch and Goyal (2007), Campbell and Thompson (2008) show that the out-of-sample predictive power of the predictors improves substantially. Cochrane (2008) and van Binsbergen and Koijen (2010) argue and show that market returns are positively and significantly predicted by dividend yields. Bollerslev et al. (2009) show that the variance risk premium is a powerful predictor of future market returns. Rapach, Strauss, and Zhou (2010) show that combining the market return forecasts of Welch and Goyal (2007) yields a large out-of-sample predictive power. Rapach, Strauss, and Zhou (2013) provide evidence that U.S. market returns predict future market returns of various industrialized countries. Huang, Jiang, Tu, and Zhou (2014), Rapach, Ringgenberg, and Zhou (2016), and Jiang, Lee, Martin, and Zhou (2019) show that investor sentiment, short interest, and manager sentiment, respectively, are powerful predictors of market returns. Martin (2016) and Martin and Wagner (2018) show that combining the SVIX measures of the market and of individual stocks yields predictors for both market returns and stock returns. Rossi (2018) and Gu et al. (2019) provide evidence that machine learning techniques are particularly valuable to predict both market returns and stock returns out-of-sample.

This paper is also closely related to the recent literature showing that the CAPM tends to hold in specific periods of time. Savor and Wilson (2014), Ben-Rephael et al. (2018), Jylha (2018), and Hendershott et al. (2018) show that the security market line is upward-sloping, as predicted by the static CAPM, during macro announcement days, in abnormally high attention periods, when borrowing constraints are weak, and overnight, respectively. Using a large panel of test assets, Hasler and Martineau (2019) show that the dynamic CAPM is supported by the data at both the daily and monthly frequency.

Our paper contributes to these two strands of literature by showing that the dynamic CAPM has strong predictive power both in- and out-of-sample. Indeed, we provide evidence that the predictive power of the market return predictor transmits to the entire cross-section of risky assets via the dynamic CAPM.

The remainder of the paper is organized as follows. Section 2 documents the predictive power of the dynamic CAPM, discusses the performance of trading strategies exploiting such predictive power, and compares the security market lines obtained for the original test assets and for the strategies exploiting predictability. Section 3 concludes, and the Appendix reports all derivations and supplementary material.

2. Empirical Results

In this section, we first provide evidence that the dynamic CAPM can strongly predict returns both in- and out-of-sample. Then, we show that strategies exploiting the out-of-sample predictive power of the dynamic CAPM yield Sharpe ratios that are up to 100% larger than the corresponding buy-and-hold strategies. Finally, we confirm existing findings that the security market line (SML) obtained with the test assets is flat. However, we show that the SML obtained with the strategies exploiting return predictability is markedly upward-sloping, as predicted by the static CAPM.

The dynamic CAPM satisfies

$$\mathbb{E}_t(r_{i,t+1}) = \beta_{it}\mathbb{E}_t(r_{M,t+1}), \tag{1}$$

where $\beta_{it} \equiv \frac{\mathrm{Cov}_t(r_{i,t+1}, r_{M,t+1})}{\mathrm{Var}_t(r_{M,t+1})}$ is the conditional beta of asset i at time t, and r_i and r_M are the excess returns of asset i and the market, respectively. Using realized excess returns to perform their tests, Hasler and Martineau (2019) provide empirical evidence that the dynamic CAPM is hard to reject and that conditional betas estimated using a 24-month rolling window of past monthly excess returns are accurate. This suggests that, conditional on having a predictor of market excess returns, the dynamic CAPM (1) should allow us to predict the next period return of an entire cross-section of risky assets.

2.1. Data

To empirically test the predictive ability of the dynamic CAPM in Equation (1), we retrieve monthly excess market returns, r_{Mt}, and risk-free rates from Kenneth French's website.[2] We follow Black et al. (1972), Savor and Wilson (2014), Hendershott et al. (2018), and Ben-Rephael et al. (2018) among others and construct ten monthly beta-sorted portfolios using U.S. common stocks that are identified in CRSP as having a share code of 10 or 11. Monthly betas for all stocks are estimated using 24-month rolling windows of past monthly excess returns. At the beginning of each month, stocks are sorted into one of the ten beta-deciles, and the monthly value-weighted excess return, r_{it}, of each decile portfolio i is computed. For each portfolio, we estimate monthly rolling betas, β_{it}, using the last 24 monthly excess returns. These ten beta-sorted portfolios represent our test assets.

As shown by Bollerslev et al. (2009), the variance risk premium is a powerful predictor of market excess returns. For this reason, we use the monthly variance risk premium, VRP_t, to estimate the conditional expectation of the next month's market excess return in Equation (1). The variance risk premium is obtained from Hao Zhou's website and is available at the monthly frequency from 01/1990 to 12/2017.[3]

2.2. In-Sample Predictability

To provide evidence that the CAPM can be used to predict returns, we start by estimating the dynamic CAPM (1) in-sample. The first step consists in obtaining market return forecasts using the variance risk premium (VRP thereafter). These

[2] Data source: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french
[3] Data source: https://sites.google.com/site/haozhouspersonalhomepage/

market return forecasts are defined as the fitted values of the following regression

$$r_{M,t+1} = c_1 + c_2 \times VRP_t + \epsilon_{t+1},$$

which we estimate in-sample.

Consistent with the findings of Bollerslev et al. (2009), Panel A of Table 1 shows that the next month's market return is positively related to the current month's VRP. The slope of the relationship is statistically significant at the 1% level and the R^2 is about 5%, which means that the VRP has high in-sample predictive power for the market return at the monthly frequency.

Panel B of Table 1 reports the estimated intercept and slope obtained by regressing the beta-sorted portfolio i's return at month $t+1$ on the product of its conditional beta at month t and the forecasted next month's market return as of month t. Consistent with the findings of Hasler and Martineau (2019), the intercept is not statistically different from zero and the slope is not statistically different from one. That is, the dynamic CAPM (1) cannot be rejected by the data. Furthermore, these results show that the dynamic CAPM is a powerful predictor of future returns in-sample, at the monthly frequency. Indeed, the R^2 is about 5% over the full sample period and about 3% when excluding the financial crisis. The rational for reporting the results by excluding the financial crisis is that the VRP drops sharply at the beginning of the crisis and bounces back up at the end of it. One could argue that this large drop and subsequent rebound generate most of the return predictability observed over the full sample. We show that this is not the case, as the results obtained by excluding the financial are similar to those obtained over the full sample.

Table 1
In-Sample Return Predictability for the Ten Beta-Sorted Portfolios

This table reports the in-sample predictions of the following regressions:

$$r_{M,t+1} = c_1 + c_2 \times VRP_t + \epsilon_{t+1} \text{ in Panel A,}$$
$$r_{i,t+1} = a + b[\beta_{i,t}\mathbb{E}_t(r_{M,t+1})] + \varepsilon_{i,t+1} \text{ in Panel B,}$$

where r_M is the excess return of the market, VRP is the variance risk premium, r_i is the excess return of the beta-sorted portfolio i, $\mathbb{E}_t(r_{M,t+1}) = c_1 + c_2 \times VRP_t$ is the in-sample forecasted market return using the VRP at month t (see Panel A for the estimation results), and β_{it} is the monthly rolling beta estimated using the past 24 monthly excess returns. The results are reported for two periods, 01/1990 to 12/2017 and from 01/1995 to 12/2017 using all months and excluding the 2008-2009 period of the financial crisis (FC). The standard errors are reported in parentheses and are calculated using Newey-West with 12 month lags in Panel A and using Driscoll-Kraay with 12 month lags in Panel B. ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively.

Panel A. Forecasting excess market return, $r_{M,t+1}$

	Full sample		Excluding the FC	
	1990-2017	1995-2017	1990-2017	1995-2017
Intercept	-0.001	-0.001	-0.000	0.000
	(0.003)	(0.003)	(0.003)	(0.004)
VRP_t	0.047***	0.049***	0.048***	0.051***
	(0.008)	(0.009)	(0.016)	(0.016)
R^2	0.05	0.06	0.03	0.04
N	335	275	311	251

Panel B. Forecasting excess portfolio return, $r_{i,t+1}$

	Full sample		Excluding the FC	
	1990-2017	1995-2017	1990-2017	1995-2017
Intercept	-0.001	-0.000	0.001	0.001
	(0.002)	(0.003)	(0.003)	(0.003)
$\beta_{i,t}\mathbb{E}_t(r_{M,t+1})$	1.054***	1.086***	0.966***	1.017***
	(0.197)	(0.200)	(0.330)	(0.352)
R^2	0.05	0.05	0.03	0.03
N	3,350	2,750	3,110	2,510

2.3. Out-of-Sample Predictability

While we have provided evidence that the CAPM has predictive power in-sample, it is now particularly important to show that the CAPM also has out-of-sample predictive power. The reason is that investors need out-of-sample predictive models to build their optimal investment strategies.

To construct an out-of-sample portfolio return predictor that relies on the CAPM, we proceed as follows. As in the previous section, we estimate the monthly rolling beta β_{it} of portfolio i using the past 24 monthly excess returns. Therefore, by definition, β_{it} is estimated out-of-sample. To obtain the out-of-sample market return forecast $\mathbb{E}_t(r_{M,t+1})$, we regress the market return $\{r_{Mu}\}_{u=2}^{t}$ onto a constant and the lagged VRP $\{VRP_u\}_{u=1}^{t-1}$. The out-of-sample market return forecast at month t is then defined as

$$\mathbb{E}_t(r_{M,t+1}) = \widehat{c}_{1t} + \widehat{c}_{2t} \times VRP_t, \tag{2}$$

where $\widehat{c}_{1t}$ and $\widehat{c}_{2t}$ are the aforementioned estimated intercept and slope, respectively. Finally, the out-of-sample return forecast of portfolio i at month t satisfies

$$\widehat{r}_{it} \equiv \mathbb{E}_t(r_{i,t+1}) = \beta_{it}\mathbb{E}_t(r_{M,t+1}) = \beta_{it}\left(\widehat{c}_{1t} + \widehat{c}_{2t} \times VRP_t\right). \tag{3}$$

Note that we choose a burn-in period of five years (from 01/1990 to 12/1994), which implies that the out-of-sample period is from 01/1995 to 12/2017.

As a preliminary illustration of the out-of-sample predictive power of the CAPM, we regress portfolio i's next month return on its forecast defined in Equation (3). The results are reported in Table 2. Column 1 shows that the intercept is statistically insignificant, the slope is statistically indistinguishable from one, and the R^2 is about

4%. This preliminary analysis, therefore, suggests that predicting portfolios' returns using the CAPM should yield fairly high out-of-sample R^2 (Welch and Goyal, 2007), as confirmed below. In Column 2 portfolio i's return is predicted using the CAPM in conjunction with the historical mean of the market return, whereas in column 3 portfolio i's return is predicted using the historical mean of portfolio i's return. Both columns shows that the intercept is economically large, the slope is insignificant, and the R^2 is zero. This shows that the CAPM in conjunction with the VRP is a far superior out-of-sample predictor of portfolio returns than both the CAPM in conjunction with the historical mean of the market return and the historical mean of the portfolio return.

To properly estimate the out-of-sample predictive ability of the CAPM, we follow Welch and Goyal (2007), Campbell and Thompson (2008), Rapach et al. (2010), and Gu et al. (2019) among others and compute the following two out-of-sample R^2 (OOS R^2) statistics

$$R^2_{OOS} = 1 - \frac{MSFE_i}{MSFE_i^0}, \qquad R^2_{OOS,Null} = 1 - \frac{MSFE_i}{\frac{1}{N}\sum_{t=1}^{N} r^2_{i,t+1}},$$

where

$$MSFE_i = \frac{1}{N}\sum_{t=1}^{N}(r_{i,t+1} - \widehat{r}_{i,t})^2, \qquad MSFE_i^0 = \frac{1}{N}\sum_{t=1}^{N}(r_{i,t+1} - \overline{r}_{i,t})^2,$$

$\widehat{r}_{i,t} \equiv \mathbb{E}_t(r_{i,t+1})$ is portfolio i's return forecast defined in Equation (3), and $\overline{r}_{it} = \frac{1}{t}\sum_{u=1}^{t} r_{iu}$ is the historical mean of portfolio i's return. While R^2_{OOS} is the statistics that has been commonly used in the literature to evaluate the out-of-sample predictability of market excess returns, $R^2_{OOS,Null}$ has been suggested by Gu et al. (2019) to evaluate the out-of-sample predictability of individual stock returns as well

Table 2
Out-of-Sample Return Predictability for the Ten Beta-Sorted Portfolios

This table reports the estimates of the following regressions:

$$
\begin{aligned}
r_{i,t+1} &= a + b[\beta_{i,t}\mathbb{E}_t(r_{M,t+1})] + \varepsilon_{i,t}, \\
r_{i,t+1} &= a + b[\beta_{i,t}\overline{r}_{Mt}] + \varepsilon_{i,t}, \\
r_{i,t+1} &= a + b\overline{r}_{it} + \varepsilon_{i,t},
\end{aligned}
$$

where r_i is the excess return of the beta-sorted portfolio i and β_{it} is the monthly rolling beta estimated using the past 24 monthly excess returns. $\overline{r}_{Mt} = \frac{1}{t}\sum_{u=1}^{t} r_{Mu}$ is the historical mean of the market return and $\overline{r}_{it} = \frac{1}{t}\sum_{u=1}^{t} r_{iu}$ is the historical mean of portfolio i's return. $\mathbb{E}_t(r_{M,t+1})] = \widehat{c}_{1t} + \widehat{c}_{2t} \times VRP_t$ is the out-of-sample forecasted market return using the VRP at month t. The coefficients $\widehat{c}_{1t}$ and $\widehat{c}_{2t}$ are estimated by regressing the market return, $\{r_{Mu}\}_{u=2}^{t}$, onto a constant and the lagged VRP, $\{VRP_u\}_{u=1}^{t-1}$. The initial estimation period (burn-in period) is from 01/1990 to 12/1994. Therefore, the results reported in this table are for the period from 01/1995 to 12/2017. The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags. ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively.

Forecasting excess portfolio return, $r_{i,t+1}$

	(1)	(2)	(3)
Intercept	0.001	0.007**	0.008
	(0.003)	(0.003)	(0.009)
$\beta_{i,t}\mathbb{E}_t(r_{M,t+1})$	1.103***		
	(0.250)		
$\beta_{i,t} \times \overline{r}_{M,t}$		0.084	
		(0.580)	
$\overline{r}_{i,t}$			-0.162
			(1.350)
R^2	0.04	-0.00	-0.00
N	2,750	2,750	2,750

as portfolio returns. The rational of computing $R^2_{OOS,Null}$ instead of R^2_{OOS} is that, for certain individual stocks and certain portfolios, using a benchmark return forecast of zero performs better than using the historical mean.

Table 3 shows that the monthly OOS R^2 obtained across all portfolios is about 4% (resp., 5.5%) when computed against the historical mean (resp., a constant value of zero) as the benchmark forecast. These values are economically large and highly statistically significant based on Clark and West (2007) one-sided test. As a reference point, Gu et al. (2019), who evaluate the out-of-sample predictive performance of several machine learning methods, find that the monthly OOS R^2 is at most 1.9% across both their test portfolios and machine learning methods. Also, Martin and Wagner (2018) show that predicting individual stock returns using $SVIX$ measures yields monthly OOS R^2 of at most 1.4%. It is important to note that, although our out-of-sample testing period is not the same as that in Gu et al. (2019) or that in Martin and Wagner (2018), the three periods are comparable. Indeed, our out-of-sample period is from 1995 to 2017, whereas those in Martin and Wagner (2018) and Gu et al. (2019) are from 1996 to 2014 and from 1987 to 2016, respectively. When considering each portfolio separately, Table 3 shows that the OOS R^2 is economically large and statistically significant for eight out of ten portfolios. The bottom two decile portfolios are the only ones that do not feature significant return predictability. Interestingly, the OOS R^2s for the high beta portfolios (i.e., deciles 6 to 10) have the tendency to be larger than those for the low beta portfolios (i.e., deciles 1 to 5).

To highlight the economic value of the OOS R^2 obtained using the CAPM, we compute the following adjusted monthly Sharpe ratios

$$SR^* = \sqrt{\frac{SR^2 + R^2_{OOS}}{1 - R^2_{OOS}}}, \qquad SR^*_{Null} = \sqrt{\frac{SR^2 + R^2_{OOS,Null}}{1 - R^2_{OOS,Null}}},$$

where SR is the monthly Sharpe ratio of the buy-and-hold strategy, and R^2_{OOS} and $R^2_{OOS,Null}$ are the out-of-sample R^2 reported in Table 3. As in Gu et al. (2019),

Table 3
Out-of-Sample R^2 for the Ten Beta-Sorted Portfolios

This table reports the out-of-sample (OOS) R^2 across all portfolios and by portfolio separately. The OOS R^2 is calculated as follows:

$$
\begin{aligned}
MSFE_i &= \frac{1}{N}\sum_{t=1}^{N}(r_{i,t+1}-\widehat{r}_{i,t})^2, & MSFE_i^0 &= \frac{1}{N}\sum_{t=1}^{N}(r_{i,t+1}-\bar{r}_{i,t})^2,\\
R^2_{i,OOS} &= 1-\frac{MSFE_i}{MSFE_i^0}, & R^2_{i,OOS,Null} &= 1-\frac{MSFE_i}{\frac{1}{N}\sum_{t=1}^{N} r^2_{i,t+1}},\\
R^2_{All,OOS} &= 1-\frac{\sum_{i=1}^{10} MSFE_i}{\sum_{i=1}^{10} MSFE_i^0}, & R^2_{All,OOS,Null} &= 1-\frac{\sum_{i=1}^{10} MSFE_i}{\sum_{i=1}^{10}\frac{1}{N}\sum_{t=1}^{N} r^2_{i,t+1}}
\end{aligned}
$$

where $\widehat{r}_{i,t}=\mathbb{E}_t(r_{i,t+1})=\beta_{it}\mathbb{E}_t(r_{M,t+1})=\beta_{it}\left(\widehat{c}_{1t}+\widehat{c}_{2t}\times VRP_t\right)$ and $\bar{r}_{it}=\frac{1}{t}\sum_{u=1}^{t} r_{iu}$. Statistical significance (based on Clark and West (2007) one-sided test) at the 1%, 5%, and 10% levels is denoted by ***, **, and *, respectively. The initial estimation period (burn-in period) is from 01/1990 to 12/1994. Therefore, the results reported in this table are for the period from 01/1995 to 12/2017.

	R^2_{OOS}	$R^2_{OOS,Null}$
All portfolios	0.0391***	0.0543***
Low beta	0.0191	0.0185
2	-0.003	0.0245
3	0.0041**	0.0417**
4	0.0178***	0.0586***
5	0.0635***	0.1004***
6	0.0506***	0.0841***
7	0.0608***	0.0812***
8	0.0465**	0.0614**
9	0.0473***	0.0558***
High beta	0.0338***	0.0381***

the adjusted Sharpe ratio is defined as the theoretical Sharpe ratio of an optimal investment strategy implemented by a mean-variance investor who exploits return predictability. The derivation of the adjusted Sharpe ratio is provided in Appendix A.

Table 4
Buy-and-Hold and Adjusted Sharpe Ratios for the Ten Beta-Sorted Portfolios

The adjusted monthly Sharpe ratio is calculated as

$$SR^* = \sqrt{\frac{SR^2 + R^2_{OOS}}{1 - R^2_{OOS}}}, \qquad SR^*_{Null} = \sqrt{\frac{SR^2 + R^2_{OOS,Null}}{1 - R^2_{OOS,Null}}},$$

where SR is the monthly Sharpe ratio of the buy-and-hold strategy, and R^2_{OOS} and $R^2_{OOS,Null}$ are the out-of-sample R^2 reported in Table 3. The adjusted Sharpe ratio is defined as the theoretical Sharpe ratio of an optimal investment strategy implemented by a mean-variance investor who exploits return predictability (see Campbell and Thompson, 2008, Gu et al., 2019). The annualized Sharpe ratios reported in this table satisfy $SR_a = \sqrt{12} \times SR$, $SR^*_a = \sqrt{12} \times SR^*$, and $SR^*_{a,Null} = \sqrt{12} \times SR^*_{Null}$. The initial estimation period (burn-in period) is from 01/1990 to 12/1994. Therefore, the results reported in this table are for the period from 01/1995 to 12/2017.

	SR_a	SR^*_a	$SR^*_{a,Null}$	$\frac{SR^*_a - SR_a}{SR_a}$	$\frac{SR^*_{a,Null} - SR_a}{SR_a}$
Low beta	0.2429	0.5421	0.5351	1.2312	1.2025
2	0.5922	0.5602	0.8130	-0.0541	0.3727
3	0.7080	0.7435	1.0224	0.0501	0.4440
4	0.7411	0.8813	1.1534	0.1891	0.5563
5	0.7212	1.1701	1.3847	0.6224	0.9200
6	0.6820	1.0628	1.2687	0.5583	0.8604
7	0.5328	1.0388	1.1703	0.9496	1.1963
8	0.4514	0.8938	1.0010	0.9802	1.2177
9	0.3487	0.8505	0.9154	1.4390	1.6250
High beta	0.2561	0.6983	0.7372	1.7266	1.8784

Table 4 shows that the annualized adjusted Sharpe ratio is typically between 20% and 170% larger than the annualized Sharpe ratio of the corresponding buy-and-hold strategy. All strategies investing in portfolios with significant return predictability (i.e., decile portfolios 3 to 10) feature annualized adjusted Sharpe ratios that are between 20% and 100% larger than the annualized Sharpe ratio of the market portfolio, the latter being 0.57 over 1995-2017 sample period. Furthermore, the adjusted Sharpe ratios of the high beta portfolios are typically larger than those of the low beta portfolios. The reason is that the OOS R^2 obtained for the high beta portfolios tend to be larger than those obtained for the low beta portfolios (see Table 3).

2.4. Trading Strategies

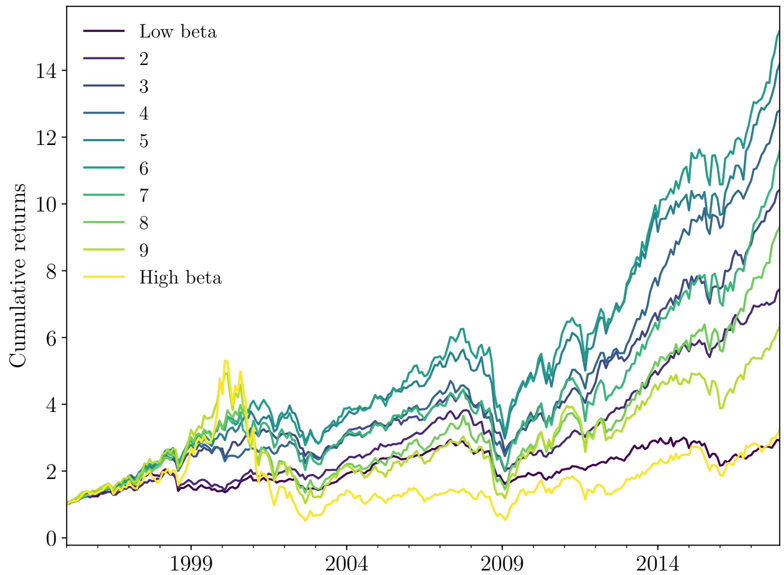
In this section, we build, implement, and discuss the performance of various trading strategies that exploit the out-of-sample return predictability documented in Section 2.3. We show that the Sharpe ratio of a trading strategy that exploits predictability is typically between 10% and 90% larger than that of the corresponding buy-and-hold strategy. The annualized Sharpe ratios of these strategies are between 0.6 and 0.8, which is larger than the Sharpe ratio of 0.57 obtained when buying and holding the market portfolio.

We start by plotting in Figure 1 the cumulative returns of buying and holding the different beta-sorted portfolios. Consistent with the findings of Frazzini and Pedersen (2014), the cumulative returns of the portfolios with low betas are typically larger than those of the portfolios with high betas. That is, from a pure risk neutral point of view, investing in low beta portfolios is more valuable than investing in high beta portfolios. Unsurprisingly, from a risk-averse point of view, the latter conclusion applies too. Indeed, the first column in panel A of Table 5 shows that the Sharpe ratios of the low beta portfolios are larger than those of the high beta portfolios.

We will now show that the relationship between the cumulative return of a simple trading strategy exploiting return predictability and the beta of the portfolio in which the strategy invests turns out to be positive. The strategy consists in going long 100% of wealth in portfolio i when the market return forecast, $\mathbb{E}_t(r_{M,t+1})$, defined in Equation (2) is above its historical (expanding) median, and invests 100% of wealth

Figure 1. Buy-and-Hold Returns of the Ten Beta-Sorted Portfolios

This figure plots the cumulative returns of the ten beta-sorted portfolios from from 1995 to 2017.



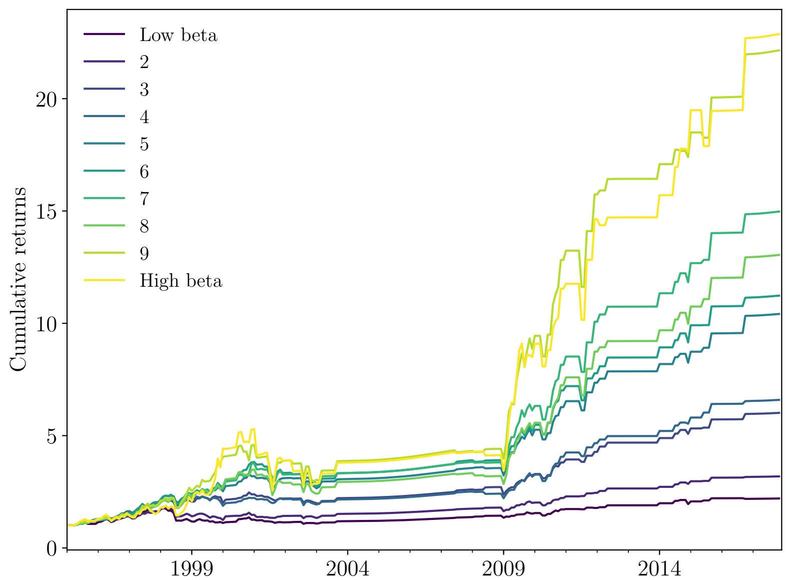
in the risk-free asset otherwise. The rational of this simple trading strategy is that investors should invest a large fraction of their wealth in the risky asset when the risky asset return forecast is high, whereas the fraction invested in the risky asset should be fairly small when the return forecast is low. Since the CAPM states that the risky asset return forecast is the product of beta and the market return forecast, the threshold used here to determine whether the risky asset return forecast is high or low is simply the historical median of the market return forecast.[4]

Figure 2 depicts the cumulative returns of the simple trading strategy described above. It shows that the cumulative returns of the strategies investing in high beta portfolios are larger than those of the strategies investing in low beta portfolios.

[4]Note that the results presented thereafter are similar if we use as a threshold the historical 33rd percentile of the market return forecast instead of the historical median.

Figure 2. Returns of a Simple Strategy Exploiting the Return Predictability of the Ten Beta-Sorted Portfolios

This figure plots the cumulative returns of a trading strategy that invests 100% of wealth in the beta portfolio i when the market return forecast, $\mathbb{E}_t(r_{M,t+1})$, defined in Equation (2) is above its historical (expanding) median, and invests 100% of wealth in the risk-free asset otherwise. The initial estimation period (burn-in period) is from 01/1990 to 12/1994. Therefore, the results reported in this table are for the period from 01/1995 to 12/2017.



More precisely, there is an almost perfect positive relationship between the strategy's average excess return and the beta of the portfolio in which it invests, as shown on the third column in panel B of Table 5. This shows that the betting against beta anomaly uncovered by Frazzini and Pedersen (2014) disappears, and the intuitive, theoretically motivated (Sharpe, 1964, Lintner, 1965b, Mossin, 1966), positive relationship between beta and the average excess return appears when return predictability is exploited. In other words, although the betting against beta anomaly clearly hurts investors who buy and hold high beta portfolios instead of low beta portfolios, an investor who properly exploits return predictability and is not afraid of taking some risk would still prefer to enter and exit high beta portfolios than low beta portfolios. The fourth column in Panel B of Table 5 also shows that the volatilities of the high beta strategies are unsurprisingly higher than those of the low beta strategies. However, consistent with the adjusted Sharpe ratios reported in the second column of Table 4, the second column in panel A of Table 5 shows that the realized Sharpe ratios of the high beta strategies tend to be larger. The reason is that the OOS R^2 obtained for the high beta portfolios are typically larger than those obtained for the low beta portfolios (see Table 3). Strategies investing in portfolios that feature significant return predictability (i.e., decile portfolios 3 to 10) have Sharpe ratios ranging between 0.57 and 0.79, which is larger than the Sharpe ratio of the market portfolio.

To provide further evidence that predicting returns using the CAPM is valuable, we consider the optimization problem faced by a mean-variance investor who has a single-period horizon and invests in both a risky asset i and a risk-free asset. The dynamic problem is

$$\max_{\{\omega_{it}\}} \mathbb{E}_t(r_{p,t+1}) - \frac{\gamma}{2}\mathrm{Var}_t(r_{p,t+1}), \tag{4}$$

where γ is the coefficient of relative risk aversion, ω_{it} is the fraction of wealth invested in risky asset i at time t, $r_{p,t+1} = r_{ft} + \omega_t r_{i,t+1}$ is the investor's portfolio return between time t and $t+1$, r_{ft} is the risk-free rate between time t and $t+1$, and $r_{i,t+1}$ is the risky asset excess return between time t and $t+1$. The first order condition yields the following optimal fraction of wealth invested in the risky asset at time t

$$\omega_{it} = \frac{\mathbb{E}_t(r_{i,t+1})}{\gamma \mathrm{Var}_t(r_{i,t+1})} \equiv \frac{\widehat{r}_{it}}{\gamma \sigma_{it}^2}. \tag{5}$$

Table 5
Trading Strategies' Summary Statistics

Panel A reports the Sharpe ratio and Panel B the excess return mean and volatility of the buy-and-hold strategies and of two trading strategies exploiting the predictive power of the CAPM. SR_A is the annualized Sharpe ratio of the buy-and-hold strategy. SR_{simple} is the annualized Sharpe ratio of a strategy that goes long 100% of wealth in portfolio i when the forecasted market return, $\mathbb{E}_t(r_{M,t+1})$, defined in Equation (2) is above its historical (expanding) median, and invests 100% of wealth in the risk-free asset otherwise. $SR_{dynamic}$ is the annualized Sharpe ratio of a strategy that invests a fraction of wealth at time t, $\omega^*_{it} = \max\left(\min\left(\frac{\widehat{r}_{it}}{\gamma\sigma^2_{it}}, 1\right), 0\right)$, in portfolio i and a fraction, $1-\omega^*_{it}$, in the risk-free asset. σ_{it} is the historical 5-year (rolling) volatility (Campbell and Thompson, 2008) of portfolio i's return. The coefficient of relative risk aversion is set to $\gamma = 3$. Similar to Campbell and Thompson (2008) and Rapach et al. (2010), we constrain the fraction of wealth invested in the risky asset to be between 0 and 1. The initial estimation period (burn-in period) is from 01/1990 to 12/1994. Therefore, the results reported in this table are for the period from 01/1995 to 12/2017.

Panel A. Beta-sorted portfolios: Sharpe ratios

	SR_a	SR_{simple}	$SR_{dynamic}$	$\frac{SR_{simple}-SR_a}{SR_a}$	$\frac{SR_{dynamic}-SR_a}{SR_a}$
Low beta	0.2429	0.1649	0.2407	-0.3211	-0.0093
2	0.5922	0.3474	0.5290	-0.4134	-0.1069
3	0.7080	0.6303	0.7800	-0.1098	0.1017
4	0.7411	0.6402	0.7520	-0.1362	0.0147
5	0.7212	0.7850	0.8426	0.0884	0.1684
6	0.6820	0.7364	0.7486	0.0798	0.0977
7	0.5328	0.7336	0.6412	0.3767	0.2033
8	0.4514	0.6514	0.5294	0.4432	0.1729
9	0.3487	0.6829	0.5233	0.9582	0.5007
High beta	0.2561	0.5687	0.4871	1.2203	0.9017

Panel B. Beta-sorted portfolios: Excess return mean and volatility

	Buy-and-hold		Simple		Dynamic	
	Mean	Std	Mean	Std	Mean	Std
Low beta	0.0027	0.0389	0.0014	0.0290	0.0021	0.0307
2	0.0060	0.0350	0.0026	0.0263	0.0042	0.0273
3	0.0072	0.0353	0.0050	0.0275	0.0064	0.0284
4	0.0081	0.0376	0.0054	0.0291	0.0067	0.0309
5	0.0086	0.0412	0.0071	0.0314	0.0081	0.0334
6	0.0090	0.0458	0.0075	0.0354	0.0080	0.0372
7	0.0085	0.0555	0.0088	0.0416	0.0081	0.0437
8	0.0082	0.0627	0.0084	0.0449	0.0072	0.0470
9	0.0078	0.0776	0.0109	0.0552	0.0080	0.0527
High beta	0.0075	0.1012	0.0122	0.0741	0.0089	0.0631

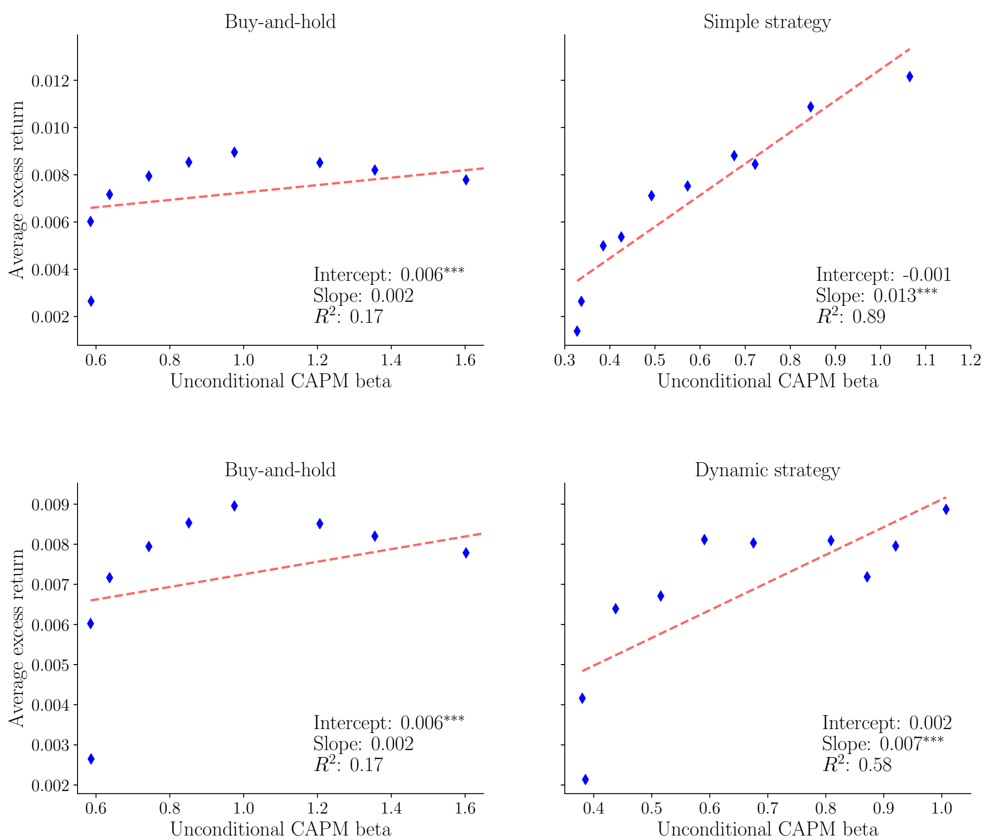
As in Campbell and Thompson (2008), Rapach et al. (2010), Huang et al. (2014), and Rapach et al. (2016) among others, the conditional excess return variance σ_{it}^2 is estimated using a 5-year rolling window of past excess returns. The coefficient of relative risk aversion is set to $\gamma = 3$. The excess return forecast $\widehat{r}_{it}$ is estimated using the CAPM, as described in Equation (3). To prevent short selling and leveraged positions when implementing the strategy, we follow Campbell and Thompson (2008), Rapach et al. (2010), and Neely, Rapach, Tu, and Zhou (2014) among others and constrain the fraction of wealth invested in the risky asset to lie between 0 and 1. That is,

$$\omega_{it}^* \equiv \max\left(\min\left(\omega_{it}, 1\right), 0\right) = \max\left(\min\left(\frac{\widehat{r}_{it}}{\gamma\sigma_{it}^2}, 1\right), 0\right). \tag{6}$$

The fifth and sixth columns in Panel B of Table 5 report the average excess return and excess return volatility of the mean-variance strategy described in Equation (6). The strategy's Sharpe ratio is reported in the third column of Panel A. Similar to the results obtained with the simple trading strategies described at the beginning of this section, the average excess return of a mean-variance strategy increases with the beta of the portfolio in which it invests. This again confirms that the betting against beta anomaly, which hurts investors who buy and hold high beta portfolios, disappears once return predictability is exploited. In other words, investors who exploit return predictability and who are not too risk averse prefer to dynamically enter and exit high beta portfolios than low beta portfolios. Except for the strategy investing in the bottom decile portfolio, all other strategies have large Sharpe ratios ranging between 0.5 and 0.85.

Figure 3. Security Market Line

This figure depicts the security market line for various sets of trading strategies over the 1995-2017 sample period. The security market line plots the average excess return of the strategies against the unconditional beta of the strategies. The two left panels plot the security market line for the ten beta-sorted portfolios (buy-and-hold strategy) on slightly different scales to make them comparable to the corresponding right panels. The upper right and lower right panels plot the security market lines for the simple strategies and mean-variance strategies, respectively. The simple and mean-variance strategies are described in Section 2.4



2.5. Security Market Line

Figure 3 depicts the security market line (SML) for the ten beta-sorted portfolios in the left panels and for the corresponding strategies exploiting return predictability in the right panels. Note that the two left panels plot the same SML but on slightly different scales to make the comparison with the right panels convenient.

The SML for the ten beta-sorted portfolios is flat, confirming existing findings (Black et al., 1972, Savor and Wilson, 2014, Ben-Rephael et al., 2018). Indeed, the intercept is economically large and statistically significant, whereas the slope is economically small and statistically insignificant. In clear contrast, the strategies exploiting the predictive power of the dynamic CAPM feature markedly upward-sloping SMLs, as predicted by the static CAPM (Sharpe, 1964, Lintner, 1965a,b, Mossin, 1966). More precisely, both SMLs on the right panels have positive and strongly statistically significant slopes together with statistically insignificant intercepts. Furthermore, we cannot reject the null hypothesis that the slope of the SML for the mean-variance strategies (bottom right panel) is equal to the average market excess return, which is 0.0071. Indeed, the test statistic yields a p-value of 0.92.

These findings can potentially explain why mutual fund investors use the static CAPM to make their capital allocation decisions (Berk and van Binsbergen, 2016). Since actively managed funds are likely to exploit return predictability, the relationship between the unconditional beta and the average return of these funds should be positive, as documented on the right panels of Figure 3. Consequently, the static CAPM should indeed be the benchmark model used by investors to properly invest into actively managed mutual funds, as documented by Berk and van Binsbergen (2016).

We also believe that the results in Figure 3 help understand the findings of Savor and Wilson (2014), Hendershott et al. (2018), and Ben-Rephael et al. (2018) that the SML is markedly upward-sloping on macroeconomic announcement days, overnight, and in abnormally high attention periods. Indeed, investors are more likely to accurately predict returns on macro announcement days, overnight, and when they are highly attentive to news because most macro announcements are released before markets open, little to no trading and especially no noise trading takes place overnight (Barclay and Hendershott, 2004), and news only influence prices if investors pay close attention to it (Huberman and Regev, 2001), respectively. As a consequence, asset returns on these days, which in equilibrium are driven by investors' trading strategies exploiting return predictability, are likely to satisfy the positive risk-return relationship depicted on the right panels of Figure 3.

2.6. Robustness: Fama-French Portfolios

In this section, we provide evidence that the predictive power of the CAPM is robust to the choice of the test assets. To do so, we consider the Fama-French 6 size-and-book-to-market-sorted portfolios, 6 size-and-investment-sorted portfolios, 6 size-and-profitability-sorted portfolios, and 6 size-and-momentum-sorted portfolios obtained from Kenneth French's website. We show that the OOS R^2 obtained using the CAPM as the predictive model range between 2.5% and 5.9%. As a result, the Sharpe ratios of optimal investment strategies exploiting return predictability are about 5% to 100% larger than the Sharpe ratios of the corresponding buy-and-hold strategies.

Table 6 shows the in-sample predictability results obtained by regressing portfolio i's next month excess return on the product of portfolio i's beta and the expected excess return of the market. As in the previous sections, the expected excess market return is the fitted value obtained from a regression of excess market returns on the 1-month lagged variance risk premium. The results for the size-and-book-to-market-sorted portfolios, the size-and-investment-sorted portfolios, the size-and-profitability-sorted portfolios, and the size-and-momentum-sorted portfolios are provided in Panels A to D, respectively.

Similar to what was obtained for the beta-sorted portfolios (see Table 1), the

Table 6
In-Sample Return Predictability for the Fama-French Portfolios

This table reports the in-sample predictions of the following regression

$$r_{i,t+1} = a + b[\beta_{i,t}\mathbb{E}_t(r_{M,t+1})] + \varepsilon_{i,t+1},$$

where r_i is the return of the Fama-French portfolio i, $\mathbb{E}_t(r_{M,t+1}) = c_1 + c_2 \times VRP_t$ is the in-sample forecasted market return using the VRP at month t (see Panel A in Table 1 for the estimation results), and β_{it} is the monthly rolling beta estimated using the past 24 monthly excess returns. The results for the size-and-book-to-market-sorted portfolios, the size-and-investment-sorted portfolios, the size-and-profitability-sorted portfolios, and the size-and-momentum-sorted portfolios are provided in Panels A to D, respectively. The results are reported for two periods, 01/1990 to 12/2017 and from 01/1995 to 12/2017 using all months and excluding the 2008-2009 period of the financial crisis (FC). The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags. ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively.

Panel A. Size-and-Book-to-Market-Sorted Portfolios
Forecasting excess portfolio return, $r_{i,t+1}$

	Full sample		Excluding the FC	
	1990-2017	1995-2017	1990-2017	1995-2017
Intercept	0.000	0.001	0.002	0.002
	(0.003)	(0.003)	(0.003)	(0.004)
$\beta_{i,t}\mathbb{E}_t(r_{M,t+1})$	1.071***	1.114***	0.969**	1.050**
	(0.210)	(0.209)	(0.411)	(0.441)
R^2	0.05	0.05	0.02	0.03
N	2,010	1,650	1,866	1,506

Panel B. Size-and-Investment-Sorted Portfolios
Forecasting excess portfolio return, $r_{i,t+1}$

	Full sample		Excluding the FC	
	1990-2017	1995-2017	1990-2017	1995-2017
Intercept	0.000	0.001	0.001	0.002
	(0.003)	(0.003)	(0.003)	(0.004)
$\beta_{i,t}\mathbb{E}_t(r_{M,t+1})$	1.064***	1.110***	1.023***	1.115***
	(0.204)	(0.205)	(0.396)	(0.421)
R^2	0.05	0.05	0.03	0.03
N	2,010	1,650	1,866	1,506

Table 6
In-Sample Return Predictability for the Fama-French Portfolios (Cont.)

Panel C. Size-and-Profitability-Sorted portfolios
Forecasting excess portfolio return, $r_{i,t+1}$

	Full sample		Excluding the FC	
	1990-2017	1995-2017	1990-2017	1995-2017
Intercept	-0.000	0.000	0.001	0.001
	(0.003)	(0.003)	(0.004)	(0.004)
$\beta_{i,t}\mathbb{E}_t(r_{M,t+1})$	1.059***	1.099***	0.992**	1.068**
	(0.209)	(0.210)	(0.406)	(0.436)
R^2	0.05	0.05	0.02	0.03
N	2,010	1,650	1,866	1,506

Panel D. Size-and-Momentum-Sorted Portfolios
Forecasting excess portfolio return, $r_{i,t+1}$

	Full sample		Excluding the FC	
	1990-2017	1995-2017	1990-2017	1995-2017
Intercept	-0.001	-0.000	0.001	0.001
	(0.003)	(0.003)	(0.003)	(0.004)
$\beta_{i,t}\mathbb{E}_t(r_{M,t+1})$	1.124***	1.154***	0.998**	1.050**
	(0.206)	(0.208)	(0.393)	(0.421)
R^2	0.05	0.05	0.02	0.03
N	2,010	1,650	1,866	1,506

intercept is not statistically different from zero and the slope of the relationship is not statistically different from one. In line with the findings of Hasler and Martineau (2019), these results show that the dynamic CAPM cannot be rejected by the data. Furthermore, the significance of the slope coefficient together with the magnitude of the R^2 suggest that the dynamic CAPM is a particularly valuable predictor of asset returns.

Table 7
Out-of-Sample R^2 for the Fama-French Portfolios

This table reports the out-of-sample (OOS) R^2 across all portfolios and by portfolio separately. The OOS R^2 is calculated as follows:

$$MSFE_i = \frac{1}{N}\sum_{t=1}^{N}(r_{i,t+1} - \widehat{r}_{i,t})^2, \qquad MSFE_i^0 = \frac{1}{N}\sum_{t=1}^{N}(r_{i,t+1} - \overline{r}_{i,t})^2,$$

$$R^2_{i,OOS} = 1 - \frac{MSFE_i}{MSFE_i^0}, \qquad R^2_{i,OOS,Null} = 1 - \frac{MSFE_i}{\frac{1}{N}\sum_{t=1}^{N} r^2_{i,t+1}},$$

$$R^2_{All,OOS} = 1 - \frac{\sum_{i=1}^{6} MSFE_i}{\sum_{i=1}^{6} MSFE_i^0}, \qquad R^2_{All,OOS,Null} = 1 - \frac{\sum_{i=1}^{6} MSFE_i}{\sum_{i=1}^{6} \frac{1}{N}\sum_{t=1}^{N} r^2_{i,t+1}}$$

where $\widehat{r}_{i,t} = \mathbb{E}_t(r_{i,t+1}) = \beta_{it}\mathbb{E}_t(r_{M,t+1}) = \beta_{it}\left(\widehat{c}_{1t} + \widehat{c}_{2t} \times VRP_t\right)$ and $\bar{r}_{it} = \frac{1}{t}\sum_{u=1}^{t} r_{iu}$. The results for the size-and-book-to-market-sorted portfolios, the size-and-investment-sorted portfolios, the size-and-profitability-sorted portfolios, and the size-and-momentum-sorted portfolios are provided in Panels A to D, respectively. Statistical significance (based on Clark and West (2007) one-sided test) at the 1%, 5%, and 10% levels is denoted by ***, **, and *, respectively. The initial estimation period (burn-in period) is from 01/1990 to 12/1994. Therefore, the results reported in this table are for the period from 01/1995 to 12/2017.

Panel A. Size-and-Book-to-Market-Sorted Portfolios

	R^2_{OOS}	$R^2_{OOS,Null}$
All portfolios	0.04***	0.0621***
Big Growth	0.0336***	0.0617***
Big Neutral	0.0336**	0.0593**
Big Value	0.0585**	0.0769**
Small Growth	0.0257**	0.0337**
Small Neutral	0.0456***	0.0774***
Small Value	0.0481**	0.0796**

Panel B. Size-and-Investment-Sorted Portfolios

	R^2_{OOS}	$R^2_{OOS,Null}$
All portfolios	0.0413***	0.0626***
Big Aggressive	0.0406**	0.053**
Big Conservative	0.0354***	0.0678***
Big Neutral	0.0458***	0.0754***
Small Aggressive	0.0328**	0.0411**
Small Conservative	0.0476***	0.07***
Small Neutral	0.0473**	0.0837**

Table 7
Out-of-Sample R^2 for the Fama-French Portfolios (Cont.)

Panel C. Size-and-Profitability-Sorted Portfolios

	R^2_{OOS}	$R^2_{OOS,Null}$
All portfolios	0.042***	0.0611***
Big Neutral	0.0479***	0.0653***
Big Robust	0.0356***	0.0696***
Big Weak	0.0466**	0.0512**
Small Neutral	0.0434**	0.0767**
Small Robust	0.0416**	0.0754**
Small Weak	0.0382***	0.0459***

Panel D. Size-and-Momentum-Sorted Portfolios

	R^2_{OOS}	$R^2_{OOS,Null}$
All portfolios	0.0393***	0.0579***
Big Down	0.0437**	0.0484**
Big Medium	0.0318***	0.0588***
Big Up	0.0384***	0.0709***
Small Down	0.0344**	0.0398**
Small Medium	0.0485**	0.0829**
Small Up	0.04***	0.0731***

Table 7 reports the estimate and statistical significance of the out-of-sample (OOS) R^2 obtained by predicting the returns of the various Fama-French portfolios using the dynamic CAPM. The OOS R^2 are particularly large, ranging between about 2.5% and 5.9% depending on the portfolio under consideration. The OOS R^2 of all portfolios are statistically significant at either the 1% or 5% level.

Table 8
Buy-and-Hold and Adjusted Sharpe Ratios for the Fama-French Portfolios

The adjusted monthly Sharpe ratio is calculated as

$$SR^* = \sqrt{\frac{SR^2 + R^2_{OOS}}{1 - R^2_{OOS}}}, \qquad SR^*_{Null} = \sqrt{\frac{SR^2 + R^2_{OOS,Null}}{1 - R^2_{OOS,Null}}},$$

where SR is the monthly Sharpe ratio of the buy-and-hold strategy, and R^2_{OOS} and $R^2_{OOS,Null}$ are the out-of-sample R^2 reported in Table 3. The adjusted Sharpe ratio is defined as the theoretical Sharpe ratio of an optimal investment strategy implemented by a mean-variance investor who exploits return predictability (see Campbell and Thompson, 2008, Gu et al., 2019). The annualized Sharpe ratios reported in this table satisfy $SR_a = \sqrt{12} \times SR$, $SR^*_a = \sqrt{12} \times SR^*$, and $SR^*_{a,Null} = \sqrt{12} \times SR^*_{Null}$. The results for the size-and-book-to-market-sorted portfolios, the size-and-investment-sorted portfolios, the size-and-profitability-sorted portfolios, and the size-and-momentum-sorted portfolios are provided in Panels A to D, respectively. The initial estimation period (burn-in period) is from 01/1990 to 12/1994. Therefore, the results reported in this table are for the period from 01/1995 to 12/2017.

Panel A. Size-and-Book-to-Market-Sorted Portfolios

	SR_a	SR^*_a	$SR^*_{a,Null}$	$\frac{SR^*_a - SR_a}{SR_a}$	$\frac{SR^*_{a,Null} - SR_a}{SR_a}$
Big Growth	0.6236	0.9053	1.0970	0.4518	0.7593
Big Neutral	0.5912	0.8826	1.0621	0.4928	0.7964
Big Value	0.5164	1.0143	1.1352	0.9642	1.1981
Small Growth	0.3326	0.6558	0.7301	0.9716	1.1949
Small Neutral	0.6486	1.0071	1.2094	0.5526	0.8646
Small Value	0.6585	1.0305	1.2284	0.5649	0.8654

Panel B. Size-and-Investment-Sorted Portfolios

	SR_a	SR^*_a	$SR^*_{a,Null}$	$\frac{SR^*_a - SR_a}{SR_a}$	$\frac{SR^*_{a,Null} - SR_a}{SR_a}$
Big Aggressive	0.4701	0.8592	0.9513	0.8275	1.0235
Big Conservative	0.6819	0.9605	1.1712	0.4084	0.7174
Big Neutral	0.7010	1.0445	1.2288	0.4900	0.7530
Small Aggressive	0.3641	0.7376	0.8079	1.0256	1.2185
Small Conservative	0.5589	0.9632	1.1131	0.7234	0.9918
Small Neutral	0.7092	1.0601	1.2826	0.4947	0.8085

Table 8
Buy-and-Hold and Adjusted Sharpe Ratios for the Fama-French Portfolios (Cont.)

Panel C. Size-and-Profitability-Sorted Portfolios

	SR_a	SR^*_a	$SR^*_{a,Null}$	$\frac{SR^*_a - SR_a}{SR_a}$	$\frac{SR^*_{a,Null} - SR_a}{SR_a}$
Big Neutral	0.5330	0.9498	1.0688	0.7821	1.0053
Big Robust	0.7357	1.0021	1.2163	0.3621	0.6532
Big Weak	0.2956	0.8235	0.8600	1.7861	1.9096
Small Neutral	0.6843	1.0168	1.2264	0.4859	0.7921
Small Robust	0.6866	1.0063	1.2200	0.4658	0.7770
Small Weak	0.3530	0.7786	0.8414	1.2055	1.3834

Panel D. Size-and-Momentum-Sorted Portfolios

	SR_a	SR^*_a	$SR^*_{a,Null}$	$\frac{SR^*_a - SR_a}{SR_a}$	$\frac{SR^*_{a,Null} - SR_a}{SR_a}$
Big Down	0.2846	0.7956	0.8339	1.7958	1.9303
Big Medium	0.6092	0.8817	1.0695	0.4474	0.7558
Big Up	0.6630	0.9676	1.1785	0.4595	0.7775
Small Down	0.2893	0.7171	0.7646	1.4788	1.6430
Small Medium	0.6773	1.0458	1.2589	0.5441	0.8587
Small Up	0.6678	0.9821	1.1948	0.4707	0.7891

Table 8 shows that the OOS R^2 reported in Table 7 translate into large investment gains, as measured by the theoretical (adjusted) Sharpe ratio SR^* of a strategy implemented by a mean-variance investor who optimally exploits return predictability. Indeed, the adjusted Sharpe ratios are between 35% and 180% larger than the Sharpe ratios of the corresponding buy-and-hold strategies.

Table 9
Trading Strategies' Sharpe Ratios for the Fama-French Portfolios

This table reports the Sharpe ratio of the buy-and-hold strategies and of two trading strategies exploiting the predictive power of the CAPM for the Fama-French Portfolios. The results for the size-and-book-to-market-sorted portfolios, the size-and-investment-sorted portfolios, the size-and-profitability-sorted portfolios, and the size-and-momentum-sorted portfolios are provided in Panels A to D, respectively. SR_A is the annualized Sharpe ratio of the buy-and-hold strategy. SR_{simple} is the annualized Sharpe ratio of a strategy that goes long 100% of wealth in portfolio i when the forecasted market return, $\mathbb{E}_t(r_{M,t+1})$, defined in Equation (2) is above its historical (expanding) median, and invests 100% of wealth in the risk-free asset otherwise. $SR_{dynamic}$ is the annualized Sharpe ratio of a strategy that invests a fraction of wealth at time t, $\omega^*_{it} = \max\left(\min\left(\frac{\widehat{r}_{it}}{\gamma\sigma^2_{it}}, 1\right), 0\right)$, in portfolio i and a fraction, $1-\omega^*_{it}$, in the risk-free asset. σ_{it} is the historical 5-year (rolling) volatility (Campbell and Thompson, 2008) of portfolio i's return. The coefficient of relative risk aversion is set to $\gamma = 3$. Similar to Campbell and Thompson (2008) and Rapach et al. (2010), we constrain the fraction of wealth invested in the risky asset to be between 0 and 1. The initial estimation period (burn-in period) is from 01/1990 to 12/1994. Therefore, the results reported in this table are for the period from 01/1995 to 12/2017.

Panel A. Size-and-Book-to-Market-Sorted portfolios

	SR_a	SR_{simple}	$SR_{dynamic}$	$\frac{SR_{simple}-SR_a}{SR_a}$	$\frac{SR_{dynamic}-SR_a}{SR_a}$
Big Growth	0.6236	0.7743	0.7191	0.2418	0.1532
Big Neutral	0.5912	0.6022	0.6434	0.0186	0.0883
Big Value	0.5164	0.5910	0.6129	0.1444	0.1869
Small Growth	0.3326	0.5183	0.4624	0.5582	0.3901
Small Neutral	0.6486	0.7376	0.7320	0.1372	0.1284
Small Value	0.6585	0.7207	0.7039	0.0944	0.0690

Panel B. Size-and-Investment-Sorted Portfolios

	SR_a	SR_{simple}	$SR_{dynamic}$	$\frac{SR_{simple}-SR_a}{SR_a}$	$\frac{SR_{dynamic}-SR_a}{SR_a}$
Big Aggressive	0.4701	0.6995	0.6093	0.4879	0.2959
Big Conservative	0.6819	0.7026	0.7365	0.0303	0.0801
Big Neutral	0.7010	0.7596	0.7712	0.0836	0.1002
Small Aggressive	0.3641	0.5255	0.4352	0.4430	0.1951
Small Conservative	0.5589	0.7280	0.6867	0.3027	0.2287
Small Neutral	0.7092	0.7718	0.7749	0.0883	0.0926

Table 9
Trading Strategies' Sharpe Ratios for the Fama-French Portfolios (Cont.)

Panel C. Size-and-Profitability-Sorted Portfolios

	SR_a	SR_{simple}	$SR_{dynamic}$	$\frac{SR_{simple}-SR_a}{SR_a}$	$\frac{SR_{dynamic}-SR_a}{SR_a}$
Big Neutral	0.5330	0.6708	0.6221	0.2586	0.1673
Big Robust	0.7357	0.7833	0.8011	0.0646	0.0888
Big Weak	0.2956	0.6049	0.4388	1.0467	0.4847
Small Neutral	0.6843	0.7294	0.7337	0.0658	0.0722
Small Robust	0.6866	0.7168	0.7040	0.0440	0.0254
Small Weak	0.3530	0.5593	0.5082	0.5843	0.4395

Panel D. Size-and-Momentum-Sorted Portfolios

	SR_a	SR_{simple}	$SR_{dynamic}$	$\frac{SR_{simple}-SR_a}{SR_a}$	$\frac{SR_{dynamic}-SR_a}{SR_a}$
Big Down	0.2846	0.4887	0.3205	0.7171	0.1264
Big Medium	0.6092	0.6509	0.6753	0.0686	0.1086
Big Up	0.6630	0.7991	0.7822	0.2053	0.1797
Small Down	0.2893	0.4612	0.3055	0.5942	0.0560
Small Medium	0.6773	0.7408	0.7270	0.0937	0.0733
Small Up	0.6678	0.8116	0.7659	0.2154	0.1468

Table 9 reports the realized Sharpe ratios of two strategies exploiting return predictability. As in the previous sections, the first simple strategy consists in going long 100% of wealth in portfolio i if the market return forecast is above its historical (expanding) median, and long 100% in the risk-free asset otherwise. The second strategy is that implemented by a mean-variance investor who can invest in portfolio i and the risk-free asset, who has a single-period horizon, and who optimally exploits return predictability (refer to Equations (4), (5), and (6) for more details).

The fourth column of Table 9 shows that the Sharpe ratios of the simple strategies are always larger than those obtained when buying and holding the corresponding portfolios. They are between 2% and 105% larger than the Sharpe ratios of the corresponding buy-and-hold strategies. Similarly, the Sharpe ratios of the optimal mean-variance strategies are all larger than those obtained when buying and holding the corresponding portfolios. These Sharpe ratios are between 3% and 50% larger than their buy-and-hold counterparts, which is substantial.

3. Conclusion

This paper provides evidence that the dynamic CAPM is a particularly valuable predictor of asset returns. We use the variance risk premium as the predictor of future market returns and estimate the conditional betas of our test assets using a 24-month rolling window of past returns. The out-of-sample R^2 obtained by predicting future monthly excess returns of various test assets using the CAPM is about 4% over the 1995-2017 sample period. The substantial predictive power of the CAPM translates into large investment gains. Indeed, the Sharpe ratios of strategies exploiting the predictive power of the CAPM are up to 100% larger than those of the corresponding buy-and-hold strategies.

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Appendix

A. Derivation of the Adjusted Sharpe Ratio

Following Campbell and Thompson (2008), let us assume the following model for the excess return of the risky asset

$$r_{t+1} = \mu + x_t + \epsilon_{t+1},$$

where μ is the unconditional expected excess return, x_t has mean zero and variance σ_x^2, ϵ_{t+1} has mean zero and variance σ_ϵ^2, and x_t in independent of ϵ_{t+1}.

A mean-variance investor who has a single-period horizon and who can invest in a risk-free asset and a risky asset solves the following problem

$$\max_{\{\omega_{it}\}} \mathbb{E}_t(r_{p,t+1}) - \frac{\gamma}{2}\mathrm{Var}_t(r_{p,t+1}),$$

where γ is the coefficient of relative risk aversion, ω_t is the fraction of wealth invested in the risky asset at time t, $r_{p,t+1} = r_{ft} + \omega_t r_{t+1}$ is the investor's portfolio return between time t and $t+1$, r_{ft} is the risk-free rate between time t and $t+1$, and $r_{i,t+1}$ is the risky asset excess return between time t and $t+1$. The first order condition yields the following optimal fraction of wealth invested in the risky asset at time t

$$\omega_t = \frac{\mathbb{E}_t(r_{t+1})}{\gamma \mathrm{Var}_t(r_{t+1})} = \frac{\mu + x_t}{\gamma \sigma_\epsilon^2}.$$

Therefore, the unconditional expected excess return of the strategy satisfies

$$\begin{aligned}\mathbb{E}(r_{p,t+1} - r_{ft}) &= \mathbb{E}(\mathbb{E}_t(\omega_t r_{t+1})) \\ &= \frac{\mathbb{E}((\mu + x_t)^2)}{\gamma \sigma_\epsilon^2} \\ &= \frac{\mu^2 + \sigma_x^2}{\gamma \sigma_\epsilon^2} \\ &= \frac{(SR)^2 + R^2}{\gamma(1 - R^2)},\end{aligned}$$

where $SR = \frac{\mu}{\sqrt{\sigma_x^2 + \sigma_\epsilon^2}}$ is the Sharpe ratio of the risky asset and $R^2 = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\epsilon^2}$ is the R^2 obtained by regressing the excess return r_{t+1} on the predictor x_t.

The expected conditional excess return variance of the strategy satisfies

$$\begin{aligned}\mathbb{E}(\mathrm{Var}_t(r_{p,t+1} - r_{ft})) &= \mathbb{E}(\mathrm{Var}_t(\omega_{it} r_{i,t+1})) \\ &= \mathbb{E}(\omega_{it}^2 \sigma_\epsilon^2) \\ &= \frac{\mu^2 + \sigma_x^2}{\gamma^2 \sigma_\epsilon^2} \\ &= \frac{(SR)^2 + R^2}{\gamma^2(1 - R^2)}.\end{aligned}$$

Gu et al. (2019) define the adjusted Sharpe ratio of the strategy as follows:

$$\begin{aligned}SR^* &\equiv \frac{\mathbb{E}(\mathbb{E}_t(r_{p,t+1} - r_{ft}))}{\mathbb{E}(\mathrm{Var}_t(r_{p,t+1} - r_{ft}))} \\ &= \sqrt{\frac{(SR)^2 + R^2}{(1 - R^2)}}.\end{aligned}$$