THE CAPM HOLDS*

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ABSTRACT

Under realistic conditions, the conditional risk premium of an asset is equal to its conditional market beta times the conditional risk premium of the market (Merton, 1972). We empirically test this CAPM relation using beta-sorted portfolios, size-and-book-to-market sorted portfolios, and industry portfolios. We show that regressing an asset excess return onto the product of its conditional beta and the market excess return yields an R^2 of about 80%, an intercept of zero, and a slope of one. These results provide strong evidence that a single factor explains both the level and the variation in the cross-section of returns.

JEL Classification: D53, G11, G12

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1. Introduction

The capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965a,b), and Mossin (1966), which allowed William F. Sharpe to win the 1990 Nobel Prize in economics, is the most famous and influential pricing relation that has ever been discovered. It states that the risk premium of an asset is equal to the asset's exposure to market risk (beta) times the risk premium of the market. As of today, the CAPM has been taught in business schools for more than fifty years, and it is commonly used by practitioners and investors to compute the cost of capital (Graham and Harvey, 2001) and to build investment strategies (Berk and van Binsbergen, 2016).

Despite its popularity, Black, Jensen, and Scholes (1972) and Fama and French (2004) document that the CAPM is actually not supported by the data. Indeed, the security market line, which plots assets' expected returns as a function of their betas, is flat, whereas the CAPM predicts that it should be positive. Interestingly, Tinic and West (1984), Cohen, Polk, and Vuolteenaho (2005), Savor and Wilson (2014), Hendershott, Livdan, and Rosch (2018), and Jylha (2018) provide evidence that the CAPM holds in January, on months of low inflation, on days of important macroeconomic announcements, overnight, and on months during which investors can borrow easily, respectively. That is, there are specific periods of time during which the CAPM cannot be rejected by the data.

In this paper, we first argue theoretically that, under some realistic conditions, the CAPM holds but in a conditional manner. Indeed, when investors' hedging demands are equal to zero, the conditional risk premium of a stock is equal to the conditional beta of the stock times the conditional risk premium of the market (Merton, 1973). Second, we empirically test the aforementioned CAPM relation and show that the data lend strong support to it. In particular, our panel regression analysis shows that regressing a stock excess return onto the product of its conditional beta and the market excess return yields *(i)* an adjusted R^2 of about 80%, *(ii)* an intercept that is economically and statistically indistinguishable from zero, and *(iii)* a slope that is economically and statistically indistinguishable from one.

Our theoretical motivation is borrowed from Merton (1973), who considers a continuous-time economy populated by agents that can invest in n stocks and one risk-free asset paying a stochastic risk-free rate. Agents have homogeneous beliefs about the instantaneous expected return and return volatility of each stock, which are assumed to be stochastic. Merton (1973) shows that if agents' hedging demands are equal to zero, then the conditional risk premium of a stock is equal to its conditional beta times the conditional risk premium of the market. Note that hedging demands are equal to zero if either agents have logarithmic preferences, or the investment opportunity set is constant, or changes in the state variables are uncorrelated to stock returns, or changes in the state variables are correlated to stock returns in such a way that the sum of all hedging components is equal to zero. If either one of these conditions is satisfied, then the model predicts that performing a panel regression of excess stock returns onto the product of the conditional betas and the market excess returns should provide an intercept equal to zero and a slope equal to one. In addition, the regression R^2 is predicted to be large if stocks' idiosyncratic volatilities are low.

We test the predictions of the model using monthly and daily U.S. stock return data from 1926 to 2017. Our test assets include ten CAPM beta-sorted portfolios, the Fama-French 25 size-and-book-to-market sorted portfolios, ten industry-sorted portfolios as well as individual stocks.

Our first empirical test examines the main implication of the model using panel regressions, as in Martin and Wagner (2018). The model predicts that regressing an asset return on the product of its conditional beta and the market excess return should provide an intercept equal to zero, a slope equal to one, and a large R^2. Using monthly returns, the product of the conditional beta and the excess return of the market, which we label as the market risk component, largely explains the cross-section of stock returns.[1] For most of our portfolios, we find an intercept that is indistinguishable from zero, and the loading on the market risk component is indistinguishable from one. Moreover, the explanatory power (R^2) of only including the market risk component to explain the cross-section of monthly stock returns is large; 87% and 78% for the ten beta-sorted value-weighted and equal-weighted portfolios, respectively. The explanatory power is also large for a variety of other portfolios ranging from 70% to 75% for monthly returns.

Our results are also supported when using daily returns. For the ten beta-sorted portfolios, we document an R^2 of 79% to 78% for the value- and equal-weighted portfolios, respectively, and a loading on the market risk component that is indistinguishable from one. The intercept for the value-weighted portfolio is not statistically different from zero (at the 5% level), whereas that for the equal-weighted portfolio is. Yet, the estimate is economically small (0.01% or 2.5% in annualized terms). For the other portfolios, the intercept estimates are all economically small but typically slightly larger for equal-weighted portfolios than for value-weighted portfolios.

We then evaluate the performance of the market risk component relative to other

[1] The conditional CAPM beta is calculated using 24 months (250 trading days) for monthly (daily) returns strictly prior to month (day) t. Our results are robust to different window lengths.

risk factors. Similarly to our market risk component, we construct additional risk components using the Fama and French (1993, 2015) and Carhart (1997) factors. More precisely, we examine individually the performance of the product of the conditional exposure to the Fama-French high-minus-low (HML), small-minus-big (SMB), robust-minus-weak (RMW), conservative-minus-aggressive (CMA), and momentum (MOM) factor and the factor return (which we label as HML, SMB, etc., risk components). We find that none of these risk factors outperform the simple market risk component. The best performance comes from the HML and SMB risk components achieving R^2 not larger than 33%. Moreover, the intercept estimates are three to seven times larger than that obtained using the market risk component only.

We confirm that our results using the market risk component are robust to including the Fama and French (1993, 2015) and Carhart (1997) risk components into the regression. When including these risk components, the loading on the market risk component decreases slightly but remains larger than 0.85. Additionally, the loadings on the other risk components decrease by more than 50% relative to their univariate estimates. Most importantly, adding the Fama-French and momentum risk components has a negligible impact on the R^2 relative to the univariate regression using only the market risk component. To summarize, the prediction that the market risk component is the main driver explaining the cross-section of stock returns is strongly supported by the data.

Finally, we examine whether our theoretical predictions hold for individual stocks using monthly returns. We find strong results for individual stocks, but unsurprisingly, not as strong as using portfolio returns. We find an R^2 varying from 27% for the largest decile stocks to 2% for the smallest decile stocks. Nonetheless, the market risk component outperforms any of the other Fama-French and momentum risk components. The intercept and estimate of the market risk component also varies across different size deciles. For eight out of the ten size decile stocks, we find an intercept that is not statistically different from zero at the 5% level. The two size deciles for which the intercept is statistically different from zero are for stocks in the two smallest market capitalization deciles. Controlling for any of the additional risk components marginally improves our results, suggesting that a simple univariate panel regression with the market risk component is powerful enough by itself to explain daily stock returns.

Our work is closely related to the growing empirical literature showing that the relation between an asset's average excess return and its beta is positive only during a specific time. Cohen et al. (2005) show that the relation between average excess stock returns and their beta is positive during months of low inflation and negative during months of high inflation. Savor and Wilson (2014) find that average excess stock returns are positively related to their beta only on days with important macroeconomic announcements (inflation, unemployment, or Federal Open Markets Committee announcements). Jylha (2018) finds that the security market line is positive during months when investors' borrowing constraints are slack and negative during months when borrowing constraints are tight. Hendershott et al. (2018) show that the CAPM performs poorly during regular trading hours (open to close), but holds during the overnight period (close to open). Ben-Rephael, Carlin, Da, and Israelsen (2018) provide empirical evidence that the Security Market Line is upward-sloping, as predicted by the CAPM, when the demand for information is high. Hong and Sraer (2016) show both theoretically and empirically that the Security Market Line is upward-sloping in low disagreement periods and hump-shaped in high disagreement periods.

Our paper is also related to Jagannathan and Wang (1996) who assume that the risk premium of the market is linear in the yield spread, and that the market return is linear in the stock index return and in the labor income growth rate. In this case, the expected return of a stock is a linear function of three betas: yield spread beta, stock index beta, and labor income beta. This three-factor model is shown to explain the cross-section of returns significantly better than the CAPM. Lewellen and Nagel (2006) obtain direct estimates of the conditional CAPM alphas and betas from short window regressions (3 months, 6 months, or 12 months). They show that the average conditional alpha is large, and therefore argue that the conditional CAPM performs as poorly as the unconditional one. The key difference between our test and theirs is that their beta is constant over each short window, whereas our beta changes every day when using daily returns and every month when using monthly returns. By correcting for the bias in unconditional alphas due to market timing, volatility timing, and overconditioning, Boguth, Carlson, Fisher, and Simutin (2011) show that momentum alphas are significantly lower than previously documented. By applying the instrumental variable method of Boguth et al. (2011) to model conditional betas, Cederburg and O'Doherty (2016) show that the betting-against-beta anomaly (Frazzini and Pedersen, 2014) disappears.

Our paper further relates to the recent work by Dessaint, Olivier, Otto, and Thesmar (2018) who argue that managers using the CAPM should overvalue low beta projects relative to the market because of the gap between CAPM-implied returns and realized returns. They show empirically that takeovers of low beta targets typically yield smaller abnormal returns for the bidders, supporting the aforementioned hypothesis. Martin and Wagner (2018) demonstrate that a stock expected return can be written as a sum of the market risk neutral variance and the stock's excess risk neutral variance relative to the average stock. Their panel regression analysis shows that the aforementioned prediction of the model is supported by the data. In their theoretical framework, Andrei, Cujean, and Wilson (2018) show that, although the CAPM is the correct model, an econometrician incorrectly rejects it because of its informational disadvantage compared with the average investor.

Our paper differs from these studies in two important aspects. First, we provide a theoretical motivation for the fact that the CAPM relation should hold but in a conditional fashion as predicted in Merton (1973). That is, under realistic assumptions, the conditional risk premium of a stock should be equal to its conditional beta times the conditional risk premium of the market. Second, we test this specific CAPM relation by regressing asset excess returns onto the product of the asset's conditional beta and the market excess return, and show that the data lend support to it.

The remainder of the paper is as follows. Section 2 provides our theoretical motivation. Section 3 describes the data and the empirical design. Section 4 discusses our empirical results and Section 6 concludes.

2. Theoretical Motivation

2.1. The CAPM

The capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965a,b), and Mossin (1966) is derived under the assumptions that agents have homogeneous beliefs, are mean-variance optimizers, and have an horizon of one period. That is, all agents solve the portfolio selection problem presented in Markowitz (1952). Under these assumptions and given the supply of each asset, the equilibrium risk premium on any stock is a linear function of its beta, which is defined as the covariance between the stock return and the market return over the variance of the market return. Specifically, the static CAPM is written

$$\mathbb{E}\left(r_i\right) - r_f = \beta_i \left[\mathbb{E}\left(r_M\right) - r_f\right], \tag{1}$$

where r_i is the return of stock i, r_f is the risk-free rate, r_M is the market return, and $\beta_i \equiv \frac{\text{Cov}(r_i, r_M)}{\text{Var}(r_M)}$ is the beta of stock i.

As argued in Merton (1973), the single period and mean-variance optimization assumptions have been subject to criticism. Therefore, Merton (1973) extends the aforementioned economic environment by allowing agents to trade continuously over their life times, and to have time-separable von Neumann-Morgenstern utility functions. Agents can invest in n stocks and one riskless asset. Importantly, the vector of (instantaneous) stock returns is allowed to have both a stochastic mean and a stochastic variance-covariance matrix. That is, the investment opportunity set is allowed to be non-constant. Formally, the dynamics of asset returns satisfy

$$\begin{aligned}
\frac{dP_{it}}{P_{it}} &= \mu_{it} dt + \sigma_{it} dz_{it}, \\
d\mu_{it} &= a_{it} dt + b_{it} dq_{it}, \\
d\sigma_{it} &= f_{it} dt + g_{it} dx_{it}, \qquad i = 1, \ldots, n+1
\end{aligned}$$

where $\frac{dP_{it}}{P_{it}}$, μ_{it}, and σ_{it} are respectively the return, expected return, and return volatility of asset i at time t. The constant correlation between the Brownian motions dz_{it} and dz_{jt} is ρ_{ij}, which implies that the variance-covariance matrix of returns is $\Omega_t \equiv [\sigma_{ij,t}] = [\sigma_{it}\sigma_{jt}\rho_{ij}]$. Although not specified here, the constant correlations between the Brownian motions dq_{it} and dq_{jt}, dx_{it} and dx_{jt}, dz_{it} and dq_{jt}, dz_{it} and dx_{jt}, and dq_{it} and dx_{jt} are allowed to be different from zero. The processes a_{it}, f_{it}, b_{it}, and g_{it} are functions of the vector of prices P_t, the vector of expected returns μ_t, and the vector of return volatilities σ_t. Asset $n+1$ is assumed to be the riskless asset, i.e., $\mu_{n+1,t} \equiv r_{ft}$ and $\sigma_{n+1,t} \equiv 0$ so that

$$\frac{dP_{n+1,t}}{P_{n+1,t}} = r_{ft}dt,$$

where r_{ft} is the risk-free rate at time t.

If investors' hedging demands are equal to zero, the equilibrium risk premium of stock i satisfies

$$\mu_{it} - r_{ft} = \beta_{it}\left[\mu_{Mt} - r_{ft}\right], \tag{2}$$

where μ_{Mt} is the expected return of the market portfolio and $\beta_{it} \equiv \frac{\text{Cov}_t\left(\frac{dP_{it}}{P_{it}}, \frac{dP_{Mt}}{P_{Mt}}\right)}{\text{Var}_t\left(\frac{dP_{Mt}}{P_{Mt}}\right)}$ is the beta of stock i. It is worth noting that hedging demands are equal to zero if either agents have logarithmic preferences, or the investment opportunity set is constant, or changes in the state variables are uncorrelated to stock returns, or changes in the state variables are correlated to stock returns in such a way that the sum of all hedging components is equal to zero.

Equation (2) shows that, when agents trade continuously and have no hedging motives, the original (static) CAPM relation (1) still holds but in a dynamic manner.

Specifically, the time-t risk premium on any stock is the product of the time-t stock's beta and the time-t risk premium of the market. Whether or not the CAPM relation (2) holds empirically crucially depends on the assumption that investors' have no or say negligible hedging motives.

2.2. Testing the CAPM empirically

The CAPM relation (2) relates expected excess stock returns to expected excess market returns. Since expected returns are unobservable, the empirical framework needed to test the CAPM relation (2) is not necessarily straightforward, and therefore requires additional details.

As in Lewellen and Nagel (2006), our empirical framework focuses on realized returns. Specifically, we consider the following model for stock i's excess return

$$\frac{dP_{it}}{P_{it}} - r_{ft}dt = adt + b\widehat{\beta}_{it}\left[\frac{dP_{Mt}}{P_{Mt}} - r_{ft}dt\right] + \tilde{\sigma}_{it}dW_{it}, \tag{3}$$

where

$$\frac{dP_{Mt}}{P_{Mt}} - r_{ft}dt \equiv \left[\mu_{Mt} - r_{ft}\right]dt + \sigma_{Mt}dW_{Mt}, \tag{4}$$

is the excess market return, μ_{Mt} is the market expected return, σ_{Mt} is the volatility of the market return, $\widehat{\beta}_{it}$ is an empirical estimate of the beta of stock i, $\tilde{\sigma}_{it}$ is the idiosyncratic volatility of stock i's return, r_{ft} is the risk-free rate, and dW_{it} and dW_{Mt} are independent Brownian motions. Note that the beta of stock i satisfies

$\beta_{it} = b\widehat{\beta}_{it}$. Substituting Equation (4) in Equation (3) yields

$$\frac{dP_{it}}{P_{it}} - r_{ft}dt = \left(a + b\widehat{\beta}_{it}\left[\mu_{Mt} - r_{ft}\right]\right)dt + b\beta_{it}\sigma_{Mt}dW_{Mt} + \tilde{\sigma}_{it}dW_{it} \tag{5}$$

$$\equiv \left[\mu_{it} - r_{ft}\right]dt + \sigma_{it}dz_{it}, \tag{6}$$

where μ_{it} is stock i's expected return, σ_{it} is the volatility of stock i's return, and dz_{it} is a Brownian motion.

Proposition 1 below provides conditions for the CAPM relation (2) to hold in our empirical framework.

Proposition 1 *Let us consider model* (3)*:*

$$\frac{dP_{it}}{P_{it}} - r_{ft}dt = adt + b\widehat{\beta}_{it}\left[\frac{dP_{Mt}}{P_{Mt}} - r_{ft}dt\right] + \tilde{\sigma}_{it}dW_{it}.$$

- *If the intercept $a = 0$, then the CAPM relation* (2) *holds:*

$$\mu_{it} - r_{ft} = b\widehat{\beta}_{it}\left[\mu_{Mt} - r_{ft}\right] = \beta_{it}\left[\mu_{Mt} - r_{ft}\right].$$

- *If both the intercept $a = 0$ and the slope $b = 1$, then the CAPM relation* (2) *holds and the empirical estimate of the beta of stock i is well defined, i.e., $\widehat{\beta}_{it} = \beta_{it}$.*

Proof*: See the derivations from Equations* (3) *and* (4) *to Equations* (5) *and* (6)*.*

In Section 4, we consider a discretized version of model (3) and empirically test the null hypothesis that both the intercept $a = 0$ and the slope $b = 1$. That is, our test takes into account the issue raised by Lewellen and Nagel (2006) that the slope

b has to be equal to one for the empirical estimate $\widehat{\beta}_{it}$ to be well defined.[2] We show that the null hypothesis cannot be rejected (at conventional confidence levels), which through Proposition 1 implies that the CAPM relation (2) holds (at conventional confidence levels).

3. Data

3.1. Stock returns and portfolio construction

We obtain stock return data from the Center for Research in Security Prices (CRSP). Our main stock market return proxy is the market return from Kenneth French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french). We also obtain from Kenneth French's website the risk-free rate and returns for the following test assets: 25 size-and book-to-market-, 25 size-and momentum-, 25 size-and-investment-, and 25 size-and-operating-profits-sorted portfolio returns, the ten and 49 industry portfolio returns. Finally, we further obtain the high-minus-low (HML), small-minus-big (SMB), robust-minus-weak (RMW), and conservative-minus-aggressive (CMA) Fama and French (1993, 2015) factors and Carhart (1997) momentum (MOM) factor. The results obtained when using the 25 size-and-investment- and the 25 size-and-operating-profits-sorted portfolios, and the 49 industry portfolios are presented in the Internet Appendix. The sample period is from July 1, 1926 to December 31, 2017. When using the more recent factors, RMW and CMA, the sample period starts on July 1, 1963.

[2]Refer to Section 5 in Lewellen and Nagel (2006) for a discussion on why their conclusions differ from those of Jagannathan and Wang (1996), Lettau and Ludvigson (2001), Lustig and Nieuwerburgh (2005), and Santos and Veronesi (2006).

We also construct ten daily and monthly beta-sorted portfolios using U.S. common stocks that are identified in CRSP as having a share code of 10 or 11 trading on the NYSE, Nasdaq, or AMEX stock exchange. We estimate stock market monthly (daily) betas for all stocks using rolling windows of 24 months (250 trading days) of monthly (daily) returns.[3] At the beginning of each month, we sort stocks into one of the ten beta-decile initially-value-weighted and initially-equal-weighted portfolios, and calculate their respective monthly and daily returns.

Our last step consists of calculating for each of the portfolios their monthly and daily market betas, $\beta^M_{i,t}$, using the last 24 months (250 trading days) rolling windows of excess returns, which we denote as β^M. Similarly, we calculate for each of the portfolios their HML, SMB, RMW, CMA, MOM betas, denoted respectively as $\beta^{HML}_{i,t}$, $\beta^{SMB}_{i,t}$, $\beta^{RMW}_{i,t}$, $\beta^{CMA}_{i,t}$, and $\beta^{MOM}_{i,t}$.

In the next section, we begin our main empirical analysis with a direct test of the conditional CAPM stated in Equation (2).

4. Main Empirical Results

4.1. Univariate panel regressions

We empirically test the CAPM relation defined in Equation (2). Formally, we estimate the following panel regression

$$R_{i,t+1} = a + b[\beta^M_{i,t} R_{M,t+1}] + e_{i,t+1}, \tag{7}$$

[3] If for a given stock the availability of returns is less than 24 months (250 days), we require at least 12 months (100 days) of returns to calculate the stock's monthly (daily) market beta.

where $R_{i,t+1}$ is the excess return of portfolio i, $R_{M,t+1}$ is the market excess return, and $\beta_{i,t}^{M}$ is the coefficient of a regression of the monthly (daily) excess return of portfolio i on the excess market return using 24 months (250 trading days) strictly prior to month (day) $t+1$.[4] That is, $\beta_{i,t}^{M}$ is known at month (day) t. We label $\beta_{i,t}^{M}(R_{M,t+1} - R_{F,t+1})$ as the *market risk component*.

[Insert Figure 1 about here]

We present in Figure 1 the estimated intercept a, the slope b associated with the market risk component, and their respective 95% confidence intervals.[5] Panel A shows the results using monthly returns and Panel B using daily returns. The figure confirms the prediction of the CAPM relation defined in Equation (2). For the monthly returns (Panel A), 8 out of 12 of the portfolios have an intercept a that is not statistically different from zero at the 5% level. For all the portfolios, the estimated loading b on the market risk component are not statistically different from one. For daily returns (Panel B), three out of 12 portfolios have an intercept a that is not statistically different from zero. The loadings b on the market risk component are all not statistically different from one. As we will later show, the explanatory power associated with the market risk component is also high both at the daily and monthly frequencies, and surpasses any other risk components (i.e., Fama-French factors).

[4]Choosing 48 months rather than 24 months when estimating the β does not alter our results.

[5]Throughout this paper, we calculate standard errors using the Driscoll and Kraay (1998) extension of the Newey-West HAC estimator using 12 (250) month (trading day) lags for monthly (daily) returns. The Driscoll-Kray procedure is a GMM technique for panels where both the cross-sectional and time dimensions are large. In principle, the standard errors are also robust to heteroscedasticity, autocorrelation, and general spatial (cross-firm) dependence.

4.2. Examining the relation between implied returns and realized returns

Despite confirming the implied values of the CAPM relation in Equation (2), the empirical model might poorly explain (i.e., low R^2) empirically the relation between realized returns and the implied returns.

[Insert Figure 2 about here]

To examine this issue, Figure 2 presents a scatter plot highlighting the relation between the realized monthly returns and the implied monthly returns for the ten beta-sorted value- and equal-weighted portfolios in Panel A and B, respectively. The implied monthly return is defined as $\beta_{i,t}^{M} R_{M,t+1}$. We also examine the relation between the realized monthly returns and the implied monthly returns using the Fama and French (1993, 2015) and Carhart (1997) factors, i.e., HML, SMB, RMW, CMA, and MOM. For these factors, the implied monthly returns are, $\beta_{i,t}^{HML} \times HML_{t+1}$ (HML risk component), $\beta_{i,t}^{SMB} \times SMB_{t+1}$ (SMB risk component), $\beta_{i,t}^{RMW} \times RMW_{t+1}$ (RMW risk component), $\beta_{i,t}^{CMA} \times CMA_{t+1}$ (CMA risk component), and $\beta_{i,t}^{MOM} \times MOM_{t+1}$ (MOM risk component). The scatter plots show that the market risk component $\beta_{i,t}^{M} R_{M,t+1}$ best explains the realized monthly returns with R^2 equal to 87% and 78% for the value-weighted and equal-weighted portfolios, respectively. The Fama-French factors and momentum factor do not fit the data as well. The HML and SMB risk components are the best explanatory variables among these factors but their R^2 are not larger than 33%.

4.3. Panel regressions: Controlling for Fama and French (1993, 2015) and Carhart (1997) factors

We now examine how robust our results presented in Section 4.1 are when controlling for the Fama-French and Carhart risk components. We estimate the following panel regression:

$$R_{i,t+1} = a + b[\beta_{i,t}^{M} R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] + s[\beta_{i,t}^{SMB} SMB_{t+1}] \\ + m[\beta_{i,t}^{MOM} MOM_{t+1}] + r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1} \tag{8}$$

We present our results, using monthly and daily returns, for both value-weighted and equal-weighted market-beta-sorted portfolios, 25 size-and-book-to-market-sorted portfolios, and 10 industry-sorted portfolios. In the Internet Appendix, we show the results for additional sorted portfolios.

4.3.1. Ten beta-sorted portfolios

[Insert Table 1 about here]

Table 1 presents the results of the model specified in Equation (8) for the ten beta-sorted portfolios for monthly returns in Panel A and for daily returns in Panel B. Columns (1) of Panels A and B show that the loadings b on the market risk component are indistinguishable from one, and the R^2 are remarkably high, ranging from 78 to 87%. For value-weighted monthly and daily returns, the intercept a is statistically indistinguishable from zero at the 5% level. Although the intercept is statistically different from zero for equal-weighted portfolios, its economic magnitude is much lower than that obtained using the other factors. Indeed, the intercept is about 3% in annualized terms for the market risk component, and ranges from 7.20 to 12% in annualized terms for the other risk components. Columns (2)-(4) and (7)-(8) show that the explanatory power of HML, SMB, MOM, RMW, and CMA is poor relative to that of the market risk component. Column (5) shows that including the HML, SMB, MOM risk components increases the R^2 by only 1% for the value-weighted portfolios and by a maximum of 10% for the equal-weighted portfolios. The last two rows of Table 1 reports the p-values of the Wald statistics testing the joint hypothesis of H_0: $a = 0$ and $b = 1$ and H_0: $\forall a_i = 0$ and $b = 1$ when the intercepts are estimated separately for each portfolio i. The reported p-values show that we do not reject the null, H_0: $a = 0$ and $b = 1$, for both value- and equal-weighted monthly portfolios and the null, H_0: $\forall a_i = 0$ and $b = 1$ for value-weighted portfolios at the 5% level. Similar results is found in Panel B using daily returns.

4.3.2. 25 size-and-book-to-market sorted portfolios

We repeat the same analysis but using the Fama-French 25 size-and-book-to-market sorted portfolios and present the results in Table 2. Our previous conclusion that the market risk component largely explains the cross-section of stock returns is confirmed. Indeed, the R^2 are 74% (59%) and 72% (59%) for the monthly (daily) value- and equal-weighted portfolios, respectively. Column (5) reports that adding the HML, SMB, and MOM factors increases the R^2 at most by 10% and 2% for the monthly and daily returns, respectively. The explanatory powers of the RMW and CMA factors are weak relative to that of the market risk component, ranging from 7 to 11% for monthly returns and from 14 to 17% for daily returns. Column (9) shows that adding all Fama-French risk factors the momentum factor improves the R^2 by only 11% and 3% for monthly and daily returns, respectively. These results show that the market risk component largely outperforms the HML, SMB, RMW, CMA, and MOM risk components in explaining the cross-section of stock returns. The reported p-values of the Wald statistics show that we do not reject the null that $a = 0$, $b = 1$ for both value- and equal-weighted monthly portfolios. However, the null that $\forall a_i = 0$ and $b = 1$ is rejected at 1% level. For daily returns portfolios, we do not reject the null that $a = 0$, $b = 1$ for value-weighted returns only.

[Insert Table 2 about here]

4.3.3. Ten industry sorted portfolios

Table 3 reports our empirical results for the ten industry-sorted portfolios. Columns (1) and (6) show that the R^2 for the market risk component in the univariate regressions ranges from 58 to 75% for monthly returns and from 66 to 77% for daily returns. Moreover, adding the HML, SMB, RMW, CMA, MOM risk components improves only marginally the explanation of the cross-section of stock returns.

[Insert Table 3 about here]

Overall, the results reported in this section paint a clear picture that the CAPM relation defined in Equation (2) is strongly supported by the data for a wide range of different portfolios. In particular, using only the market risk component to explain the cross-section of portfolio returns yields very large R^2, ranging from 58 to 87%.

4.4. Individual stocks

Our results so far show that the market risk component accurately explains the returns of a wide cross-section of stock portfolios. We next evaluate the ability of the market risk component to explain individual stocks returns using the regression specified in Equations (7) and (8).

We present the regression results in Table 4 by firm size deciles. Firm size deciles are assigned to each stock based on their market capitalization calculated at the end of June preceding the month or day t. For simplicity, we report the results only for the loading b on the market risk component. Panel A reports the univariate regression results, and Panel B further controls for the HML, SMB, and MOM risk components for the time period of 1926 to 2017. Panel C provides the univariate regression results but for the time period of 1963 to 2017, and Panel D reports the results controlling for the HML, SMB, MOM, CMA, and RMW risk components, also from 1963 to 2017.

[Insert Table 4 about here]

As the systematic to total risk ratio is larger for large stocks than for small stocks, one expects the market risk component's ability to explain the cross-section of stock returns to be superior for large stocks. Results presented in Table 4 confirms this intuition. Across all panels, as we go from the smallest to the largest stocks, the intercept a decreases towards zero and the loading b on the market risk component increases towards 1. For eight out of the ten size decile stocks, we find an intercept that is not statistically different from zero at the 5% level. The two size deciles for which the intercept is statistically different from zero are for the stocks in the two smallest size deciles. Panel A reports an R^2 ranging from 27% for the largest decile stocks to 2% for the smallest size decile stocks. The results are similar in the sub-sample of 1963 to 2017. Finally, Panels B and D show that controlling for the additional risk components marginally improves the explanatory power (R^2) and marginally lowers the intercept estimate a.

In summary, consistent with our previous analysis, using individual stocks produces results supporting our theoretical predictions that the CAPM holds dynamically.

5. The Security Market Line

Figure 3 depicts the Security Market Line for the 10 beta-sorted portfolios in Panel A, for the 25 size-and-book-to-market-sorted portfolios in Panel B, and for the 10 industry-sorted portfolios in Panel C. As the dashed and solid lines show, the empirical relation between average excess returns and unconditional betas is consistently much flatter than predicted by the static CAPM. This observation is particularly striking for the 10 beta-sorted portfolios. Indeed, their average excess returns are independent of their unconditional betas at both the monthly and daily frequency, whereas the static CAPM predicts a steep, positive relation. That is, the static CAPM is strongly rejected by the data, whereas the results of the previous section show that the dynamic CAPM cannot be rejected.

[Insert Figure 3 about here]

To understand this perhaps surprising result, let us assume from now on that the dynamic CAPM holds:

$$E_t(R_{i,t+1}) = \beta_{it} E_t(R_{M,t+1}) \equiv \beta_{it} m_{Mt},$$

where R_i and R_M denote respectively the excess return of stock i and the excess return of the market, $m_{Mt} \equiv E_t(R_{M,t+1})$ is the time-t risk premium of the market, and $\beta_{it} \equiv \frac{\text{Cov}_t\left(R_{i,t+1}, R_{M,t+1}\right)}{\text{Var}_t\left(R_{M,t+1}\right)}$ is the time-t beta of stock i. Taking unconditional expectations yields

$$\begin{aligned}
E(R_{i,t+1}) =& \beta_i^u E(R_{M,t+1}) \\
&+ \text{Cov}(\beta_{it}, m_{Mt}) \left[1 + \frac{E(R_{M,t+1})^2}{\text{Var}(R_{M,t+1})}\right] \\
&+ \left[E(\beta_{it})E(v_{Mt}) + E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}) - \text{Cov}(\beta_{it}, m_{Mt}^2)\right] \frac{E(R_{M,t+1})}{\text{Var}(R_{M,t+1})},
\end{aligned} \tag{9}$$

$$\equiv \beta_i^u E(R_{M,t+1}) + \alpha_i^u$$

where $\beta_i^u \equiv \frac{\text{Cov}(R_{i,t+1}, R_{M,t+1})}{\text{Var}(R_{M,t+1})}$ is the unconditional beta of stock i, α_i^u is the unconditional alpha of stock i, v_{Mt} is time-t market return variance, v_{it} is time-t return variance of stock i, $\epsilon_{i,t+1}$ is the unexpected white noise shock in stock i's return at time $t+1$, and $\epsilon_{M,t+1}$ is the unexpected white noise shock in the market return at time $t+1$. Refer to Appendix A for the derivation of Equation (9).

The comovement terms observed on the second and third rows of Equation (9) imply that the relation between a stock's expected excess return and its unconditional beta is ambiguous, even though the dynamic CAPM holds. Indeed, the static CAPM would hold, and therefore the security market line would have an intercept equal to zero and a slope equal to the expected return of the market, only if the sum of the comovement terms were equal to zero. If, in contrast, the sum of the comovement terms is larger for stocks with low unconditional betas than for stocks with high unconditional betas, then the static CAPM does not hold because the security market line is flatter than predicted by the latter model.

The dash-dotted lines depicted on Figure 3 represent the empirical estimates of the sum of the comovement terms, or in other words, the empirical estimates of the unconditional alphas. The estimated alphas are high for portfolios with low unconditional betas and low for portfolios with high unconditional betas. While a complete model for the conditional expected excess return of each portfolio and the market as well as for the conditional excess return variance of each portfolio and the market would be needed to explain the magnitude of alpha, its asymmetric relation with the unconditional beta can be understood by investigating the dynamics of the conditional betas estimated in Section 4. Indeed, the correlation among conditional portfolio betas is positive and strong (ranging between about 0.5 and 0.9) for portfolios having unconditional betas that are close to each other. In contrast, the correlation among conditional betas is negative and strong (ranging between about -0.5 and -0.3) for portfolios having unconditional betas that are far from each other. This result implies that the covariance between the conditional beta and the conditional market expected excess return observed on the second row of Equation (9) flips sign as the unconditional beta of the portfolio increases, thereby explaining the asymmetric relation between the unconditional alpha and the unconditional beta depicted on Figure 3.

As in Martin and Wagner (2018), we next examine the relationship between portfolios average excess returns and average implied returns. We perform this analysis on the ten beta-, 25 size-and-book-to-market-, and ten industry-sorted portfolios. As shown in Section 4, the implied return on portfolio i at time $t+1$ is defined as $\beta_{i,t} R_{M,t+1}$. On each month (day), we sort the 45 aforementioned portfolios implied returns into ten deciles indexed by j. We denote by R_{jt} and M_{jt} the time-t cross-sectional average excess return and implied return within each decile j, respectively. We then compute the time series average of R_{jt} and M_{jt}, which we denote by $\overline{R_j}$ and $\overline{M_j}$, respectively. Figure 4 depicts the relationship between the average returns $\overline{R_j}$ and average implied returns $\overline{M_j}$. The figure shows a positive relationship between the implied returns and average returns for both monthly and daily returns. The decile portfolio average returns fit well the theoretical relationship (solid line) predicted by the implied returns.

[Insert Figure 4 about here]

6. Conclusion

When investors' hedging demands are equal to zero, the CAPM holds in a dynamic manner (Merton, 1973). That is, the time-t risk premium of a stock is equal to the product of its time-t beta and the time-t risk premium of the market. We test this dynamic CAPM relation by performing a panel regression of excess stock returns onto the product of their conditional beta and the excess return of the market. We find that *(i)* the intercept is indistinguishable from zero, *(ii)* the slope is indistinguishable from one, and *(iii)* the adjusted R^2 is about 80%. To summarize, the CAPM holds dynamically.

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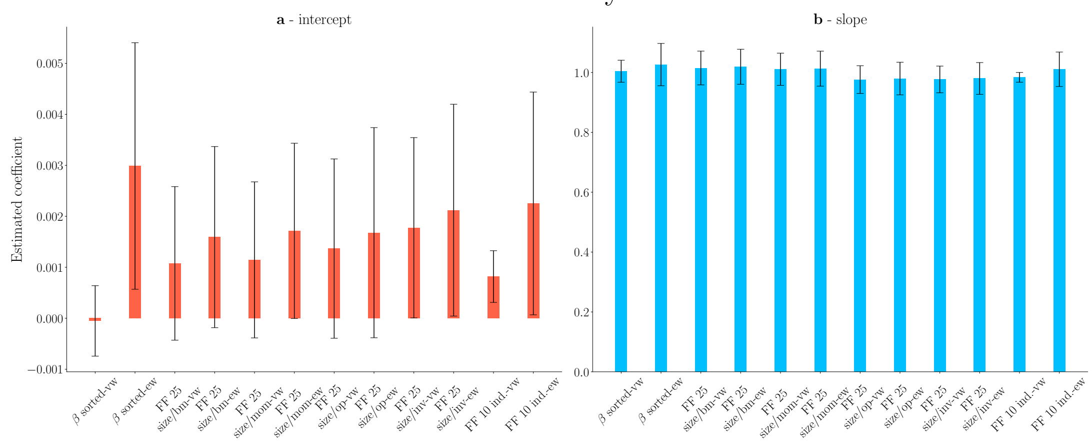
Figure 1. The Intercept and Slope Estimates of the CAPM

This figure shows the estimated coefficients a (the intercept) and b (the slope) of the following regression:

$$R_{i,t+1} = a + b[\beta^M_{i,t} R_{M,t+1}] + e_{i,t+1},$$

where $R_{i,t+1}$ is the excess return of portfolio i, $R_{M,t+1}$ is the market excess return, and $\beta^M_{i,t}$ is the coefficient of a regression of the monthly (daily) excess return of portfolio i on the excess market return using 24 months (250 trading days) strictly prior to month (day) $t+1$. The portfolios are the value-weighted (vw) and equally-weighted (ew) ten beta-sorted portfolios, the Fama-French 25-size-and-book-to-market, size-and-momentum, size-and-operating profits, size-and-investment sorted portfolios, and the ten industry portfolios. The black vertical lines represent the estimates respective 95% confidence intervals. The standard errors are calculated using Driscoll-Kraay with 12 month lags for monthly returns and 250 trading days for daily returns. Monthly and daily returns are presented in Panel A and B, respectively.

Panel A. Monthly returns



Panel B. Daily returns

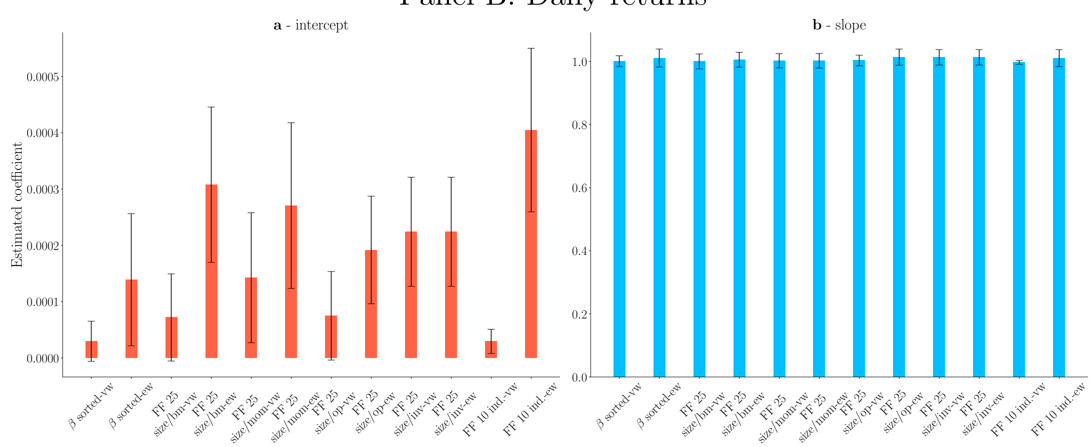
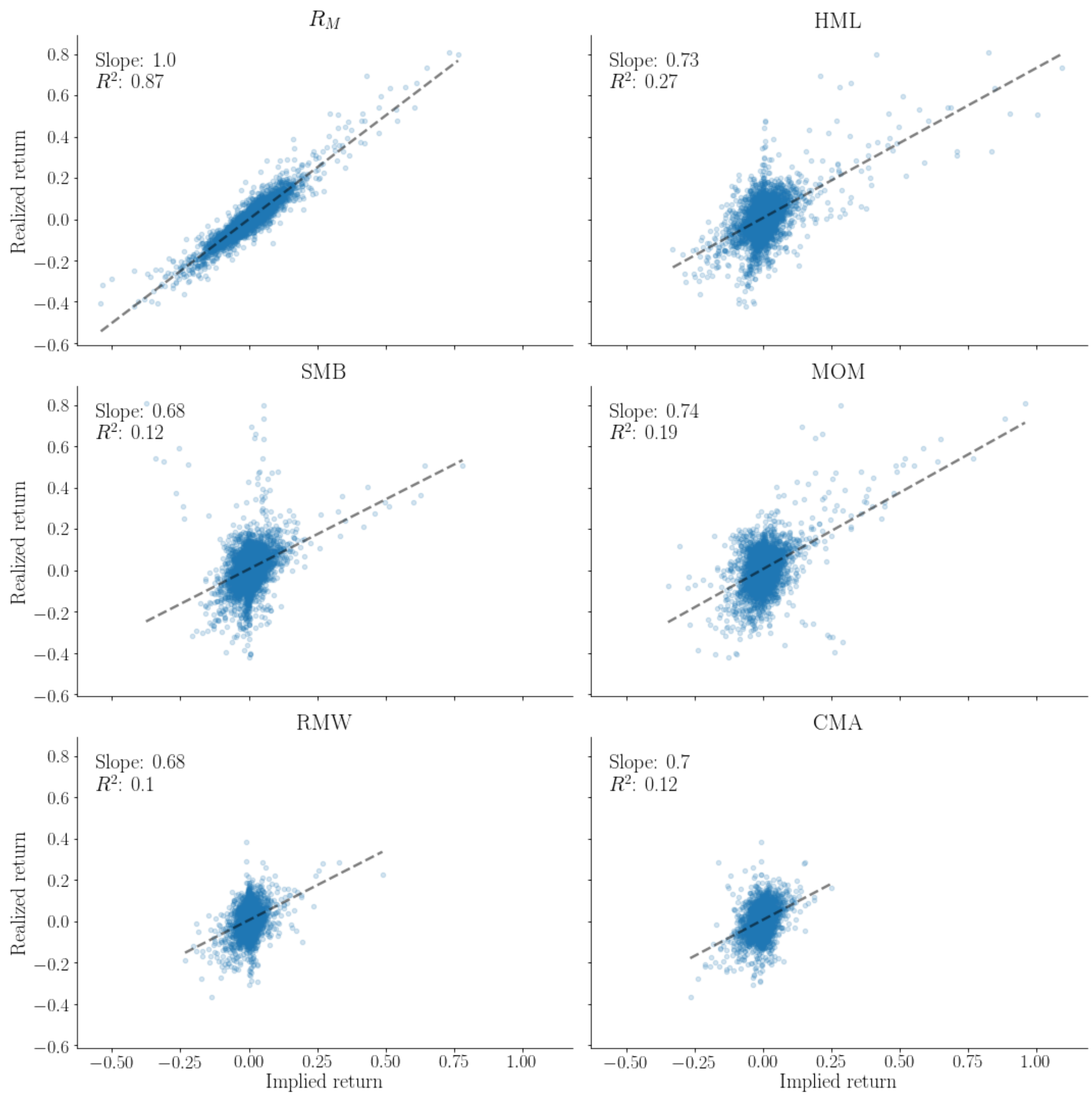


Figure 2. Scatter plots: Realized vs. Implied Returns

This figure shows scatter plots highlighting the relation between realized monthly excess returns and implied monthly excess returns for the 10 beta value-weighted and equal-weighted sorted portfolios in Panel A and B, respectively. The implied monthly excess returns is defined as $\beta_{i,t}^{M} R_{M,t+1}$, where $R_{i,t+1}$ is the excess return of portfolio i, $R_{M,t+1}$ is the market excess return, and $\beta_{i,t}^{M}$ is the coefficient of a regression of the monthly excess return of portfolio i on the excess market return using 24 months strictly prior to month $t+1$. We also, examine the relation between the realized excess returns and the implied monthly returns using the Fama and French (1993) and Carhart (1997) factors HML, SMB, MOM, RMW, and CMA. For these factors, the implied monthly returns are, $\beta_{i,t}^{HML} \times HML_{t+1}$, $\beta_{i,t}^{SMB} \times SMB_{t+1}$, $\beta_{i,t}^{MOM} \times MOM_{t+1}$, $\beta_{i,t}^{RMW} \times RMW_{t+1}$, and $\beta_{i,t}^{CMA} \times CMA_{t+1}$. The dashed line is the line that best fit the relation. We further report the estimated slope of the best-fit line and the R^2.

Panel A. Value-weighted portfolios



Panel B. Equal-weighted portfolios

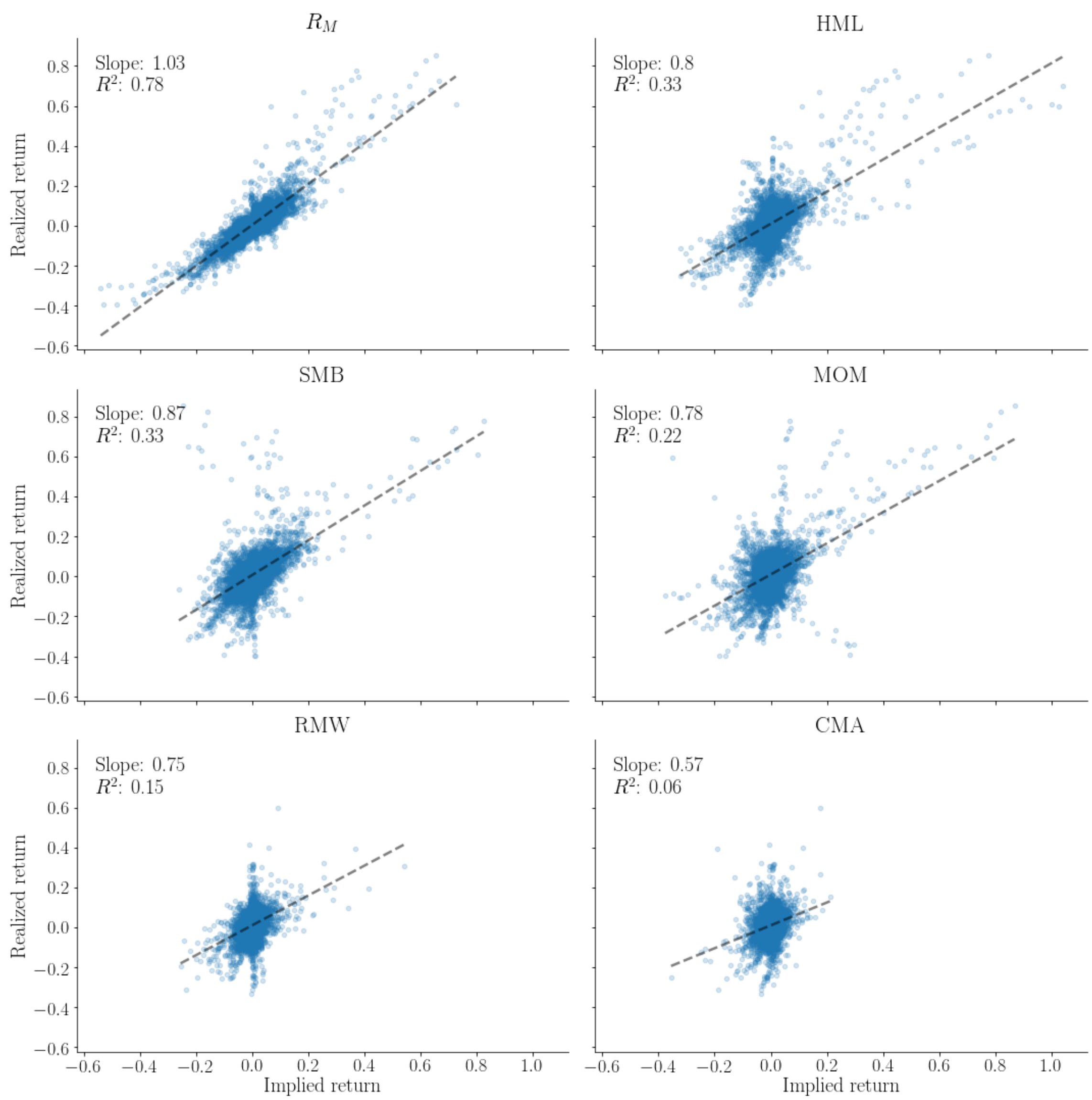
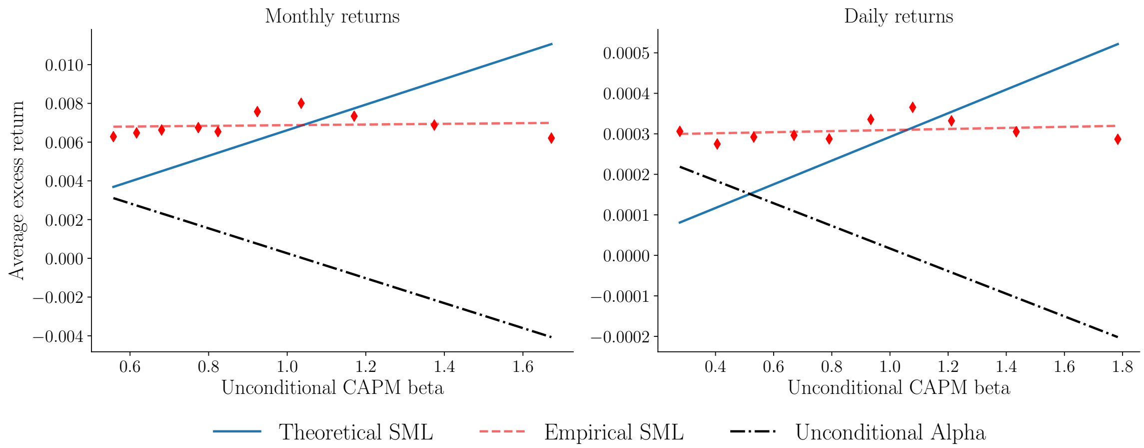


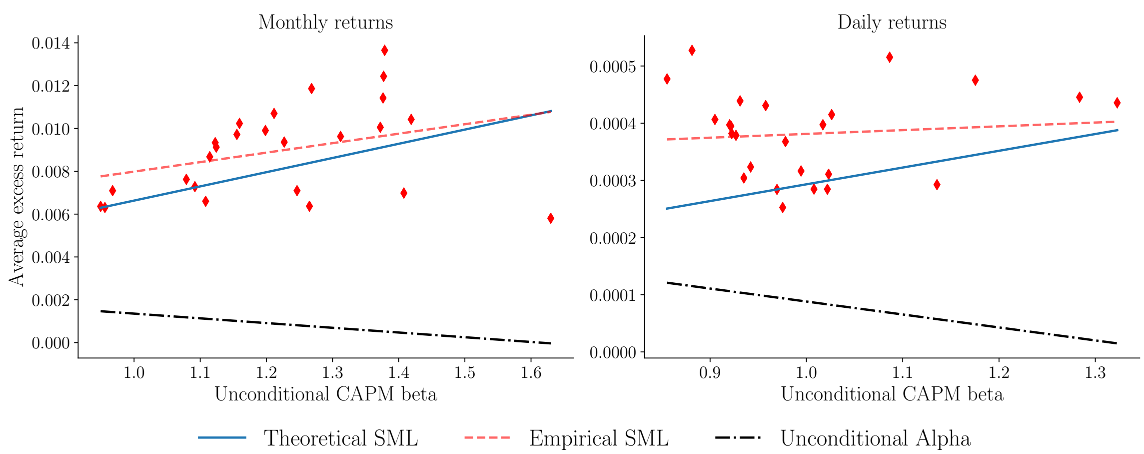
Figure 3. Security Market Line

This figure shows the empirical relation between the unconditional CAPM beta and average monthly excess return. The test assets are the 10 beta-, 25 size-and-book-to-market-, and the ten 10 industry-sorted value-weighted portfolios in Panels A to C, respectively. The solid line depicts the theoretical security market line predicted by the static CAPM, the dashed line depicts the empirical security market line, and the dash-dotted depicts the unconditional alpha, i.e., $\alpha_i^u \equiv E(R_i) - \beta_i^u E(R_M)$.

Panel A. Ten beta-sorted portfolio



Panel B. 25 size-and-book-to-market-sorted portfolios



Panel C. Ten industry-sorted portfolios

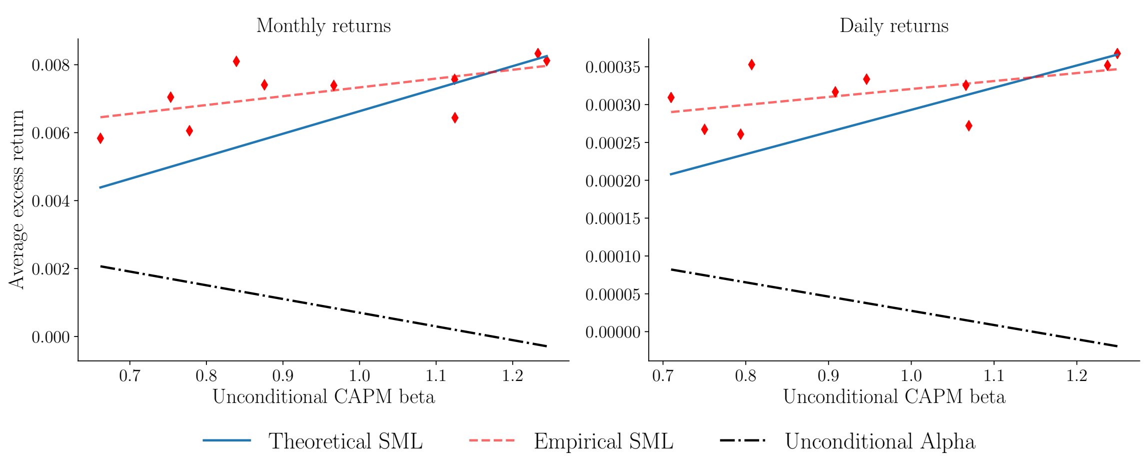


Figure 4. Average Return vs. Average Implied Return

This figure shows the empirical relation between the average excess return $\overline{R_j}$ and the average implied return $\overline{M_j}$ for ten decile sorts on the implied return. We sort on each month (day) when using monthly (daily) returns the ten beta-, 25 size-and-book-to-market-, and ten-industry-sorted portfolios. The solid line depicts the theoretical relation predicted by the dynamic CAPM.

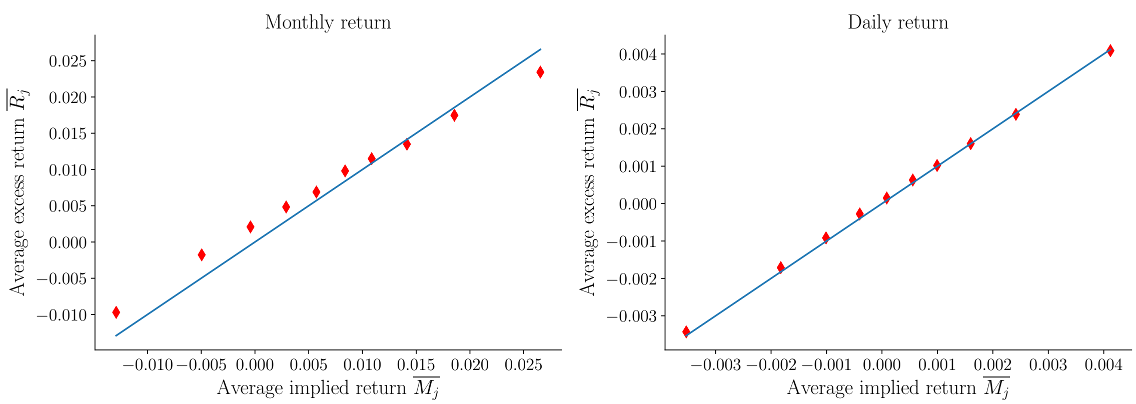


Table 1
Panel Regressions: Ten Beta-Sorted Portfolios

This table presents results from regression of portfolio equity excess returns on month (day) $t+1$ on the implied excess returns for the market risk component and the Fama and French (1993, 2015) and Carhart (1997) risk components on month (day) $t+1$ for the ten beta-sorted portfolios. Specifically, we estimate:

$$R_{i,t+1} = a + b[\beta_{i,t}^{M} R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] + s[\beta_{i,t}^{SMB} SMB_{t+1}] \\ + m[\beta_{i,t}^{MOM} MOM_{t+1}] + r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1},$$

Each β coefficients are estimated using the 24 months (250 trading days) strictly prior to month (day) $t+1$ for each portfolio i and for each of the respective factor. Panels A and B report the results using monthly and daily returns, respectively for both value- and equal-weighted portfolios. The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags when using monthly returns and 250 trading day lags when using daily returns. The table further reports the adjusted R^2, the number of observations (N), and the p-values of the Wald statistics testing the joint hypothesis of H_0: $a = 0$ and $b = 1$ and H_0: $\forall a_i = 0$ and $b = 1$ when the intercepts are estimated separately for each portfolio i. ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively. The sample period is from January 1, 1926 to December 31, 2017 in Columns (1) to (5) and from July 1, 1963 to December 31, 2017 in Columns (6) to (9).

Panel A. Monthly returns

Value-weighted

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	-0.000	0.006***	0.006**	0.006**	-0.000	-0.000	0.006**	0.008***	0.000
	(0.000)	(0.002)	(0.002)	(0.002)	(0.000)	(0.000)	(0.002)	(0.002)	(0.000)
R_M (b)	1.004***				0.934***	0.981***			0.896***
	(0.016)				(0.017)	(0.021)			(0.019)
HML (h)		0.727***			0.085***				0.075**
		(0.072)			(0.019)				(0.027)
SMB (s)			0.675***		0.136***				0.234***
			(0.121)		(0.027)				(0.027)
MOM (m)				0.741***	0.062**				0.096*
				(0.103)	(0.024)				(0.045)
RMW (r)							0.678***		0.105***
							(0.122)		(0.029)
CMA (c)								0.699***	0.104**
								(0.104)	(0.033)
R^2	0.87	0.27	0.12	0.19	0.88	0.81	0.10	0.12	0.84
N	10,500	10,500	10,500	10,500	10,500	6,300	6,300	6,300	6,300
p-value a=0, b=1	0.965				0.003	0.540			0.001
p-value $\forall a_i$=0, b=1	0.120				0.009	0.198			0.002

Equal-weighted

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.003**	0.009***	0.007***	0.010***	0.002**	0.003*	0.011***	0.010***	0.003**
	(0.001)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.003)	(0.001)
R_M (b)	1.026***				0.820***	0.982***			0.797***
	(0.031)				(0.023)	(0.033)			(0.032)
HML (h)		0.805***			0.116**				-0.027
		(0.075)			(0.045)				(0.054)
SMB (s)			0.870***		0.442***				0.565***
			(0.058)		(0.045)				(0.037)
MOM (m)				0.781***	0.142**				0.066
				(0.111)	(0.054)				(0.087)
RMW (r)							0.748***		0.187***
							(0.108)		(0.052)
CMA (c)								0.573***	0.072
								(0.139)	(0.054)
R^2	0.78	0.33	0.33	0.22	0.88	0.68	0.15	0.06	0.84
N	10,500	10,500	10,500	10,500	10,500	6,300	6,300	6,300	6,300
p-value a=0, b=1	0.056				<0.001	0.150			0.001
p-value $\forall a_i$=0, b=1	0.028				0.001	0.039			0.004

Table 1
Panel Regressions (continue)

Panel B. Daily returns
Value-weighted

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.0000*	0.0003***	0.0003***	0.0002**	0.0000**	0.0000	0.0003***	0.0003***	0.0001**
	(0.0000)	(0.0001)	(0.0001)	(0.0001)	(0.0000)	(0.0000)	(0.0001)	(0.0001)	(0.0000)
R_M (b)	1.0017***				0.9363***	0.9996***			0.8915***
	(0.0075)				(0.0096)	(0.0120)			(0.0101)
HML (h)		0.9897***			0.1226***				0.0859**
		(0.0281)			(0.0178)				(0.0286)
SMB (s)			0.8786***		0.0619***				0.1157***
			(0.0464)		(0.0187)				(0.0280)
MOM (m)				0.7879***	0.0542***				0.0801***
				(0.0659)	(0.0150)				(0.0215)
RMW (r)							0.9940***		0.1547***
							(0.0677)		(0.0197)
CMA (c)								0.9587***	0.1106**
								(0.0635)	(0.0414)
R^2	0.79	0.29	0.13	0.16	0.80	0.80	0.16	0.18	0.82
N	237,400	237,400	237,400	237,400	237,400	132,160	134,700	134,700	134,700
p-value a=0, b=1	0.190				<0.001	0.296			<0.001
p-value $\forall a_i$=0, b=1	0.028				<0.001	0.260			<0.001

Equal-weighted

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.0001**	0.0004***	0.0004***	0.0003**	0.0002**	0.0002*	0.0005***	0.0004***	0.0002**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
R_M (b)	1.0115***				0.8674***	1.0148***			0.8487***
	(0.0127)				(0.0158)	(0.0178)			(0.0173)
HML (h)		0.9963***			0.1952***				0.0859**
		(0.0246)			(0.0243)				(0.0314)
SMB (s)			0.9115***		0.3164***				0.4125***
			(0.0416)		(0.0613)				(0.0562)
MOM (m)				0.7886***	0.0965***				0.1509***
				(0.0650)	(0.0294)				(0.0368)
RMW (r)							1.0027***		0.2061***
							(0.0658)		(0.0493)
CMA (c)								0.9318***	0.0440
								(0.0599)	(0.0307)
R^2	0.78	0.33	0.15	0.17	0.80	0.76	0.20	0.17	0.82
N	237,400	237,400	237,400	237,400	237,400	132,160	134,700	134,700	134,700
p-value a=0, b=1	0.071				<0.001	0.152			<0.001
p-value $\forall a_i$=0, b=1	0.001				<0.001	0.004			<0.001

Table 2
Panel Regressions: 25 Size-and-Book-to-Market-Sorted Portfolios

This table presents results from regression of portfolio equity excess returns on month (day) $t+1$ on the implied excess returns for the market risk component and the Fama and French (1993, 2015) and Carhart (1997) risk components on month (day) $t+1$ for the 25 size-and-book-to-market sorted portfolios. Specifically, we estimate:

$$R_{i,t+1} = a + b[\beta_{i,t}^{M} R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] + s[\beta_{i,t}^{SMB} SMB_{t+1}] \\ + r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1}$$

Each β coefficients are estimated using the 24 months (250 trading days) strictly prior to month (day) $t+1$ for each portfolio i and for each of the respective factor. Panels A and B report the results using monthly and daily returns, respectively for both value- and equal-weighted portfolios. The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags when using monthly returns and 250 trading day lags when using daily returns. The table further reports the adjusted R^2, the number of observations (N), and the p-values of the Wald statistics testing the joint hypothesis of H_0: $a = 0$ and $b = 1$ and H_0: $\forall a_i = 0$ and $b = 1$ when the intercepts are estimated separately for each portfolio i. ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively. The sample period is from January 1, 1926 to December 31, 2017 in Columns (1) to (4) and from July 1, 1963 to December 31, 2017 in Columns (5) to (8).

Panel A. Monthly returns
Value-weighted

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.001	0.007***	0.006***	0.008***	0.001	0.002*	0.008***	0.009***	0.001
	(0.001)	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)
R_M (b)	1.015***				0.829***	0.973***			0.843***
	(0.027)				(0.021)	(0.024)			(0.028)
HML (h)		0.799***			0.156***				0.122***
		(0.067)			(0.037)				(0.035)
SMB (s)			0.839***		0.416***				0.500***
			(0.066)		(0.043)				(0.027)
MOM (m)				0.750***	0.107***				0.029
				(0.117)	(0.038)				(0.035)
RMW (r)							0.677***		0.113***
							(0.080)		(0.033)
CMA (c)								0.617***	-0.028
								(0.092)	(0.037)
R^2	0.74	0.32	0.27	0.19	0.82	0.72	0.11	0.07	0.83
N	26,700	26,700	26,700	26,700	26,700	15,750	15,750	15,750	15,750
p-value a=0, b=1	0.322				<0.001	0.193			<0.001
p-value $\forall a_i$=0, b=1	0.001				<0.001	<0.001			<0.001

Equal-weighted

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.002*	0.008***	0.006***	0.009***	0.001	0.002*	0.009***	0.009***	0.001
	(0.001)	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)
R_M (b)	1.019***				0.824***	0.978***			0.837***
	(0.028)				(0.023)	(0.028)			(0.029)
HML (h)		0.796***			0.133***				0.079*
		(0.073)			(0.042)				(0.040)
SMB (s)			0.844***		0.432***				0.491***
			(0.067)		(0.054)				(0.034)
MOM (m)				0.776***	0.137***				0.080
				(0.108)	(0.046)				(0.074)
RMW (r)							0.672***		0.107**
							(0.105)		(0.049)
CMA (c)								0.598***	0.054
								(0.109)	(0.045)
R^2	0.72	0.31	0.28	0.20	0.82	0.70	0.10	0.07	0.81
N	26,700	26,700	26,700	26,700	26,700	15,750	15,750	15,750	15,750
p-value a=0, b=1	0.194				<0.001	0.183			<0.001
p-value $\forall a_i$=0, b=1	<0.001				<0.001	<0.001			<0.001

Table 2
Panel Regressions: 25 Size-and-Book-to-Market-Sorted Portfolios (continue)

Panel B. Daily returns
Value-weighted

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.0001*	0.0004***	0.0004***	0.0003**	0.0001**	0.0001**	0.0004***	0.0004***	0.0001**
	(0.0000)	(0.0001)	(0.0001)	(0.0001)	(0.0000)	(0.0000)	(0.0001)	(0.0001)	(0.0000)
R_M (b)	1.0008***				0.8535***	1.0045***			0.8766***
	(0.0115)				(0.0133)	(0.0101)			(0.0113)
HML (h)		0.9796***			0.2181***				0.1294***
		(0.0215)			(0.0232)				(0.0362)
SMB (s)			0.9003***		0.3235***				0.3433***
			(0.0414)		(0.0518)				(0.0571)
MOM (m)				0.7566***	0.0624**				0.0973***
				(0.0675)	(0.0242)				(0.0171)
RMW (r)							0.9728***		0.1359***
							(0.0573)		(0.0422)
CMA (c)								0.8975***	-0.0187
								(0.0539)	(0.0333)
R^2	0.59	0.25	0.13	0.12	0.61	0.79	0.17	0.14	0.82
N	593,500	593,500	593,500	593,500	593,500	330,400	336,750	336,750	336,750
p-value a=0, b=1	0.165				<0.001	0.129			<0.001
p-value $\forall a_i$=0, b=1	0.010				<0.001	<0.001			<0.001

Equal-weighted

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.0003***	0.0006***	0.0006***	0.0005***	0.0003***	0.0002***	0.0005***	0.0005***	0.0002***
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0000)	(0.0001)	(0.0001)	(0.0001)
R_M (b)	1.0062***				0.8571***	1.0140***			0.8854***
	(0.0115)				(0.0167)	(0.0132)			(0.0104)
HML (h)		0.9840***			0.2135***				0.0937***
		(0.0232)			(0.0275)				(0.0301)
SMB (s)			0.8929***		0.3220***				0.3195***
			(0.0411)		(0.0591)				(0.0609)
MOM (m)				0.7811***	0.0821**				0.1217***
				(0.0647)	(0.0296)				(0.0306)
RMW (r)							0.9887***		0.1422***
							(0.0720)		(0.0380)
CMA (c)								0.9139***	0.0310
								(0.0660)	(0.0261)
R^2	0.59	0.25	0.12	0.13	0.61	0.78	0.17	0.14	0.81
N	593,500	593,500	593,500	593,500	593,500	330,400	336,750	336,750	336,750
p-value a=0, b=1	<0.001				<0.001	0.001			<0.001
p-value $\forall a_i$=0, b=1	<0.001				<0.001	<0.001			<0.001

Table 3
Panel Regressions: Ten Industry-Sorted Portfolios

This table presents results from regression of portfolio equity excess returns on month (day) $t+1$ on the implied excess returns for the market risk component and the Fama and French (1993, 2015) and Carhart (1997) risk components on month (day) $t+1$ for the ten industry-sorted portfolios. Specifically, we estimate:

$$R_{i,t+1} = a + b[\beta_{i,t}^{M} R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] + s[\beta_{i,t}^{SMB} SMB_{t+1}] \\ + r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1}$$

Each β coefficients are estimated using the 24 months (250 trading days) strictly prior to month (day) $t+1$ for each portfolio i and for each of the respective factor. Panels A and B report the results using monthly and daily returns, respectively for both value- and equal-weighted portfolios. The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags when using monthly returns and 250 trading day lags when using daily returns. The table further reports the adjusted R^2, the number of observations (N), and the p-values of the Wald statistics testing the joint hypothesis of H_0: $a = 0$ and $b = 1$ and H_0: $\forall a_i = 0$ and $b = 1$ when the intercepts are estimated separately for each portfolio i. ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively. The sample period is from January 1, 1926 to December 31, 2017 in Columns (1) to (4) and from July 1, 1963 to December 31, 2017 in Columns (5) to (8).

Panel A. Monthly returns

Value-weighted

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.001***	0.006***	0.006***	0.006***	0.001***	0.001**	0.006***	0.007***	0.001
	(0.000)	(0.002)	(0.002)	(0.002)	(0.000)	(0.000)	(0.002)	(0.002)	(0.000)
R_M (b)	0.984***				0.955***	0.972***			0.917***
	(0.007)				(0.010)	(0.007)			(0.012)
HML (h)		0.650***			0.041*				0.150***
		(0.090)			(0.022)				(0.030)
SMB (s)			0.518***		0.050				0.154***
			(0.140)		(0.032)				(0.028)
MOM (m)				0.668***	0.045*				0.122**
				(0.122)	(0.022)				(0.046)
RMW (r)							0.541***		-0.004
							(0.118)		(0.037)
CMA (c)								0.651***	0.053
								(0.085)	(0.030)
R^2	0.75	0.17	0.05	0.13	0.76	0.66	0.05	0.09	0.68
N	10,680	10,680	10,680	10,680	10,680	6,300	6,300	6,300	6,300
p-value a=0, b=1	0.012				0.001	0.006			<0.001
p-value $\forall a_i$=0, b=1	0.078				0.023	0.023			0.002

Equal-weighted

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.002**	0.008***	0.006***	0.009***	0.002**	0.002	0.009***	0.010***	0.002*
	(0.001)	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)
R_M (b)	1.011***				0.839***	0.972***			0.779***
	(0.025)				(0.023)	(0.032)			(0.032)
HML (h)		0.753***			0.079				0.051
		(0.082)			(0.050)				(0.046)
SMB (s)			0.845***		0.440***				0.553***
			(0.060)		(0.048)				(0.037)
MOM (m)				0.730***	0.140**				0.128
				(0.109)	(0.052)				(0.081)
RMW (r)							0.694***		0.139***
							(0.094)		(0.040)
CMA (c)								0.598***	0.094*
								(0.121)	(0.051)
R^2	0.70	0.25	0.28	0.17	0.79	0.58	0.12	0.07	0.73
N	10,680	10,680	10,680	10,680	10,680	6,300	6,300	6,300	6,300
p-value a=0, b=1	0.116				<0.001	0.249			<0.001
p-value $\forall a_i$=0, b=1	0.172				0.005	0.142			0.006

Table 3

Panel Regressions: Ten Industry-Sorted Portfolios (continue)

Panel B. Daily returns

Value-weighted

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.0000**	0.0003***	0.0003***	0.0002**	0.0000**	0.0000	0.0003***	0.0004***	0.0000**
	(0.0000)	(0.0001)	(0.0001)	(0.0001)	(0.0000)	(0.0000)	(0.0001)	(0.0001)	(0.0000)
R_M (b)	0.9978***				0.9514***	0.9981***			0.9110***
	(0.0027)				(0.0070)	(0.0038)			(0.0114)
HML (h)		0.9647***			0.0586***				0.0524***
		(0.0275)			(0.0127)				(0.0123)
SMB (s)			0.8590***		0.0773***				0.1302***
			(0.0590)		(0.0191)				(0.0282)
MOM (m)				0.7924***	0.0576***				0.0801***
				(0.0562)	(0.0125)				(0.0183)
RMW (r)							0.9517***		0.0998***
							(0.0471)		(0.0234)
CMA (c)								0.9214***	0.1038***
								(0.0444)	(0.0248)
R^2	0.77	0.25	0.13	0.15	0.77	0.73	0.11	0.14	0.74
N	237,400	237,400	237,400	237,400	237,400	132,160	134,700	134,700	134,700
p-value H_0 :a=0, b=1	0.032				<0.001	0.221			<0.001
p-value H_0 :$\forall a_i$=0, b=1	0.115				<0.001	0.355			<0.001

Equal-weighted

	1926-2017					1963-2017			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept (a)	0.0004***	0.0007***	0.0006***	0.0006***	0.0004***	0.0004***	0.0007***	0.0007***	0.0004***
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
R_M (b)	1.0112***				0.8762***	1.0154***			0.8484***
	(0.0119)				(0.0187)	(0.0182)			(0.0247)
HML (h)		0.9823***			0.1675***				0.0608
		(0.0272)			(0.0265)				(0.0481)
SMB (s)			0.8897***		0.3004***				0.3840***
			(0.0435)		(0.0709)				(0.0616)
MOM (m)				0.7960***	0.1316***				0.2125***
				(0.0577)	(0.0318)				(0.0384)
RMW (r)							0.9725***		0.1752***
							(0.0604)		(0.0477)
CMA (c)								0.9159***	0.0833
								(0.0598)	(0.0457)
R^2	0.68	0.27	0.12	0.16	0.71	0.66	0.16	0.14	0.71
N	237,400	237,400	237,400	237,400	237,400	132,160	134,700	134,700	134,700
p-value H_0 :a=0, b=1	<0.001				<0.001	0.003			<0.001
p-value H_0 :$\forall a_i$=0, b=1	<0.001				<0.001	0.001			<0.001

Table 4
Individual Stock Excess Return Panel Regressions

This table presents results from regression of individual stock stock excess monthly returns on month $t+1$ on the implied excess returns for the market risk component and the Fama and French (1993, 2015) and Carhart (1997) risk components and MOM, by firm size decile. Specifically, we estimate:

$$\begin{aligned} R_{i,t+1} = a &+ b[\beta_{i,t}^{M} R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] \\ &+ s[\beta_{i,t}^{SMB} SMB_{t+1}] + m[\beta_{i,t}^{MOM} MOM_{t+1}] \\ &+ r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1}. \end{aligned}$$

Each β coefficients are estimated using the 24 months strictly prior to month $t+1$ for each portfolio i and for each of the respective factor. The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags. Firm size deciles are calculated based on stocks market capitalization at the end of June of each year. The table further reports the adjusted R^2 and the number of observations (N). ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively. The sample period is from July 1, 1926 in Panel A and B and from July 1, 1963 in Panel C and D to December 31, 2017.

Panel A. Univariate regression (1926 to 2017)

	Small	2	3	4	5	6	7	8	9	Large
Intercept (a)	0.016**	0.007**	0.006*	0.004	0.003	0.003	0.002	0.002	0.002	0.001*
	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)
R_M (b)	0.518***	0.624***	0.688***	0.731***	0.752***	0.789***	0.800***	0.818***	0.844***	0.877***
	(0.055)	(0.044)	(0.038)	(0.034)	(0.029)	(0.027)	(0.026)	(0.025)	(0.022)	(0.014)
N	260,863	269,001	269,749	274,146	275,382	282,812	288,034	297,782	309,848	327,153
R^2	0.02	0.04	0.06	0.08	0.10	0.13	0.15	0.18	0.21	0.27

Panel B. Controlling for HML, SMB, and MOM (1926 to 2017)

	Small	2	3	4	5	6	7	8	9	Large
Intercept (a)	0.015***	0.006**	0.005*	0.003	0.002	0.002	0.001	0.002	0.002	0.001
	(0.003)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
R_M (b)	0.373***	0.493***	0.554***	0.594***	0.616***	0.659***	0.690***	0.717***	0.759***	0.815***
	(0.034)	(0.028)	(0.026)	(0.022)	(0.021)	(0.019)	(0.022)	(0.021)	(0.020)	(0.015)
N	260,562	268,700	269,471	273,877	275,115	282,514	287,740	297,478	309,566	326,818
R^2	0.04	0.07	0.09	0.12	0.14	0.17	0.18	0.21	0.23	0.28

Panel C. Univariate regression (1963 to 2017)

	Small	2	3	4	5	6	7	8	9	Large
Intercept (a)	0.017**	0.008*	0.006*	0.005	0.004	0.003	0.002	0.002	0.002	0.001*
	(0.003)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
R_M (b)	0.372***	0.514***	0.606***	0.672***	0.703***	0.750***	0.767***	0.792***	0.823***	0.865***
	(0.043)	(0.036)	(0.035)	(0.033)	(0.030)	(0.028)	(0.028)	(0.027)	(0.024)	(0.015)
N	226,183	234,769	236,564	241,333	242,688	249,122	254,244	263,215	275,004	291,215
R^2	0.01	0.02	0.04	0.06	0.08	0.11	0.13	0.15	0.19	0.24

Panel D. Controlling for HML, SMB, MOM, CMA, and RMW (1963 to 2017)

	Small	2	3	4	5	6	7	8	9	Large
Intercept (a)	0.017**	0.007**	0.005*	0.004	0.003	0.002	0.001	0.002	0.002	0.002
	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
R_M (b)	0.289***	0.418***	0.496***	0.547***	0.572***	0.616***	0.650***	0.683***	0.726***	0.787***
	(0.035)	(0.028)	(0.028)	(0.024)	(0.022)	(0.020)	(0.025)	(0.024)	(0.023)	(0.016)
N	224,081	232,674	234,115	238,553	239,468	245,285	250,361	259,058	270,650	286,543
R^2	0.02	0.05	0.07	0.09	0.12	0.15	0.16	0.19	0.21	0.26

Appendix

A. Derivation of the Security Market Line

Let us assume that the dynamic CAPM holds:

$$E_t(R_{i,t+1}) = \beta_{it} E_t(R_{M,t+1}) \equiv \beta_{it} m_{Mt}, \tag{10}$$

where R_i and R_M denote respectively the excess return of stock i and the excess return of the market, and $E_t(R_{M,t+1}) \equiv m_{Mt}$. We can write

$$R_{i,t+1} = \beta_{it} m_{Mt} + \sqrt{v_{it}}\epsilon_{i,t+1},$$

$$\beta_{it} = E(\beta_{it}) + u_{it},$$

$$R_{M,t+1} = m_{Mt} + \sqrt{v_{Mt}}\epsilon_{M,t+1},$$

where $E_t(\epsilon_{i,t+1}) = E_t(\epsilon_{M,t+1}) = E(u_{it}) = 0$, $\text{Var}_t(\epsilon_{i,t+1}) = \text{Var}_t(\epsilon_{M,t+1}) = 1$, $\text{Var}_t(R_{i,t+1}) = v_{it}$, and $\text{Var}_t(R_{M,t+1}) = v_{Mt}$.

This implies that

$$
\begin{aligned}
\text{Cov}(R_{i,t+1}, R_{M,t+1}) =& \text{Cov}((E(\beta_{it}) + u_{it})m_{Mt} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
=& \text{Cov}((E(\beta_{it}) + u_{it})(R_{M,t+1} - \sqrt{v_{Mt}}\epsilon_{M,t+1}) + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1})
\end{aligned}
$$

$$
\begin{aligned}
=& E(\beta_{it})\text{Var}(R_{M,t+1}) \\
& + \text{Cov}(u_{it}m_{Mt} - E(\beta_{it})\sqrt{v_{Mt}}\epsilon_{M,t+1} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
=& E(\beta_{it})\text{Var}(R_{M,t+1}) \\
& + E(u_{it}m_{Mt}R_{M,t+1}) - E(u_{it}m_{Mt})E(R_{M,t+1}) \\
& - \text{Cov}(E(\beta_{it})\sqrt{v_{Mt}}\epsilon_{M,t+1} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
=& E(\beta_{it})\text{Var}(R_{M,t+1}) \\
& + E(u_{it}m_{Mt}^2) - E(u_{it}m_{Mt})E(R_{M,t+1}) \\
& - \text{Cov}(E(\beta_{it})\sqrt{v_{Mt}}\epsilon_{M,t+1} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
=& E(\beta_{it})\text{Var}(R_{M,t+1}) \\
& + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - \text{Cov}(E(\beta_{it})\sqrt{v_{Mt}}\epsilon_{M,t+1} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
=& E(\beta_{it})\text{Var}(R_{M,t+1}) \\
& + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it})E(\sqrt{v_{Mt}}\epsilon_{M,t+1}R_{M,t+1}) + E(\beta_{it})E(\sqrt{v_{Mt}}\epsilon_{M,t+1})E(R_{M,t+1})
\end{aligned}
$$

$$
\begin{aligned}
& - E(\sqrt{v_{it}}\epsilon_{i,t+1}R_{M,t+1}) + E(\sqrt{v_{it}}\epsilon_{i,t+1})E(R_{M,t+1}) \\
=& E(\beta_{it})\text{Var}(R_{M,t+1}) \\
& + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it})E(\sqrt{v_{Mt}}\epsilon_{M,t+1}R_{M,t+1}) - E(\sqrt{v_{it}}\epsilon_{i,t+1}R_{M,t+1})
\end{aligned}
$$

$$
\begin{aligned}
\text{Cov}(R_{i,t+1}, R_{M,t+1}) =& E(\beta_{it})\text{Var}(R_{M,t+1}) \\
& + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it})\text{Cov}(\sqrt{v_{Mt}}\epsilon_{M,t+1}, R_{M,t+1}) - \text{Cov}(\sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
=& E(\beta_{it})\text{Var}(R_{M,t+1}) \\
& + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it})\text{Cov}(\sqrt{v_{Mt}}\epsilon_{M,t+1}, m_{Mt} + \sqrt{v_{Mt}}\epsilon_{M,t+1}) \\
& - \text{Cov}(\sqrt{v_{it}}\epsilon_{i,t+1}, m_{Mt} + \sqrt{v_{Mt}}\epsilon_{M,t+1}) \\
=& E(\beta_{it})\text{Var}(R_{M,t+1}) \\
& + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it}) \left[E(\sqrt{v_{Mt}}\epsilon_{M,t+1}m_{Mt}) + E(v_{Mt}\epsilon_{M,t+1}^2)\right] \\
& - E(\sqrt{v_{it}}\epsilon_{i,t+1}m_{Mt}) - E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}) \\
=& E(\beta_{it})\text{Var}(R_{M,t+1}) \\
& + \text{Cov}(u_{it}, m_{Mt}^2) - \text{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it})E(v_{Mt}) - E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}) \\
=& E(\beta_{it})\text{Var}(R_{M,t+1}) \\
& + \text{Cov}(\beta_{it}, m_{Mt}^2) - \text{Cov}(\beta_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it})E(v_{Mt}) - E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}),
\end{aligned} \tag{11}
$$

where last equality comes from the fact that $u_{it} = \beta_{it} - E(\beta_{it})$. Equation (11) implies that the unconditional beta of stock i, β_i^u, satisfies

$$\begin{aligned}\beta_i^u =&\frac{\text{Cov}(R_{i,t+1}, R_{M,t+1})}{\text{Var}(R_{M,t+1})}\\ =&E(\beta_{it})\\ &+ \frac{\text{Cov}(\beta_{it}, m_{Mt}^2) - \text{Cov}(\beta_{it}, m_{Mt})E(R_{M,t+1})}{\text{Var}(R_{M,t+1})}\\ &- \frac{E(\beta_{it})E(v_{Mt}) + E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1})}{\text{Var}(R_{M,t+1})}\end{aligned} \tag{12}$$

Applying unconditional expectations to Equation (10) and substituting Equation (12) into it yields

$$\begin{aligned}E(R_{i,t+1}) =&\text{Cov}(\beta_{it}, m_{Mt}) + E(\beta_{it})E(m_{Mt})\\ =&\text{Cov}(\beta_{it}, m_{Mt}) + E(\beta_{it})E(R_{M,t+1})\\ =&\beta_i^u E(R_{M,t+1})\\ &+\text{Cov}(\beta_{it}, m_{Mt}) - E(R_{M,t+1})\frac{\text{Cov}(\beta_{it}, m_{Mt}^2) - \text{Cov}(\beta_{it}, m_{Mt})E(R_{M,t+1})}{\text{Var}(R_{M,t+1})}\\ &+E(R_{M,t+1})\frac{E(\beta_{it})E(v_{Mt}) + E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1})}{\text{Var}(R_{M,t+1})}\\ =&\beta_i^u E(R_{M,t+1})\\ &+\text{Cov}(\beta_{it}, m_{Mt})\left[1 + \frac{E(R_{M,t+1})^2}{\text{Var}(R_{M,t+1})}\right]\\ &+\frac{E(R_{M,t+1})}{\text{Var}(R_{M,t+1})}\left[E(\beta_{it})E(v_{Mt}) + E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}) - \text{Cov}(\beta_{it}, m_{Mt}^2)\right].\end{aligned}$$