

The CAPM Holds *

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ABSTRACT

When investors' hedging demands are equal to zero, the conditional risk premium of an asset is equal to its conditional market beta times the conditional risk premium of the market (Merton, 1973). We empirically test this dynamic CAPM relation on portfolios and individual stocks using both daily and monthly returns. We show that regressing an asset excess return onto the product of its conditional beta and the market excess return yields an intercept of zero, a slope of one, and an R^2 of about 80%. That is, the data lend support to the null hypothesis that the dynamic CAPM holds.

JEL Classification: D53, G11, G12

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1. Introduction

The capital asset pricing model (CAPM) of [Sharpe \(1964\)](#), [Lintner \(1965a,b\)](#), and [Mossin \(1966\)](#), which allowed William F. Sharpe to win the 1990 Nobel Prize in economics, is the most famous and influential pricing relation that has ever been discovered. It states that the risk premium of an asset is equal to the asset's exposure to market risk (beta) times the risk premium of the market. As of today, the CAPM has been taught in business schools for more than fifty years, and it is commonly used by practitioners and investors to compute the cost of capital ([Graham and Harvey, 2001](#)) and to build investment strategies ([Berk and van Binsbergen, 2016](#)).

Despite its popularity, [Black, Jensen, and Scholes \(1972\)](#) and [Fama and French \(1992, 2004\)](#) document that the CAPM is actually not supported by the data. Indeed, the security market line, which plots assets' expected returns as a function of their betas, is flat, whereas the CAPM predicts that it should be positive. Interestingly, [Tinic and West \(1984\)](#), [Cohen, Polk, and Vuolteenaho \(2005\)](#), [Savor and Wilson \(2014\)](#), [Hendershott, Livdan, and Rosch \(2018\)](#), and [Jylha \(2018\)](#) provide evidence that the CAPM holds in January, on months of low inflation, on days of important macroeconomic announcements, overnight, and on months during which investors can borrow easily, respectively. That is, there are specific periods during which the data can not reject the CAPM.

In this paper, we first argue theoretically that, under some condition, the CAPM holds but in a dynamic manner. Indeed, when investors' hedging demands are equal to zero, the time- t risk premium of an asset is equal to its time- t beta times the time- t risk premium of the market ([Merton, 1973](#)). Second, we empirically test this dynamic CAPM relation and show that the data lend strong support to it. In particular, our panel regression analysis shows that regressing an asset excess return onto the

product of its conditional beta and the market excess return yields (i) an intercept that is economically and statistically indistinguishable from zero, (ii) a slope that is economically and statistically indistinguishable from one, and (iii) an adjusted R^2 of about 80%. In other words, the data cannot reject the null hypothesis that the time- t expected future excess return of an asset is equal to the time- t beta of the asset times the time- t expected future excess return of the market.

Our theoretical motivation is borrowed from [Merton \(1973\)](#), who considers a continuous time economy populated by agents that can invest in n stocks and one risk-free asset paying a stochastic risk-free rate. Agents have homogeneous beliefs about the instantaneous expected return and return volatility of each stock, which are assumed to be stochastic. [Merton \(1973\)](#) shows that if agents' hedging demands are equal to zero, then the conditional risk premium of a stock is equal to its conditional beta times the conditional risk premium of the market. Note that hedging demands are equal to zero if either agents have logarithmic preferences, or the investment opportunity set is constant, or changes in the state variables are uncorrelated to stock returns, or changes in the state variables are correlated to stock returns in such a way that the sum of all hedging components is equal to zero. If either one of these conditions is satisfied, then the model predicts that performing a panel regression of excess stock returns onto the product of the conditional betas and the market excess returns should provide an intercept equal to zero and a slope equal to one. In addition, the regression R^2 is predicted to be large if stocks' idiosyncratic volatilities are low.

We test the predictions of the model using monthly and daily U.S. stock return data from 1926 to 2017. Our test assets include ten CAPM beta-sorted portfolios, the Fama-French 25 size-and-book-to-market-sorted portfolios, ten industry-sorted portfolios, among other test assets, as well as individual stocks.

As in [Martin and Wagner \(2018\)](#), our empirical tests are performed using panel regressions. The model predicts that regressing an asset return onto the product of its conditional beta and the market excess return should provide an intercept equal to zero and a slope equal to one. Using monthly returns, the product of the conditional beta and the excess return of the market, which we label as the market risk component, largely explains the cross-section of portfolio returns.¹ We find intercepts that are indistinguishable from zero, and loadings on the market risk component that are indistinguishable from one. Moreover, the explanatory power (R^2) of only including the market risk component to explain the cross-section of monthly portfolio returns is large; it is 87, 74, and 75% for the ten beta-, 25 size-and-book-to-market-, and ten industry-sorted value-weighted portfolios, respectively.

Our results are also supported when using daily returns. Indeed, intercepts are economically small and not statistically different from zero, the loadings on the market risk component are indistinguishable from one, and the R^2 are in the neighborhood of 70%.

We then evaluate the performance of the market risk component relative to other risk factors. Similarly to our market risk component, we construct additional risk components using the [Fama and French \(1993, 2015\)](#) and [Carhart \(1997\)](#) factors. More precisely, we examine individually the performance of the product of the conditional exposure to the Fama-French high-minus-low (HML), small-minus-big (SMB), robust-minus-weak (RMW), conservative-minus-aggressive (CMA), and momentum (MOM) factor and the factor return (which we label as HML, SMB, RMW, CMA, and MOM risk components). We find that none of these risk factors outperform the simple market risk component. Indeed, the intercept estimates are three to seven

¹The conditional CAPM beta is calculated using 24 months (250 trading days) for monthly (daily) returns strictly before month (day) t . Our results are robust to different window lengths.

times larger than that obtained using the market risk component only, and R^2 are never larger than 33%.

We confirm that our results using the market risk component are robust to including the [Fama and French \(1993, 2015\)](#) and [Carhart \(1997\)](#) risk components into the regression. When including these risk components, the intercept and loading on the market risk component remain close to zero and one, respectively. The loadings on the other risk components, however, decrease two to tenfold relative to their univariate estimates, depending on the risk component, the frequency of returns, and portfolios under consideration. Relative to the univariate regression performed using the market risk component only, adding the Fama-French and momentum risk components has a negligible impact on the R^2 ; the increase in the R^2 ranges from 1 to 11%. To summarize, the prediction that the market risk component is the main determinant of portfolio returns is strongly supported by the data.

We then examine whether our theoretical predictions hold for individual stocks using monthly returns. We find strong results for individual stocks as well. For eight out of the ten size-decile stocks, we find an intercept that is not statistically different from zero at the 5% level. The intercepts of the two smallest size-decile stocks are not statistically different from zero at the 1%. That is, the dynamic CAPM is also hard to reject when tested on individual stocks. In addition, we find an R^2 varying from 27% for the largest decile stocks to 2% for the smallest decile stocks.

Finally, a direct implication of dynamic CAPM is that the relation between an asset's unconditional expected return and its unconditional expected market risk component should be linear, with an intercept equal to zero and a slope equal to one. Note that this unconditional relation is analogous to the Security Market Line. We test this unconditional relation on both daily and monthly portfolio returns and provide evidence that the data lend support to it.

To summarize, this paper highlights the importance of directly interacting conditional betas with market excess returns to explain asset excess returns. The data lend support to the null hypothesis that the dynamic CAPM holds for monthly and daily returns on both equity portfolios and individual stocks. In addition, our results show that the market risk component alone explains a large fraction of the variation in asset returns.

Our work is closely related to the growing empirical literature showing that the relation between an asset's average excess return and its beta is positive only during specific time periods. [Cohen et al. \(2005\)](#) show that the relation between average excess stock returns and their beta is positive during months of low inflation and negative during months of high inflation. [Savor and Wilson \(2014\)](#) find that average excess stock returns are positively related to their beta only on days with important macroeconomic announcements (inflation, unemployment, or Federal Open Markets Committee announcements). [Jylha \(2018\)](#) finds that the security market line is positive during months when investors' borrowing constraints are slack and negative during months when borrowing constraints are tight. [Hendershott et al. \(2018\)](#) show that the CAPM performs poorly during regular trading hours (open to close), but holds during the overnight period (close to open). [Ben-Rephael, Carlin, Da, and Israelsen \(2018\)](#) provide empirical evidence that the Security Market Line is upward-sloping, as predicted by the CAPM, when the demand for information is high. [Hong and Sraer \(2016\)](#) show both theoretically and empirically that the Security Market Line is upward-sloping in low disagreement periods and hump-shaped in high disagreement periods.

Our paper is also related to [Jagannathan and Wang \(1996\)](#) who assume that the risk premium of the market is linear in the yield spread, and that the market return is linear in the stock index return and in the labor income growth rate. In this case,

the expected return of a stock is a linear function of three betas: yield spread beta, stock index beta, and labor income beta. This three-factor model is shown to explain the cross-section of returns significantly better than the CAPM. [Lewellen and Nagel \(2006\)](#) obtain direct estimates of the conditional CAPM alphas and betas from short window regressions (3 months, 6 months, or 12 months). They show that the time-series average conditional alpha is large, and therefore argue that the conditional CAPM performs as poorly as the unconditional one. There are two main differences between our test and theirs. First, their beta is constant over each short window, whereas our beta changes every month (or every day when using daily data) and is directly interacted with the market excess return to form the explanatory variable. Second, their tests are performed separately on each of their portfolios, whereas ours are conducted by pooling portfolios and therefore by following the panel regression approach of [Martin and Wagner \(2018\)](#). By correcting for the bias in unconditional alphas due to market timing, volatility timing, and overconditioning, [Boguth, Carlson, Fisher, and Simutin \(2011\)](#) show that momentum alphas are significantly lower than previously documented. By applying the instrumental variable method of [Boguth et al. \(2011\)](#) to model conditional betas, [Cederburg and O'Doherty \(2016\)](#) show that the betting-against-beta anomaly of [Frazzini and Pedersen \(2014\)](#) disappears. [Choi \(2013\)](#) shows that the asset beta and leverage of value firms vary significantly over the business cycle, while growth firms are less levered and feature a much more stable asset beta than value firms. This heterogeneity among value firms and growth firms can explain about 40% of the unconditional value premium.

Our paper further relates to the recent work by [Dessaint, Olivier, Otto, and Thesmar \(2018\)](#) who argue that managers using the CAPM should overvalue low beta projects relative to the market because of the gap between CAPM-implied returns and realized returns. They show empirically that takeovers of low beta targets typi-

cally yield smaller abnormal returns for the bidders, supporting the aforementioned hypothesis. [Martin and Wagner \(2018\)](#) demonstrate that a stock expected return can be written as a sum of the market risk neutral variance and the stock's excess risk neutral variance relative to the average stock. Their panel regression analysis shows that the aforementioned prediction of the model is supported by the data. In their theoretical framework, [Andrei, Cujean, and Wilson \(2018\)](#) show that, although the CAPM is the correct model, an econometrician incorrectly rejects it because of its informational disadvantage compared with the average investor.

Our paper differs from these studies in two aspects. First, we provide a theoretical motivation for the fact that the CAPM relation should hold but in a dynamic fashion. That is, when investors' hedging demands are equal to zero, the time- t risk premium of an asset should be equal to its time- t beta times the time- t risk premium of the market ([Merton, 1973](#)). Second, we test this specific dynamic CAPM relation by regressing an asset excess return onto the product of its conditional beta and the market excess return, and show that the data lend support to it.

The remainder of the paper is as follows. Section 2 provides our theoretical motivation. Section 3 describes the data and the empirical design. Section 4 discusses our empirical results and Section 5 concludes.

2. Theoretical Motivation

This section presents the CAPM, discusses the condition for the dynamic CAPM to hold, and specifies the model that will be tested empirically.

2.1. The CAPM

The capital asset pricing model (CAPM) of [Sharpe \(1964\)](#), [Lintner \(1965a,b\)](#), and [Mossin \(1966\)](#) is derived under the assumptions that agents have homogeneous beliefs, are mean-variance optimizers, and have a horizon of one period. That is, all agents solve the portfolio selection problem presented in [Markowitz \(1952\)](#). Under these assumptions and given the supply of each asset, the equilibrium risk premium on any stock is a linear function of its beta, which is defined as the covariance between the stock return and the market return over the variance of the market return. Specifically, the static CAPM is written

$$\mathbb{E}(r_i) - r_f = \beta_i [\mathbb{E}(r_M) - r_f], \quad (1)$$

where r_i is the return of stock i , r_f is the risk-free rate, r_M is the market return, and $\beta_i \equiv \frac{\text{Cov}(r_i, r_M)}{\text{Var}(r_M)}$ is the beta of stock i .

As argued in [Merton \(1973\)](#), the single period and mean-variance optimization assumptions have been subject to criticism. Therefore, [Merton \(1973\)](#) extends the aforementioned economic environment by allowing agents to trade continuously over their lifetimes, and to have time-separable von Neumann-Morgenstern utility functions. Agents can invest in n stocks and one riskless asset. Importantly, the vector of (instantaneous) stock returns is allowed to have both a stochastic mean and a stochastic variance-covariance matrix. That is, the investment opportunity set is allowed to

be non-constant. Formally, the dynamics of asset returns satisfy

$$\begin{aligned}\frac{dP_{it}}{P_{it}} &= \mu_{it}dt + \sigma_{it}dz_{it}, \\ d\mu_{it} &= a_{it}dt + b_{it}dq_{it}, \\ d\sigma_{it} &= f_{it}dt + g_{it}dx_{it}, \quad i = 1, \dots, n + 1\end{aligned}$$

where $\frac{dP_{it}}{P_{it}}$, μ_{it} , and σ_{it} are respectively the return, expected return, and return volatility of asset i at time t . The constant correlation between the Brownian motions dz_{it} and dz_{jt} is ρ_{ij} , which implies that the variance-covariance matrix of returns is $\Omega_t \equiv [\sigma_{ij,t}] = [\sigma_{it}\sigma_{jt}\rho_{ij}]$. Although not specified here, the constant correlations between the Brownian motions dq_{it} and dq_{jt} , dx_{it} and dx_{jt} , dz_{it} and dq_{jt} , dz_{it} and dx_{jt} , and dq_{it} and dx_{jt} are allowed to be different from zero. The processes a_{it} , f_{it} , b_{it} , and g_{it} are functions of the vector of prices P_t , the vector of expected returns μ_t , and the vector of return volatilities σ_t . Asset $n + 1$ is assumed to be the riskless asset, i.e., $\mu_{n+1,t} \equiv r_{ft}$ and $\sigma_{n+1,t} \equiv 0$ so that

$$\frac{dP_{n+1,t}}{P_{n+1,t}} = r_{ft}dt,$$

where r_{ft} is the risk-free rate at time t .

If investors' hedging demands are equal to zero, the equilibrium risk premium of stock i satisfies

$$\mu_{it} - r_{ft} = \beta_{it} [\mu_{Mt} - r_{ft}], \quad (2)$$

where μ_{Mt} is the expected return of the market portfolio and $\beta_{it} \equiv \frac{\text{Cov}_t\left(\frac{dP_{it}}{P_{it}}, \frac{dP_{Mt}}{P_{Mt}}\right)}{\text{Var}_t\left(\frac{dP_{Mt}}{P_{Mt}}\right)}$ is the beta of stock i . It is worth noting that hedging demands are equal to zero if either

agents have logarithmic preferences, or the investment opportunity set is constant, or changes in the state variables are uncorrelated to stock returns, or changes in the state variables are correlated to stock returns in such a way that the sum of all hedging components is equal to zero.

Equation (2) shows that when agents trade continuously and have no hedging motives, the original (static) CAPM relation (1) still holds but in a dynamic manner. Specifically, the time- t risk premium on any stock is the product of the time- t stock's beta and the time- t risk premium of the market.

2.2. Testing the dynamic CAPM empirically

The dynamic CAPM relation (2) relates expected excess stock returns to expected excess market returns. Since expected returns are unobservable, the empirical framework needed to test the dynamic CAPM relation (2) is not necessarily straightforward, and therefore requires additional details.

As in [Lewellen and Nagel \(2006\)](#) and [Choi \(2013\)](#) among others, our empirical framework focuses on realized returns. Specifically, we consider the following model for stock i 's excess return

$$\frac{dP_{it}}{P_{it}} - r_{ft}dt = a dt + b\hat{\beta}_{it} \left[\frac{dP_{Mt}}{P_{Mt}} - r_{ft}dt \right] + \tilde{\sigma}_{it}dW_{it}, \quad (3)$$

where

$$\frac{dP_{Mt}}{P_{Mt}} - r_{ft}dt \equiv [\mu_{Mt} - r_{ft}] dt + \sigma_{Mt}dW_{Mt}, \quad (4)$$

is the excess market return, μ_{Mt} is the market expected return, σ_{Mt} is the volatility of the market return, $\hat{\beta}_{it}$ is an empirical estimate of the beta of stock i , $\tilde{\sigma}_{it}$ is the

idiosyncratic volatility of stock i 's return, r_{ft} is the risk-free rate, and dW_{it} and dW_{Mt} are independent Brownian motions. Note that the beta of stock i satisfies $\beta_{it} \equiv \frac{\text{Cov}_t\left(\frac{dP_{it}}{P_{it}}, \frac{dP_{Mt}}{P_{Mt}}\right)}{\text{Var}_t\left(\frac{dP_{Mt}}{P_{Mt}}\right)} = b\widehat{\beta}_{it}$. Substituting Equation (4) in Equation (3) yields

$$\frac{dP_{it}}{P_{it}} - r_{ft}dt = \left(a + b\widehat{\beta}_{it} [\mu_{Mt} - r_{ft}]\right) dt + b\widehat{\beta}_{it}\sigma_{Mt}dW_{Mt} + \tilde{\sigma}_{it}dW_{it} \quad (5)$$

$$\equiv [\mu_{it} - r_{ft}] dt + \sigma_{it}dz_{it}, \quad (6)$$

where μ_{it} is stock i 's expected return, σ_{it} is the volatility of stock i 's return, and dz_{it} is a Brownian motion.

Proposition 1 below provides conditions for the dynamic CAPM relation (2) to hold in our empirical framework.

Proposition 1 *Let us consider model (3):*

$$\frac{dP_{it}}{P_{it}} - r_{ft}dt = adt + b\widehat{\beta}_{it} \left[\frac{dP_{Mt}}{P_{Mt}} - r_{ft}dt \right] + \tilde{\sigma}_{it}dW_{it}.$$

- *If the intercept $a = 0$, then the dynamic CAPM relation (2) holds:*

$$\mu_{it} - r_{ft} = b\widehat{\beta}_{it} [\mu_{Mt} - r_{ft}] = \beta_{it} [\mu_{Mt} - r_{ft}].$$

- *If both the intercept $a = 0$ and the slope $b = 1$, then the dynamic CAPM relation (2) holds and the empirical estimate of the beta of stock i is well defined, i.e., $\widehat{\beta}_{it} = \beta_{it}$.*

Proof: *See the derivations from Equations (3) and (4) to Equations (5) and (6).*

In Section 4, we consider a discretized version of model (3) and empirically test the null hypothesis that both the intercept $a = 0$ and the slope $b = 1$. That is,

our test takes into account the issue raised by [Lewellen and Nagel \(2006\)](#) that the slope b has to be equal to one for the empirical estimate $\widehat{\beta}_{it}$ to be well defined.² We show that the null hypothesis cannot be rejected (at conventional confidence levels), which through Proposition 1 implies that the dynamic CAPM relation (2) cannot be rejected and the empirical estimate of beta is well defined.

3. Data

This section describes the data used to perform empirical tests.

3.1. Stock returns and portfolio construction

From Kenneth French’s website³, we obtain the excess market return, the risk-free rate, and value-weighted returns for the following test assets: the 25 size-and-book-to-market-, the 25 size-and-momentum-, the 25 size-and-investment-, the 25 size-and-operating-profits-, the ten and 49 industry sorted portfolios. From the same source, we download the high-minus-low (HML), small-minus-big (SMB), robust-minus-weak (RMW), and conservative-minus-aggressive (CMA) [Fama and French \(1993, 2015\)](#) factors and [Carhart \(1997\)](#) momentum (MOM) factor. The sample period is from July 1, 1926 to December 31, 2017. When using the more recent factors, RMW and CMA, the sample period starts on July 1, 1963.

We also construct ten monthly and daily beta-sorted portfolios using U.S. common stocks that are identified in CRSP as having a share code of 10 or 11 trading on the NYSE, Nasdaq, or AMEX stock exchange. We estimate monthly (daily) market

²Refer to Section 5 in [Lewellen and Nagel \(2006\)](#) for a discussion on why their conclusions differ from those of [Jagannathan and Wang \(1996\)](#), [Lettau and Ludvigson \(2001\)](#), [Lustig and Nieuwerburgh \(2005\)](#), and [Santos and Veronesi \(2006\)](#).

³Data source: <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french>

betas for all stocks using 24-month (250-trading day) rolling windows of past monthly (daily) returns.⁴ At the beginning of each month, we sort stocks into one of the ten beta-deciles, and calculate their respective monthly and daily value-weighted returns.

Note that we follow [Hou, Xue, and Zhang \(2018\)](#) and use exclusively value-weighted returns. As argued in detail in [Hou et al. \(2018\)](#), wealth effects, transactions costs, and microstructure frictions make value-weighted returns preferred to equal-weighted returns when used to test asset pricing models.

Our last step consists in calculating for each of the portfolios their monthly and daily market betas, $\beta_{i,t}^M$, using the last 24 months (250 trading days) of monthly (daily) excess returns.⁵ Similarly, we calculate for each of the portfolios their HML, SMB, RMW, CMA, and MOM factor loadings, which are denoted as $\beta_{i,t}^{HML}$, $\beta_{i,t}^{SMB}$, $\beta_{i,t}^{RMW}$, $\beta_{i,t}^{CMA}$, and $\beta_{i,t}^{MOM}$, respectively.

In what follows, we provide a direct test of the dynamic CAPM stated in Equation (2) using both monthly and daily returns on the 10 beta-sorted portfolios, the 25 size-and-book-to-market-sorted portfolios, the ten industry-sorted portfolios, and individual stocks. The results obtained using the 25 size-and-investment-sorted portfolios, the 25 size-and-operating-profits-sorted portfolios, and the 49 industry-sorted portfolios are presented in the Internet Appendix.

4. Main Empirical Results

This section empirically tests the dynamic CAPM, as described in Proposition 1. Our empirical tests are conducted using both monthly and daily value-weighted returns on

⁴If for a given stock the availability of returns is less than 24 months (250 days), we require at least 12 months (100 days) of returns to calculate the stock's monthly (daily) market beta.

⁵Note that the results presented thereafter are robust to changes in the length of the rolling window under two conditions. First, the window must not be too short as betas would become too noisy. Second, the window must not be too long as betas would become time invariant.

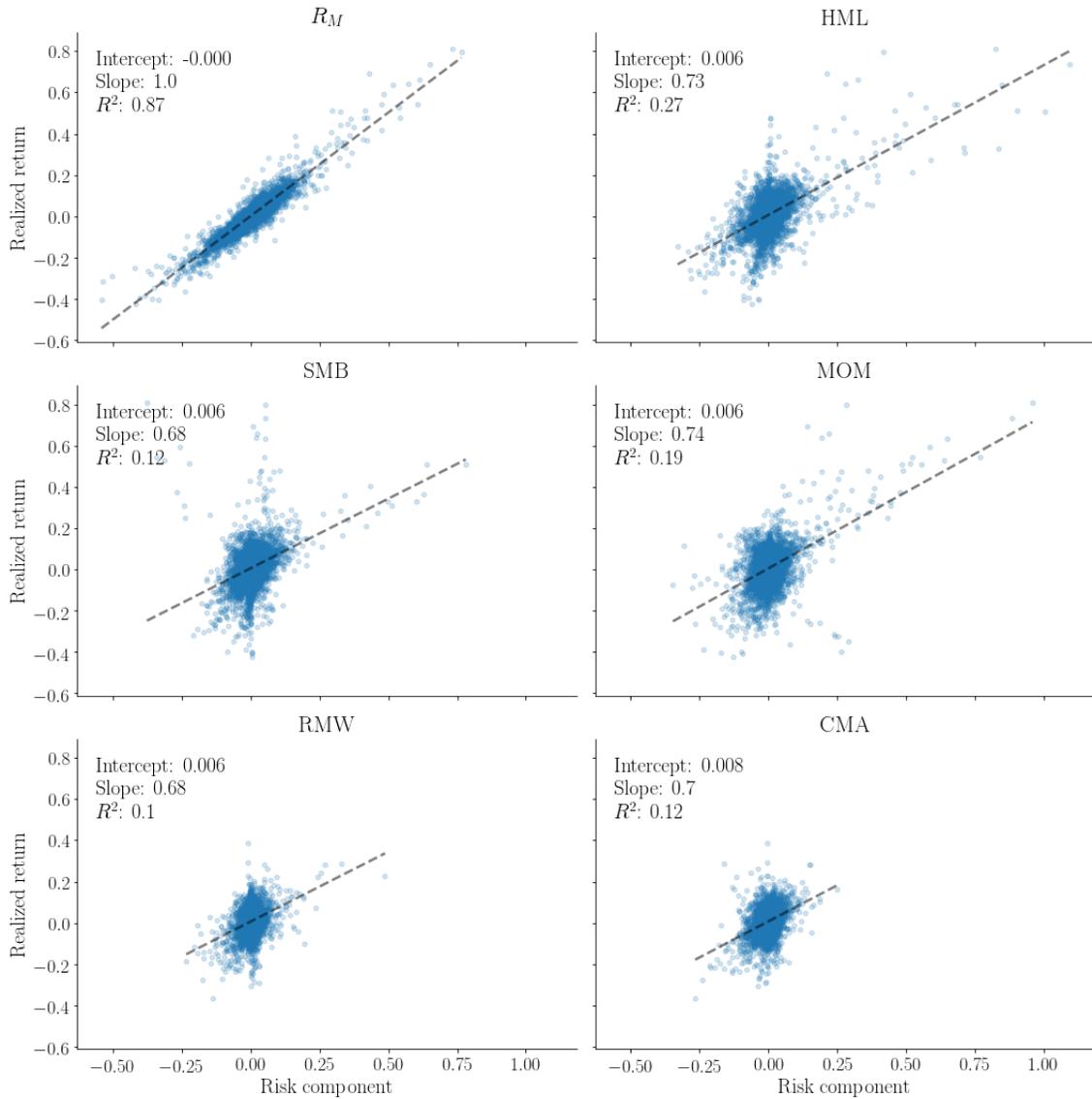
the ten beta-, 25 size-and-book-to-market-, and ten industry-sorted portfolios as well as on individual stocks. This section provides empirical evidence that the dynamic CAPM cannot be rejected by the data.

4.1. An illustration of the dynamic CAPM

As a preliminary illustration, Figure 1 presents a scatter plot highlighting the relation between realized excess returns and dynamic-CAPM-implied excess returns for the ten beta-sorted value-weighted portfolios. The dynamic-CAPM-implied excess return, $\beta_{i,t}^M R_{M,t+1}$, is labeled as the *market risk component*. We also examine the relation between realized excess returns and implied excess returns obtained using the [Fama and French \(1993, 2015\)](#) and [Carhart \(1997\)](#) factors, i.e., HML, SMB, MOM, RMW, and CMA. For these factors, the implied excess returns are, $\beta_{i,t}^{HML} \times HML_{t+1}$ (*HML risk component*), $\beta_{i,t}^{SMB} \times SMB_{t+1}$ (*SMB risk component*), $\beta_{i,t}^{MOM} \times MOM_{t+1}$ (*MOM risk component*), $\beta_{i,t}^{RMW} \times RMW_{t+1}$ (*RMW risk component*), and $\beta_{i,t}^{CMA} \times CMA_{t+1}$ (*CMA risk component*). $\beta_{i,t}^{HML}$, $\beta_{i,t}^{SMB}$, $\beta_{i,t}^{MOM}$, $\beta_{i,t}^{RMW}$, and $\beta_{i,t}^{CMA}$ are portfolio i 's factor loadings on HML, SMB, MOM, RMW, and CMA, respectively. Regressing realized excess returns onto the market risk component yields an intercept and a slope that are economically close to zero and one, respectively. This suggest that, from an economic point of view, the dynamic CAPM is hard to reject and the estimate of beta is well defined (see Proposition 1). Furthermore, the scatter plots show that the market risk component best explains the realized excess returns with R^2 equal to 87%. The Fama-French and momentum risk components do not fit the data as well. The HML and SMB risk components are the best explanatory variables among these risk factors, but their R^2 are not larger than 33%.

Figure 1. Realized Returns vs. Risk Components

This figure shows scatter plots highlighting the relation between realized monthly excess returns and the different risk components for the 10 beta-sorted value-weighted portfolios. The market risk component is defined as $\beta_{i,t}^M R_{M,t+1}$, where $R_{i,t+1}$ is the excess return of portfolio i , $R_{M,t+1}$ is the market excess return, and $\beta_{i,t}^M$ is the coefficient of a regression of the monthly excess return of portfolio i on the excess market return using 24 months strictly prior to month $t + 1$. The Fama and French (1993, 2015) and Carhart (1997) risk components are, $\beta_{i,t}^{HML} \times HML_{t+1}$, $\beta_{i,t}^{SMB} \times SMB_{t+1}$, $\beta_{i,t}^{MOM} \times MOM_{t+1}$, $\beta_{i,t}^{RMW} \times RMW_{t+1}$, and $\beta_{i,t}^{CMA} \times CMA_{t+1}$, where $\beta_{i,t}^{HML}$, $\beta_{i,t}^{SMB}$, $\beta_{i,t}^{MOM}$, $\beta_{i,t}^{RMW}$, and $\beta_{i,t}^{CMA}$ are portfolio i 's factor loadings on HML, SMB, MOM, RMW, and CMA, respectively. The dashed line is the linear function that best fits the relation. We further report the estimated intercept, slope, and R^2 of the linear fit.



4.2. Panel regressions

We now examine how statistically robust the results presented previously are when considering different test assets, different return frequencies, and when controlling for the Fama-French and Carhart risk components. Formally, we estimate the following panel regression:

$$\begin{aligned} R_{i,t+1} = & a + b[\beta_{i,t}^M R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] + s[\beta_{i,t}^{SMB} SMB_{t+1}] \\ & + m[\beta_{i,t}^{MOM} MOM_{t+1}] + r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1}. \end{aligned} \tag{7}$$

We present our result using monthly and daily returns for the 10 beta-, 25 size-and-book-to-market-, 10 industry-sorted value-weighted portfolios as well as for individual stocks. The results for the 25 size-and-investment-sorted portfolios, the 25 size-and-operating-profits-sorted portfolios, and the 49 industry-sorted portfolios are provided in the Internet Appendix; they are consistent with the conclusion drawn from what follows that the data lend support to the dynamic CAPM.

4.2.1. Ten beta-sorted portfolios

Table 1 presents the results of the model specified in Equation (7) for the ten beta-sorted portfolios. Panels A and B consider monthly returns and daily returns, respectively. Columns (1)-(5) consider the sample from 1926 to 2017, while columns (6)-(9) consider the sample from 1963 to 2017. Columns (1) and (6) for monthly returns show that the intercept is not statistically different from zero. Columns (1) and (6) for daily returns show that the intercept is not statistically different from zero at the 5% and 10% level, respectively. According to Proposition 1, this result provides evidence that the dynamic CAPM cannot be rejected at conventional statistical lev-

Table 1
Panel Regressions: 10 Beta-Sorted Portfolios

This table presents results from a regression of equity portfolio excess returns on month (day) $t + 1$ on the market risk, Fama and French (1993, 2015), and Carhart (1997) risk components on month (day) $t + 1$ for the ten beta-sorted value-weighted portfolios. Specifically, we estimate:

$$R_{i,t+1} = a + b[\beta_{i,t}^M R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] + s[\beta_{i,t}^{SMB} SMB_{t+1}] + m[\beta_{i,t}^{MOM} MOM_{t+1}] + r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1},$$

Each factor loading β coefficients are estimated using the 24 months (250 trading days) strictly prior to month (day) $t + 1$ for each portfolio i and for each of the respective factor. Panels A and B report the results using monthly and daily returns, respectively. The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags when using monthly returns and 250 trading day lags when using daily returns. The table further reports the adjusted R^2 , the number of observations (N), and the p -values of the Wald statistics testing the joint hypothesis of $H_0: a = 0$ and $b = 1$ and $H_0: \forall a_i = 0$ and $b = 1$ when the intercepts are estimated separately for each portfolio i . ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively. The sample period is from January 1, 1926 to December 31, 2017 in Columns (1) to (5) and from July 1, 1963 to December 31, 2017 in Columns (6) to (9).

Panel A. Monthly returns

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	-0.000 (0.000)	0.006*** (0.002)	0.006** (0.002)	0.006** (0.002)	-0.000 (0.000)	-0.000 (0.000)	0.006** (0.002)	0.003*** (0.002)	0.000 (0.000)
R_M (b)	1.004*** (0.016)				0.934*** (0.017)	0.981*** (0.021)			0.896*** (0.019)
HML (h)		0.727*** (0.072)			0.085*** (0.019)				0.075** (0.027)
SMB (s)			0.675*** (0.121)		0.136*** (0.027)				0.234*** (0.027)
MOM (m)				0.741*** (0.103)	0.062** (0.024)				0.096* (0.045)
RMW (r)							0.678*** (0.122)		0.105*** (0.029)
CMA (c)								0.699*** (0.104)	0.104** (0.033)
R^2	0.87	0.27	0.12	0.19	0.88	0.81	0.10	0.12	0.84
N	10,500	10,500	10,500	10,500	10,500	6,300	6,300	6,300	6,300
p -value $H_0: a=0, b=1$	0.965					0.540			
p -value $H_0: \forall a_i=0, b=1$	0.120					0.198			

Panel B. Daily returns

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.0000* (0.0000)	0.0003*** (0.0001)	0.0003*** (0.0001)	0.0002** (0.0001)	0.0000** (0.0000)	0.0000 (0.0000)	0.0003*** (0.0001)	0.0003*** (0.0001)	0.0001** (0.0000)
R_M (b)	1.0017*** (0.0075)				0.9363*** (0.0096)	0.9996*** (0.0120)			0.8915*** (0.0101)
HML (h)		0.9897*** (0.0281)			0.1226*** (0.0178)				0.0859** (0.0286)
SMB (s)			0.8786*** (0.0464)		0.0619*** (0.0187)				0.1157*** (0.0280)
MOM (m)				0.7879*** (0.0659)	0.0542*** (0.0150)				0.0801*** (0.0215)
RMW (r)							0.9940*** (0.0677)		0.1547*** (0.0197)
CMA (c)								0.9587*** (0.0635)	0.1106** (0.0414)
R^2	0.79	0.29	0.13	0.16	0.80	0.80	0.16	0.18	0.82
N	237,400	237,400	237,400	237,400	237,400	132,160	134,700	134,700	134,700
p -value $H_0: a=0, b=1$	0.190					0.296			
p -value $H_0: \forall a_i=0, b=1$	0.028					0.260			

els. Columns (1) and (6) show that the loadings on the market risk component are indistinguishable from one. That is, the estimated conditional betas are well defined (see Proposition 1). The R^2 are all remarkably high, ranging from 79% to 87%.

Comparing column (5) and (9) to respective columns (1)-(4) and (6)-(8) shows that the loading on the market risk component remains close to one, whereas the loadings on the other risk components decrease five to tenfold. That is, the market risk component strongly dominates all of the other risk components in explaining changes in portfolio returns. In terms of economic magnitude, a one percentage point increase in the monthly market risk component implies an increase in monthly portfolio returns of at least 0.9 percentage point, whereas a one percentage point increase in any of the other risk component implies an increase in portfolio returns ranging from only 0.06 to 0.23 percentage point. In addition, columns (2)-(4) and (7)-(8) show that the individual explanatory powers of HML, SMB, MOM, RMW, and CMA are weak relative to that of the market risk component. Comparing column (1) to column (5) shows that including the HML, SMB, and MOM risk components increases the R^2 by only 1%, whereas comparing column (6) to column (9) shows that adding the HML, SMB, MOM, RMW, and CMA risk components into the regression increases the R^2 by only 3%.

The last two rows of Table 1 reports the p -values of the Wald statistics testing the joint hypothesis $H_0: a = 0$ and $b = 1$ and $H_0: \forall a_i = 0$ and $b = 1$, where a_i is defined as portfolio i 's fixed effect. The reported p -values show that we do not reject the null, $H_0: a = 0$ and $b = 1$, for both monthly and daily returns at the 10% level. Note that the null hypothesis, $H_0: \forall a_i = 0$ and $b = 1$ is particularly hard not to reject because it suffices that a single coefficient departs from the null hypothesis for the test to be rejected. Yet, the last row of Panels A and B show that the null is not rejected at the 10% level when using either monthly returns or daily returns from 1963 to 2017.

Table 2
Panel Regressions: 25 Size-and-Book-to-Market-Sorted Portfolios

This table presents results from a regression of equity portfolio excess returns on month (day) $t + 1$ on the market risk, Fama and French (1993, 2015), and Carhart (1997) risk components on month (day) $t+1$ for the 25 size-and-book-to-market-sorted value-weighted portfolios. Specifically, we estimate:

$$R_{i,t+1} = a + b[\beta_{i,t}^M R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] + s[\beta_{i,t}^{SMB} SMB_{t+1}] \\ + r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1}$$

Each factor loading β coefficients are estimated using the 24 months (250 trading days) strictly prior to month (day) $t + 1$ for each portfolio i and for each of the respective factor. Panels A and B report the results using monthly and daily returns, respectively. The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags when using monthly returns and 250 trading day lags when using daily returns. The table further reports the adjusted R^2 , the number of observations (N), and the p -values of the Wald statistics testing the joint hypothesis of $H_0: a = 0$ and $b = 1$ and $H_0: \forall a_i = 0$ and $b = 1$ when the intercepts are estimated separately for each portfolio i . ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively. The sample period is from January 1, 1926 to December 31, 2017 in Columns (1) to (5) and from July 1, 1963 to December 31, 2017 in Columns (6) to (9).

Panel A. Monthly returns

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.001 (0.001)	0.007*** (0.002)	0.006*** (0.002)	0.008*** (0.002)	0.001 (0.001)	0.002* (0.001)	0.008*** (0.002)	0.009*** (0.002)	0.001 (0.001)
R_M (b)	1.015*** (0.027)				0.829*** (0.021)	0.973*** (0.024)			0.843*** (0.028)
HML (h)		0.799*** (0.067)			0.156*** (0.037)				0.122*** (0.035)
SMB (s)			0.839*** (0.066)		0.416*** (0.043)				0.500*** (0.027)
MOM (m)				0.750*** (0.117)	0.107*** (0.038)				0.029 (0.035)
RMW (r)						0.677*** (0.080)			0.113*** (0.033)
CMA (c)							0.617*** (0.092)	-0.028 (0.037)	
R^2	0.74	0.32	0.27	0.19	0.82	0.72	0.11	0.07	0.83
N	26,700	26,700	26,700	26,700	26,700	15,750	15,750	15,750	15,750
p -value $H_0: a=0, b=1$	0.322					0.193			
p -value $H_0: \forall a_i=0, b=1$	0.001					<0.001			

Panel B. Daily returns

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.0001* (0.0000)	0.0004*** (0.0001)	0.0004*** (0.0001)	0.0003** (0.0001)	0.0001** (0.0000)	0.0001** (0.0000)	0.0004*** (0.0001)	0.0004*** (0.0001)	0.0001** (0.0000)
R_M (b)	1.0008*** (0.0115)				0.8535*** (0.0133)	1.0045*** (0.0101)			0.8766*** (0.0113)
HML (h)		0.9796*** (0.0215)			0.2181*** (0.0232)				0.1294*** (0.0362)
SMB (s)			0.9003*** (0.0414)		0.3235*** (0.0518)				0.3433*** (0.0571)
MOM (m)				0.7566*** (0.0675)	0.0624** (0.0242)				0.0973*** (0.0171)
RMW (r)						0.9728*** (0.0573)			0.1359*** (0.0422)
CMA (c)							0.8975*** (0.0539)	-0.0187 (0.0333)	
R^2	0.59	0.25	0.13	0.12	0.61	0.79	0.17	0.14	0.82
N	593,500	593,500	593,500	593,500	593,500	330,400	336,750	336,750	336,750
p -value $H_0: a=0, b=1$	0.165					0.129			
p -value $H_0: \forall a_i=0, b=1$	0.010					<0.001			

When using daily returns from 1926 to 2017, the null is not rejected at the 1% level.

4.2.2. 25 size-and-book-to-market sorted portfolios

We repeat the same analysis but using the Fama-French 25 size-and-book-to-market sorted portfolios and present the results in Table 2. Our previous conclusions that the dynamic CAPM cannot be rejected by the data, the estimated betas are well defined, and the explanatory power of the market risk component is large are all confirmed.

Columns (1) and (6) show that the intercept is not statistically different from zero at the 10% level when using monthly returns from 1926 to 2017, it is not statistically different from zero at the 5% level when using monthly returns from 1963 to 2017 and daily returns from 1926 to 2017, and it is not statistically different from zero at the 1% level when using daily returns from 1963 to 2017. This provides evidence that the dynamic CAPM cannot be rejected at conventional statistical levels (see Proposition 1). In addition, the loadings on the market risk components are all indistinguishable from one, which according to Proposition 1 confirms that the estimated betas are well defined. The market risk component explains a large fraction of the variation in portfolios' returns, with R^2 ranging from 72 to 74% when using monthly returns and from 59 to 79% when using daily returns.

Columns (5) and (9) show that adding risk components into the regression has a minor impact on the loading on the market risk component, whereas it has a strong impact on the other risk components' loadings. Indeed, the loading on the market risk component remains close to one, while the other loadings decrease two to sevenfold. To understand the strength of the economic impact of the market risk component relative to that of the other risk components, let us consider a one percentage point increase in the market risk component. This yields an increase in portfolio returns of about 0.85 percentage point, irrespective of the period and

frequency of returns considered. In contrast, a one percentage point increase in any of the other risk component implies an increase in portfolio returns ranging from 0 to about 0.4 percentage point, depending on the risk component and return frequency considered. In addition, columns (2)-(4) and (7)-(8) show that the explanatory powers of the HML, SMB, MOM, RMW, and CMA risk components are weak relative to that of the market risk component, ranging from 7 to 32% for monthly returns and from 14 to 25% for daily returns. Comparing columns (5) and (9) to columns (1) and (6) shows that adding the HML, SMB, MOM, RMW, and CMA risk components increases the R^2 by at most 11% and 3% for monthly and daily returns, respectively.

The reported p -values of the Wald statistics show that we do not reject the null hypothesis that $a = 0$ and $b = 1$ at the 10% level for both monthly and daily returns. When considering portfolios' fixed effects, the null hypothesis that $\forall a_i = 0$ and $b = 1$ cannot be rejected at the 1% level for daily returns from 1926 to 2017. It is, however, rejected at the 1% level for monthly returns and for daily returns from 1963 to 2017.

4.2.3. Ten industry sorted portfolios

Table 3 reports our empirical results for the ten industry-sorted portfolios. Columns (1) and (6) show that the intercept is not statistically different from zero at the 10% level for daily returns from 1963 to 2017, and it is not statistically different from zero at the 1% level for daily returns from 1926 to 2017 and for monthly returns from 1926 to 2017. Irrespective of the frequency of returns and the sample period considered, the loading on the market risk component is indistinguishable from one. According to Proposition 1, this provides empirical evidence that the dynamic CAPM can not again be rejected at the conventional level, and that the estimated betas are well defined. The explanatory power of the market risk component in the univariate regressions ranges from 66 to 75% for monthly returns and from 73 to 77% for daily

Table 3
Panel Regressions: Ten Industry-Sorted Portfolios

This table presents results from a regression of equity portfolio excess returns on month (day) $t + 1$ on the market risk, Fama and French (1993, 2015), and Carhart (1997) risk components on month (day) $t+1$ for the ten industry-sorted value-weighted portfolios. Specifically, we estimate:

$$R_{i,t+1} = a + b[\beta_{i,t}^M R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] + s[\beta_{i,t}^{SMB} SMB_{t+1}] \\ + r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1}$$

Each factor loading β coefficients are estimated using the 24 months (250 trading days) strictly prior to month (day) $t + 1$ for each portfolio i and for each of the respective factor. Panels A and B report the results using monthly and daily returns, respectively. The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags when using monthly returns and 250 trading day lags when using daily returns. The table further reports the adjusted R^2 , the number of observations (N), and the p -values of the Wald statistics testing the joint hypothesis of $H_0: a = 0$ and $b = 1$ and $H_0: \forall a_i = 0$ and $b = 1$ when the intercepts are estimated separately for each portfolio i . ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively. The sample period is from January 1, 1926 to December 31, 2017 in Columns (1) to (5) and from July 1, 1963 to December 31, 2017 in Columns (6) to (9).

Panel A. Monthly returns

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.001*** (0.000)	0.006*** (0.002)	0.006*** (0.002)	0.006*** (0.002)	0.001*** (0.000)	0.001** (0.000)	0.006*** (0.002)	0.007*** (0.002)	0.001 (0.000)
R_M (b)	0.984*** (0.007)				0.955*** (0.010)	0.972*** (0.007)			0.917*** (0.012)
HML (h)		0.650*** (0.090)			0.041* (0.022)				0.150*** (0.030)
SMB (s)			0.518*** (0.140)		0.050 (0.032)				0.154*** (0.028)
MOM (m)				0.668*** (0.122)	0.045* (0.022)				0.122** (0.046)
RMW (r)						0.541*** (0.118)			-0.004 (0.037)
CMA (c)								0.651*** (0.085)	0.053 (0.030)
R^2	0.75	0.17	0.05	0.13	0.76	0.66	0.05	0.09	0.68
N	10,680	10,680	10,680	10,680	10,680	6,300	6,300	6,300	6,300
p -value $H_0: a=0, b=1$	0.012					0.006			
p -value $H_0: \forall a_i=0, b=1$	0.078					0.023			

Panel B. Daily returns

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.0000** (0.0000)	0.0003*** (0.0001)	0.0003*** (0.0001)	0.0002** (0.0001)	0.0000** (0.0000)	0.0000 (0.0000)	0.0003*** (0.0001)	0.0004*** (0.0001)	0.0000** (0.0000)
R_M (b)	0.9978*** (0.0027)				0.9514*** (0.0070)	0.9981*** (0.0038)			0.9110*** (0.0114)
HML (h)		0.9647*** (0.0275)			0.0586*** (0.0127)				0.0524*** (0.0123)
SMB (s)			0.8590*** (0.0590)		0.0773*** (0.0191)				0.1302*** (0.0282)
MOM (m)				0.7924*** (0.0562)	0.0576*** (0.0125)				0.0801*** (0.0183)
RMW (r)						0.9517*** (0.0471)			0.0998*** (0.0234)
CMA (c)								0.9214*** (0.0444)	0.1038*** (0.0248)
R^2	0.77	0.25	0.13	0.15	0.77	0.73	0.11	0.14	0.74
N	237,400	237,400	237,400	237,400	237,400	132,160	134,700	134,700	134,700
p -value $H_0: a=0, b=1$	0.032					0.221			
p -value $H_0: \forall a_i=0, b=1$	0.115					0.355			

returns.

Comparing columns (5) and (9) to respective columns (1) and (6) shows that the economic impact of the market risk component is unaffected by the inclusion of the other risk components; the loading on the market risk component remains close to one. The economic impact of the other risk components, however, decrease more than tenfold when considering all risk components simultaneously. That is, the market risk component is the main driver of the variation in portfolio returns. To fix ideas, a one percentage point increase in the market risk component implies an increase in portfolios' returns ranging from 0.91 to 0.95, depending on the frequency of returns and sample period considered. In contrast, a one percentage point increase in any of the other risk component implies an increase in portfolios' returns ranging from 0 to a maximum of 0.15. Moreover, adding the HML, SMB, RMW, CMA, and MOM risk components into the regression increases the R^2 only marginally. Indeed, the R^2 increases by at most 2% when using monthly returns, and by at most 1% when using daily returns.

Overall, the results reported in this section paint a clear picture that the dynamic CAPM relation defined in Equation (2) is supported by the data for a wide range of portfolios. Moreover, the data show that the estimated betas are well defined and that the market risk component alone explains a large fraction of the variation in portfolio returns.

4.2.4. Individual stocks

Our results so far have shown that the market risk component accurately explains equity portfolio returns. We next evaluate the ability of the market risk component to explain individual stock returns using the regression specified in Equation (7).

We present the regression results in Table 4 by firm size deciles. Firm size deciles

Table 4
Panel Regressions: Individual Stocks

This table presents results from a regression of individual stock excess returns on month $t + 1$ on the market risk, Fama and French (1993, 2015), and Carhart (1997) risk components, by firm size decile. Specifically, we estimate:

$$R_{i,t+1} = a + b[\beta_{i,t}^M R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] \\ + s[\beta_{i,t}^{SMB} SMB_{t+1}] + m[\beta_{i,t}^{MOM} MOM_{t+1}] \\ + r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1}.$$

Each factor loading β coefficients are estimated using the 24 months strictly prior to month $t + 1$ for each portfolio i and for each of the respective factor. The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags. Firm size deciles are calculated based on stocks market capitalization at the end of June of each year. The table further reports the adjusted R^2 and the number of observations (N). ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively. The sample period is from July 1, 1926 in Panel A and B and from July 1, 1963 in Panel C and D to December 31, 2017.

Panel A. Univariate regression (1926 to 2017)

	Small	2	3	4	5	6	7	8	9	Large
Intercept (a)	0.016** (0.003)	0.007** (0.002)	0.006* (0.002)	0.004 (0.002)	0.003 (0.002)	0.003 (0.001)	0.002 (0.001)	0.002 (0.001)	0.002 (0.001)	0.001* (0.000)
R_M (b)	0.518*** (0.055)	0.624*** (0.044)	0.688*** (0.038)	0.731*** (0.034)	0.752*** (0.029)	0.789*** (0.027)	0.800*** (0.026)	0.818*** (0.025)	0.844*** (0.022)	0.877*** (0.014)
N	260,863	269,001	269,749	274,146	275,382	282,812	288,034	297,782	309,848	327,153
R^2	0.02	0.04	0.06	0.08	0.10	0.13	0.15	0.18	0.21	0.27

Panel B. Controlling for HML, SMB, and MOM (1926 to 2017)

	Small	2	3	4	5	6	7	8	9	Large
Intercept (a)	0.015*** (0.003)	0.006** (0.002)	0.005* (0.002)	0.003 (0.001)	0.002 (0.001)	0.002 (0.001)	0.001 (0.001)	0.002 (0.001)	0.002 (0.001)	0.001 (0.001)
R_M (b)	0.373*** (0.034)	0.493*** (0.028)	0.554*** (0.026)	0.594*** (0.022)	0.616*** (0.021)	0.659*** (0.019)	0.690*** (0.022)	0.717*** (0.021)	0.759*** (0.020)	0.815*** (0.015)
N	260,562	268,700	269,471	273,877	275,115	282,514	287,740	297,478	309,566	326,818
R^2	0.04	0.07	0.09	0.12	0.14	0.17	0.18	0.21	0.23	0.28

Panel C. Univariate regression (1963 to 2017)

	Small	2	3	4	5	6	7	8	9	Large
Intercept (a)	0.017** (0.003)	0.008* (0.003)	0.006* (0.002)	0.005 (0.002)	0.004 (0.002)	0.003 (0.002)	0.002 (0.001)	0.002 (0.001)	0.002 (0.001)	0.001* (0.001)
R_M (b)	0.372*** (0.043)	0.514*** (0.036)	0.606*** (0.035)	0.672*** (0.033)	0.703*** (0.030)	0.750*** (0.028)	0.767*** (0.028)	0.792*** (0.027)	0.823*** (0.024)	0.865*** (0.015)
N	226,183	234,769	236,564	241,333	242,688	249,122	254,244	263,215	275,004	291,215
R^2	0.01	0.02	0.04	0.06	0.08	0.11	0.13	0.15	0.19	0.24

Panel D. Controlling for HML, SMB, MOM, CMA, and RMW (1963 to 2017)

	Small	2	3	4	5	6	7	8	9	Large
Intercept (a)	0.017** (0.003)	0.007** (0.002)	0.005* (0.002)	0.004 (0.002)	0.003 (0.002)	0.002 (0.001)	0.001 (0.001)	0.002 (0.001)	0.002 (0.001)	0.002 (0.001)
R_M (b)	0.289*** (0.035)	0.418*** (0.028)	0.496*** (0.028)	0.547*** (0.024)	0.572*** (0.022)	0.616*** (0.020)	0.650*** (0.025)	0.683*** (0.024)	0.726*** (0.023)	0.787*** (0.016)
N	224,081	232,674	234,115	238,553	239,468	245,285	250,361	259,058	270,650	286,543
R^2	0.02	0.05	0.07	0.09	0.12	0.15	0.16	0.19	0.21	0.26

are assigned to each stock based on their market capitalization calculated at the end of June preceding the month or day t . For ease of exposition, we report the results only for the intercept and the loading on the market risk component. Panel A reports the univariate regression results for the time period of 1926 to 2017, and Panel B further controls for the HML, SMB, and MOM risk components. Panel C provides the univariate regression results but for the period of 1963 to 2017, and Panel D reports the results controlling for the HML, SMB, MOM, CMA, and RMW risk components.

Because small stocks are typically much more illiquid and feature significantly higher transactions and short-selling costs than big stocks, the no friction assumption of the CAPM framework is expected to be strongly violated in the case of small stocks. In the case of big stocks, however, the no friction assumption is likely to be fairly realistic. As a result, one expects the dynamic CAPM to price big stocks much better than small stocks. Results presented in Table 4 confirm this intuition. Across all panels, as we go from the smallest to the largest stocks, the intercept decreases towards zero and the loading on the market risk component increases towards one. Over the sample period 1926 to 2017, eight out of ten size-deciles stocks feature an intercept that is not statistically different from zero at the 5% level. For the two smallest size-deciles stocks, the intercept is not statistically different from zero at the 1% level. Over the sample period 1963 to 2017, nine out of ten size-deciles stocks feature an intercept that is not statistically different from zero at the 5% level, while the intercept of the smallest size-decile stocks is not statistically different from zero at the 1% level. That is, the dynamic CAPM typically can not be rejected either when tested on individual stocks (see Proposition 1).

In contrast to what was obtained with portfolio returns, the loadings on the market risk component are significantly smaller than one when estimated using individual

stock returns. This suggests that the true beta, which is equal to the product of the loading b and the empirically estimated beta, is smaller than the empirically estimated beta (see Proposition 1). Panel A reports an R^2 ranging from 27% for the largest size-decile stocks to 2% for the smallest size-decile stocks. The results are similar in the sub-sample of 1963 to 2017. Finally, Panels B and D show that controlling for the additional risk components only marginally decreases both the intercept a and the loading b .

To summarize, consistent with our previous analysis on equity portfolios, the null hypothesis that the dynamic CAPM holds is also largely supported when tested on individual stocks.

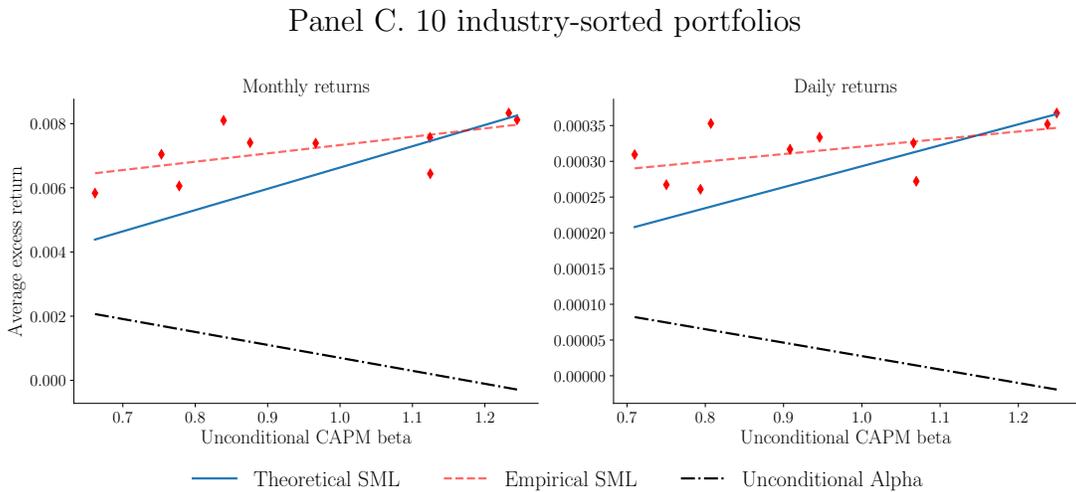
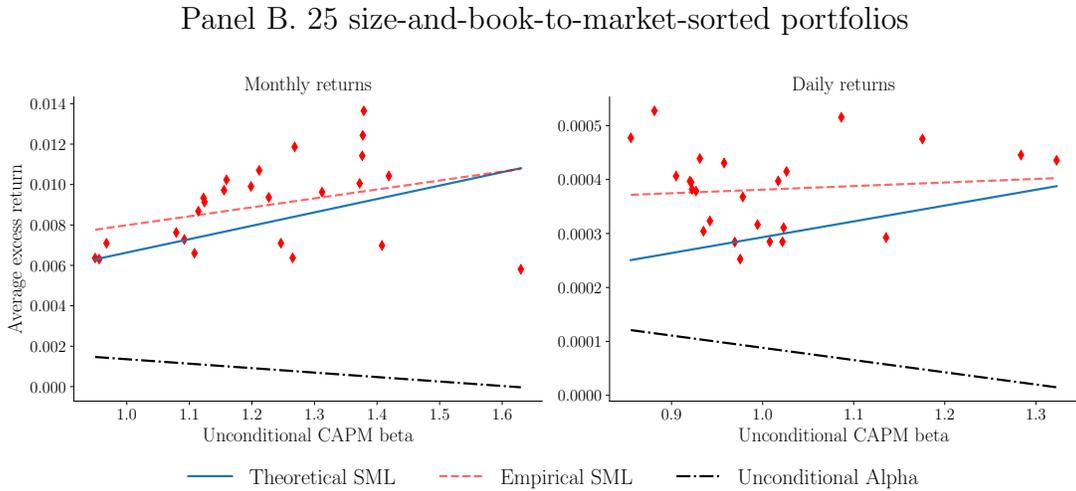
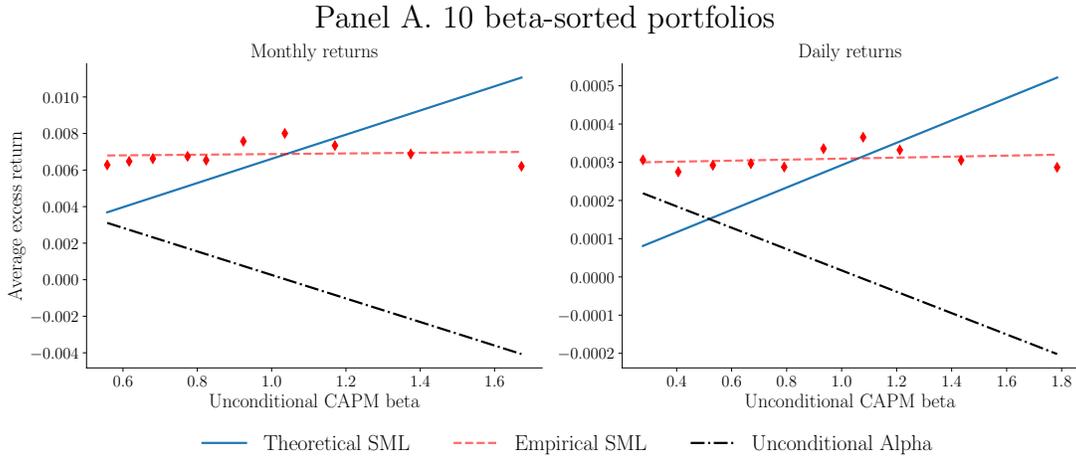
4.3. The security market line

Most of the existing literature on the CAPM focuses on testing whether the intercept and slope of the Security Market Line are equal to zero and to the unconditional expected excess market return, respectively (see, for instance, [Black et al., 1972](#), [Savor and Wilson, 2014](#), [Jylha, 2018](#), [Hendershott et al., 2018](#), [Ben-Rephael et al., 2018](#), among others). In this section, we discuss the Security Market Line as well as the relation between an asset's unconditional expected return and its unconditional expected market risk component. In particular, we show empirically that the unconditional expected asset return is a linear function of the unconditional expected market risk component, with an intercept equal to zero and a slope equal to one. This unconditional relation is a direct implication of the fact that the dynamic CAPM is supported by the data.

Figure 2 depicts the Security Market Line for the 10 beta-sorted portfolios in Panel A, for the 25 size-and-book-to-market-sorted portfolios in Panel B, and for the 10 industry-sorted portfolios in Panel C. As the dashed and solid lines show, the empirical

Figure 2. Security Market Line

This figure shows the empirical relation between the unconditional CAPM beta and average monthly excess return. The test assets are the 10 beta-, 25 size-and-book-to-market-, and the 10 industry-sorted value-weighted portfolios in Panels A to C, respectively. The solid line depicts the theoretical security market line predicted by the static CAPM, the dashed line depicts the empirical security market line, and the dash-dotted line depicts the unconditional alpha, i.e., $\alpha_i^u \equiv E(R_i) - \beta_i^u E(R_M)$.



relation between average excess returns and unconditional betas is consistently much flatter than predicted by the static CAPM. This observation is particularly striking for the 10 beta-sorted portfolios. Indeed, their average excess returns are independent of their unconditional betas at both the monthly and daily frequencies, whereas the static CAPM predicts a steep, positive relation. That is, the static CAPM (1) is strongly rejected by the data, whereas the results of the previous section show that the dynamic CAPM (2) cannot be rejected.

To understand this perhaps surprising result, let us assume from now on that the dynamic CAPM holds:

$$E_t(R_{i,t+1}) = \beta_{it}E_t(R_{M,t+1}) \equiv \beta_{it}m_{Mt}, \quad (8)$$

where R_i and R_M denote respectively the excess return of stock i and the excess return of the market, $m_{Mt} \equiv E_t(R_{M,t+1})$ is the time- t risk premium of the market, and $\beta_{it} \equiv \frac{\text{Cov}_t(R_{i,t+1}, R_{M,t+1})}{\text{Var}_t(R_{M,t+1})}$ is the time- t beta of stock i . Taking unconditional expectations yields

$$\begin{aligned} E(R_{i,t+1}) &= \beta_i^u E(R_{M,t+1}) \\ &+ \text{Cov}(\beta_{it}, m_{Mt}) \left[1 + \frac{E(R_{M,t+1})^2}{\text{Var}(R_{M,t+1})} \right] \\ &+ \left[E(\beta_{it})E(v_{Mt}) + E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}) - \text{Cov}(\beta_{it}, m_{Mt}^2) \right] \frac{E(R_{M,t+1})}{\text{Var}(R_{M,t+1})}, \end{aligned} \quad (9)$$

$$\equiv \beta_i^u E(R_{M,t+1}) + \alpha_i^u$$

where $\beta_i^u \equiv \frac{\text{Cov}(R_{i,t+1}, R_{M,t+1})}{\text{Var}(R_{M,t+1})}$ is the unconditional beta of stock i , α_i^u is the unconditional alpha of stock i , v_{Mt} is time- t market return variance, v_{it} is time- t return

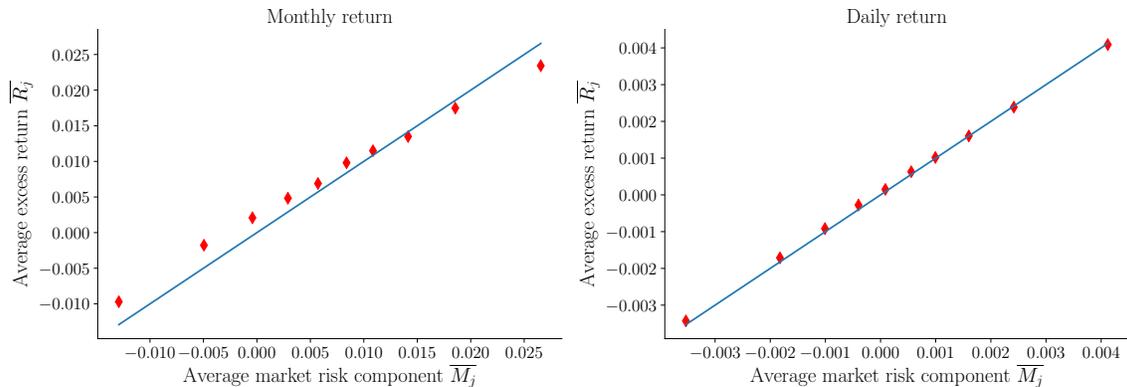
variance of stock i , $\epsilon_{i,t+1}$ is the unexpected white noise shock in stock i 's return at time $t + 1$, and $\epsilon_{M,t+1}$ is the unexpected white noise shock in the market return at time $t + 1$. Refer to Appendix A for the derivation of Equation (9).

The comovement terms observed on the second and third rows of Equation (9) imply that the relation between a stock's unconditional expected excess return and its unconditional beta is ambiguous, even though the dynamic CAPM holds. Indeed, the static CAPM would hold, and therefore the security market line would have an intercept equal to zero and a slope equal to the expected return of the market, only if the sum of the comovement terms were equal to zero. If, in contrast, the sum of the comovement terms is larger for stocks with low unconditional betas than for stocks with high unconditional betas, then the static CAPM does not hold because the security market line is flatter than predicted by the latter model.

The dash-dotted lines depicted in Figure 2 represent the empirical estimates of the sum of the comovement terms, or in other words, the empirical estimates of the unconditional alphas. The estimated alphas are high for portfolios with low unconditional betas and low for portfolios with high unconditional betas. While a complete model for both the conditional expected excess return and the conditional excess return variance of each portfolio and of the market would be needed to explain the magnitude of alpha, its asymmetric relation with the unconditional beta can be understood by investigating the dynamics of the conditional betas estimated in Section 4. Indeed, the correlation among conditional portfolio betas is positive and strong (ranging between about 0.5 and 0.9) for portfolios having unconditional betas that are close to each other. In contrast, the correlation among conditional betas is negative and strong (ranging between about -0.5 and -0.3) for portfolios having unconditional betas that are far from each other. This result implies that the covariance between the conditional beta and the conditional market expected excess return observed on

Figure 3. Average Return vs. Average Market Risk Component

This figure shows the empirical relation between the average excess return \overline{R}_j and the average market risk component \overline{M}_j for ten decile sorts on the market risk component. We sort on each month (day) when using monthly (daily) returns on the 10 beta-, 25 size-and-book-to-market-, and 10 industry-sorted value-weighted portfolios. The solid line depicts the theoretical relation predicted by the dynamic CAPM.



the second row of Equation (9) flips sign as the unconditional beta of the portfolio increases, thereby explaining the asymmetric relation between the unconditional alpha and the unconditional beta depicted on Figure 2.

Analogous to the Security Market Line, we next examine the relation between a portfolio's unconditional expected excess return and its unconditional expected dynamic-CAPM-implied return.⁶ Using our terminology, the latter is simply the unconditional expected market risk component. We perform this analysis on the ten beta-, 25 size-and-book-to-market-, and ten industry-sorted portfolios. Assuming that the dynamic CAPM holds, taking unconditional expectations on both sides of Equation (8) yields

$$E(R_{i,t+1}) = E(\beta_{it}R_{M,t+1}). \quad (10)$$

⁶Martin and Wagner (2018) perform a similar analysis in the context of their SVIX measures (see Figure 5 in their paper).

To test this unconditional relation, on each month (day) $t + 1$ we sort the 45 aforementioned portfolios' market risk components, $\beta_{it}R_{M,t+1}$, into ten deciles indexed by j . We denote by R_{jt} and M_{jt} the time- t cross-sectional average excess return and market risk component, respectively, within each decile j . We then compute the time series average of R_{jt} and M_{jt} , which we denote by \overline{R}_j and \overline{M}_j , respectively. Figure 3 depicts the empirical relation between the average return \overline{R}_j and the average market risk component \overline{M}_j . The empirical relation is well approximated by an affine function with slope equal to one and intercept equal to zero for both monthly and daily returns. That is, the data lend strong support to the theoretical relation (solid line) predicted by Equation (10).

5. Conclusion

When investors' hedging demands are equal to zero, the CAPM holds in a dynamic manner (Merton, 1973). That is, the time- t risk premium of a stock is equal to the product of its time- t beta and the time- t risk premium of the market. We test this dynamic CAPM relation by performing a panel regression of excess asset returns onto the product of their conditional beta and the excess return of the market. We find that (i) the intercept is indistinguishable from zero, (ii) the slope is indistinguishable from one, and (iii) the adjusted R^2 is about 80%. To summarize, the data lend support to the prediction that the dynamic CAPM holds.

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Appendix

A. Derivation of the Security Market Line

Let us assume that the dynamic CAPM holds:

$$E_t(R_{i,t+1}) = \beta_{it}E_t(R_{M,t+1}) \equiv \beta_{it}m_{Mt}, \quad (11)$$

where R_i and R_M denote respectively the excess return of stock i and the excess return of the market, and $E_t(R_{M,t+1}) \equiv m_{Mt}$. We can write

$$R_{i,t+1} = \beta_{it}m_{Mt} + \sqrt{v_{it}}\epsilon_{i,t+1},$$

$$\beta_{it} = E(\beta_{it}) + u_{it},$$

$$R_{M,t+1} = m_{Mt} + \sqrt{v_{Mt}}\epsilon_{M,t+1},$$

where $E_t(\epsilon_{i,t+1}) = E_t(\epsilon_{M,t+1}) = E(u_{it}) = 0$, $\text{Var}_t(\epsilon_{i,t+1}) = \text{Var}_t(\epsilon_{M,t+1}) = 1$, $\text{Var}_t(R_{i,t+1}) = v_{it}$, and $\text{Var}_t(R_{M,t+1}) = v_{Mt}$.

This implies that

$$\begin{aligned}
\text{Cov}(R_{i,t+1}, R_{M,t+1}) &= \text{Cov}((E(\beta_{it}) + u_{it})m_{Mt} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
&= \text{Cov}((E(\beta_{it}) + u_{it})(R_{M,t+1} - \sqrt{v_{Mt}}\epsilon_{M,t+1}) + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
&= E(\beta_{it})\text{Var}(R_{M,t+1}) \\
&\quad + \text{Cov}(u_{it}m_{Mt} - E(\beta_{it})\sqrt{v_{Mt}}\epsilon_{M,t+1} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
&= E(\beta_{it})\text{Var}(R_{M,t+1}) \\
&\quad + E(u_{it}m_{Mt}R_{M,t+1}) - E(u_{it}m_{Mt})E(R_{M,t+1}) \\
&\quad - \text{Cov}(E(\beta_{it})\sqrt{v_{Mt}}\epsilon_{M,t+1} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
&= E(\beta_{it})\text{Var}(R_{M,t+1}) \\
&\quad + E(u_{it}m_{Mt}^2) - E(u_{it}m_{Mt})E(R_{M,t+1}) \\
&\quad - \text{Cov}(E(\beta_{it})\sqrt{v_{Mt}}\epsilon_{M,t+1} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
&= E(\beta_{it})\text{Var}(R_{M,t+1}) \\
&\quad + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
&\quad - \text{Cov}(E(\beta_{it})\sqrt{v_{Mt}}\epsilon_{M,t+1} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
&= E(\beta_{it})\text{Var}(R_{M,t+1}) \\
&\quad + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
&\quad - E(\beta_{it})E(\sqrt{v_{Mt}}\epsilon_{M,t+1}R_{M,t+1}) + E(\beta_{it})E(\sqrt{v_{Mt}}\epsilon_{M,t+1})E(R_{M,t+1}) \\
&\quad - E(\sqrt{v_{it}}\epsilon_{i,t+1}R_{M,t+1}) + E(\sqrt{v_{it}}\epsilon_{i,t+1})E(R_{M,t+1}) \\
&= E(\beta_{it})\text{Var}(R_{M,t+1}) \\
&\quad + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
&\quad - E(\beta_{it})E(\sqrt{v_{Mt}}\epsilon_{M,t+1}R_{M,t+1}) - E(\sqrt{v_{it}}\epsilon_{i,t+1}R_{M,t+1})
\end{aligned}$$

$$\begin{aligned}
\text{Cov}(R_{i,t+1}, R_{M,t+1}) &= E(\beta_{it})\text{Var}(R_{M,t+1}) \\
&\quad + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
&\quad - E(\beta_{it})\text{Cov}(\sqrt{v_{Mt}}\epsilon_{M,t+1}, R_{M,t+1}) - \text{Cov}(\sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
&= E(\beta_{it})\text{Var}(R_{M,t+1}) \\
&\quad + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
&\quad - E(\beta_{it})\text{Cov}(\sqrt{v_{Mt}}\epsilon_{M,t+1}, m_{Mt} + \sqrt{v_{Mt}}\epsilon_{M,t+1}) \\
&\quad - \text{Cov}(\sqrt{v_{it}}\epsilon_{i,t+1}, m_{Mt} + \sqrt{v_{Mt}}\epsilon_{M,t+1}) \\
&= E(\beta_{it})\text{Var}(R_{M,t+1}) \\
&\quad + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
&\quad - E(\beta_{it}) [E(\sqrt{v_{Mt}}\epsilon_{M,t+1}m_{Mt}) + E(v_{Mt}\epsilon_{M,t+1}^2)] \\
&\quad - E(\sqrt{v_{it}}\epsilon_{i,t+1}m_{Mt}) - E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}) \\
&= E(\beta_{it})\text{Var}(R_{M,t+1}) \\
&\quad + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
&\quad - E(\beta_{it})E(v_{Mt}) - E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}) \\
&= E(\beta_{it})\text{Var}(R_{M,t+1}) \\
&\quad + \text{Cov}(\beta_{it}, m_{Mt}^2) - \text{Cov}(\beta_{it}, m_{Mt})E(R_{M,t+1}) \\
&\quad - E(\beta_{it})E(v_{Mt}) - E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}), \tag{12}
\end{aligned}$$

where last equality comes from the fact that $u_{it} = \beta_{it} - E(\beta_{it})$. Equation (12) implies

that the unconditional beta of stock i , β_i^u , satisfies

$$\begin{aligned}
\beta_i^u &= \frac{\text{Cov}(R_{i,t+1}, R_{M,t+1})}{\text{Var}(R_{M,t+1})} \\
&= E(\beta_{it}) \\
&\quad + \frac{\text{Cov}(\beta_{it}, m_{Mt}^2) - \text{Cov}(\beta_{it}, m_{Mt})E(R_{M,t+1})}{\text{Var}(R_{M,t+1})} \\
&\quad - \frac{E(\beta_{it})E(v_{Mt}) + E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1})}{\text{Var}(R_{M,t+1})} \tag{13}
\end{aligned}$$

Applying unconditional expectations to Equation (11) and substituting Equation (13) into it yields

$$\begin{aligned}
E(R_{i,t+1}) &= \text{Cov}(\beta_{it}, m_{Mt}) + E(\beta_{it})E(m_{Mt}) \\
&= \text{Cov}(\beta_{it}, m_{Mt}) + E(\beta_{it})E(R_{M,t+1}) \\
&= \beta_i^u E(R_{M,t+1}) \\
&\quad + \text{Cov}(\beta_{it}, m_{Mt}) - E(R_{M,t+1}) \frac{\text{Cov}(\beta_{it}, m_{Mt}^2) - \text{Cov}(\beta_{it}, m_{Mt})E(R_{M,t+1})}{\text{Var}(R_{M,t+1})} \\
&\quad + E(R_{M,t+1}) \frac{E(\beta_{it})E(v_{Mt}) + E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1})}{\text{Var}(R_{M,t+1})} \\
&= \beta_i^u E(R_{M,t+1}) \\
&\quad + \text{Cov}(\beta_{it}, m_{Mt}) \left[1 + \frac{E(R_{M,t+1})^2}{\text{Var}(R_{M,t+1})} \right] \\
&\quad + \frac{E(R_{M,t+1})}{\text{Var}(R_{M,t+1})} \left[E(\beta_{it})E(v_{Mt}) + E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}) - \text{Cov}(\beta_{it}, m_{Mt}^2) \right].
\end{aligned}$$