

Persuading Multiple Audiences: An Information Design Approach to Banking Regulation *

Nicolas Inostroza

Rotman School of Management, University of Toronto

August 31, 2019

Please find the latest version [HERE](#)

Abstract

A policy-maker concerned with the potential default of a bank conducts an *asset quality review* and a *liquidity stress test* under the scrutiny of multiple types of market participants (audiences). Surprisingly, the optimal *comprehensive assessment* is opaque when the bank has high-quality assets, and transparent when the bank has poor-quality assets. Additionally, the policy-maker imposes debt buybacks and contingent recapitalizations. I find that without the latter, disclosure of information about the bank's fundamentals may backfire. When the policy-maker lacks the technology to test the bank's private information, she designs a liquidity-provision program whereby the government offers to buy assets from the bank in exchange for cash and a public disclosure of the bank's liquidity position. Interventions display a non-monotone pecking order: the private sector funds banks with either high or poor-quality assets, while institutions with assets of intermediate quality participate in the government's liquidity program. My results shed light on the optimal way to disclose information in environments with multiple audiences and multi-dimensional fundamentals.

JEL classification: D83, G28, G33.

Keywords: Multiple Audiences, Stress Tests, Asset Quality Reviews, Information Design, Security Design, Mechanism Design.

*Email: nicolas.inostroza@utoronto.ca. I am deeply indebted to my advisors Alessandro Pavan, Mike Fishman, and Jeff Ely for their continuous support and encouragement. This paper has greatly benefitted from extended conversations with Nicolas Figueroa. I also thank Gara Afonso, Eddie Dekel, Piotr Dworzak, Redouane Elkamhi, William Fuchs, Maryam Farboodi, Camille Hébert, Michael Lee, Konstantin Milbradt, Stephen Morris, Aaron Pancost, Jonathan Parker, Antoinette Schoar, Michael Sockin, Haoxiang Zhu, Junyuan Zou, and seminar participants at Boulder Colorado, Fed NY, Fed Richmond, MIT Sloan, Northwestern, UT Austin McCombs, Toronto Rotman, and at the "Information and Coordination" Conference for their valuable feedback. The usual disclaimer applies.

1 Introduction

Information disclosure has become a prominent tool in banking supervision since the global financial crisis. In February 2009, the Federal Reserve introduced the Supervisory Capital Assessment Program (SCAP), commonly known as the Fed’s *stress test*. The objective of the program was to assess whether the capital buffers of the 19 largest bank holding companies were enough to sustain lending in the event of an unexpectedly severe recession, and to communicate these results to the public (Hirtle and Lehnert [2015]). The supervisors’ disclosure came at a time when informational asymmetries between inside and outside market participants regarding the soundness of the banking system had disrupted credit channels, leading to unprecedented interbank lending rates, abrupt haircuts in the repo market, and the freeze of capital markets for banks (Morgan et al. [2014]). Many scholars and policy-makers believe that the disclosure of stress tests results was a critical inflection point in the financial crisis because it provided market participants with credible information about potential losses at banks which helped restore market confidence (Bernanke [2013]).

Since their introduction, *stress tests* and *asset quality reviews* have been regularly conducted both in the US and in the Eurozone.¹ Despite the consensus that transparency may impose market discipline on the otherwise opaque banking system (Morgan [2002], Flannery et al. [2013]),² there still exists disagreement concerning the amount of information that should be disclosed, and the set of policies that should accompany such disclosures. While the *stress tests* conducted by the Fed, for example, have combined granular data with a pass/fail grade,³ the European Central Bank decided in 2016 to not assign grades to banks in order to avoid stigmatization. Moreover, while both regulatory authorities complement their disclosures with capital requirements, American regulators have chosen to publicly announce their decisions while their European counterparts have opted for private recommendations.^{4, 5}

A crucial difficulty associated with the design of such targeted disclosures is the complexity of the interactions among the multiple market participants involved. To illustrate, observe that when a policy-maker discloses information about a bank, it speaks to multiple *audiences* who care about different aspects of the bank’s private information. Namely, potential investors interested

¹See Morgan et al. [2014], Flannery et al. [2017] and Petrella and Resti [2013] for evidence on the effect of such disclosure policies. The first two papers show that the stress tests conducted in the US provided information not previously available to the rest of market participants. The last paper provides similar evidence for the tests conducted in the EU.

²See Babus and Farboodi [2018] for a theory where opacity endogenously emerges as part of banks’ strategy to create information asymmetry with external investors.

³In 2018, the Fed introduced for the first time an intermediate third grade: *conditional non-objection*, assigned to Goldman Sachs and Morgan Stanley. Both bank holding companies had to cut by half the amount they intended to distribute among shareholders in order to avoid failing the test.

⁴The privacy policy does not apply to those companies publicly listed for which capital requirements count as inside information and must be disclosed.

⁵Goldstein and Sapra [2014] offer an excellent review of the costs associated with information disclosures.

in the quality of the bank's assets; short-term creditors concerned by the bank's liquidity position; speculators interested in the fate of the bank; counterparties exposed to a potential default; taxpayers concerned with the use of public funds if a bailout takes place; the bank itself, which strategically chooses its funding strategy in response to the information publicly disclosed, among others. As a result, a policy-maker who wishes to help a bank under distress by disclosing information, necessarily has to account for the strategic reactions that disclosures induce on these multiple audiences.

Despite the recent attention that stress tests (and more generally, disclosure policies as regulatory tools) have drawn from the theoretical literature, the natural question concerning the *optimal* degree of transparency of these disclosures remains essentially unanswered. The reason behind this surprising observation is the standard assumption, usually encountered in the literature, of a single audience (i.e., a single receiver) for the policy-maker's disclosure, which, to a large extent, simplifies the policy-maker's problem.⁶ When this is the case, the optimal policy is *opaque* and consists of an action recommendation to the single audience.⁷ In most cases this can be reinterpreted as a pass/fail test (e.g., a recommendation to keep pledging to the bank, or not). In contrast, when multiple audiences are considered, disclosures intended for one particular audience are *observed* simultaneously by the rest of the audiences, generating an endogenous reaction. It is thus not longer clear what the optimal degree of transparency of such disclosures should be, and how to design them. Put differently, a crucial ingredient to discuss about the optimal degree of transparency of such disclosures is accounting for the strategic interaction between the multiple audiences concerned about the banks' information. This paper aims to provide an answer to this question and to inform the debate on the optimal design of such disclosures.

The questions that motivate this work are: (i) What is the optimal degree of transparency of disclosure policies when accounting for the interaction among multiple audiences? (ii) What is the role of companion regulatory policies such as imposing recapitalizations or debt buybacks? (iii) Are these policies complements to information disclosures or substitutes to them? (iv) Acknowledging that different audiences worry about variables that are determined at different points in time, how do early disclosures affect later disclosures, and vice versa? (v) If the designer lacks the ability to measure the bank's private information, how do information disclosure and recapitalization requirements interact with the effectiveness of the policy-maker's elicitation capacity?

These questions are not restricted to the design of intervention policies. Rather, they apply to a broad set of environments. As a matter of fact, they are expected to arise in any context where a firm is subject to a maturity mismatch between short-term liabilities and long-term assets, a common theme in the corporate finance literature. Think, for example, of a firm that wants to undertake a socially desirable project. The project promises to pay off in the future but requires an initial investment and, most likely, some liquidity injections before it starts delivering dividends. If the firm

⁶An important exception is the paper ?, where the authors consider the optimal design of stress test in an environment of heterogeneously informed short-term creditors.

⁷This is a simple manifestation of the revelation principle.

is cash-constrained it may need to sell claims on the project's cash-flows to fund its operations. In such circumstances, how do information disclosures affect the firm's ability to raise funds?

To answer these questions I consider the minimal model that preserves the richness of the problem. The model consists of a bank, a policy-maker and two audiences: short-term creditors and long-term investors. The bank has private information about two dimensions, namely, (i) the quality of its assets and (ii) its liquidity position. Throughout the paper I refer to these two variables as the fundamentals. Uncertainty about the bank's fundamentals is gradually resolved. While the quality of the bank's assets is determined early, the amount of liquid funds is determined at a later stage after a shock materializes. The timing is meant to reflect the idea that the quality of the bank's assets depends on investment decisions made in the past, while the liquidity position of the bank is subject to shocks and may vary precipitously. The policy-maker's technology allows her to learn the realization of these variables as soon as they are determined, and to make public announcements as a function of them.⁸

The rich environment proposed in this paper emphasizes the strategic interaction among the multiple audiences, who care about different aspects of the bank's fundamentals. Long-term investors, on the one hand, are primarily interested in the long-term profitability of the bank's assets (e.g., the amount of non-performing loans). Short-term creditors, on the other hand, are concerned by the bank's liquidity position and its ability to repay short-term debt. Nevertheless, long-term investors also care about the disclosure of information regarding the bank's liquidity position, as such information affects short-term creditors' beliefs about the bank's liquidity buffers and, hence, their decisions of whether to keep rolling-over the bank's debt. Given that short-term creditors' claims are senior to those of long-term investors, the latter understand that they may be wiped out if short-term creditors decide to stop pledging to the bank. Hence, they are indirectly affected by disclosures about the bank's liquidity position. In turn, short-term creditors also care about the level of funds the bank is able to raise, which in turn depends on the information about the long-term profitability of the bank's assets disclosed by the policy-maker, as such information determines how much funds long-term investors are willing to provide for claims on the bank's assets.

My first result characterizes the equilibrium of the *fund-raising game* played by the informed bank and long-term investors, in the absence of government intervention. The bank issues claims on its assets in exchange for funds which helps it meet former obligations and creates a precautionary buffer against potential liquidity shocks. I show that in the current environment, when investors' prior beliefs about the subsequent liquidity shock are pessimistic, the existence of a bank type with poor-quality assets is enough to induce market freezing, regardless of the aggregate quality of the assets. When the aggregate quality of the assets falls below the minimal amount of capital required to dissuade creditors from running, market freezing is the unique equilibrium of the fund-raising game. The bank is then left unprotected against short-term liquidity shortages, which induce the

⁸As is standard in the *information design* literature, I assume that the policy-maker has commitment power and chooses the information disclosure policy before observing the true realization of the bank's fundamentals.

bank to default in case they materialize.

To prevent the bank's default, the policy-maker may disclose information about the quality of the bank's assets and its liquidity position. I fully characterize the optimal *comprehensive assessment*. The policy-maker first examines the long-term profitability of the bank's assets by conducting an *asset quality review*. When the profitability of the bank's assets is above a threshold, the asset quality review assigns a unique passing grade. Conditional on passing the asset quality review, no further disclosures about the bank's liquidity position are necessary. When the quality of the assets, instead, falls below such a threshold, the asset quality review assigns one of multiple failing grades. The optimal asset quality review has a monotone partitional structure in which adjacent quality levels are pooled together under the same grade. To improve the bank's chances of survival, and conditional on the bank having failed the asset quality review, the policy-maker conducts a *liquidity stress test*. When the liquidity position of the bank is sufficiently good, the bank is assigned a *pass* grade, which convinces short-term creditors to keep rolling over the bank's debt. In the opposite case, the bank is given a *failing* grade, which prompts short-term creditors to run.

The asymmetrical structure of the optimal disclosure policy stems from the strategic interaction of the two audiences. When the expected value of the bank's assets is low, there exists an endogenous amplification effect associated with increasing the perceived quality of the assets sold by the bank. Namely, more valuable assets induce the first audience, long-term investors, to pledge a bigger amount of funds to the bank. This increases the probability that the bank survives a run of the second audience, short-term creditors, as the set of liquidity shocks that induce default has shrunk. The increase in the probability of survival then allows long-term investors to offer a higher price for the bank's assets (or equivalently to charge a lower premium). The additional increase in the price offered to the bank feeds back and induces a larger probability of survival, and so on. As a result, the interaction between both audiences implies that the probability that the bank survives increases more than proportionally with increments in the value of the securities placed by the bank. In other words, the probability of survival is *convex* in the perceived quality of the bank's assets. As a result, when the profitability of the assets is low, the policy-maker prefers finer disclosure policies over coarser rules, just like a risk-lover decision-maker prefers lotteries over certain (i.e., deterministic) outcomes.

In contrast, when the quality of the bank's assets is high, the bank may prevent default altogether by raising enough funds to persuade short-term creditors to keep rolling over the bank's debt. Using a more transparent disclosure policy in this case does not generate any benefits and, in fact, may hinder risk-sharing among banks with heterogeneous asset qualities. Thus, when the long-term profitability of the bank's assets is sufficiently good, the optimal asset quality review assigns a unique and hence opaque passing grade.

Crucially, I find that imposing contingent recapitalizations is instrumental to implementing the optimal policy. I show that without forced recapitalizations, information disclosure about the bank's fundamentals may be ineffective and the regulator may fail to help the bank raise funds. As a matter

of fact, a disclosure rule that is not complemented with recapitalizations may backfire and prove worse than a *laissez faire* policy⁹. Moreover, I show that policy proposed in the paper implements the optimal solution to a broader mechanism design problem in which the policy-maker possesses the authority to dictate the type of securities and price the bank should choose when approaching long-term investors. I show that conferring this authority to the policy-maker is not necessary because the same outcome can be implemented by combining appropriately designed information disclosures with forced recapitalizations.

The intuition behind the former result is that, in the absence of government intervention, the threat of a run of short-term creditors serves as a discipline device toward the possibility that types with different asset qualities *separate* during the fund-raising stage, and hence may promote risk-sharing.¹⁰ In fact, if the probability of default is small enough, banks with high quality assets may signal their private information by retaining a larger fraction of their assets on their balance sheets and, consequently, raising less precautionary funds to minimize default risk. The possibility of disclosing information about the bank's liquidity position reduces the subsequent probability of default and, as a result, increases the incentives to signal. This has a negative impact on risk-sharing. Recapitalizations thus substitute for the disciplining role served by creditors' run, by threatening the bank to reduce the dividends that can be distributed among shareholders in case it fails to raise the funds specified by the policy-maker.

In certain environments, the policy-maker may not be able to measure the variables that are private information to the bank. In the second part of the paper I consider a richer setting where the regulator cannot conduct a liquidity stress test in a timely manner, before short-term creditors make their decision whether to run on the bank. The policy-maker implements a liquidity-provision program that asks the bank to self-report the magnitude of the liquidity shortage. The regulator may purchase claims on the bank's assets and, additionally, may publicly communicate part of the information learned while dealing with bank to the rest of market participants.

The problem of designing a liquidity-provision program that elicits information about the bank's buffers is similar to the problem considered in Philippon and Skreta [2012] and Tirole [2012], in that a bank's outside options are endogenous to the choice of the government's program. A bank that refuses to participate in the program faces short-term creditors whose beliefs depend on the government's mechanism. The novelty with respect to those earlier models is that the policy-maker may provide *privacy* to the bank and may engage in strategic information disclosure about the information elicited from the bank. These additional properties drastically change the set of equilibrium outcomes.

The optimal liquidity-provision program asks the bank to (confidentially) report its liquidity

⁹I assume that the policy-maker cannot commit ex-ante to the liquidity stress test before the liquidity shock materializes. That is, in case a liquidity shock occurs, the policy-maker runs the liquidity stress test that maximizes the probability of survival.

¹⁰The disciplining role served by short-term creditors has been described in the literature going back to Calomiris and Kahn [1991]. For recent developments, see Cheng and Milbradt [2012] and Eisenbach [2017].

position and promises in return to assign a pass-fail grade. Contingent on assigning the pass grade, the policy-maker purchases claims on the bank's assets. When the regulator announces the bank has passed the test, short-term creditors find it in their best interest to keep rolling over the bank's debt. In turn, when the policy-maker fails the bank, short-term creditors willingly stop pledging to it.

To induce all liquidity types to truthfully report their liquidity positions, the policy-maker needs to compensate those types that are passed with lower probability. This compensation is done by offering them higher prices for their securities. The optimal liquidity-provision program offers a passing grade to most illiquid banks with low probability but compensates them with higher prices for their assets, while more liquid (but still vulnerable) banks are assigned a pass grade with higher probability and lower prices for their remaining claims on their assets. In this manner, the government improves the average liquidity position of banks receiving the passing grade, which persuades creditors to keep pledging to the bank.

I use the characterization of the optimal-liquidity-provision program to show that interventions that involve simultaneous pledging by both the private (i.e. long-term investors) and the public sector are suboptimal. To prove this result, I show that imposing recapitalizations undermines the effectiveness of the government's liquidity programs. In fact, a bank that retains a smaller fraction of its assets can be promised fewer funds by the government under the natural constraint that the latter does not pay more than the fair price of the securities purchased. Given that the effectiveness of the liquidity-provision program relies on compensating extremely vulnerable banks (which receive a passing grade less often than more liquid banks) with higher prices for the remaining claims on their assets, requiring that the bank sells a fraction of such assets to long-term investors decreases the elicitation capacity of the policy-maker once the liquidity shock materializes. Additionally, having the bank raise funds from external investors intensifies incentive compatibility issues in the regulator's elicitation program. As a result, if a liquidity-provision program is implemented, recapitalizations are minimized.

The policy-maker is thus confronted with the dilemma of choosing between private-sector financing, which maximizes the price of the bank's securities by selling them before the liquidity shock occurs, and the government's liquidity-provision program, which asks the bank to report information about its liquidity buffers (and hence it is more effective after a liquidity shock materializes) and then reveals information to its creditors. I show that optimal comprehensive interventions display a non-monotone pecking order. Institutions with high-quality assets are given a pass grade by the asset quality review and are required to raise enough capital from long-term investors. Banks with intermediate-quality assets are assigned one of multiple failing grades and are funded with the government's liquidity-provision program. Finally, institutions with extremely poor-quality assets are failed with one of multiple failing grades and are induced to seek private-sector funding. As a result, the paper shows that the non-monotonicity in funding strategies need not be proof of suboptimality. In fact, the non-monotone pecking order naturally arises when accounting for the strategic interaction of the multiple audiences.

The rest of the paper is organized as follows. Below, I wrap up the introduction with a brief review of the most pertinent literature. Section 2 presents the model. Section 3 describe the equilibria in the absence of government intervention. Section 4 studies the optimal comprehensive disclosure policy. Section 5 studies the case where the policy-maker designs an elicitation mechanism to learn the liquidity position of the bank. Proofs omitted in the text are in the Appendix or in the Supplementary Material.

Related literature. The paper is related to several strands of the literature. The first strand is the literature on *regulatory disclosures*. Close in spirit to this paper is the work by Faria-e Castro et al. [2016] who consider a model of information disclosure in an environment with runnable liabilities and asymmetric information. The paper focuses on the trade-off between the government’s fiscal capacity and the degree of transparency of stress tests. Crucially, that paper assumes that there exists a one-to-one relationship between liquidity position and asset quality. In contrast, in the present paper, I relax the assumption that liquidity and asset quality are perfectly correlated, which allows me to examine the role of disclosure of multi-dimensional fundamentals to multiple audiences. In the second part of the paper, where I allow the policy-maker to purchase claims on the bank’s assets, I find that the degree of transparency of the stress test affects the amount of funds the policy-maker can commit to use, generating a trade-off between coarser disclosure policies (which allow banks to raise, on average, more funds) and the effectiveness of the regulator’s program at eliciting information from the bank. In other words, I show that stronger financial capacity need not come with more information disclosure, contrary to what is established in Faria-e Castro et al. [2016].

Goldstein and Leitner [2018] consider the stress test design problem of a regulator who wishes to facilitate risk sharing among banks endowed with assets of heterogeneous qualities. My model complements theirs by analyzing an environment where (i) the amount of additional funds needed by the bank is endogenously determined by the disclosure policy selected by the policy-maker and the interaction between the multiple audiences, and (ii) the fundamentals are multidimensional and comprise the quality of the bank’s assets and its liquidity buffers. Orlov et al. [2017] consider the joint design of stress tests and capital requirements in a setting where multiple banks have correlated exposure to an exogenous shock. Inostroza and Pavan [2018] explore optimal disclosure policies when the policy-maker faces multiple receivers endowed with heterogeneous information, under an adversarial approach. They show that optimal stress tests need not generate conformism in beliefs among market participants, but generate perfect coordination among their actions. Alvarez and Barlevy [2015] study the incentives of banks to disclose balance sheet (hard) information in a setting where the market is not able to observe the exposure to counterparty risks. In my model, banks cannot disclose hard information but may try to signal information through their funding strategy. Bouvard et al. [2015] study a credit rollover setting where a policy maker must choose between transparency (full disclosure) and opacity (no disclosure) but cannot commit to a disclosure policy. In contrast, I assume the policy maker can fully commit to her disclosure policy and allow for fully flexible information structures.

Optimal government interventions in markets plagued by adverse selection have been studied in Philippon and Skreta [2012], Tirole [2012], and Fuchs and Skrzypacz [2015]. These papers share the common feature that government interventions affect post-intervention outcomes and vice versa. The first two papers consider a static setting, and show that the policy-maker optimally chooses to purchase low quality assets to *jump-start* a frozen market, permitting banks with better assets to receive funding from the private sector. The third paper considers a dynamic model in which low quality assets are sold first, which gradually improves the pool of legacy assets. The paper shows that the regulator should subsidize trade early in the model, and then impose prohibitively high taxes that essentially shut-down the asset market. In the second part of the paper, I propose a model that shares the common feature of these papers. Namely, that the policy-maker's liquidity-provision program generates endogenous participation constraints. In my model, however, the policy-maker may also engage in information design when trading with the bank, and some banks are funded directly by the government, instead of the private sector.

The present paper also contributes to the extensive literature on security design under adverse selection, as in Myers and Majluf [1984], DeMarzo and Duffie [1999], and DeMarzo and Fishman [2007], among others. I adopt the framework of Nachman and Noe [1994], who consider the problem of a seller with private (but imperfect) information about the profitability of her assets, and who issues claims on them in exchange for funds that help her meet a former liability. I modify their setting by introducing a probability of default, which is determined in equilibrium. In contrast to their celebrated result, which shows existence of a unique equilibrium where all types of sellers pool over the same debt-like security, I show that in the current environment there exist multiple equilibria, and that when investors' prior beliefs about the subsequent liquidity shock are pessimistic, the existence of a bank type with poor-quality assets is enough to induce market freezing, regardless of the aggregate quality of the assets. Recent developments along these lines include Daley et al. [2016], who consider the effect of ratings on security issuance; Yang [2015], who studies security design when the buyer may acquire information about asset quality at a cost; Szydlowski [2018], who considers the problem of a firm that seeks financing and chooses both its information disclosure policy and the type of security it offers to external investors; and Azarmsa and Cong [2018] who study the role of information in relationship finance.

Finally, this paper relates to the literature on *information design*. This literature can be traced back to Myerson [1986], who introduced the idea that, in a general class of dynamic games of incomplete information, the designer can restrict attention to private incentive-compatible action recommendations to agents. Recent developments include Kamenica and Gentzkow [2011], Kamenica and Gentzkow [2016], and Ely [2017]. These papers consider persuasion with a single receiver. Persuasion with multiple receivers is less studied. Calzolari and Pavan [2006a] consider an auction setting in which the sender is the initial owner of a good and where the different receivers are bidders in an upstream market who then resell in a downstream market. Related to this paper is Dworzak [2016], who offers an analysis of persuasion in other mechanism design environments with

aftermarkets. Alonso and Camara [2016a] and Bardhi and Guo [2017] consider persuasion in a voting context, whereas Mathevet et al. [2016] and Taneva [2016] study persuasion in more general multi-receiver settings. Bergemann and Morris [2016a] and Bergemann and Morris [2016b] characterize the set of outcome distributions that can be sustained as Bayes-Nash equilibria under arbitrary information structures consistent with a given common prior. Alonso and Camara [2016b] study public persuasion in a setting with multiple receivers with heterogeneous priors. Kolotilin et al. [2017] consider a screening environment whereby the designer elicits the agents' private information prior to disclosing further information. Basak and Zhou [2017] and Doval and Ely [2017] study dynamic games in which the designer can control both the agents' information and the timing of their actions.

2 Model

Players and Actions. The economy is populated by a bank, short-term creditors, external investors, and a policy-maker. There are 3 periods, $T \equiv \{1, 2, 3\}$. The bank is cash-less, risk-neutral and has two legacy assets: a risky asset, and a safe zero-coupon bond. Both assets mature at period $t = 3$. The risky asset delivers an observable stochastic cash flow, $y \in \mathbb{R}_+$, while the bond has a face value of R , but can be (partially) liquidated early in period 2. During the first period, in order to increase the amount of liquid funds available at the second period, the bank may sell claims on its assets to a competitive, risk-neutral set of external investors. At the beginning of period 2, the bank may suffer a liquidity shock, described in detail below, that prevents the bank from selling a fraction or the totality of her bond ¹¹. Finally, a continuum of short-term creditors of mass one, uniformly distributed over $[0, 1]$, has a claim of \$1 if redeemed *early*, during the second period, and equal to R if *rolled over* until $t = 3$. Let $a_i \in \{0, 1\}$ denote the action chosen by creditor i , where $a_i = 0$ represents the action of rolling over the bank's debt, and $a_i = 1$ the decision of withdrawing by the end of the second period. I denote by $A \in [0, 1]$ the fraction of creditors who chooses to stop pledging to the bank.

Fundamentals. The fundamentals of the bank's balance sheet are captured by the vector (ω, y) . The variable y represents the asset's cash flows which are drawn from the full-support cdf F^y over \mathbb{R}_+ . The variable ω represents the bank's short-term liquidity. More specifically, $\omega \in \Omega \equiv [0, 1]$ represents the fraction of the bond that the bank can sell during the second period in order to obtain additional funds to repay its obligations. A value of $\omega < 1$ can be interpreted as an unexpected liquidity shock which prevents the bank from selling the totality of the bond (e.g., off-balance sheet items or the imposition of haircuts). I will frequently refer to ω as the bank's liquidity shock. I assume that the fraction of the bond that is not liquidated becomes available at $t = 3$ and can be used to repay late creditors.

Default. If the fraction of creditors who decide not to roll over the bank's debt is large enough with respect to the bank's available cash, bankruptcy is triggered. In that case, the bank's risky asset is confiscated along with any available cash the bank possesses at that moment. For simplicity I suppose that the recovery rate associated with bankruptcy is 0.

Precautionary Fund Raising. To reduce the probability of default, the bank may raise funds at $t = 2$ by selling claims on its risky asset to external investors. If the bank raises P units of funds, the amount of cash available to repay early withdrawals is given by $\omega + P$. Let $r \in \{0, 1\}$ represent the event of whether the bank defaults, with $r = 1$ in case of default, and $r = 0$ otherwise. I assume that the fate of the bank is determined by the linear rule $r = 1_{\{P+\omega \geq A\}}$.

Exogenous Information. We assume that there is *gradual resolution of uncertainty*. At $t = 1$,

¹¹Alternatively, we may think that there exists a stochastic obligation that needs to be paid during the second period in addition to the fraction of early withdrawals. Importantly, the bank will suffer a liquidity shortage with positive probability.

the bank's long-term cash flows, y , are drawn from F^y . The bank then learns a signal θ about y , and forms beliefs about the realization of y according to the conditional cdf F_θ^y (resp., pdf f_θ^y), where θ belongs to the set $\Theta = \{\theta_L, \theta_H\}$, with $\theta_H > \theta_L$. I will refer to θ as the bank's asset quality type. I assume that the conditional pdf f_θ^y satisfies log-supermodularity in (y, θ) (or, equivalently, that the realization of cash-flows of different types θ are ordered according to MLRP). The cash flow realization cannot be observed by any market participant until $t = 3$. The liquidity shock ω is drawn from $F^\omega \in \Delta\Omega$ at the beginning of the second period and is observed by the bank. The rest of market participants only learn whether the shock materialized or not (i.e., whether $\omega = 1$ or $\omega \in [0, 1)$). These assumptions are made to reflect the idea that the profitability of the bank's asset depends on investment decision made in the past, while the bank's liquidity position is subject to unexpected contingencies and may vary precipitously. All market participants anticipate at $t = 1$ the possibility that a liquidity shock takes place in period 2 but do not know its severity. All agents in the economy share the prior belief F^ω about the bank's liquidity position. The policy-maker, investors and short-term creditors share a common prior $\mu_\theta \in \Delta\Theta$ about the bank's asset type.

Payoffs. For simplicity, I assume no discounting. If the bank raises P units of money during the second period, draws a liquidity shock ω , and a fraction A of creditors withdraws early, it survives as long as the available funds are greater than its obligations: $\omega + P \geq A$. In such a case, the bank may use the remaining cash to buy a bond and obtain a payoff of $R \times (P + \omega - A)$ at $t = 3$. Thus, the bank's payoff when it raises P units of cash in period 2, cash flows are \tilde{y} during the third period, the liquidity shock is ω , and faces a fraction A of early withdrawals, is given by:

$$\begin{aligned}
U(P, \tilde{y}, \omega, A) &= \left(R(P + \omega - A) + \left(R - \underbrace{\omega R}_{\text{liquidated early}} - \underbrace{(1 - A)R}_{\text{late withdrawals}} \right) + \tilde{y} \right) \times \\
&\quad \times 1_{\{\omega + P \geq A\}} \\
&= (PR + \tilde{y}) \times 1_{\{\omega + P \geq A\}}.
\end{aligned} \tag{1}$$

The creditors' payoffs depend on their actions. I normalize the utility from withdrawing early to 0 and let $u_i(\tilde{\omega}, A)$ be the utility of a creditor who decides to pledge to the bank, when the total amount of available cash held by the bank at $t = 2$ is $\tilde{\omega} = \omega + P$ and the fraction of early withdrawals is A . Observe that when the bank survives, a creditor who withdraws early obtains \$1 w.p. 1, while he would have received R had he chosen to roll over the bank's debt. We denote by $g \equiv R - 1 > 0$ the positive utility differential from rolling over the bank's debt in case the bank does not default. When the bank defaults, a creditor who chooses to pledge to the bank at $t = 2$ receives a payoff $b(\tilde{\omega}, A)$ at $t = 3$. The function $b(\tilde{\omega}, A)$ is negative, non-decreasing in $\tilde{\omega}$, and non-increasing in A .¹² That is,

¹²A natural example of such a function might be $u_i(\tilde{\omega}, A) \equiv \left(\frac{\tilde{\omega}}{A} - 1\right) 1_{\tilde{\omega} \leq A} + (R - 1) 1_{\tilde{\omega} > A}$. That is, if the amount of withdrawals exceeds the bank's liquidity position, $\tilde{\omega}$, the bank defaults and short-term creditors obtain a proportional fraction of their claim $\frac{\tilde{\omega}}{A}$. Otherwise, if the bank survives, they receive R .

$$u_i(\tilde{\omega}, A) = \begin{cases} g & \text{if } r = 0 \\ b(\tilde{\omega}, A) & \text{if } r = 1. \end{cases}$$

Finally, I assume large social costs associated with the default of the systemically important bank. The policy-maker's objective can then be stated as lowering the default probability. Equivalently, policy-maker's obtains a positive payoff $W_0(A)$ when default is successfully avoided, and a payoff of 0 when that is not the case, with $W_0(\cdot)$ non-increasing.

$$U^P(\tilde{\omega}, A) = W_0(A) \times 1_{\{\tilde{\omega} > A\}}$$

Asset Market. After observing its asset quality type, $\theta \in \Theta$, in period 1 the bank proposes to the external investors a security $s[\theta]$, which corresponds to a claim on future cash-flow realizations of the risky asset and belongs to $S \equiv \{s : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \text{ s.t.: (LL),(M),(MR)}\}$ where:

$$\begin{aligned} \text{(LL)} \quad & 0 \leq s(y) \leq y \quad \forall y \geq 0 \\ \text{(M)} \quad & s(y) \quad \text{non-decreasing} \\ \text{(MR)} \quad & y - s(y) \quad \text{non-decreasing.} \end{aligned}$$

These assumptions are standard in the literature of security design.¹³ The market observes the security $s[\theta]$ and prices it according to the available public information. Importantly, I assume that the claims promised to long-term investors are subordinated to the ones of short-term creditors, and hence are paid only if the bank avoids default.¹⁴

Intervention Policies. The policy-maker concerned with the possibility that the bank defaults may choose to intervene. The policy maker possesses a technology that allows her to disclose information to all market participants and to give recommendations to the bank about the amount of capital to raise from external investors. The assumption of *gradual resolution of uncertainty* implies that the designer may disclose information about the cash-flows at $t = 1$, after y has been determined, but can disclose information about the liquidity shock ω only at $t = 2$, after ω has been drawn. I denote by Γ^y the disclosure policy about the profitability of the bank's assets, y , and by Γ^ω the liquidity examination conducted in the second period about the bank's liquidity position. I will refer to Γ^y as the bank's *asset quality review* and to Γ^ω as the *liquidity stress test*. In addition to the information

¹³The first constraint represents *limited liability* and states that a security $s \in S$ is in fact a sharing rule of the asset's cash-flows. The second constraint, the *monotonicity* condition, requires that the security is non-decreasing in the the cash-flows, since otherwise the bank would have the option of asking for (risk free) credit to a third party to boost the cash-flow realization and thus decrease the amount owed to the initial investors. Finally, the last constraint imposes that the share of cash-flows kept by the bank is non-decreasing for, otherwise, the bank would have incentives to burn part of the cash-flows to improve her payoff.

¹⁴See Ahnert et al. [2018] for a model of asset encumbrance where banks may choose the fraction of secured funding they request from external investors.

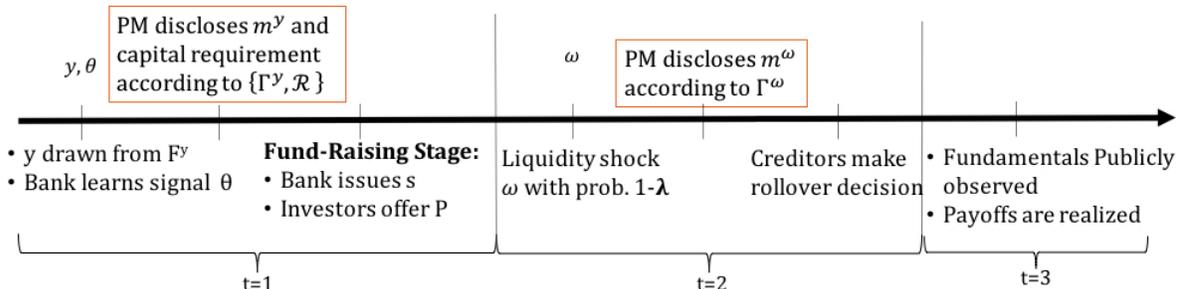


Figure 1: Timing.

revealed by the asset quality review, the policy-maker may impose minimal recapitalization requirements according to the rule \mathcal{R} which specifies the amount that can be distributed as dividends as a function of the information disclosed by Γ^y . I make the implicit assumption that the technology needed to conduct the asset quality review Γ^y is time-demanding and cannot be postponed until the liquidity shock takes place, since then the policy-maker might not be able to disclose information on time, before short-term creditors make their decisions. Moreover, I assume that any information learned while conducting the asset quality review Γ^y becomes public. That is, the policy-maker cannot choose to learn information about y and not share it with market participants¹⁵. Finally, I assume that the policy-maker cannot commit¹⁶ at $t = 1$ to the liquidity stress test Γ^ω she will conduct in period 2.

Timing. The sequence of events is as follows:

Period 0. The policy maker chooses a policy $\{\Gamma^y, \mathcal{R}\}$, and publicly announces it.

Period 1. (a) y is drawn from F^y . (b) The bank observes a private signal θ about y . (c) The policy-maker discloses information m^y and recapitalizations according to the joint policy $\{\Gamma^y, \mathcal{R}\}$. (d) The bank sells a security $s \in S$ to external investors at price $P \geq 0$. I refer to (d) as the *fund-raising stage*.

Period 2. (a) ω is drawn from F^ω . (b) The policy-maker conducts a liquidity stress test Γ^ω and discloses information m^ω . (c) Short-term creditors observe P and all information available with respect to ω , and decide whether to keep pledging to the bank. (e) The bank liquidates a fraction of her bond and its fate is determined according to whether $\omega + P$ is greater than the fraction of early withdrawals, A . Any excess of liquid funds is reinvested.

Period 3. Conditional on the bank's survival, (a) y is materialized and $s(y)$ is paid to investors. (b) The fraction of bond not liquidated early, and any amount reinvested at period 2, is collected with interest and late creditors are paid back.

¹⁵Any information produced by the regulator that is kept hidden from the rest of agents, always leaks and, therefore, if the policy-maker wants the rest of market participants not to learn some information she should not produce it in the first place. A similar assumption is made by Faria-e Castro et al. [2016].

¹⁶I assume the policy-maker *can* commit to disclose information according to asset quality review Γ^y (resp. Γ^ω), in period 1 (resp. 2), but not to the critical stress test to conduct in period 2, at $t = 1$.

3 Laissez Faire

3.1 Raising Capital to Persuade Creditors

I first study the case where the policy maker does not intervene. In this case, the bank observing its asset quality type, θ , enters the *fund-raising stage* by approaching investors to whom it offers claims on its asset in order to raise funds that allow it to pay its obligations and, hence, avoid an eventual default triggered by a creditors' run. I follow an adversarial approach, and assume that when multiple action profiles are consistent with equilibrium play, creditors coordinate on the most aggressive outcome from the perspective of the bank.

Let $\mathbb{E}(u(P, \omega, A))$ be the expected utility of a creditor who chooses to pledge when the fraction of early withdrawals is given by A , the seller has successfully raised P units of capital, and the liquidity shock is ω . The adversarial approach then implies that all creditors choose to stop rolling over the bank's debt whenever withdrawing early is a best response to everyone withdrawing early. That is, each creditor withdraws early when

$$\mathbb{E}(u(\omega, P, 1)) \equiv \int_0^1 (g \times 1_{\{P+\omega>1\}} + b(P+\omega, 1) \times 1_{\{P+\omega \leq 1\}}) F^\omega(d\omega) \leq 0. \quad (2)$$

Define then $A(P)$ as the most aggressive fraction of early withdrawals, for a given recapitalization level, P . In what follows I assume that λ , the probability with which no liquidity shock occurs, is small enough so that, if the bank does not raise additional funds, creditors withdraw early (i.e., inequality (2) holds for $P = 0$). Next, let $K \geq 0$ be the minimum amount of capital that the bank needs to raise in order to persuade short-term creditors to keep rolling over its debt. That is,

$$K \equiv \sup \{P \geq 0 : \mathbb{E}(u(\omega, P, 1)) \leq 0\}.$$

From the definition of K above, we have that $A(P) = 1\{P \leq K\}$. To make the problem interesting we assume that the low type bank has an asset with expected cash-flows below K , while the expected cash-flows of the asset of type H are above it.

Assumption 1. $\frac{1}{R}\mathbb{E}_L(y) < K \leq \frac{1}{R}\mathbb{E}_H(y)$.

The bank understands that the only way to convince short-term creditors that it is liquid is by raising K units of capital in the asset market. By the end of period 2, short-term creditors observe the recapitalization secured by the bank and decide whether or not to rollover the bank's debt. If the bank raises at least K units of capital, then no short-term creditor withdraws early, allowing the bank to survive and to re-invest the funds buying a 1-period bond. On the other hand, if the amount raised is smaller than K , then all creditors withdraw early, in which case the survival of the bank depends on the amount of capital raised and on the realization of the liquidity shock ω . Given the above observation, the maximal price that external investors are willing to pay for any security s and is given by:

$$P(s; \mu) \equiv \sup \left\{ p \geq 0 : \frac{\mathbb{E}_\mu(s)}{R} \times \mathbb{P} \{ \omega + p \geq A(p) \} \geq p \right\} \quad (3)$$

where $\mathbb{E}_\mu(s)$ is the expected value of security s when the market holds beliefs $\mu \in \Delta\Theta$ about the bank's type. Note that the definition of $P(s, \mu)$ implies that, in case the equation

$$\frac{\mathbb{E}_\mu(s)}{R} \times \mathbb{P} \{ \omega + p \geq A(p) \} = p, \quad (4)$$

admits multiple solutions, the selected one is the one associated with the largest price¹⁷. The next assumption will be used for certain results, for it favors tractability.

Assumption 2. *The prior distribution of the liquidity level ω , F^ω , is concave.*

Assumption 2 reflects the idea that the liquidity problem is severe. Intuitively, when F^ω is concave (i.e., when the density f^ω is non-increasing), low liquidity levels are more likely to occur. When this is the case, and additionally $\lambda = 0$ (that is, there is no mass point at $\omega = 1$), investors refuse to fund any project with NPV below K . To see this, note that in this case the LHS of inequality in (3), $\frac{\mathbb{E}_\mu(s)}{R} \times \mathbb{P} \{ \omega + p \geq A(p) \}$, is smaller than the RHS, p , meaning that the expected payoff an investor obtains from purchasing security s is no greater than what he pays. As a consequence, the market refuses to purchase security s because it expects a high probability of default. The intuition behind this result is that, under an adversarial approach, investors believe that short-term creditors will overreact to the inability of the bank of raising enough capital. This generates a negative feedback cycle since it invites the market to offer a lower price for the security issued by the bank. The bank's inability to raise funds then makes a massive early withdrawal more likely, which in turn implies a higher probability of default and thus a lower price. Hence, when $\lambda = 0$ and assumption 2 holds, the bank survives only if the price collected is at least K .

3.2 Solution Concept

I assume that renegotiation between short-term creditors and the bank is not feasible. Given the speed of events and the dispersion of short-term creditors, such an option is, in most cases, unviable in practice.¹⁸ The government most preferred outcome consists of having all bank types issuing securities that allow them to survive the liquidity shock, and hence avoid bankruptcy. As is usually

¹⁷This selection has a game-theoretic foundation similar in spirit to the one encountered in *Bertrand* competition models. Namely, if the market reached a price $\tilde{P} < P(s; \mu)$ satisfying 4, any buyer could deviate and offer a greater price \hat{P} for which the LHS of equation 4 is strictly greater than the RHS, and obtain a positive gain in the process. Such deviation would be willingly accepted by the bank. As a result, \tilde{P} would be inconsistent with equilibrium play. $P(s; \mu)$ is thus the unique price consistent with competitive markets and immune to such deviations.

¹⁸Landier and Ueda [2009] make a similar assumption. They argue that: *"Although the proposed debt-for-equity swap is the first-best solution, it is often a difficult solution to implement in practice. A major reason is the speed of events, which leaves no time for renegotiation. The possibility of a deposit run calls for speedy resolution, while dispersion of bank debt holders requires a lengthy negotiation process."*

the case with signaling games, the fund raising game may be plagued with multiple equilibria. In order to focus on equilibria which take into account the propensity of bank types to deviate, I restrict attention to PBE satisfying the D1 criterion, and I refer to them hereafter as equilibria.

Let $V(P, s, \theta)$ be the utility of a bank of type θ , selling a security s and receiving funds in the amount of P . Without government intervention, the bank's payoff can be written as:

$$\begin{aligned} V(P, s, \theta) &\equiv \mathbb{E}((PR + y - s) \times 1_{\{\omega + P \geq A(P)\}}) \\ &= (PR + \mathbb{E}_\theta(y - s)) \mathbb{P}\{\omega \geq A(P) - P\}. \end{aligned} \quad (5)$$

I will say that $\{\{s_\theta^*\}_{\theta \in \Theta}, \mu^*, P^*, A^*\}$ is an equilibrium of the fund-raising game if:

$$\begin{aligned} \text{[Sequential Rationality]:} \quad & s_\theta^* \in \arg \max_s V(P^*(s), \theta, s) \\ \text{[Competitive Investors]:} \quad & P^*(s) = \sup \left\{ P : \frac{\mathbb{E}_{\mu^*}(s)}{R} \times \mathbb{P}\{\omega + P \geq A^*(P)\} \geq P \right\} \\ \text{[Adversarial Coordination]:} \quad & A^*(P) = 1_{\{P < K\}}, \forall P \geq 0 \\ \text{[Belief Consistency]:} \quad & \mu^*(s) \text{ computed according to Bayes rule on-path} \end{aligned}$$

Additionally, I impose that off-path beliefs associated with securities not observed in equilibrium, assign all probability weight to the asset quality type with the greatest propensity to deviate to them. For a formal definition of this refinement, please see the Appendix.

3.3 Equilibrium Characterization.

In what follows I characterize the set of equilibria that arise in the fund-raising game. My first proposition shows that, in any *pooling* equilibrium, both bank types issue debt. When this is the case, the price obtained by the bank is no larger than K . Then I show that separating equilibria may exist only if the expected cash-flows of type L are sufficiently large, in which case type H chooses to raise less funds than needed to avoid default with certainty, and hence remains exposed to rollover risk. I prove this proposition in the Appendix for general distribution of the fundamentals; This will permit me to invoke the result also in the next sections, when additional information about y may be revealed by the policy-maker. The result is an adaptation of the results in Nachman & Noe (94) to the setting under examination where I incorporate the probability of default to the pricing of securities.

Proposition 1. (i) Let $\{\{s_\theta^{pool} = s\}_{\theta \in \Theta}, \mu, P, A\}$ be a pooling equilibrium outcome of the fund-raising game. Then necessarily, $s = \min\{y, D\}$ for some $D > 0$. Moreover, $P(s) \leq K$. (ii) Let $\{\{s_\theta^{sep}\}_{\theta \in \Theta}, \mu, P, A\}$ be a separating equilibrium of the fund-raising game. Then, $\mathbb{E}_H(s_H^{sep}) < \mathbb{E}_L(y)$.

My second result characterizes the set of equilibria that arise when $\lambda = 0$ (i.e., when the liquidity shock occurs with probability one) and Assumption 2 holds. I first show that the only type of

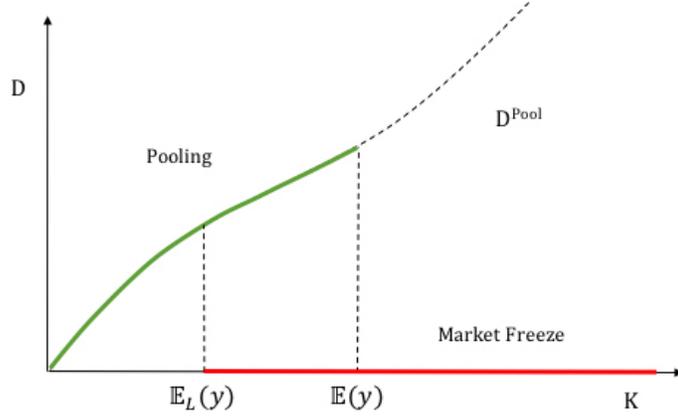


Figure 2: Face value of equilibrium debt contracts, under Assumption 2 and $\lambda = 0$.

equilibria in the fund-raising game are pooling equilibria, where both bank types issue debt contracts. I then prove that if the expected profitability of the asset of the L-type bank is low enough, then there exists an equilibrium where the asset market freezes and no security is issued. Furthermore, I show that when, in addition, the average quality of the bank's asset is low, then market freezing is the unique equilibrium outcome of the fund-raising game. Given that these results obtain under the assumption that a liquidity shock occurs with certainty ($\lambda = 0$), under such conditions the bank defaults with probability 1. Finally, I show that when the expected profitability of a type L-bank is good enough, the unique equilibrium of the game has both types of bank placing a debt contract which collects enough funds to dissuade creditors from running. This last result is simply a manifestation of Nachman and Noe [1994]'s celebrated uniqueness result.

Proposition 2. *Suppose Assumption (2) holds and $\lambda = 0$. Then,*

1. *In any equilibrium of the fund-raising game, $s_\theta^* = \min\{y, D\}$ for all $\theta \in \Theta$, and for some $D \geq 0$.*
2. *(Market freeze) If $\frac{1}{R}\mathbb{E}_L(y) < K$, there exists an equilibrium where $s_\theta = \mathbf{0}$ for all $\theta \in \Theta$. Moreover, if $\frac{1}{R}\mathbb{E}(y) < K$, this is the unique equilibrium.*
3. *(Pooling) If $\frac{1}{R}\mathbb{E}(y) \geq K$, there exists an equilibrium where $s_\theta = \min\{y, D^{pool}\}$ with D^{pool} defined as the unique solution to $\frac{1}{R}\mathbb{E}(\min\{y, D^{pool}\}) = K$. Moreover, if $\frac{1}{R}\mathbb{E}_L(y) \geq K$, this is the unique equilibrium.*

An immediate implication of proposition 2 is that, when bank expects a severe liquidity shock, the presence of a bank type with sufficiently poor assets is enough to guarantee the existence of an equilibrium where the market for the bank's assets freezes, preventing the bank from raising funds to avoid the imminent run of short-term creditors. The investors' ability to foresee the possibility of a run, and to price assets accordingly, together with the incentives of the banks of type H H to

separate from L , induce a *fire sale* so severe the bank is unable to raise any funds. As a consequence of this property, any security which a *type H-bank* type may try to issue is also issued by the type L-bank, generating contagion among bank types and provoking the freeze of the asset market.

4 Disclosure Policies

The policy-maker, concerned with the potential freeze of the asset market may choose to intervene by conducting a *comprehensive assessment*. In period 1, the policy-maker has the possibility of conducting an *asset quality review*, $\Gamma^y = \{M^y, \pi^y\}$, characterized by a disclosure policy, $\pi^y : \mathbb{R}_+ \rightarrow \Delta M^y$, where M^y is an arbitrary set of messages. Conditional on the information $m^y \in M^y$ disclosed by the asset quality review, the policy-maker specifies a *recapitalization* rule $\mathcal{R}(\cdot|m^y) : \mathbb{R}_+ \rightarrow [0, 1]$, where for any $P \in \mathbb{R}_+$, $\mathcal{R}(P|m^y)$ represents the maximal fraction of the bank's payoff the bank is allowed to distribute as dividends, as a function of the the level of capital raised during the fund-raising stage, P .^{19, 20}

Consider then recapitalization rules satisfying

$$\mathcal{R}(P) = \begin{cases} 1 & P > C \\ \alpha & P \leq C, \end{cases}$$

with $\alpha, C \in [0, 1]$. I interpret any such rule as imposing a *minimal recapitalization requirement*, C , which if not met limits the amount of dividends the bank is allowed to distribute to a fraction α of the bank's total profit. As I show below, the optimal recapitalization policy can be described as a minimal recapitalization requirement. The decision of allowing shareholders to distribute only a fraction of the bank's profit if the bank does not comply with the recapitalization specified by the rule serves the purpose of enforcing the policy-maker's recommendation.²¹ Note that although I confer the designer the authority to impose recapitalizations, I do not allow her to repudiate any contract the bank agrees upon with the investors. That is, investors preserve their claims on the future cash-flows of the asset even if the bank does not comply with the recapitalization requirement. Importantly, the government commits to not inject any type of funds to insulate creditors from the liquidity shock and, hence, taxpayers' money is not at stake. I relax this assumption in the next section. As I show below, imposing recapitalizations serves as a *discipline device* to control separation incentives among bank types during the fund-raising game.

¹⁹The assumption that $\mathcal{R}(P|m^y)$ takes only deterministic values is without loss of optimality as it will become clear later on.

²⁰Assuming that \mathcal{R} does not depend directly on y is wlog. I make this assumption so that the induced beliefs about the quality of the bank's asset depend only on m^y , and not on \mathcal{R} . A similar assumption is made in Orlov et al. [2017].

²¹Imposing recapitalizations can be interpreted in different ways in this *one-shot* framework (as opposed to a repeated game setup). The favored interpretation is that it represents a limit on the amount that can be distributed as dividends if the bank fails to raise the required level of capital. It could also represent the decision of selling the firm to another institution, and α in that case, represents the discount applied to the value of the bank.

In period 2, the policy-maker conducts a liquidity stress test, $\Gamma^\omega = \{M^\omega, \pi^\omega [P]\}$, and discloses information about the bank's liquidity shock according to the rule²² $\pi^\omega [P] : \Omega \rightarrow \Delta M^\omega$. Hereafter, I refer to the combination of an asset quality review, a recapitalization rule, and a liquidity stress test $\Psi = \{\Gamma^y, \mathcal{R}, \Gamma^\omega\}$ as a *comprehensive assessment*.

4.1 Period 1

During the first period, y is determined. The policy-maker then discloses information m^y according to the policy π^y . Given m^y the policy then specifies a recapitalization rule $\mathcal{R}[m^y]$. The bank then approaches external investors and offers a security s . The latter, after observing the security issued by the bank, form beliefs $\mu \in \Delta\Theta$ about its asset quality type. I denote by $P_\mu(s; m^y)$ the competitive price offered to the bank. Suppose that investors, which hold beliefs F^ω about the seller's liquidity position, expect the designer to disclose information about ω according to $\Gamma^\omega(P) = \{M^\omega, \pi^\omega [P]\}$. Then

$$P_\mu(s; m^y) \equiv \sup \left\{ P : \frac{\mathbb{E}_\mu(s; m^y)}{R} \times \int_\Omega \left(\sum_{m^\omega \in M^\omega} \mathbb{P}\{\omega + P \geq A(P, m^\omega) | m^\omega\} \pi^\omega(m^\omega | \omega; P) \right) F^\omega(d\omega) \geq P \right\}, \quad (6)$$

where $A(P, m^\omega)$ represents the most aggressive fraction of early withdrawals when the seller is able to raise P units of additional capital and the designer discloses information m^ω about ω .

4.2 Period 2

After the liquidity shock ω materializes and the amount of capital raised by the bank, P , has been observed by all market participants, the designer conducts the liquidity stress test Γ^ω . Assume that message $m^\omega \in M^\omega$ is publicly disclosed as a result of the exercise. Let $F^\omega(\cdot | m^\omega)$ be the posterior measure characterizing the beliefs about the liquidity shock ω , of an arbitrary creditor who observes the public information m^ω . That is,

$$F^\omega(\Lambda | m^\omega) = \frac{\int_\Lambda \pi^\omega(m^\omega | \omega) F^\omega(d\omega)}{\int_\Omega \pi^\omega(m^\omega | \omega) F^\omega(d\omega)}, \quad \forall \Lambda \subseteq \Omega.$$

The most aggressive fraction of early withdrawals faced by the bank is then given by

$$A^{\Gamma^\omega}(P, m^\omega) = 1_{\{P < K^{\Gamma^\omega}(m^\omega)\}}.$$

where $K^{\Gamma^\omega}(m^\omega)$ is defined as the minimal amount of capital needed to persuade creditors to keep pledging, when receiving m^ω . That is,

$$K^{\Gamma^\omega}(m^\omega) \equiv \sup \{P \geq 0 : \mathbb{E}(u(\omega, P, 1) | m^\omega; \Gamma^\omega) \leq 0\}.$$

²²Given that the ownership of asset's claims and the true realization of y are irrelevant for short-term creditors, who care about the liquidity shock and the amount of funds collected by the bank, P , restricting attention to policies π^ω that only depend on P is without loss.

In other words,

$$A^{\Gamma^\omega}(P, m^\omega) = 1_{\{\mathbb{E}(u(\omega, P, 1) | m^\omega) \leq 0\}}.$$

This implies that for every recapitalization amount, P , there exists a critical liquidity level, $\hat{\omega}^{\Gamma^\omega}(P, m^\omega)$, above which the bank survives the creditors run. That is,

$$\{\omega : \omega \geq A^{\Gamma^\omega}(P, m^\omega) - P\} = \{\omega : \omega \geq \hat{\omega}^{\Gamma^\omega}(P, m^\omega)\}.$$

As a result, the payoff that a bank of type θ obtains when it issues a security s at price P , information m^y is disclosed at $t = 1$, and recapitalizations are specified by the policy \mathcal{R} , is given by:

$$\begin{aligned} V(s, P, \theta; m^y, \mathcal{R}) &= \mathcal{R}(P | m^y) \times (PR + \mathbb{E}_\theta(y - s | m^y)) \times \\ &\times \left(\int_{\Omega} \left(\sum_{m^\omega} \mathbb{P}\{\omega \geq \hat{\omega}^{\Gamma^\omega}(P, m^\omega) | m^\omega\} \pi^\omega(m^\omega | \omega, P) \right) F^\omega(d\omega) \right) \end{aligned}$$

4.3 Stress tests as convex functions

In what follows I characterize the optimal comprehensive policy $\Psi = \{\Gamma^y, \mathcal{R}, \Gamma^\omega\}$. I proceed by backward induction. To find the optimal liquidity stress test Γ^ω that follows the choice of an arbitrary policy $\{\Gamma^y, \mathcal{R}\}$, and the subsequent interaction among the bank and long-term investors, I assume that an amount P is raised during the fund-raising game. The approach I follow borrows from Gentzkow and Kamenica [2016] who characterize arbitrary disclosures policies by the distribution of posterior expectations induced. The approach is described in detail in Appendix A.

Consider *any* liquidity stress test $\Gamma^\omega = \{M^\omega, \pi^\omega\}$. Each message m^ω disclosed by stress test Γ^ω induces a posterior distribution over ω , $F^\omega(\cdot | m^\omega)$. Hence, every message m^ω disclosed with positive probability generates a posterior expectation of $u(\omega, P, 1)$, $\mathbb{E}(u(\omega, P, 1) | m^\omega)$, the expected payoff of a short-term creditor, who expects all other short-term creditors to attack, after message m^ω has been disclosed. That is, each message m^ω induces a new assessment:

$$\mathbb{E}(u(\omega, P, 1) | m^\omega) = \int_{\Omega} (g \times 1_{\{\omega \geq 1 - P\}} + b(\omega + P, 1) \times 1_{\{\omega < 1 - P\}}) F^\omega(d\omega | m^\omega).$$

I show that the optimal liquidity stress test can be characterized by the distribution of posterior means of $u(\omega, P, 1)$ that Γ^ω induces. I denote by G^{Γ^ω} the distribution of posterior expectations $\mathbb{E}(u(\omega, P, 1) | m^\omega)$ induced by an arbitrary stress test Γ^ω . Let $G_{\text{FD}}^\omega(\cdot; P)$ be the distribution of posterior means of $u(\omega, P, 1)$ induced by policy the full-disclosure policy (i.e., the policy that follows the rule $\Gamma_{\text{FD}}^\omega = \{M^\omega = \Omega, \pi_{\text{FD}}^\omega\}$, with $\pi_{\text{FD}}^\omega(m^\omega | \omega) = 1_{\{m^\omega = \omega\}}$). The next proposition shows that the problem of finding the optimal liquidity stress test is equivalent to the one of finding the distribution of short-term creditor's payoff, under adversarial beliefs, that maximizes the weight assigned to the event $\{\mathbb{E}(u(\omega, P, 1) | m^\omega) > 0\}$.

Proposition 3. Fix the amount raised by the bank during the fund-raising game, $P \geq 0$. The problem of finding the stress test that maximizes the designer's payoff :

$$\begin{aligned} \max_{\Gamma^\omega = \{\pi^\omega, M^\omega\}} \quad & \mathbb{E} (W_0(A) \times 1_{\{\omega + P \geq A(P, m^\omega)\}}) \\ \text{s.t.} \quad & A(P, m^\omega) = 1_{\{\mathbb{E}(u(\omega, P, 1) | m^\omega) \leq 0\}}, \end{aligned}$$

is equivalent to the problem of finding the distribution G^{Γ^ω} among all mean preserving contractions of the prior distribution G_{FD}^ω that maximizes the probability that the expected payoff of a short-term creditor, who expects all other short-term creditors to attack, is positive. That is,

$$\begin{aligned} \max_{G^{\Gamma^\omega}} \quad & 1 - G^{\Gamma^\omega}(0) \\ \text{s.t.} \quad & G_{FD}^\omega \succeq_{MPS} G^{\Gamma^\omega} \end{aligned}$$

Next, for any stress test, Γ^ω , and any amount raised by the bank during the fund-raising game, $P \geq 0$, define the integral function $\mathcal{G}^{\Gamma^\omega}(t; P) \equiv \int_{\tilde{u}=u(0, P, 1)}^t G^{\Gamma^\omega}(\tilde{u}; P) d\tilde{u}$. Let \mathcal{G}_{FD}^ω and $\mathcal{G}_\emptyset^\omega$ be the integral functions associated with the full-disclosure policy, Γ_{FD}^ω , and no-disclosure policy, Γ_\emptyset^ω , respectively. The set of feasible critical stress tests Γ^ω , coincides with the set of convex functions that lie between \mathcal{G}_{FD}^ω and $\mathcal{G}_\emptyset^\omega$.

Lemma 1. Consider an arbitrary liquidity stress test Γ^ω . Then $\mathcal{G}^{\Gamma^\omega}(t; P)$ is convex and satisfies $\mathcal{G}_{FD}^\omega(t) \geq \mathcal{G}^{\Gamma^\omega}(t) \geq \mathcal{G}_\emptyset^\omega(t)$ for all $t \in [u(0, P, 1), u(1, P, 1)]$. Conversely, any convex function $h(\cdot)$, satisfying $\mathcal{G}_{FD}^\omega(t) \geq h(t) \geq \mathcal{G}_\emptyset^\omega(t)$ for all $t \in [u(0, P, 1), u(1, P, 1)]$ corresponds to the integral distribution function of some disclosure policy Γ^ω .

The designer's problem is thus equivalent to finding the policy Γ^ω which generates the convex function $\mathcal{G}^{\Gamma^\omega}$, between $\mathcal{G}_\emptyset^\omega$ and \mathcal{G}_{FD}^ω , with minimal slope at $t = 0$. As can be seen from Figure 3, the solution to the designer's problem is thus given by the monotone-binary policy $\Gamma_\star^\omega = (\{0, 1\}, \pi_\star^\omega)$ so that:

$$\pi_\star^\omega(0 | \omega) = 1_{\{u(\omega, P, 1) \geq u(\bar{\omega}(P), P, 1) \equiv \bar{u}(\tau)\}} = 1_{\{\omega \geq \bar{\omega}(P)\}},$$

where $\bar{u}(\tau)$ corresponds to the point at which \mathcal{G}_{FD}^ω is tangent to the line with minimal slope to the left of 0, which respects the convexity of $\mathcal{G}^{\Gamma_\star^\omega}$. The value of $\bar{u}(\tau)$ can also be characterized by the liquidity level that it induces, $\bar{\omega}(\tau)$, which can alternatively be defined as the liquidity cutoff for which²³

$$\mathbb{E}(u(\omega, P, 1) | \omega \geq \bar{\omega}(P)) = 0. \tag{7}$$

²³More precisely,

$$\bar{\omega}(P) \equiv \inf \{\tilde{\omega} \in [0, 1] : \mathbb{E}(u(\omega, P, 1) | \omega \geq \tilde{\omega}) \geq 0\}.$$

This implies, in particular, that $\bar{\omega}(P) = 0$ for all $P \geq K$.

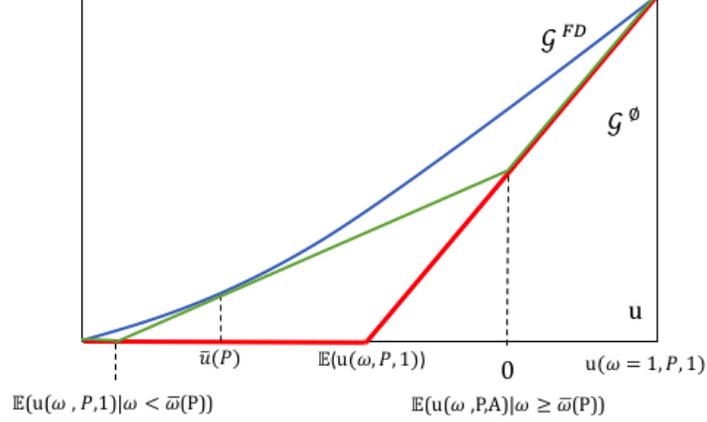


Figure 3: Optimal Liquidity Stress Test.

To see this last point, note that the policy Γ_{\star}^{ω} induces a distribution of posterior means $G^{\Gamma_{\star}^{\omega}}$ which assigns positive probability to only two points, which coincide with the points at which $\mathcal{G}^{\Gamma_{\star}^{\omega}}$ changes slope. Finally, to see that the first point at which $\mathcal{G}^{\Gamma_{\star}^{\omega}}$ changes slope coincides with

$$\mathbb{E}(u(\omega, P, 1) | \omega < \bar{\omega}(P)),$$

note that the tangency condition implies that $G^{\Gamma_{\star}^{\omega}}(\bar{u}(P)) = G_{\text{FD}}^{\omega}(\bar{u}(P))$ where the RHS corresponds to $\mathbb{P}\{u(\omega, P, 1) \leq \bar{u}(P)\}$, or equivalently, $\mathbb{P}\{\omega \leq \bar{\omega}(P)\}$.

The optimal stress test can thus be interpreted as a pass-fail announcement, where given the level of recapitalization, P , the policy-maker assigns a *pass* grade when the liquidity of the bank is above the cutoff $\bar{\omega}(P)$. Proposition 4 summarizes the above findings.

Proposition 4. *Fix the amount of capital $P \geq 0$ raised by the bank at $t = 1$. Then the optimal liquidity stress test Γ_{\star}^{ω} consists of a monotone pass-fail test. That is, there exists $\bar{\omega}(p)$, such that $\Gamma_{\star}^{\omega}(P) = (\{0, 1\}, \pi_{\star}^{\omega}(P))$, with $\pi_{\star}^{\omega}(0|\omega; P) = 1_{\{\omega \geq \bar{\omega}(P)\}}$.*

When the government announces that the bank passed the liquidity stress test (i.e., when $\Gamma^{\omega}(P)$ discloses $m^{\omega} = 0$), all creditors keep rolling over the bank's debt, and hence survival occurs with certainty. When instead the bank fails the liquidity stress test Γ^{ω} (i.e., when $\Gamma^{\omega}(P)$ discloses $m^{\omega} = 1$), all creditors withdraw early from the bank. Whether the bank defaults then depend on whether $\omega + P$ is larger than 1, which under the optimal policy never occurs since $\bar{\omega}(P) < 1 - P$ and the liquidity stress test has announced that $\omega \leq \bar{\omega}(P)$.

4.4 Asset Quality Review

We now proceed to characterize the optimal policy $\{\Gamma^y, \mathcal{R}\}$ conducted in period 1, taking into account the optimal liquidity stress test Γ^{ω} . We will see that the optimal policy $\{\Gamma^y, \mathcal{R}\}$ takes a very simple form: it combines a recommendation to the bank about the minimal amount of capital to raise

during the first period, along with some disclosure about y . To make sure that the recapitalization is followed, the policy-maker imposes a constraint on the bank's ability to distribute dividends if the amount of capital falls short of the minimal level required.

As the next result shows, the policy-maker asks the bank to raise an amount equivalent to the minimum between the capital cutoff which prevents posterior runs, K , and the expected price of the entire asset $\bar{P}(\mathbb{E}(y|m^y))$, where

$$\bar{P}(z) \equiv \sup \left\{ P \geq 0 : \frac{z}{R} \times \mathbb{P} \{ \omega \geq \bar{\omega}(P) \} \geq P \right\} \quad (8)$$

represents the maximal fair price consistent with selling a security with expected cash-flows $z \geq 0$, taking into account the probability of default. Given the authority's commitment to limit the bank's ability to distribute dividends when the bank does not meet the capital cutoff, the game played by the bank and external investors becomes similar to the one in Proposition 2 for values of α small enough and, therefore, under the best continuation equilibrium, both asset quality types pool and offer a debt security s_\star^{pool} satisfying:

$$\frac{1}{R} \mathbb{E} \left(s_\star^{\text{pool}} | m^y \right) = \min \{ K, \bar{P}(\mathbb{E}(y|m^y)) \}$$

On-path, recapitalization requirements are always obeyed.

At $t = 1$, the designer discloses information about the realization of future cash-flows y according to the rule $\pi^y : \mathbb{R}_+ \rightarrow \Delta M^y$. Proposition 5 below shows that recapitalizations are necessary to minimize the probability of default. By introducing recapitalizations the policy-maker mitigates separation incentives among bank types during the fund-raising game. In fact, high-asset quality types have an incentive to separate from low-asset quality ones, so as to avoid *underpricing*. If the probability of default is low, high-quality banks may prefer to expose themselves to rollover risk, by raising less funds than K , and signal their type. The imposition of a minimal recapitalization requirement makes this type of strategies unprofitable for high-asset quality types. The following proposition shows that, whenever possible, the optimal policy asks the bank to raise at least K so as to persuade creditors to keep rolling over the bank's debt. Whenever this is not possible (i.e., whenever the value of the assets falls below K) the regulator asks that the bank to sell the whole asset.

Proposition 5. *For any m^y disclosed with positive probability under the asset quality review $\Gamma^y = \{M^y, \pi^y\}$, the policy-maker imposes a recapitalization requirements according to the rule:*

$$\mathcal{R}(P|m^y) = \begin{cases} 1, & P > \min \{ K, \bar{P}(\mathbb{E}(y|m^y)) \} \\ \alpha, & \text{otherwise} \end{cases} \quad (9)$$

for some $\alpha \in (0, 1)$ small enough.

Recapitalizations are instrumental to implement the optimal comprehensive policy. Contrary to what might be conjectured based on Proposition 2, an asset quality review that reveals that

the asset's expected cash-flows are greater than K (i.e., a test that discloses information m^y , such that $\frac{1}{R}\mathbb{E}(y|m^y) \geq K$), but does not impose recapitalizations, need not prevent the freeze of the asset market. In fact, in the absence of recapitalizations, market freezing may occur with positive probability, across all equilibria, even if without government intervention the bank would have survived with certainty. The reason is that short-term creditors, who may stop rolling over the bank's debt if the amount of capital raised is insufficient, impose *market discipline* on the bank during the fund-raising stage and mitigate, to some extent, separation incentives. Indeed, when the bank and external investors expect the policy-maker to disclose information about the liquidity shock, their assessment about creditors' expected response becomes more optimistic. This, in turn, makes it easier for high-quality types to separate from low-quality ones, since rollover risk is mitigated. As a result, risk-sharing incentives dissipate. Imposing contingent recapitalizations substitute for the disciplining role of creditors' run by limiting the bank's dividends if the minimal capital cut-off is not met. This implies that disclosing information about the liquidity shock, without imposing recapitalizations, may prove ineffective and even counterproductive at preventing the disruption of capital markets. The next example shows that a policy-maker endowed with a technology to conduct liquidity stress tests may fare worse than a policy-maker that does not intervene at all.

Assumption 3. Creditors' conditional payoffs in case they choose to pledge, b and g , are constant.

Example 1. Suppose Assumption 3 holds and let $\gamma \equiv \sqrt{\frac{g}{g+|b|}}$. Assume that $\mathbb{E}(y) \geq 1 - \gamma$ and that

$$f^\omega(\omega) = \begin{cases} 2 \times (\gamma - \omega) & \omega \leq \gamma \\ 2 \times \left(\frac{1+\gamma}{1-\gamma}\right) \times (\omega - \gamma) & \omega > \gamma. \end{cases}$$

Then, without recapitalizations, default occurs with positive probability under the (sequentially) optimal asset quality review and liquidity stress test. In contrast, under a *laissez-faire* policy, the probability of default reduces to 0.

To prevent separation among asset quality types during the fund-raising game, the policy-maker has to punish banks that, despite being able to raise K , choose not to do so. If the policy-maker were able to commit to the liquidity stress test Γ^ω when designing the asset quality review Γ^y , she might threaten the bank to conduct an adversarial liquidity stress test if the latter were to raise less funds than what she envisions. These threats, however, would require the policy-maker to minimize the probability of survival if the bank failed to raise enough capital. The approach followed in this paper (which assumes that the policy-maker cannot commit to the liquidity stress test Γ^ω when designing the asset quality review Γ^y) implies that the optimal policy will not be sustained with non-credible threats. By imposing recapitalizations, the policy-maker retains the benefits of having a technology to conduct liquidity stress tests, and avoids the costs of dissipating pooling incentives.

4.5 Optimal Comprehensive Assessment

The analysis conducted so far shows how to choose the optimal recapitalization policy $\mathcal{R}[m^y]$ for any information m^y disclosed by the asset quality review Γ^y . We now proceed to the characterization of the optimal asset quality review Γ^y taking into account the optimal policies $\{\mathcal{R}, \Gamma^\omega\}$ that follow. Any information m^y disclosed with positive probability induces a posterior probability distribution over y , $\mathbb{E}(y|m^y)$. As a result of Proposition 5, the optimal recapitalization policy specifies requirements that depend on the posterior mean of cash-flows, $\mathbb{E}(y|m^y)$. Let G^{Γ^y} be the distribution of posterior means induced by policy Γ^y . The set of possible distributions of posterior means that can be induced with a disclosure policy coincides with the set of distributions which are a mean-preserving contraction of the prior F^y (Gentzkow and Kamenica [2016]).

Next, fix a message m^y and the induced expected value of the bank's asset $\mathbb{E}(y|m^y)$. Proposition 5 implies that the policy-maker may choose recapitalizations so that the cutoff defining whether the bank survives or not is given by $\bar{\omega}(\bar{P}(\mathbb{E}(y|m^y)))$. Recall that the function $\bar{\omega}$ identifies the critical value of the liquidity shock below which the bank defaults when the capital raised at $t = 1$ is equal to P . This value is equal to 0 for any $P \geq K$ since at these prices the probability of default is 0. To ease notation, let $\bar{\omega}(z) \equiv \bar{\omega}(\bar{P}(z))$. The designer's objective function can thus be written as:

$$\mathbb{E}(W_0(1\{\omega < \bar{\omega}(\mathbb{E}(y|m^y))\}) \times 1\{\omega \geq \bar{\omega}(\mathbb{E}(y|m^y))\}),$$

or equivalently,

$$W_0(0) \times (1 - F^\omega(\bar{\omega}(\mathbb{E}(y|m^y)))).$$

Thus, the policy-maker's problem reduces to:

$$\begin{aligned} \max_{G^{\Gamma^y}} \quad & \int_0^\infty (1 - F^\omega(\bar{\omega}(\tau))) G^{\Gamma^y}(d\tau) \\ \text{s.t:} \quad & F^y \succeq_{\text{MPS}} G^{\Gamma^y}. \end{aligned}$$

Define Z^{Γ^y} as the auxiliary function that allows to take mean-preserving contractions of F^y . That is, for any mean-preserving contraction G^{Γ^y} , Z^{Γ^y} is defined so that $G^{\Gamma^y} = F^y + Z^{\Gamma^y}$. Any such Z^{Γ^y} must respect the condition below:

Condition 1. Z^{Γ^y} is such that $F^y + Z^{\Gamma^y}$ is (i) positive, (ii) non-decreasing, and (iii) right-continuous. Additionally, (iv) Z^{Γ^y} belongs to the set:

$$\mathcal{Z} \equiv \left\{ Z : \mathbb{R}_+ \rightarrow \mathbb{R} : \int_0^{\tilde{y}} Z(y)dy \leq 0 \ (\forall \tilde{y} \geq 0), \int_0^\infty Z(y)dy = 0, Z(\infty) = 0 \right\}.$$

We can thus rewrite the designer's period 1 problem in terms of Z^{Γ^y} as follows:

$$\begin{aligned} \max_{Z^{\Gamma^y}} \quad & \int_0^\infty (1 - F^\omega(\bar{\omega}(\tau))) Z^{\Gamma^y}(d\tau) \\ \text{s.t:} \quad & Z^{\Gamma^y} \text{ satisfies condition (1)} \end{aligned}$$

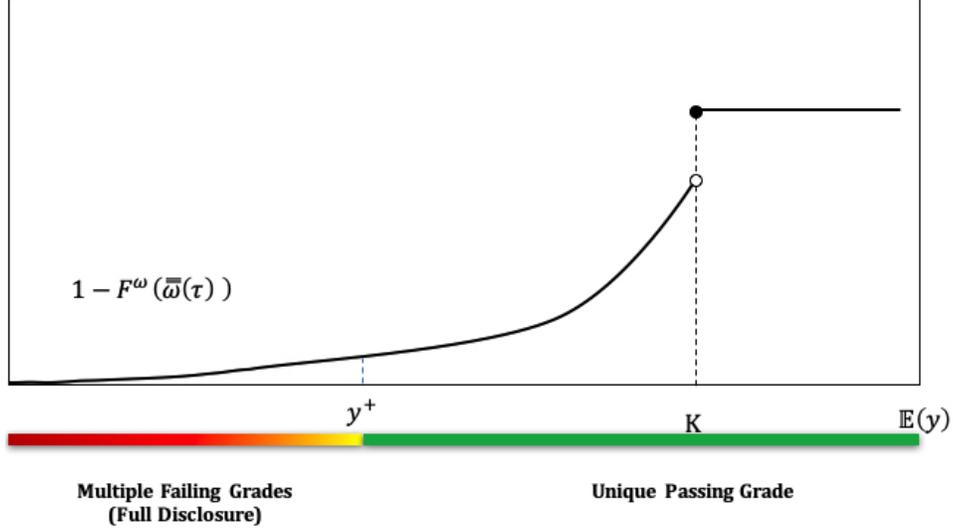


Figure 4: Optimal Asset Quality Review under Assumption 2.

As the next theorem shows, the optimal asset quality review consists of a monotone partition signal, where different values of y are pooled (if at all) with adjacent realizations (i.e., within the same interval). Moreover, I show that under Assumptions 2 and 3, the optimal review Γ^y takes a simple form. Namely, it fully discloses the realization of y for any realization below a cutoff y^+ , and pool all realizations above y^+ under a single message, say m_+^y . The posterior mean induced by message m_+^y satisfies $\frac{1}{R}\mathbb{E}(y|y \geq y^+) \geq P^+$, where P^+ corresponds to the level of funds for which no further disclosure is required in the next period. That is, P^+ is the smallest amount for which $\bar{\omega}(P^+) = 0$. Equivalently, it corresponds to the lowest amount of capital that persuades creditors to rollover the bank's debt, under the prior beliefs characterized by F^ω . In other words,

$$P^+ = K.$$

Theorem 1. *The optimal asset quality review consists of a monotone partitional signal. That is, there exists a monotone partition $\mathcal{P} = \{(y_i, y_{i+1}]\}_{i \in I}$ of \mathbb{R}_+ such that the optimal asset quality review $\Gamma^y = \{\{m_i^y\}_{i \in I}, \pi^y\}$ satisfies $\mathbb{E}(y|m_i^y) < \mathbb{E}(y|m_j^y)$ for all $i < j$. Moreover, the highest cell in the partition always include $K \times R$. Furthermore, under Assumptions 2 and 3, the optimal asset quality review is given by $\Gamma^y = \{\{[0, y^+] \cup m_{pass}^y\}, \pi^y\}$, with $\pi^y(\tilde{y}|y) = 1_{\{\tilde{y}=y\}}$ and $\pi^y(m_{pass}^y|y) = 1_{\{y \geq y^+\}}$ for all $\tilde{y} \in [0, y^+]$, and all $y \geq 0$, where y^+ is defined by:*

$$y^+ \equiv \inf \left\{ y \geq 0 : \int_y^{\max\{KR, \mathbb{E}(y)\}} (F^y(y) - F^y(\tau)) d\tau + \int_{\max\{KR, \mathbb{E}(y)\}}^\infty (1 - F^y(\tau)) d\tau \geq 0 \right\}. \quad (10)$$

Theorem 1, along with the former results, imply that under assumptions 2 and 3, the optimal comprehensive policy $\Psi = \{\Gamma^y, \mathcal{R}, \Gamma^\omega\}$ has a simple structure. The policy Ψ assigns a single grade to

all banks that meet a minimum standard in terms of profitability of their assets. This grade should be thought of as passing the policy-maker test on the quality of the bank's asset. Any bank failing to meet this minimal standard receives a grade that fully reveals the quality of its assets. The policy Ψ also specifies a recapitalization rule that asks the bank to either raise enough funds to prevent a creditors' run, or to sell the whole asset to external investors when its quality is low. Finally, the optimal policy entails a follow-up stress test on the bank's liquidity position which takes the form of a monotone pass-fail test that fails all banks with a liquidity position below an optimal cut-off, and passes the other.

Corollary 1. *The optimal comprehensive policy $\Psi = \{\Gamma^y, \mathcal{R}, \Gamma^\omega\}$ can be sequentially implemented by:*

(1) *Conducting an asset quality review which (i) assigns a passing grade (m_{pass}^y) to all banks with assets generating cash-flow above y^+ , and assigns a failing grade m_i^y to any assets delivering cash-flows $y \in (y_i, y_{i+1}]$, and (ii) imposing recapitalizations which dictates that the bank raises K when receiving the passing grade, and to sell the asset when falling below cut-off y^+ .*

(2) *Conducting a liquidity stress test that informs creditors of whether the liquidity shock is above the cut-off $\bar{\omega} (\bar{P}(\mathbb{E}(y|m^y)))$.*

The optimal asset quality review Γ^y pools all cash-flows realizations above y^+ so that the induced posterior mean, $\mathbb{E}(y|\{y > y^+\})$, is greater than K and, hence, all creditors are dissuaded from running. Using a more transparent disclosure policy for high values of y does not generate any benefits and, in fact, may hinder risk-sharing among banks with heterogeneous asset qualities. Thus, when the long-term profitability of the assets of a bank is above y^+ , the optimal asset quality review assigns an *opaque* (and unique) pass grade.

In contrast, when the profitability of the assets, y , falls below y^+ , the optimal policy specifies multiple failing grades. The intuition for this result is that there exist two forces that shape the bank's probability of survival: (a) an endogenous amplification effect associated with increasing the perceived value of the bank's assets due to the interaction between multiple audiences, and (b) the prior distribution of the liquidity shocks, characterized by F^ω . To understand the the first effect, note that by promising more valuable securities to investors, the latter may pledge a bigger amount of funds to the bank. This increases the probability of survival since the set of liquidity shocks that induce default shrinks. The increase in the probability of survival then allows external investors to offer a higher price for the bank's securities. The additional increase in the price offered to the bank feeds back and induces a larger probability of survival, and so on. This implies that, starting from a uniform prior distribution about the liquidity shock, the posterior probability of survival increases more than proportionally with increments in the value of the securities placed by the bank. In other words, the probability of survival is *convex* in the perceived quality of the bank's assets. The second effect, which is given by the prior distribution of the liquidity shock, may then reinforce the first effect or dissipate it.

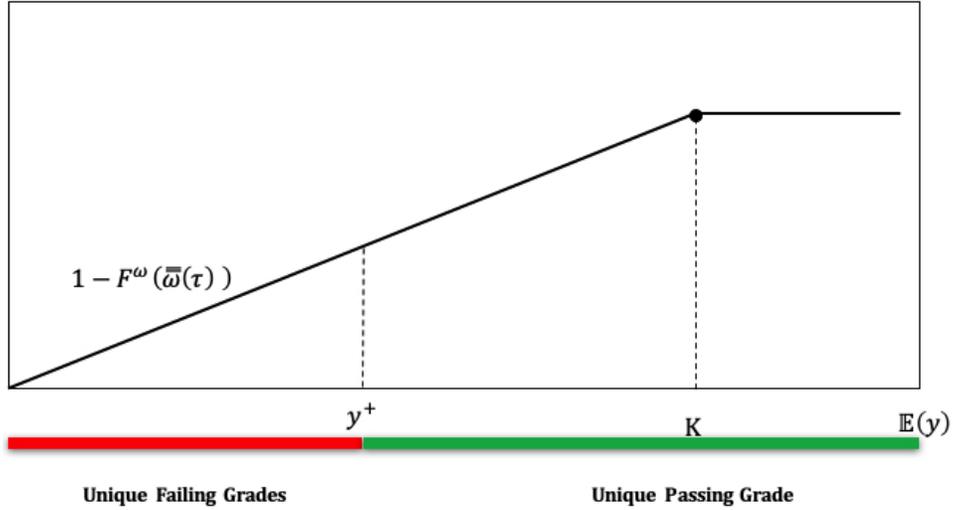


Figure 5: Optimal Asset Quality Review with a Single Audience.

If the probability density function of the liquidity shock, f^ω , does not increase too fast, as it is the case for instance under Assumption 2, the amplification effect in (a) dominates and the induced probability of survival is convex. Whenever this is the case, the policy maker prefers to separate asset profitability levels rather than pooling them together. That is, the policy-maker's objectives prefers finer information disclosures. As a result, and perhaps surprisingly, the optimal asset quality review is *more transparent* when banks have poor quality assets.

4.6 Comparison with Single Audience - Environment

Below I provide an example that shows that the optimality of multiple failing grades is a consequence of the interaction between multiple audiences. I prove that when external investors are protected against the potential default of the bank (which dissipates the amplification effect in (a)) and the prior distribution of the liquidity shock is uniform over $[0, 1]$, the optimal asset quality review is given by a monotone pass-fail test.

Example 2. Suppose that long-term investors' claims are ring-fenced, or *encumbered*, so that they remain available to investors even in the event of default. Moreover, assume that the prior distribution of the liquidity shock, F^ω , is uniformly distributed over $[0, 1]$. Then the optimal asset quality review is characterized by the monotone pass-fail test $\Gamma_{P-F}^y = \{ \{0, 1\}, \pi_{P-F}^y \}$, with $\pi_{P-F}^y(1|y) \equiv 1 \{y < y_{P-F}^+\}$ and where $\mathbb{E} \{y|y \geq y_{P-F}^+\} = \min \{ \mathbb{E} \{y\}, KR \}$.

When the bank is able to ring-fence the claims promised to long-term investors, the amplification effect described above, induced to the interaction of both audiences, evaporates. That is, long-term

investors are no longer concerned about short-term creditors' beliefs about the bank's liquidity position since their claims are protected even in the case of default. If, in addition, the prior distribution of the liquidity shock assigns equal weight to all possible realizations, the bank's probability of survival becomes linear on the value of the claims promised to long-term investors. That is, increasing the value of the security issued by the bank increases the probability of survival proportionally. This, in turn, implies that the policy-maker is indifferent between pooling different profitability levels together, under a unique failing grade, or using a more transparent disclosure policy. As a result, the *opaque* policy Γ_{P-F}^y is optimal. In the presence of a single audience (or alternatively multiple unrelated audiences), the optimal disclosure rule consists of a pass-fail message.

In the remainder of the paper I extend the analysis to cases where the policy-maker is not able to measure all the aspects that are private information to the bank. If the policy-maker desires this information to be disclosed to the rest of market participants, she must incentivize the bank to self-report it.

5 Screening Liquidity Position

In this section I consider interventions wherein the policy-maker does not have access to a disclosure technology that allows her to respond to liquidity shocks in a timely manner. Instead, she is forced to rely on information directly reported by the bank when such shocks occur. I relax the assumption that the policy-maker cannot use public funds to help the bank survive an adversarial liquidity shock. I assume instead that the policy-maker may purchase securities from the bank using taxpayers' money, but under the natural constraint that the price paid not exceed the fair price of the securities, taking into consideration the probability of default. This is a natural constraint oftenly found in practice that prevents policy-makers from giving away taxpayers' funds.

The timing of the game remains identical to the one in Section 4, with the single modification that instead of allowing the policy-maker to conduct the *critical* stress test Γ^ω at $t = 2$, the policy-maker runs a screening mechanism $\Upsilon^{\omega,\theta}$ which asks the bank to self-report its private information (ω, θ) and, conditional on the report, offers funding in exchange for a claim on the bank's asset, and specifies a public disclosure about the bank's information $\pi^{\omega,\theta} : \Omega \times \Theta \rightarrow \Delta M^{\omega,\theta}$. I further assume that the policy maker cannot force the bank to accept the deals she offers. This assumption is made to rule out solutions that involve *confiscation* by the policy-maker. An intervention $\Psi = \{\Gamma^y, \Upsilon^{\omega,\theta}\}$ thus consists of an asset quality review Γ^y , and a screening mechanism $\Upsilon^{\omega,\theta}$. Hereafter, I refer to Ψ as a *persuasion mechanism*.

5.1 Period 2: Screening Mechanism

Suppose that the policy-maker has disclosed information m^y in period 1 according to the asset quality review Γ^y , and that the bank has successfully raised P units of capital. Recall that if the

recapitalization level, P , is such that

$$\int_0^{1-P} b(\omega + P, 1) f^\omega(\omega) d\omega + g \times (1 - F^\omega(\omega)) \leq 0, \quad (11)$$

creditors withdraw early in the absence of any disclosure by the policy-maker. In this case, the policy-maker offers the bank the screening mechanism $\Upsilon^{\omega, \theta} = \{\{M^{\omega, \theta}, \pi^{\omega, \theta}\}, t, s\}$, which asks the bank to report its asset quality type θ and its liquidity position ω and, as a function of the report $(\tilde{\theta}, \tilde{\omega})$, offers to purchase a claim on the bank's asset $s[\tilde{\omega}, \tilde{\theta}]$, with $s(y|\tilde{\omega}, \tilde{\theta}) \in [0, y - s^*(y)]$, at a price $t(\tilde{\omega}, \tilde{\theta}) \geq 0$. In addition, $\Upsilon^{\omega, \theta}$ discloses a message $m^{\omega, \theta}$ to all market participants according to the disclosure policy $\pi^{\omega, \theta}[\tilde{\omega}, \tilde{\theta}] \in \Delta M^{\omega, \theta}$. The mechanism $\Upsilon^{\omega, \theta}$ must be (interim) (i) *incentive compatible* and (ii) *individually rational*. That is, (i) the bank must be at least as well-off by disclosing its private information than by reporting any other value of θ and ω , and (ii) the bank must be at least as well-off by participating in the designer's mechanism than by opting-out of it. Given that the designer can always induce the same conditions that the bank would face when opting-out of the program, it is wlog to assume that all bank types participate in the policy-maker's program²⁴.

An argument similar to the one establishing the *Revelation Principle* implies that it is without loss of optimality to restrict attention to mechanisms where the messages sent to the creditors take the form of action recommendations that creditors are willing to follow. This means that we can restrict the analysis to disclosure mechanisms with $M^{\omega, \theta} = \{0, 1\}$, where message $m^{\omega, \theta} = 0$ is interpreted as the recommendation to rollover the bank's debt, and $m^{\omega, \theta} = 1$ as the recommendation to stop pledging funds. We will distinguish between the security and price offered by the designer when disclosing message $m^{\omega, \theta} = 0$, $(t_0(\tilde{\omega}, \tilde{\theta}), s_0(\tilde{\omega}, \tilde{\theta}))$, and the contract offered when recommending $m^{\omega, \theta} = 1$, $(t_1(\tilde{\omega}, \tilde{\theta}), s_1(\tilde{\omega}, \tilde{\theta}))$. Obedience requires that when the policy-maker discloses recommendation message $m \in \{0, 1\}$, it must be the case that

$$\mathbb{E}(u(P + t_0(\omega, \theta), \omega, 1) | m = 0) = \frac{\sum_{\theta} \mu_{\theta} \times \int_{\Omega} u(\omega, P + t_0(\omega, \theta), 1) \pi^{\omega}(0|\omega, \theta) F^{\omega}(d\omega)}{\sum_{\theta} \mu_{\theta} \times \int_{\Omega} \pi^{\omega}(0|\omega, \theta) F^{\omega}(d\omega)} > 0, \quad (12)$$

and

$$\mathbb{E}(u(P + t_1(\omega, \theta), \omega, 1) | m = 1) = \frac{\sum_{\theta} \mu_{\theta} \times \int_{\Omega} u(\omega, P + t_1(\omega, \theta), 1) \pi^{\omega}(1|\omega, \theta) F^{\omega}(d\omega)}{\sum_{\theta} \mu_{\theta} \times \int_{\Omega} \pi^{\omega}(1|\omega, \theta) F^{\omega}(d\omega)} \leq 0. \quad (13)$$

Hereafter I refer to conditions (12) and (13) as *obedience constraints*. As shown in the former section, the policy-maker's optimal disclosure policy, absent incentive compatibility constraints, consists of

²⁴Philippon and Skreta [2012] and Tirole [2012] study a similar problem, but focus on indirect mechanisms where the bank has to decide whether to participate in the government program or not. This leads them to a mechanism design problem with endogenous participation constraints. In contrast, this paper follows a direct mechanism approach, where it is wlog to assume participation by all types. This distinction is obviously inconsequential for the allocations that are induced on-path. What makes my analysis fundamentally different from these works is that I enrich the designer's problem by allowing her to disclose information in addition to purchasing assets.

failing all banks with a liquidity position below the cutoff $\bar{\omega}(P)$, so that banks with liquidity positions above $\bar{\omega}(P)$ may survive. However, no bank vulnerable to runs (i.e., a bank with $\omega < 1 - P$) would ever choose to report its true type if this leads the designer to recommend short-term creditors to attack with certainty. In order to solve this conflict, the policy-maker may offer less liquid banks to purchase their assets at better terms in exchange of a lower *passing* probability. This implies that more liquid (but still vulnerable) banks have to receive lower prices for their remaining claims on the asset. The fact that these banks would default in the absence of a deal with the policy-maker then makes the mechanism incentive compatible. In what follows I provide a proof to the arguments explained above.

Let $U_P(\tilde{\omega}, \tilde{\theta}, \omega, \theta)$ be the utility of a bank with private information (ω, θ) which has successfully raised P units of capital in period 1, and chooses to report $(\tilde{\omega}, \tilde{\theta})$. Thus,

$$U_P(\tilde{\omega}, \tilde{\theta}, \omega, \theta) = \sum_{m \in \{0,1\}} \pi^\omega(m|\tilde{\omega}, \tilde{\theta}) \times 1 \left\{ \omega + P + t_m(\tilde{\omega}, \tilde{\theta}) \geq A(P, m) \right\} \times \\ \times \left((P + t_m(\tilde{\omega}, \tilde{\theta})) R + \mathbb{E}_\theta(y - s^* - s_m(\tilde{\omega}, \tilde{\theta})) \right)$$

where $A(\tau, m)$ corresponds to the most aggressive fraction of early withdrawals consistent with observing the bank raising P units of capital and the policy-maker disclosing message $m \in \{0, 1\}$. Note then that the obedience constraints (12) and (13) imply that $A(\tau, m) = m, m \in \{0, 1\}$. That the mechanism satisfies incentive compatibility then translates to:

$$(\omega, \theta) \in \arg \max_{\tilde{\omega}, \tilde{\theta}} U_P(\tilde{\omega}, \tilde{\theta}, \omega, \theta).$$

Next, observe that offering to purchase claims on the bank's asset when the policy-maker has assigned the failing grade only makes obedience constraint (13) and incentive compatibility constraints harder to satisfy and does not provide any benefit. Thus, it is without loss of optimality to set $t_1(\omega, \theta) = 0$ and $s_1[\omega, \theta] = \mathbf{0}$ for all ω and θ . Moreover, given that any vulnerable bank will fail if not helped by the government, we can restrict attention to mechanisms which set $s_0[\omega, \theta_L] = y - s^*$ for all $\omega < 1 - P$, since this allows the policy-maker to offer higher prices for the bank's securities. This property need not be satisfied for a type-H bank. To see this, note that it might be in the interest of the policy-maker to offer type-H banks to retain a fraction of their asset. This might be useful to alleviate incentive constraints. Also observe that the precise type of securities purchased by the policy maker is irrelevant. The only thing that matters is the fraction of the expected value of the security retained by the bank.

Let $z_\theta \equiv \mathbb{E}_\theta(y - s_\theta^*)$ be the value of the claims on the asset of a type- θ bank net the cash flows promised to external investors under security s_θ^* . Let $\phi_H(\omega)$ denote the fraction of z_H the bank retains on its balance sheet when its type is θ . Next, note that incentive compatibility requires that banks do not have incentives to pretend to have neither a different liquidity position nor a different

asset quality type. This implies that the utility of vulnerable banks must be equalized across all $\omega < 1 - P$, for a given asset quality type, since otherwise the bank would report the message that yields best terms. That is,

$$\pi(0|\omega, \theta_L) \times ((P + t_0(\omega, \theta_L))R) = V_L, \quad \forall \omega \leq 1 - P, \quad (14)$$

and

$$\pi(0|\omega, \theta_H) \times ((P + t_0(\omega, \theta_H))R + \phi_H(\omega)z_H) = V_H, \quad \forall \omega \leq 1 - P. \quad (15)$$

At the same time, banks must not have incentives to deviate in both dimension. That is, to pretend to have a different asset quality type and liquidity type. This means that a vulnerable type L-bank must not want to pretend to be a type H bank, for any level of liquidity:

$$V_L \geq \pi(0|\omega, \theta_H) \times ((P + t_0(\omega, \theta_H))R + \phi_H(\omega)z_L), \quad \forall \omega \leq 1 - P. \quad (16)$$

Similarly a vulnerable type H-bank must not have incentives to mimic a type L-bank:

$$V_H \geq V_L, \quad \forall \omega \leq 1 - P. \quad (17)$$

We now characterize *global* incentive constraints. Namely, we make sure that safe banks do not want to mimic vulnerable ones (those with $\omega < 1 - P$), and vice versa. We start with the former case. A liquid bank with high quality assets (i.e., a bank with $\omega > 1 - P$ and $\theta = \theta_H$) would never accept any deal to sell any security \hat{s} on its assets at a price less than $\mathbb{E}_H(\hat{s}|m^y)$. Any deal that pays a security \hat{s} at least $\mathbb{E}_H(\hat{s}|m^y)$ would prompt safe banks with low-quality assets and vulnerable banks of both asset quality to pretend to be safe and having a high-quality asset, unless they are also offered an equally attractive deal. The *fair-price* constraint mentioned above (and made explicit below) however implies that the policy-maker cannot afford to pay type- L as if it were type- H. The combination of the IC constraints with the fair-price constraint then imply that the policy-maker must not buy any security from safe banks. That is, $s_m[\omega, \theta] = \mathbf{0}$ and $t_m(\omega, \theta) = 0$ for any $m \in \{0, 1\}$, any $\omega \geq 1 - P$, and any $\theta \in \Theta$.

Additionally, if the designer were to pass *safe* banks with probability one, then all vulnerable bank types would claim to be safe. In particular, vulnerable banks with high-quality assets would claim to be safe, thus avoiding default and being pooled with low-quality types. To overcome this problem the policy-maker must fail safe banks with positive probability. Let π_s be the probability with which the policy-maker passes a safe bank. Incentive compatibility then requires that $V_L \geq \pi_s \times (P + z_L)R$, and that $V_H \geq \pi_s \times (P + z_H)R$, so that no vulnerable bank type has incentives to claim to be safe.

²⁵We can restate both inequalities as:

$$\pi_s \leq \min \left\{ \frac{V_L}{PR + z_L}, \frac{V_H}{PR + z_H} \right\}. \quad (18)$$

²⁵Note that π_s does not depend on θ since if it were to differ across different asset quality types, vulnerable types, would end up mimicking the one with the highest passing probability.

The fact that liquid banks cannot be offered the passing grade with high probability makes obedience constraint (13) hard to satisfy.

Next, consider the conditions guaranteeing that safe banks do not pretend to be vulnerable. The incentives problem is most severe for safe banks with low quality assets. By pretending to be vulnerable such banks would which receive the payment t_0 in case they receive a pass grade, and irrespective of the grade would never fail. For a safe type L-bank to not have incentives to claim to be vulnerable it must be that:

$$PR + z_L \geq \max_{\omega} \pi(0|\omega, \theta_L) \times (P + t_0(\omega, \theta_L))R + (1 - \pi(0|\omega, \theta_L))(PR + z_L),$$

and

$$PR + z_L \geq \max_{\omega} \pi(0|\omega, \theta_H) \times ((P + t_0(\omega, \theta_H))R + \phi(\omega, \theta_H)z_L) + (1 - \pi(0|\omega, \theta_L))(PR + z_L).$$

These constraints impose a bound on the amount that the policy-maker can pay to vulnerable banks. In fact, the above constraints together, imply that

$$t_0(\omega, \theta_L) \leq \frac{z_L}{R}, \quad \forall \omega < 1 - P, \quad (19)$$

and

$$t_0(\omega, \theta_H) \leq (1 - \phi_H(\omega)) \frac{z_L}{R}, \quad \forall \omega < 1 - P. \quad (20)$$

Finally, consider the requirement that the price paid by the policy-maker not exceed the fair price of the security purchased. This means that:

$$t_0(\omega, \theta) \leq \frac{\mathbb{E}_{\mu}(s[\omega, \theta; m^y] | m^y)}{R}, \quad (21)$$

where μ represents the policy-maker's beliefs about the bank's asset quality type induced by the screening mechanism. That is, when if the mechanism is discriminatory and offers different deals to different type of banks, μ represents a degenerate distribution over Θ . Instead, when the policy-maker pool different types under the same contract μ is computed according to Bayes rule. Note that (21) uses the property that the mechanism is *obedient*, and hence the probability of default equals 0 when a passing grade is given.

Summarizing, the policy-maker's problem can be reduced to finding a passing probability $\pi(0|\cdot, \cdot)$ and transfer $t_0(\cdot, \cdot)$, which maximize the probability of passing vulnerable banks, subject to the obedience constraints (12) and (13), incentive constraints among vulnerable banks (14), (15), incentive compatibility constraints guaranteeing that safe banks do not want to mimic vulnerable ones and vice versa (18), (19), and (20), and the constraint that imposes that the policy-maker does not pay more than the fair price (21) for the security she purchases from the bank:

$$\begin{aligned} \max_{\{\{0,1\}, \pi^{\omega}, t_0\}} \quad & \sum_{\theta \in \Theta} \mu_{\theta} \times \left(\int_0^{1-P} \pi^{\omega}(0|\omega, \theta) F^{\omega}(d\omega) \right) \\ \text{s.t:} \quad & (12), \dots, (21). \end{aligned} \quad (22)$$

Let $\bar{U}_{\text{LF}}(P) \equiv \int_0^{1-P} b(\omega + P, 1) f^\omega(\omega) d\omega + g \times (1 - F^\omega(1 - P))$ be a creditor's (ex-ante) expected payoff under the *Laissez Faire* regime, when the bank successfully raises P units of capital during the fund raising stage and the rest of short-term creditors choose to stop rolling over the bank's debt. We focus attention on the case where the expected quality of the bank's asset is depressed to the point that the policy-maker cannot set recapitalization levels to dissuade short-term creditors from running on the bank.

Assumption 4. $\mathbb{E}(y) \leq K$.

Assumption (4) means that $\bar{U}_{\text{LF}}(P) < 0$ for any $P < \mathbb{E}(y)$. That is, the bank is unable to persuade short-term creditors to keep rolling over its debt even if it sold the whole asset. This implies, in particular, that imposing forced recapitalizations in period 1 will not suffice to prevent bank failure if a liquidity shock materializes during $t = 2$. Under assumption (4), the policy-maker's ability to prevent the bank's default thus depends on her capacity to elicit information about the bank's liquidity position, and her ability to persuade short-term creditors to keep pledging to the bank.

My next result characterizes the optimal mechanism under an alternative (relaxed) setting wherein the policy-maker perfectly *observes* the bank's asset quality type during the second period when conducting the screening mechanism. The optimal mechanism under the new setting will be instrumental to characterize the optimal persuasion mechanism under the original environment.

5.2 Observable Asset Quality Type

Suppose that at $t = 2$ the policy-maker is able to (privately) observe the bank's asset quality type.²⁶ The optimal liquidity screening mechanism under this new setting has interest on its own as it sheds light on the trade-off faced by a policy-maker that elicits information before engaging in liquidity provision and strategic disclosure of information, abstracting from the difficulties associated with screening additional private information on the asset quality dimension. Let $\Upsilon_{\text{OAQ}}^{\omega, \theta}[\theta]$ represent the optimal screening mechanism when the policy-maker observes that the bank's asset quality type is θ . Clearly, the policy-maker's (ex-ante) expected payoff under $\Upsilon_{\text{OAQ}}^{\omega, \theta}$ (weakly) dominates the payoff under the original setting. This is a consequence of the fact that the set of incentive compatibility constraints shrinks. The next result characterizes the optimal screening mechanism under this alternative setting, $\Upsilon_{\text{OAQ}}^{\omega, \theta}[\theta]$. As I show below, the characterization of $\Upsilon_{\text{OAQ}}^{\omega, \theta}[\theta]$ will be instrumental to find the optimal persuasion mechanism under the original environment.

²⁶Under the new setting the policy-maker may observe the bank's asset quality type and the information she learns does not leak to external investors. This assumption contrasts with the assumption made in the previous section that any information the policy-maker learns during the first period about the quality of the bank's asset cannot be concealed from the market. I show below that the optimal persuasion mechanism under this new setting is constant across asset quality types, turning this assumption innocuous.

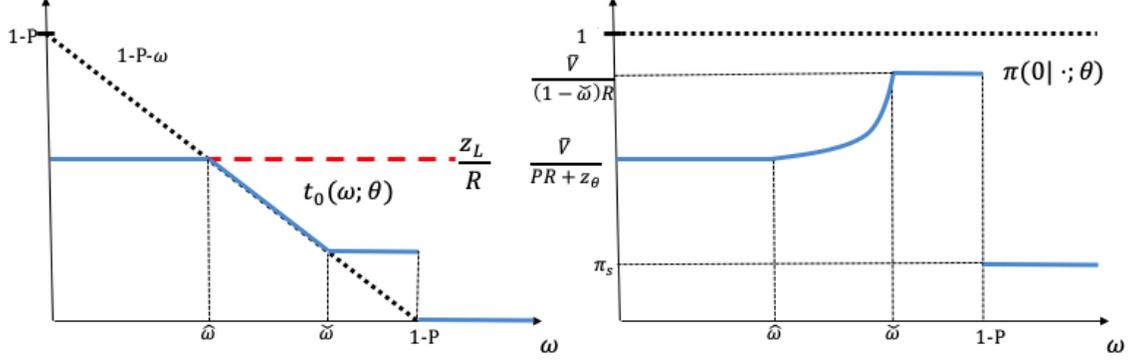


Figure 6: Optimal *liquidity screening* mechanism under perfect observability of asset quality.

Proposition 6. *Assume that the policy-maker perfectly observes θ at $t = 2$. Suppose that the bank raises $P < K$ after the asset quality review Γ^y discloses m^y . Then, the optimal screening mechanism when the bank's asset quality type is θ , $\Upsilon_{OAQ}^{\omega, \theta}[\theta; P]$, is characterized by:*

$$t_{OAQ}(\omega; \theta) = \begin{cases} \frac{z_L}{R} & \omega < \hat{\omega} \\ 1 - P - \omega & \omega \in [\hat{\omega}, \check{\omega}] \\ 1 - P - \check{\omega} & \omega \in (\check{\omega}, 1 - P) \\ 0 & \omega \geq 1 - P \end{cases}, \quad \pi_{OAQ}(0|\omega; \theta) = \begin{cases} \frac{\bar{V}_\theta}{PR+z_\theta} & \omega < \hat{\omega} \\ \frac{\bar{V}_\theta}{(1-\omega)R} & \omega \in [\hat{\omega}, \check{\omega}] \\ \frac{\bar{V}_\theta}{(1-\check{\omega})R} & \omega \in (\check{\omega}, 1 - P) \\ \frac{\bar{V}_\theta}{PR+z_\theta} & \omega \geq 1 - P \end{cases}$$

where $\hat{\omega} \equiv 1 - P - \frac{z_L}{R}$,

$$\bar{V}_\theta \equiv \min \left\{ (1 - \check{\omega}) R, \frac{|\bar{U}_{LF}(P)|}{\int_0^{1-P} \frac{b(\omega+P, 1) F^\omega(d\omega)}{(P+t_0(\omega; \theta))R} - g \times \frac{(1-F^\omega(1-P))}{PR+z_\theta}} \right\},$$

and $\check{\omega}$ is chosen so that:

$$\int_{\hat{\omega}}^{\check{\omega}} \frac{F^\omega(d\omega)}{(1-\omega)R} + \frac{F^\omega(1-P) - F^\omega(\check{\omega})}{(1-\check{\omega})R} = \frac{1}{g} \times \int_0^{\hat{\omega}} \frac{b(\omega+P, 1) |F^\omega(d\omega)}{PR + \tilde{B}_\theta} - \frac{(1 - F^\omega(1-P))}{PR + z_\theta}.$$

The optimal screening mechanism characterized in Proposition 6 is illustrated in Figure 6. To persuade creditors to follow the recommendation to rollover the bank's debt, the policy-maker has to modify the likelihood of the bank's survival. The bound on the price that can be pledged by the policy-maker implies that banks with a buffer smaller than $\hat{\omega} + P + \frac{z_\theta}{R}$ default when all creditors withdraw early. The policy-maker then minimizes the passing probability assigned to these liquidity types and compensates them by paying them the maximal price consistent with constraints (19, 20). All banks with liquidity positions above $\hat{\omega}$ receive enough funds to prevent default under an adversarial withdrawal. Incentive compatibility among vulnerable banks impose a negative relationship

between the passing probability and the price paid by the policy-maker. Banks with a liquidity shock $\omega \in [\hat{\omega}, \tilde{\omega})$ receive the smallest price that allows them to survive a massive withdrawal in order to maximize the probability of assigning a passing grade. The level $\tilde{\omega}$ is chosen so that obedience constraint is satisfied. Intuitively, the smaller the value of $\tilde{\omega}$, the more liquidity-types receive the maximal passing probability and, hence, the larger the aggregate survival probability. The optimal liquidity screening mechanism chooses the minimal value of $\tilde{\omega}$ consistent with obedience constraint (12).

We use the construction of the optimal screening mechanism when asset quality is observable to the policy-maker (but not to the asset market), $\Upsilon_{\text{OAQ}}^{\omega, \theta}$, to characterize the optimal screening mechanism under the original setting. I show below that the latter has a simple characterization. In fact, at the optimum, the policy-maker does not screen the quality of the bank's asset, and only elicits information about the bank's liquidity position. I make precise the last statement below.

Definition. A screening mechanism $\Upsilon^{\omega, \theta} = \{\{\{0, 1\}, \pi\}, t\}$ is said to be a *non-discriminatory liquidity screening* (NDLS) mechanism if:

$$t_0(\omega, \theta_L) = t_0(\omega, \theta_H) = t_0(\omega), \quad \pi(0|\omega, \theta_L) = \pi(0|\omega, \theta_H) = \pi(0|\omega), \quad \phi_H(\cdot) = \mathbf{0} \quad \forall \omega \in \Omega. \quad (23)$$

The optimal screening mechanism will be non-discriminatory. Observe that a NDLS mechanism need not satisfy *incentive compatibility*. As a matter of fact, a NDLS mechanism might satisfy local incentive constraints but will most likely fail to satisfy global incentive constraints (18), (19), and (20). These are constraints that prevent safe liquidity types to mimick vulnerable types, and vice versa. I show that the optimal screening mechanism corresponds to a NDLS mechanism that respects all incentive constraints. Intuitively, the bank's private information regarding the quality of its asset hurts the policy-maker's ability to run its liquidity provision program. In fact, as the next lemma shows, a policy-maker concerned with the potential default of the bank would do strictly better if the bank did not have private information regarding the quality of its asset in the first place. In order to avoid that safe banks with poor quality assets mimick vulnerable banks, the policy-maker needs to constraint the price she pays within its liquidity provision program. Moreover, private information about the quality of the assets implies that the policy-maker needs to decrease the probability with which she passes safe banks, since otherwise vulnerable banks with high quality assets would claim to be safe, as can be seen in (18). As a result, the policy-maker is strictly better off if banks do not possess private information.

Rigorously, let $\Upsilon^{\text{OAK}}[\emptyset] = \{\{\{\{0, 1\}, \pi_{\text{OAK}}[\emptyset]\}, t_{\text{OAK}}[\emptyset]\}\}$ be the optimal screening mechanism when the bank does not possess additional information with respect to the quality of its asset in excess of what is publicly known at $t = 2$. The next lemma shows that the NDLS mechanism $\Upsilon^{\text{OAK}}[\emptyset]$ represents an upper bound of what can be accomplished under the original setting.

Lemma 2. Let $\hat{\Upsilon}^{\omega, \theta} = \{\{\{\{0, 1\}, \hat{\pi}\}, \hat{t}\}\}$ be any feasible mechanism satisfying (12)-(21), then $\Upsilon_{\text{OAK}}^{\omega, \theta}[\emptyset] \succeq_{PM} \hat{\Upsilon}^{\omega, \theta}$. That is:

$$\int_0^{1-P} \pi_{O AQ}(0|\omega; \emptyset) F^\omega(d\omega) \geq \sum_{\theta \in \Theta} \mu_\theta \times \left(\int_0^{1-P} \hat{\pi}(0|\omega, \theta) F^\omega(d\omega) \right).$$

I show in the next section that the policy-maker can always implement a screening mechanism that reaches the same likelihood of survival than $\Upsilon_{O AQ}^{\omega, \theta}[\emptyset]$ in an incentive compatible manner.

5.3 Period 1: Asset Quality Review

In this section I study the joint design of the optimal asset quality redview Γ^y , and recapitalization requirements, that precede the choice of the screening mechanism $\Upsilon^{\omega, \theta}$. As I show below, the policy-maker faces an important trade-off when designing the recapitalizations rule to impose in the first period: On the one hand, smaller recapitalizations allow the bank to retain a greater fraction of the asset on its balance sheet. In turn, this increases the price that the policy-maker may offer to the bank, thus, enhancing the effectiveness of the liquidity provision program $\Upsilon^{\omega, \theta}$. On the other hand, more stringent recapitalizations permit the bank to raise capital before the liquidity shock materializes. This helps decrease the premium the bank has to pay to compensate for rollover risk.

In order to implement successful liquidity provision programs (i.e., for the policy-maker to be able to assign informative grades about the bank's liquidity position), the bank needs to own remaining claims on its asset with a value above a minimum threshold at the end of $t = 1$. When this cut-off is not met, the regulator can not induce the bank to self-report private information regarding its liquidity buffers. As discussed previously, the key trade-off that allows the regulator to induce the bank to self report its liquidity position involves a negative relation between the amount of funds offered to the bank, and the probability of assigning a passing grade. When the value of the remaining claims on the bank's asset is small, the maximal amount than can be pledged by the policy-maker is too low to discourage most vulnerable banks from mimicking more liquid ones and, hence, information elicitation about ω does not take place. Let \underline{E} be the minimal expected value of the bank's remaining claims necessary for information elicitation.

$$\underline{E} \equiv \inf_E \left\{ E \geq 0 : \int_0^{1-\frac{E}{R}} \frac{b(1, 1)}{E} F(d\omega) + \int_{1-\frac{E}{R}}^1 \frac{g}{(1-\omega)R} F(d\omega) \geq 0 \right\}. \quad (24)$$

Theorem 2 characterizes the optimal recapitalization policy and liquidity provision program for any message disclosed by the asset quality review $\Gamma^y = \{M^y, \pi^y\}$. I show that for intermediate ranges of asset quality y the policy-maker induces the bank to report its liquidity position and discloses information to the bank's creditors according to a stochastic rule which assigns a pass grade in a monotone manner (that is, more liquid banks are passed with higher probability). The price the policy-maker pays for the bank's assets is decreasing in the bank's liquidity. Moreover, in this case the regulator does not impose recapitalizations during the first period, and effectively asks the bank not to approach external investors. For low values of y the policy-maker, instead, is unable to elicit information about the bank's liquidity position. In that case, the policy-maker recommends

the bank to raise capital from external investors before the liquidity shock materializes, which helps the bank maximize the amount of funds it gets in exchange for claims on its asset. Similarly, when the value of y is large, the policy-maker asks the bank to seek private sector financing (i.e., from external investors). In this case the bank is asked to raise enough funds to persuade short-term creditors to rollover.

Theorem 2. *Fix a message m^y disclosed with positive probability under Γ^y . The optimal recapitalization policy and liquidity-provision program can be characterized as a function of the expected value of the asset's cash-flows, $\bar{z} \equiv \mathbb{E}(y|m^y)$, as follows:*

(i) *If $\bar{z} \geq K$, the optimal recapitalization policy is given by $R_\alpha(P) = 1 \{P < K\}$ for some $\alpha > 0$, and no liquidity-provision program is required.*

(ii) *If $\underline{E} < \bar{z} < K$, then the bank is asked to not raise capital from external investors, and the policy-maker uses the following liquidity-provision program to solicit information about ω :*

$$t_0^*(\omega; \bar{z}) \equiv \begin{cases} \frac{\bar{z}}{R} & \omega < 1 - \frac{\bar{z}}{R} \\ 1 - \omega & \omega \in [1 - \frac{\bar{z}}{R}, \tilde{\omega}] \\ 1 - \tilde{\omega} & \omega \in (\tilde{\omega}, 1] \end{cases}, \quad \pi^*(0|\omega; \bar{z}) \equiv \begin{cases} \frac{(1-\tilde{\omega})R}{z} & \omega < 1 - \frac{\bar{z}}{R} \\ \frac{1-\tilde{\omega}}{(1-\omega)} & \omega \in [1 - \frac{\bar{z}}{R}, \tilde{\omega}] \\ 1 & \omega \in (\tilde{\omega}, 1] \end{cases} \quad (25)$$

with $\tilde{\omega}$ implicitly defined by:

$$g \times \left(\int_{1-\frac{\bar{z}}{R}}^{\tilde{\omega}} \frac{f^\omega(\omega)}{(1-\omega)R} d\omega + \frac{1 - F^\omega(\tilde{\omega})}{(1-\tilde{\omega})R} \right) = \frac{\int_0^{1-\frac{\bar{z}}{R}} b\left(\frac{\bar{z}}{R}, 1\right) f^\omega d\omega}{\bar{z}}.$$

(iii) *If $z \leq \underline{E}$, the bank is asked to seek funding from external investors and the recapitalization policy $R_{\tilde{\alpha}}(P) = 1 \{P < \bar{P}(z)\}$ for some $\tilde{\alpha} > 0$ is imposed.*

Theorem 2 shows that interventions inducing simultaneous pledging by the market and the government are sub-optimal. The intuition behind this result, as explained above, is that inducing the bank to raise capital from external investors reduces the effectiveness of the policy-maker's liquidity-provision program. Recall that a bank that retains a smaller fraction of its asset can be offered less funds by the government under the *fair price* constraint. Given that the effectiveness of the liquidity-provision program relies on compensating extremely vulnerable banks, which are passed less often than more liquid ones, with higher prices for the remaining claims on their assets, requiring that the bank sells a fraction of its asset to external investors decreases the elicitation capacity of the policy-maker once the liquidity shock materializes. Additionally, having the bank raising funds from external investors intensifies incentive compatibility issues in the regulator's elicitation program. In fact, any amount of capital $P > 0$ raised during the fund-raising game makes the bank safe against runs for all $\omega > 1 - P$, regardless of the policy-maker's program. The larger P is, the larger the set of liquidity shocks under which the bank survives. Furthermore, the larger P is, the smaller the amount of cash the policy-maker can pay to vulnerable banks and the smaller the probability a pass grade can be assigned to highly liquid safe banks. At the optimum, the policy-maker either maximizes P

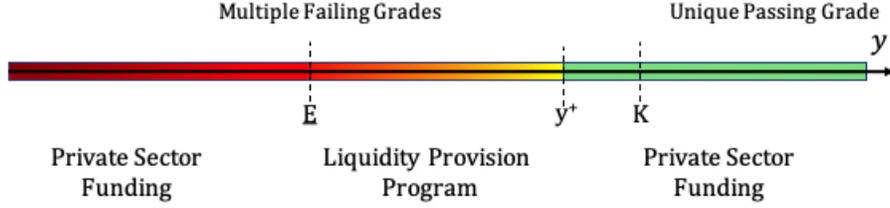


Figure 7: Structure optimal comprehensive intervention.

and then forgoes using a liquidity-provision program, or sets $P = 0$ (thus asking the bank to refrain from raising funds from external investors) and then uses a non discriminatory liquidity screening mechanism.

The formal proof that the optimal intervention has this *bang-bang* structure is in the Appendix. The strategy used to prove this result consists of solving a relaxed version of the policy-maker's problem where the bank does not receive private information about the quality of its asset. As shown above in lemma (2), the solution to this relaxed problem (weakly) dominates the solution under the original problem. I show that the solution to the alternative problem either maximizes the capital raised from the private sector, or sets $P = 0$ and then uses a liquidity-provision program. Whenever the policy-maker chooses the latter, setting $P = 0$ implies that the optimal liquidity-provision program satisfies all incentive compatibility constraints under the original problem and, hence, must be optimal.

The next theorem completes the analysis by characterizing the structure of the optimal persuasion mechanism as a function of the quality of the bank's asset.

Theorem 3. *The optimal comprehensive policy $\hat{\Psi} = (\Gamma^y, \mathcal{R}, \Upsilon^\omega)$ is characterized by a monotone partition $\mathcal{P} = \{(y_i, y_{i+1}]\}_{i \in I}$ of \mathbb{R}_+ such that the optimal asset quality review $\Gamma^y = \{\{m_i^y\}_{i \in I}, \pi^y\}$ satisfies $\mathbb{E}(y|m_i^y) < \mathbb{E}(y|m_j^y)$ for all $i < j$. Moreover, the highest interval always include y^+ . Furthermore,*

(1) *If $y \geq y^+$, the policy-maker passes the bank and sets recapitalizations according to the policy $R_\alpha(P) = 1 \{P < K\}$ for some $\alpha > 0$.*

(2) *If $y \in (y_i, y_{i+1}]$ with $\mathbb{E}(y|m_i^y) \in (E, K)$, either the bank is funded only by the private sector, in which case $R_\alpha(P) = 1 \{P < \bar{P}(z)\}$, or the bank is funded only by the government only the government through the liquidity-provision program characterized by $t_0^*(\omega; \mathbb{E}(y|m_i^y)), \pi_0^*(\omega; \mathbb{E}(y|m_i^y))$, where t_0^* and π_0^* are as defined in (25).*

(3) *If $y \in (y_i, y_{i+1}]$ with $\mathbb{E}(y|m_i^y) \leq E$, the bank is asked to seek external funding, the government imposes recapitalizations according to $R_{\tilde{\alpha}}(P) = 1 \{P < \bar{P}(z)\}$ for some $\tilde{\alpha} > 0$, and no liquidity program is used.*

Theorem 3 shows that the optimal comprehensive policy features a non-monotone pecking order. Institutions with high-quality assets are given a passing grade by the asset quality review Γ^y , and are required to raise enough capital from the private sector to persuade short-term creditors to rollover its debt. Banks with intermediate-quality assets, in turn, are assigned one of multiple failing grades and are funded with the government's optimal liquidity provision program. Finally, institutions with extremely poor-quality assets, are failed with multiple failing grades and are induced to seek funding from the private sector.

Theorem 2 informs the policy debate by showing that non-monotone relations between long-term asset profitability and the source of funding that institutions receive, need not be a proof of sub-optimality. In fact, they are expected to arise in these type of environments. In contrast, as highlighted before, simultaneous pledging by both the public and the private sector is, indeed, evidence of sub-optimality. Furthermore, the analysis shows that recapitalization rules need not be part of an optimal policy. In fact, in opposition to the results found in section 4 that advocate for the use of recapitalization policies, theorems 2 and 3 offer a message of caution. If the policy-maker believes she will not be able to react in a quickly manner to liquidity events, and implement a liquidity stress test to alleviate pessimistic assessment of short-term creditors, then recapitalization policies are costly and undesirable. Such rules deplete the amount of assets that the bank may use as collateral to obtain emergency lending from liquidity provision programs run by the policy-maker, negatively affecting her capacity to elicit information about the bank's liquidity needs, and therefore her ability to persuade short-term creditors to keep pledging to the bank.

6 Conclusions

In this paper, I study government interventions aimed at stabilizing financial institutions subject to rollover risk. I consider a rich environment which emphasizes the interaction among multiple audiences who care about different aspects of the bank's multi-dimensional fundamentals. I show that complementing disclosure policies with minimal recapitalizations is instrumental to maximizing the probability of the bank's survival. By combining appropriately designed information disclosures with recapitalizations, the policy-maker is able to implement the optimal solution to a broader mechanism design problem where she has the authority to dictate the type of securities and the price the bank should choose when approaching external investors. Conferring such authority to the policy-maker is however not necessary. Perhaps surprisingly, the optimal review is *opaque* when the institution has high-quality assets and assigns a unique pass grade. In contrast, the optimal review is more *transparent* with banks with low-quality assets, in which case multiple failing grades are assigned to the bank as a function of the precise quality of the assets, which also triggers a follow-up stress test on the bank's liquidity position.

When the policy-maker lacks the ability to examine the bank's liquidity position and, hence, needs to elicit information from the bank, the initial asset quality review is followed by a liquidity-provision program, whereby the government offers to buy assets from the bank, in exchange of cash and a public disclosure of the bank's liquidity position. I show that, in this case, imposing recapitalizations undermines the effectiveness of the government's liquidity program. I also show that simultaneous pledging by the government and the private sector is suboptimal. I find that optimal comprehensive policies feature a non-monotone pecking order: Institutions with high-quality assets are given a pass grade by the asset quality review that assess the long-term profitability of the bank's assets and are required to raise enough capital from the private sector to persuade short-term creditors to rollover its debt. Banks with intermediate-quality assets, in turn, are assigned one of multiple failing grades, and are funded with the government's liquidity-provision program. Finally, institutions with extremely poor-quality assets are failed with multiple failing grades and are induced to seek private sector financing.

The above results are worth extending in several directions. The analysis in the present paper assumes the policy maker knows the distribution of future liquidity shocks when she designs the optimal comprehensive policy. Such knowledge may come from previous experience with banks of similar fundamentals. While this is a natural starting point, there are many environments in which it is more appropriate to assume that the designer lacks information about the joint distribution of the underlying fundamentals. In future work, it would be interesting to investigate the optimal disclosure policy in such situations. One idea is to apply a robust approach to the policy-maker's problem, whereby the designer expects nature to select the information structure that minimizes her payoff. The characterization of the optimal policy in this environment is highly relevant both from a theoretical standpoint and for the associated policy implications.

The analysis in the present paper assumes that uncertainty regarding the bank's liquidity is resolved after the bank approaches external investors. However, creditors' runs are intrinsically dynamic phenomena. If the fundamentals are partially persistent over time, the optimal policy must specify the timing of information disclosures. In future work, it would be interesting to extend the analysis in this direction.

Appendix A: Laissez Faire

D1 Refinement. Define first the set of *best response* to an arbitrary security s , $BR(s)$, as the set of prices which are consistent with rationality of the investors under some belief about the asset quality type of the bank²⁷:

$$BR(s) \equiv \left\{ P : \frac{\mathbb{E}_H(s)}{R} \times \mathbb{P}\{\omega + P \geq A^*(P)\} \geq P \right\}.$$

Define then,

$$\mathcal{D}(\theta|s) \equiv \{P \in BR(s) : V(P, s, \theta) > V(P^*(s_\theta^*), s_\theta^*, \theta)\}$$

$$\mathcal{D}^0(\theta|s) \equiv \{P \in BR(s) : V(P, s, \theta) = V(P^*(s_\theta^*), s_\theta^*, \theta)\}.$$

The profile $\{\{s_\theta^*\}_{\theta \in \Theta}, \mu^*, P^*, A^*\}$ satisfies the D1 criterion if for any security $s \in S$ with $s \neq s_*(\theta)$ all $\theta \in \Theta$, $\mu_*(s)$ is such that $\forall \theta, \theta' (\mathcal{D}(\theta|s) \cup \mathcal{D}^0(\theta|s) \subset \mathcal{D}(\theta'|s)) \Rightarrow \mu_*(\theta|s) = 0$.

Definition 1. We say a function $g : Y \subseteq \mathbb{R} \rightarrow \mathbb{R}$ satisfies *single crossing from above* (SCFA), if the following holds true: if there exists some $y \in Y$ such that $g(y) < 0$, then $\forall \tilde{y} > y, g(\tilde{y}) \leq 0$. Similarly, we say that $h : Y \subseteq \mathbb{R} \rightarrow \mathbb{R}$ satisfies *single crossing from below* (SCFB), if the following holds true: if there exists some $y \in Y$ such that $h(y) > 0$, then $\forall \tilde{y} > y, h(\tilde{y}) \geq 0$.

Lemma 3. Suppose that $g : Y \subseteq \mathbb{R} \rightarrow \mathbb{R}$ satisfies SCFA and that $f(y, t)$ is log-supermodular for all $(y, t) \in Y \times T \subseteq \mathbb{R}^2$. Define $\phi(t) \equiv \int_Y g(y) f(y, t) dy$ and let $y_0 \equiv \inf \{y \in Y : g(y) < 0\}$. Then, $\forall \tilde{t} > t \in T$:

$$\phi(\tilde{t}) = 0 \Rightarrow \phi(t) > 0.$$

Proof. That $f(y, t)$ is log-SM implies that $\frac{f(\cdot, t)}{f(\cdot, \tilde{t})}$ is non-increasing. Then,

$$\begin{aligned} \phi(t) &= \int_Y 1_{\{y \leq y_0\}} g(y) \frac{f(y, t)}{f(y, \tilde{t})} f(y, \tilde{t}) dy + \int_Y 1_{\{y > y_0\}} g(y) \frac{f(y, t)}{f(y, \tilde{t})} f(y, \tilde{t}) dy \\ &\geq \left(\frac{f(y_0, t)}{f(y_0, \tilde{t})} \right) \phi(\tilde{t}) \end{aligned}$$

which implies the result. □

Lemma 4. Assume that $s_D(\cdot) = \min\{\cdot, D\}$ and that $s \in S$ satisfies $\mathbb{E}_L(s_D) \leq \mathbb{E}_L(s)$. Then, $0 \geq \mathbb{E}_L(s_D - s) > \mathbb{E}_H(s_D - s)$

Proof. See Nachman & Noe (94), Lemma A.3. □

²⁷First-order stochastic dominance (which is implied by MLRP) means that

$$\left\{ P > 0 : \frac{\mathbb{E}_H(s)}{R} \times \mathbb{P}\{\omega + P \geq A^*(P)\} \geq P \right\} = \bigcup_{\mu \in \Delta_\Theta} \left\{ P > 0 : \frac{\mathbb{E}(s; \mu)}{R} \times \mathbb{P}\{\omega + P \geq A^*(P)\} \geq P \right\}$$

Proof of Proposition 1.

The proof is divided in two parts. First, I show that in any pooling equilibrium sellers place a debt security. The proof is general in that it applies regardless of whether the designer has disclosed information about the fundamentals (y, ω) by conducting stress tests or not. We assume that the probability that the bank survives can be written as $\mathbb{P}\{\omega \geq \omega^\sharp(\tau)\}$, where $\omega^\sharp(\cdot)$ represents a decreasing function of the capital raised by the bank, P . In the context of section 3, $\omega^\sharp = A^*(P) - P$, while in the context of section 4, $\omega^\sharp = \bar{\omega}$. Define $\Phi(\mathbb{E}_\mu(s))$ as the set of prices which induce a non-negative profit to any investor when a security of value $\mathbb{E}_\mu(s)$ is purchased. That is

$$\Phi(\mathbb{E}_\mu(s)) \equiv \left\{ P \geq 0 : \frac{\mathbb{E}_\mu(s)}{R} \times \mathbb{P}\{\omega \geq \omega^\sharp(P)\} \geq P \right\}.$$

Claim 1: If $\Phi(\mathbb{E}(y)) = \{0\}$, then the unique equilibrium of the game is $s_H^* = s_L^* = \mathbf{0}$.

$\Phi(\mathbb{E}(y)) = \{0\}$ implies that $\mathbb{E}(y) < K$. We prove first that $s_H^* = s_L^* = \mathbf{0}$ is, in fact, an equilibrium. Consider the deviation to any security \hat{s} satisfying $\frac{1}{R}\mathbb{E}_H(\hat{s}) \geq K$ (which is the only relevant case since $\Phi(\mathbb{E}(y)) = \emptyset$). Observe that $BR(\hat{s}) = [K, \frac{1}{R}\mathbb{E}_{\theta_H}(\hat{s})]$, since any price below K induces default with certainty when assumption (2) holds, and any $P \geq K$ dissuades all creditors from running, and hence prevents default w.p. 1. As a consequence, a low-quality type can profitably deviate and place security \hat{s} for any price $P \in BR(\hat{s})$:

$$V(P, \theta_L, \hat{s}) = (PR + \mathbb{E}_L(y - \hat{s})) \times \underbrace{\mathbb{P}\{\omega \geq \omega^\sharp(P)\}}_{=1} > 0.$$

Thus, $\mathcal{D}(\theta_L; \hat{s}) = BR(\hat{s})$, implying that market beliefs that assign $\mu(\theta_L, s) = 1$ for any such $s \in S$ are consistent with D1. This amounts to say that any feasible deviation is always attributed to type L, and therefore no bank type gets funded. Uniqueness follow from the fact that $\mathbb{E}(y) < K$ and, hence, even if bank sell the whole asset funds are not enough to secure positive funds. Moreover, any security issued by type H that obtains a positive price may always be mimicked by type L and, therefore, cannot occur in equilibrium.

Claim 2. $\Phi(\mathbb{E}(y)) \neq \{0\}$ implies that pooling may only occur over debt contracts.

Suppose that there exists an equilibrium of the fund-raising game, $\{\{\sigma_\theta\}_\theta, \mu, P, A\}$, and a security $\hat{s} \in S$ with $\sigma_\theta(\hat{s}) > 0$, for all $\theta \in \Theta$. We prove that any such security needs to be a debt contract. To see this, suppose that \hat{s} is not a debt contract. Define the debt security $s_D \equiv \min\{y, D\}$ where D is such that $\mathbb{E}_H(s_D - \hat{s}) = 0$. Note that $s_D - \hat{s}$ satisfies *single crossing from above* (SCFA) and hence lemma 3 implies that $\mathbb{E}_L(s_D - \hat{s}) > 0 = \mathbb{E}_H(s_D - \hat{s})$. Thus,

$$\mathbb{E}_H(y - s_D) - \mathbb{E}_L(y - s_D) > \mathbb{E}_H(y - \hat{s}) - \mathbb{E}_L(y - \hat{s}) \quad (26)$$

Next, define $\Delta V_\theta(P)$ as the difference in payoffs, for seller θ , obtained by switching to security s_D ,

and sell it at price P , instead of issuing security \hat{s} and receiving the market price $\hat{P}(\hat{s})$. That is,

$$\begin{aligned}\Delta V_\theta(P) &= V(P, s_D, \theta) - V(\hat{P}(\hat{s}), \hat{s}, \theta) \\ &= (PR + \mathbb{E}_\theta(y - s_D)) \times \mathbb{P}\{\omega \geq \omega^\#(P)\} - (\hat{P}(\hat{s})R + \mathbb{E}_\theta(y - \hat{s})) \times \mathbb{P}\{\omega \geq \omega^\#(P(\hat{s}))\},\end{aligned}$$

Inequality (26) together with the fact that $y - s_D$ and $y - \hat{s}$ are monotone then imply that:

$$\begin{aligned}\Delta V_H(\tau) - \Delta V_L(\tau) &= (\mathbb{E}_H(y - s_D) - \mathbb{E}_L(y - s_D)) \times \mathbb{P}\{\omega \geq \omega^\#(P)\} \\ &\quad - (\mathbb{E}_H(y - \hat{s}) - \mathbb{E}_L(y - \hat{s})) \times \mathbb{P}\{\omega \geq \omega^\#(\hat{P}(\hat{s}))\} \\ &> 0, \quad \forall P \geq \hat{P}(\hat{s}).\end{aligned}\tag{27}$$

Note next that

$$\Phi(\mathbb{E}(\hat{s})) \subset \Phi(\mathbb{E}_H(\hat{s})) = BR(s_D),$$

where the last equality arises from $\mathbb{E}_H(s_D) = \mathbb{E}_H(\hat{s})$. By definition, we have that

$$P(\hat{s}) = \sup \Phi(\mathbb{E}(\hat{s})),$$

and hence, $\hat{P}(\hat{s}) \in \text{int}(BR(s_D))$.

Finally, notice that $\mathbb{E}_L(y - \hat{s}) > \mathbb{E}_L(y - s_D)$, and therefore $\Delta V_L(\hat{P}(\hat{s})) < 0$. On the other hand, $\Delta V_H(\hat{P}(\hat{s})) = 0$, and thus $\Delta V_H(\hat{P}(\hat{s}) + \epsilon) > 0 > V_L(\hat{P}(\hat{s}) + \epsilon)$ for $\epsilon > 0$ small enough so that $\hat{P}(\hat{s}) + \epsilon \in BR(s_D)$. As a result, $\mathcal{D}(\theta_L|s_D) \cup \mathcal{D}^0(\theta_L|s_D) \subset \mathcal{D}(\theta_H|s_D)$, and consequently market beliefs consistent with D1 must necessarily assign $\mu(\theta_H|s_D) = 1$. This implies that $P(s_D) > \hat{P}(\hat{s})$, since bank H is not pooled with L when placing s_D , and therefore by definition of s_D we have that $\Delta V_H(P(s_D)) > 0$, which contradicts the assumption that $\{\{\sigma_\theta\}_\theta, \mu, P, A\}$ is an equilibrium.

Next, we prove that the price of any debt security, s_d , which is placed by both type of banks in equilibrium, cannot be larger than K . Assume by contradiction that $P(s_d \equiv \min\{y, d\}) > K$ (and hence $P(s_d) > K$). Consider the alternative debt contract $s_\epsilon = \min\{y, d - \epsilon\}$ with $\epsilon > 0$ small. We show that type H can always profitably deviate and issue s_ϵ instead. Observe that $s_d - s_\epsilon$ is an increasing function. FOSD then implies that:

$$\mathbb{E}_H(s_d - s_\epsilon) > \mathbb{E}_L(s_d - s_\epsilon),$$

or equivalently,

$$\mathbb{E}_H(y - s_\epsilon) - \mathbb{E}_L(y - s_\epsilon) > \mathbb{E}_H(y - s_d) - \mathbb{E}_L(y - s_d).\tag{28}$$

Similar to what we did above, let $\Delta V_\theta(P) = V(P, s_\epsilon, \theta) - V(\hat{P}(s_d), s_d, \theta)$. Inequality 28 implies that:

$$\begin{aligned}\Delta V_H(P) - \Delta V_L(P) &= (\mathbb{E}_H(y - s_\epsilon) - \mathbb{E}_L(y - s_\epsilon)) \times \mathbb{P}\{\omega \geq \omega^\#(P)\} \\ &\quad - (\mathbb{E}_H(y - s_d) - \mathbb{E}_L(y - s_d)) \times \underbrace{\mathbb{P}\{\omega \geq \omega^\#(\hat{P}(s_d))\}}_{=1} \\ &> 0, \quad \forall P \geq K.\end{aligned}\tag{29}$$

For small values of ϵ we have:

$$\Phi(\mathbb{E}(s_d)) \subset \Phi(\mathbb{E}_H(s_\epsilon)) = BR(s_\epsilon),$$

and hence $P(s_d)$ which is the maximal element in $\Phi(\mathbb{E}(s_d))$ is contained in $BR(s_\epsilon)$. Moreover, given that s_ϵ is smaller than s_d , we must have that $\Delta V_\theta(P(s_d)) > 0$ for both $\theta \in \Theta$. Finally, by choosing ϵ small enough, and using inequality 29, we obtain that there must exist some $\tilde{P} \in (K, P(s_d))$ for which $\Delta V_H(\tilde{P}) > 0 > V_L(\tilde{P})$. Thus, $\mathcal{D}(\theta_L|s_\epsilon) \cup \mathcal{D}^0(\theta_L|s_\epsilon) \subset \mathcal{D}(\theta_H|s_\epsilon)$, and consequently market beliefs consistent with D1 must necessarily assign $\mu(\theta_H|s_D) = 1$, which implies that type H can profitably deviate and separate from type L. This is a contradiction and therefore any debt contract under which both types pool must have a price no larger than K.

Claim 3. $\Phi(\mathbb{E}(y)) \neq \{0\}$ implies that in any equilibrium in which there exists a security, s_H , only issued by type H (i.e., $\sigma_H(s_H) > 0 = \sigma_L(s_H)$), we must have that $\mathbb{E}_H(s_H) \leq \frac{\mathbb{E}_L(y)}{R}$.

To see this, assume by contradiction that $P(s_H) > \frac{1}{R}\mathbb{E}_L(y)$. Denote by s_L any security issued with positive probability by type L. Observe that the separating nature of the equilibrium requires that:

$$P(s_L) = \sup \Phi(s_L) \leq \frac{\mathbb{E}_L(s_L)}{R}.$$

Hence, the amount collected by type H must be such that:

$$P(s_H)R \geq P(s_L)R + \mathbb{E}_L(y - s_L). \quad (30)$$

As a result, type L has incentives to mimic type H. To see this last point, let $P(s_\theta)$ obtained when issuing and observe that:

$$\begin{aligned} V(\tau(s_H), s_H, \theta_L) - V(\tau(s_L), s_L, \theta_L) &= (\tau(s_H)R + \mathbb{E}_L(y - s_H)) \times \mathbb{P}\left\{\omega \geq \omega^\sharp(\tau(s_H))\right\} \\ &\quad - (\tau(s_L)R + \mathbb{E}_L(y - s_L)) \times \mathbb{P}\left\{\omega \geq \omega^\sharp(\tau(s_L))\right\} \\ &> (\tau(s_H)R + \mathbb{E}_L(y - s_H) - (\tau(s_L)R + \mathbb{E}_L(y - s_L))) \times \\ &\quad \times \mathbb{P}\left\{\omega \geq \omega^\sharp(\tau(s_L))\right\} \\ &> 0, \end{aligned}$$

where the first inequality arises from the fact that $P(s_H) > P(s_L)$ and that ω^\sharp is a decreasing function of the capital raised by the bank. The second inequality, in turn, is a consequence of equation (30). This is a contradiction and hence $P(s_H) \leq \frac{1}{R}\mathbb{E}_L(y|m^y)$. \square

Proof of Proposition 2.

To prove (1) we show that under assumption 2 and $\lambda = 0$, there cannot be any separating, nor semi-separating equilibrium. Assume that type H is the only type which chooses a particular security s_H

with positive probability. If $\frac{1}{R}\mathbb{E}_H(s_H) \geq K$, then $P(s_H) = \frac{1}{R}\mathbb{E}_H(s_H)$ since at this price default is avoided. Then either of the two following cases must be true: (i) both types place different securities and no pooling occurs, or (ii) there exists a different security \tilde{s} which is placed by both sellers with positive probability. Let $\omega^\sharp(P)$ be the cutoff liquidity level for which the bank defaults if it raises P from external investors. In the first case, type L has a strict incentive to deviate and pretend to be type H, since at any security placed by L with positive probability we have that:

$$\begin{aligned}
V(P(s_L), s_L, \theta_L) - V(P(s_H), s_H, \theta_L) &= (P(s_L)R + \mathbb{E}_L(y - s_L)) \times \mathbb{P}\{\omega \geq \omega^\sharp(P(s_L))\} \\
&\quad - \underbrace{(P(s_H)R + \mathbb{E}_L(y - s_H)) \times \mathbb{P}\{\omega \geq \omega^\sharp(P(s_H))\}}_{=1} \\
&< (P(s_L)R + \mathbb{E}_L(y - s_L)) - (P(s_H)R + \mathbb{E}_L(y - s_H)) \\
&< -(\mathbb{E}_H(s_H) - \mathbb{E}_L(s_H)) \\
&< 0,
\end{aligned}$$

where the second inequality obtains from $P(s_L)R = \mathbb{E}_L(s_L) \times \mathbb{P}\{\omega \geq \omega^\sharp(P(s_L))\}$, and the last one from FOSD and the fact that s_H is non-decreasing. In the second case, in turn, type H strictly prefers to deviate and relocate all the the weight assigned to \tilde{s} to s_H instead. In fact,

$$\begin{aligned}
V(P(\tilde{s}), \tilde{s}, \theta_H) - V(P(s_H), s_H, \theta_L) &= (P(\tilde{s})R + \mathbb{E}_H(y - \tilde{s})) \times \mathbb{P}\{\omega \geq \omega^\sharp(P(\tilde{s}))\} \\
&\quad - \underbrace{(P(s_H)R + \mathbb{E}_H(y - s_H)) \times \mathbb{P}\{\omega \geq \omega^\sharp(P(s_H))\}}_{=1} \\
&< (P(\tilde{s})R + \mathbb{E}_H(y - s_H)) - (P(s_H)R + \mathbb{E}_H(y - s_H)) \\
&= -(\mathbb{E}_H(s_H) - \mathbb{E}_L(s_H)) \\
&< 0,
\end{aligned}$$

where I have used that $P(\tilde{s})R < \mathbb{E}_H(\tilde{s})$ from FOSD. As a result, type H has a strict incentive to deviate. This proves that the only type of equilibria that prevail in the fund-raising game are pooling equilibria. Proposition 1 then implies that both types must pool under debt contracts only, which completes the proof of (1).

We next prove (2). Suppose first that $\frac{1}{R}\mathbb{E}_L(y) < K \leq \mathbb{E}(y)$. Consider the deviation to any security \hat{s} satisfying $\frac{1}{R}\mathbb{E}_{\theta_H}(\hat{s}) \geq K$, which is the only relevant case since the market would never fund the low type. Observe that $BR(s) = [K, \frac{1}{R}\mathbb{E}_{\theta_H}(s)]$, since any price below K induces default with certainty when assumption (2) holds, and any $P \geq K$ dissuades all creditors from running, and hence prevents default w.p. 1. As a consequence, bank L can profitably deviate and place security \hat{s} for any price $\tau \in BR(s)$:

$$V(\tau, \theta_L, \hat{s}) = (\tau R + \mathbb{E}_{\theta_L}(y - s)) \times \underbrace{\mathbb{P}\{\omega \geq \hat{\omega}(\tau)\}}_{=1} > 0.$$

Thus, $\mathcal{D}(\theta_L; s) = BR(s)$, implying that market beliefs that assign $\mu(\theta_L, s) = 1$ for any such $s \in S$ are consistent with D1. This amounts to say that any feasible deviation is always attributed to type L , and therefore no seller type gets funded. If $\frac{1}{R}\mathbb{E}(y) < K$ instead, then claim 1 in the proof of Proposition 1 implies that $s_\theta = \mathbf{0}$ for all $\theta \in \Theta$ is the unique equilibrium.

Finally, assume then that $\frac{1}{R}\mathbb{E}(y) \geq K$. The result follows directly from Theorem 4 in [Nachman & Noe (94)].

Appendix B: Comprehensive Assessment

Proof of Proposition 3.

Below I prove a sequence of lemmas that induce the result.

Lemma 5. *Fix the amount raised by the bank during the fund-raising game, $P \geq 0$. The problem of maximizing the designer's payoff :*

$$\begin{aligned} \max_{\Gamma^\omega = \{\pi^\omega, M^\omega\}} \quad & \mathbb{E}(W_0(A) \times 1_{\{\omega + P \geq A(P, m^\omega)\}}) \\ \text{s.t:} \quad & A(P, m^\omega) = 1_{\{\mathbb{E}(u(\omega, P, 1) | m^\omega) \leq 0\}}, \end{aligned}$$

is equivalent to the problem of maximizing the probability that creditors keep pledging to the bank under the most aggressive equilibrium outcome, $\mathbb{P}\{\mathbb{E}(u(\omega, P, 1); \Gamma^\omega) > 0\}$. The policy-maker's problem can thus be written as

$$\max_{\Gamma^\omega = \{\pi^\omega, M^\omega\}} \sum_{m^\omega \in M^\omega} 1_{\{\mathbb{E}(u(\omega, P, 1) | m^\omega) > 0\}} \times \underbrace{\int_{\Omega} \pi^\omega(m^\omega | \omega) F^\omega(d\omega)}_{\equiv \pi^\omega(m^\omega)}. \quad (31)$$

Proof. Consider an arbitrary policy $\Gamma^\omega = \{\pi^\omega, M^\omega\}$. Assume that there exists some message \bar{m} disclosed with positive probability under Γ^ω for which (i) $A(P, \bar{m}) = 1$, and (ii)

$$\mathbb{P}\{\{\omega : \omega + P \geq 1\} \cap \{\omega : \pi^\omega(\bar{m} | \omega) > 0\}\} > 0.$$

That is, message \bar{m} induces all creditors to stop pledging to the bank and satisfies that the set of realizations of ω in which the bank survives even if all creditors choose to withdraw early has positive measure. Consider then the alternative policy $\hat{\Gamma}^\omega = \{\hat{\pi}^\omega, \hat{M}^\omega = M^\omega \cup \{\bar{m}_0, \bar{m}_1\}\}$ constructed as follows: for any $m \in M^\omega$ different from \bar{m} , $\hat{\pi}^\omega(m | \cdot) = \pi^\omega(m | \cdot)$. Additionally, $\hat{\pi}^\omega(\bar{m}_0 | \omega) = \pi^\omega(\bar{m} | \omega) \times 1_{\{\omega + P \geq 1\}}$ and $\hat{\pi}^\omega(\bar{m}_1 | \omega) = \pi^\omega(\bar{m} | \omega) \times 1_{\{\omega + P < 1\}}$ for all $\omega \in \Omega$. Policy $\hat{\Gamma}^\omega$ preserves the probability that the bank survives and decreases the number of creditors who withdrawing early. Hence, $\hat{\Gamma}^\omega$ weakly dominates Γ^ω . As a result, assuming that the optimal policy maximizes the probability that creditors refrain from attacking is without loss. \square

This lemma shows that the problem of maximizing the policy-maker's payoff by means of a policy Γ^ω is equivalent to maximizing the probability that short-term creditors keep pledging to the bank. We thus focus on maximizing the expression in (31). Consider then *any* liquidity stress test $\Gamma^\omega = \{M^\omega, \pi^\omega\}$. Each message m^ω disclosed by stress test Γ^ω induces a posterior distribution over ω , $F^\omega(\cdot|m^\omega)$. Hence, every message m^ω disclosed with positive probability generates a posterior expectation of $u(\omega, P, 1)$, the utility a creditor who pledges to the bank obtains when the latter raises P units of capital and when all other creditors withdraw early. That is, each message m^ω induces a new assessment:

$$\mathbb{E}(u(\omega, P, 1)|m^\omega) = \int_{\Omega} (g \times 1_{\{\omega \geq 1-P\}} + b(\omega + P, 1) \times 1_{\{\omega < 1-P\}}) F^\omega(d\omega|m^\omega).$$

The optimal liquidity stress test Γ^ω can then be characterized by the distribution of posterior means of $u(\omega, P, 1)$ it induces. Let $G^{\Gamma^\omega}(\cdot; P)$ be the distribution of posterior means of $u(\omega, P, 1)$ induced by policy Γ^ω . The next lemma shows that any liquidity stress test Γ^ω , corresponds to a mean-preserving contraction of the distribution associated with the full-disclosure policy $\Gamma_{\text{FD}}^\omega$, G_{FD}^ω , and a mean-preserving spread of the no-disclosure policy, G_\emptyset^ω . That is, $G_{\text{FD}}^\omega \succeq_{\text{MPS}} G^{\Gamma^\omega} \succeq_{\text{MPS}} G_\emptyset^\omega$, where the partial order \succeq_{MPS} is defined as follows:

Definition 2. Let F and G be distribution functions with support in $X \subseteq \mathbb{R}$. We say that F dominates H in the MPS order, $F \succeq_{\text{MPS}} H$, if $\int_X \phi(x)F(dx) \geq \int_X \phi(x)G(dx)$ for any convex function ϕ in X .

Lemma 6. [Blackwell] Let $\Gamma_1^\omega = (M_1^\omega, \pi_1^\omega)$ and $\Gamma_2^\omega = (M_2^\omega, \pi_2^\omega)$ be two liquidity stress tests. Assume that there exists $z : M_1^\omega \times M_2^\omega \rightarrow [0, 1]$ such that:

- (i) $\pi_2^\omega(m_2|\omega) = \sum_{M_1^\omega} z(m_1, m_2) \pi_1^\omega(m_1|\omega)$, $\forall \omega \in [0, 1], \forall m_2 \in M_2^\omega$
- (ii) $\sum_{M_2^\omega} z(m_1, m_2) = 1$, $\forall m_1 \in M_1^\omega$.

Then the distributions of posterior expected utility of creditors, $\mathbb{E}(u(\omega, P, 1))$, induced by Γ_1^ω and Γ_2^ω are such that $G^{\Gamma_1^\omega} \succeq_{\text{MPS}} G^{\Gamma_2^\omega}$.

Proof. Let $f^{m_i} \in \Delta[0, 1]$ be the posterior pdf after observing message $m_i \in M_i^\omega$, and $\pi_i^\omega(m_i) = \int \pi_i^\omega(m_i|\omega) f^\omega(\omega) d\omega$ the total probability of observing disclosure m_i , under policy Γ_i^ω , $i \in \{1, 2\}$. Observe that *bayesian updating* together with property (i) imply that for any message $m_2 \in M_2^\omega$ with $\pi_2^\omega(m_2) > 0$ we have: □

$$f^{m_2}(\omega) = \sum_{m_1 \in M_1^\omega} \left(\frac{\pi_1^\omega(m_1) z(m_1, m_2)}{\pi_2^\omega(m_2)} \right) f^{m_1}(\omega).$$

This implies that for any convex function ϕ :

$$\begin{aligned}
\sum_{m_2 \in M_2^\omega} \pi_2^\omega(m_2) \phi \left(\int_0^1 \omega f^{m_2}(\omega) d\omega \right) &= \sum_{m_2 \in M_2^\omega} \pi_2^\omega(m_2) \phi \left(\sum_{m_1 \in M_1^\omega} \left(\frac{\pi_1^\omega(m_1) z(m_1, m_2)}{\pi_2^\omega(m_2)} \right) \int_0^1 \omega f^{m_1}(\omega) d\omega \right) \\
&\leq \sum_{m_2 \in M_2^\omega} \sum_{m_1 \in M_1^\omega} \pi_1^\omega(m_1) z(m_1, m_2) \phi \left(\int_0^1 \omega f^{m_1}(\omega) d\omega \right) \\
&= \sum_{m_1 \in M_1^\omega} \pi_1^\omega(m_1) \phi \left(\int_0^1 \omega f^{m_1}(\omega) d\omega \right),
\end{aligned}$$

where the second inequality arises from Jensen's inequality and the last equality from using property (ii). As a result, $G^{\Gamma_1^\omega} \succeq_{MPS} G^{\Gamma_2^\omega}$. \square

Lemma 6 shows that disclosure policies that are more informative (in the Blackwell sense) induce distributions of posterior expected utility of pledging creditors, $\mathbb{E}(u(\omega, P, 1))$, that dominate in the MPS order defined above. As a result, $G_{FD}^\omega \succeq_{MPS} G^{\Gamma^\omega} \succeq_{MPS} G_\emptyset^\omega$.

Consider then the problem of maximizing the likelihood that creditors keep pledging to the bank. Using lemmas (5)-(6), the policy-maker's problem can be reformulated as maximizing

$$\mathbb{P} \{ \mathbb{E}(u(\omega, P, 1); \Gamma^\omega) > 0 \} = 1 - G^{\Gamma^\omega}(0; P)$$

among all possible disclosure policies over ω . That is,

$$\begin{aligned}
\max_{G^{\Gamma^\omega}} \quad & 1 - G^{\Gamma^\omega}(0) \\
\text{s.t.} \quad & G_{FD}^\omega \succeq_{MPS} G^{\Gamma^\omega}.
\end{aligned}$$

This concludes the proof of Proposition 3. \square

Proof of Lemma 1

Proof. Under full-disclosure, each message generates a degenerate posterior distribution with all weight assigned to $u(\omega, P, 1)$ when ω is realized, which also coincides with the posterior mean induced by the message. As a result, $\mathcal{G}_{FD}^\omega(t; P) = \int_{u(0, P, 1)}^t G_{FD}^\omega(\tilde{u}; P) d\tilde{u}$, where

$$G_{FD}^\omega(\tilde{u}; P) = \int_{u(0, P, A=1)}^{\tilde{u}} \frac{f_\omega(u^{-1}(z; P, 1))}{\partial_\omega u(u^{-1}(z; P, 1), \tau, 1)} dz$$

corresponds to the distribution of $u(\omega, P, 1)$ under full-disclosure. Next, notice that under no-disclosure, the posterior mean remains unchanged and equal to $\mathbb{E}(u(\omega, P, 1) | \emptyset)$. Thus, $\mathcal{G}_\emptyset^\omega(t; P) = \int_{u(0, P, 1)}^t 1 \{ \tilde{u} \geq \mathbb{E}(u(\omega, P, 1) | \emptyset) \} d\tilde{u}$. To save on notation, hereafter we will omit the dependence on P of all disclosure policies and associated distributions. Any disclosure policy Γ^ω , induces a function $\mathcal{G}^{\Gamma^\omega}(t) \equiv \int_{u(0, P, 1)}^t G^{\Gamma^\omega}(\tilde{u}) d\tilde{u}$. That $G_{FD}^\omega \succeq_{MPS} G^{\Gamma^\omega} \succeq_{MPS} G_\emptyset^\omega$ implies that $\mathcal{G}_{FD}^\omega(t) \geq \mathcal{G}^{\Gamma^\omega}(t) \geq \mathcal{G}_\emptyset^\omega(t)$ for all $t \in [u(0, P, 1), u(1, P, 1)]$, which can be seen from applying the definition of \succeq_{MPS} to the convex function $\max\{\omega - t, 0\}$. Moreover, $\mathcal{G}^{\Gamma^\omega}$ is convex since G^{Γ^ω} is non-decreasing. Conversely, any

non-decreasing, convex function h in $[u(0, P, 1), u(1, P, 1)]$, which satisfies that $\mathcal{G}_{\text{FD}}^\omega(t) \geq h(t) \geq \mathcal{G}_\emptyset^\omega(t)$ can be induced by some policy Γ^ω . To see this note that h is differentiable almost everywhere and its right derivative is always well-defined since it is convex. Let $G(\tilde{u}) \equiv h'(\tilde{u}^+)$ be the right-derivative of h at \tilde{u} . Observe next that $\lim_{\tilde{u} \rightarrow \infty} G(\tilde{u}) = 1$, and thus G is a distribution. Finally, note that $\mathcal{G}_{\text{FD}}^\omega$ is a mean-preserving spread of G and therefore there must exist a policy that induces it by Strassen's Theorem (See Theorem 1.5.20 in Müller and Stoyan [2002]). \square

Proof of Theorem 1.

Define $\phi(\tau) \equiv 1 - F^\omega(\bar{\omega}(\bar{P}(\tau)))$. We first prove that ϕ satisfies the following properties: ϕ is (a) continuous, (b) non-decreasing, and (c) satisfies $\phi(0) = 0$, and $\phi(\tau) = 1$ for all $\tau \geq KR$. That ϕ is continuous comes from the fact that (i) $\bar{\omega}(\cdot)$ is continuously differentiable, (ii) $F^\omega(\cdot)$ admits a density and has at most one mass point at $\omega = 1$, and (iii) \bar{P} is continuous. To see this last point, we apply the *maximum theorem* to the definition of \bar{P} :

$$\begin{aligned} \bar{P}(\tau) &= \max P \\ \text{s.t. } P &\in \Gamma(\tau) \equiv \left\{ P \geq 0 : \frac{\tau}{R} \times \mathbb{P}\{\omega \geq \bar{\omega}(P)\} \geq P \right\} \end{aligned}$$

where $\Gamma(\cdot)$ is a compact valued and continuous correspondence. To see (b), we note that \bar{P} is non-decreasing and that $\bar{\omega}$ is non-increasing which implies the result. Finally, (c) is by definition of functions \bar{P} and $\bar{\omega}$. Conditions (a)-(c) guarantee that ϕ satisfies the *regularity* assumption in ?. That the optimal disclosure policy consists of monotone partitions thus follows proposition 2 in their paper. Next, to prove that the highest partition includes KR , we observe that using integration by parts, we can rewrite the policy-maker's objective function as:

$$\int_0^\infty \phi(\tau) Z(d\tau) = - \left(\left(1 - \lim_{\tau \rightarrow KR^-} \phi(\tau) \right) \times Z(KR) + \int_0^\infty \phi'(\tau) Z(\tau) d\tau \right).$$

As a result, the designer's problem is equivalent to:

$$\begin{aligned} \min_Z & \left(1 - \lim_{\tau \rightarrow KR^-} \phi(\tau) \right) \times Z(KR) + \int_0^\infty \phi'(\tau) Z(\tau) d\tau \\ \text{s.t. } & Z \text{ satisfies condition (1)}. \end{aligned}$$

Conditions (b) and (c) then imply that it is optimal to choose $Z(\tau) = 1 - F^y(\tau)$ for all $\tau \geq KR$. This implies that KR will be included in the highest partition cell.

Next, we show the second part of the theorem. Under assumptions 2 and 3, (d) ϕ is convex in $[0, KR]$. To see (d) assume first that $\bar{P}(\tau) > 0$ for positive values of τ (this is always the case if $\lambda > 0$). We can then use equation 7, which implicitly defines $\bar{\omega}$, and compute:

$$\phi'(\tau) = \underbrace{\left(\frac{g + |b|}{|b|} \right)}_{\equiv c} f^\omega(1 - \bar{P}(\tau)) \bar{P}'(\tau) = \frac{c f^\omega(1 - \bar{P}(\tau)) \phi(\tau)}{R - c f^\omega(1 - \bar{P}(\tau)) \tau} \geq 0, \quad \forall \tau \in [0, KR] \quad (32)$$

where \bar{P}' can be obtained from its definition in equation (8) and equals:

$$\bar{P}'(\tau) = \frac{1}{R} \times (\tau\phi'(\tau) + \phi(\tau)).$$

Differentiating equation (32) once more and using assumptions 2 and 3 we obtain that the sign of ϕ'' coincides with the sign of:

$$1 - \frac{c}{R} f^\omega (1 - \bar{P}(\tau)) \tau$$

which is positive for any $\tau < KR$ since otherwise ϕ would decrease with τ , proving (d).

Finally, observe that the constraint $F^y + Z \leq 1$ (everywhere), together with the requirement that $\int_0^\infty Z(\tau)d\tau = 0$, impose a lower bound on the value that $\int_0^{KR} Z(\tau)d\tau$ may take. In fact, we must have that $\int_{KR}^\infty Z(\tau)d\tau \leq \int_{KR}^\infty (1 - F^y(\tau)) d\tau$, and hence $\int_0^{KR} Z(\tau)d\tau \geq -\int_{KR}^\infty (1 - F^y(\tau)) d\tau$. That ϕ' is non decreasing in $[0, KR]$ and equal to 0 for any $\tau \geq KR$ then implies that the optimal choice of Z is given by:

$$Z(\tau) = \begin{cases} 0 & \tau \leq y^+ \\ F^y(y^+) - F^y(\tau) & \tau \in (y^+, \max\{KR, \mathbb{E}(y)\}) \\ 1 - F^y(\tau) & \tau \geq \max\{KR, \mathbb{E}(y)\} \end{cases}$$

where y^+ is chosen so that $\int_{y^+}^\infty Z(\tau)d\tau = 0$ whenever $\varsigma \equiv \int_0^{KR} (F^y(y) - F^y(\tau)) d\tau + \int_{KR}^\infty (1 - F^y(\tau)) d\tau \leq 0$. Whenever instead $\varsigma > 0$, $y^+ = 0$. More precisely,

$$y^+ = \left\{ y \geq 0 : \int_y^{KR} (F^y(y) - F^y(\tau)) d\tau + \int_{KR}^\infty (1 - F^y(\tau)) d\tau \geq 0 \right\}.$$

That $Z(\tau) = 0$ for all $\tau \leq y^+$ implies that $G(\tau) = F^y(\tau)$ for such τ , or equivalently, that G coincides with the full-disclosure policy for all $y \leq y^+$. On the other hand, that $G(\tau) = F^y(y^+) - F^y(\tau)$ for all $\tau \in (y^+, \max\{KR, \mathbb{E}(y)\})$, and $G(\tau) = 1$ for all $\tau \geq \max\{KR, \mathbb{E}(y)\}$, means that the optimal policy pools all the realizations of y above y^+ under a single message, so that the induced posterior mean is at least KR . \square

Appendix C: Elicitation Mechanisms

Proof of Proposition 6.

Fix a message m^y disclosed with positive probability under Γ^y . Suppose that during the second period the policy-maker perfectly observes that the bank's asset quality type. The policy-maker's

(ex-ante) problem in (22) can then be written as:

$$\begin{aligned}
& \max_{\{V_\theta, t_0(\cdot; \theta), \pi(0|\cdot; \theta), \pi_\theta^s\}_{\theta \in \Theta}} && \mathbb{E}_\theta \left(\int_0^{1-P} \pi(0|\omega; \theta) F^\omega(d\omega) \right) \\
& \text{s.t:} && (i) \quad \mathbb{E}_\theta \left(\int_0^{1-P} ((b-g) \times 1_{\{P+t_0(\omega; \theta)+\omega < 1\}} + g) \pi(0|\omega; \theta) F^\omega(d\omega) + \right. \\
& && \quad \left. + \pi_\theta^s \times g \times (1 - F^\omega(1-P)) \right) \geq 0 \\
& && (ii) \quad \mathbb{E}_\theta \left(V_\theta \times \left(\int_0^{1-P} |b| \pi(0|\omega; \theta) F^\omega(d\omega) - g \pi_\theta^s \times (1 - F^\omega(1-P)) \right) \right) \leq |\bar{U}_{\text{LF}}(P)| \\
& && (iii) \quad \pi(0|\omega; \theta) \times ((P + t_0(\omega; \theta)) R) = V_\theta, \quad \forall \omega \leq 1 - P \\
& && (iv) \quad \pi_\theta^s \leq \frac{V_\theta}{PR + z_\theta} \\
& && (v) \quad t_0(\omega; \theta) \leq \frac{z_\theta}{R}
\end{aligned}$$

where the first two constraints are the obedience constraints associated with messages 0 and 1, respectively, and the last three correspond to incentive compatibility constraints: (iii) imposes that the payoff of any bank reporting a liquidity position below $1 - P$ must be the same, (iv) guarantees that vulnerable banks do not have incentives to mimic safe banks, and (v) requires that safe banks do not want to be thought of as vulnerable banks, and at the same time imposes that the funds respect the regulator's budget constraint. Observe that the solution to this problem strictly dominates the optimal screening mechanism under the original setting where the policy-maker does not observe θ .

Let $\hat{\omega}_\theta \equiv 1 - P - \tilde{B}_\theta$. Define next the auxiliary variable ρ_θ as follows:

$$\rho_\theta \equiv \int_0^{\hat{\omega}_\theta} \frac{|b|}{g} \times F^\omega(d\omega) - \frac{1 - F^\omega(1-P)}{PR + z_\theta}.$$

We will characterize the optimal screening mechanism as a function of the value of $\rho \equiv \mathbb{E}_\theta(\rho_\theta)$. Assume first that

$$\rho \in \left(\mathbb{E}_\theta \left(\frac{F^\omega(1-P) - F^\omega(\hat{\omega}_\theta)}{PR + \tilde{B}_\theta} \right), \mathbb{E}_\theta \left(\int_{\hat{\omega}_\theta}^{1-P} \frac{F^\omega(d\omega)}{(1-\omega)R} \right) \right).$$

We note next that inequality (iv) must bind since this relaxes (i) and (ii), does not affect neither (iii) nor (v), and therefore allows to improve the policy-maker's objective function. Next, constraint (iii) implies that we can write the policy-maker's problem as a function only of V_θ and t_0 . Thus, the set of relevant constraints is given by:

$$\begin{aligned}
(i') \quad & \mathbb{E}_\theta \left(\int_0^{1-P} \left(\frac{(b-g) \times 1_{\{P+t_0(\omega; \theta)+\omega < 1\}} + g}{(P + t_0(\omega; \theta)) R} \right) F^\omega(d\omega) + g \times \frac{(1 - F^\omega(1-P))}{PR + z_\theta} \right) \geq 0 \\
(ii') \quad & \mathbb{E}_\theta \left(V_\theta \times \int_0^{1-P} \frac{|b| \times F^\omega(d\omega)}{(P + t_0(\omega; \theta)) R} \right) \leq |\bar{U}_{\text{LF}}(P)| + \mathbb{E}_\theta \left(V_\theta \times g \times \frac{(1 - F^\omega(1-P))}{PR + z_\theta} \right) \\
(v) \quad & t_0(\omega; \theta) \leq \frac{z_\theta}{R} \\
(vi) \quad & \frac{V_\theta}{(P + t_0(\omega; \theta)) R} \leq 1 \quad \forall \omega \leq 1 - P.
\end{aligned}$$

where the new constraint (vi) is added so that probabilities are well defined.

Claim 1: $t_0(\omega; \theta) = \frac{z_\theta}{R}$ for all $\omega < \hat{\omega}$.

To see this, let $\Upsilon^{\omega, \theta} = \{t_m, \pi(m|\cdot)\}_{m \in \{0,1\}}$ be the optimal screening mechanism and suppose by contradiction that the claim is not true. We show that we can find another mechanism which strictly improves upon $\Upsilon^{\omega, \theta}$. Consider the alternative program Υ^ϵ which offers the alternative price t_0^ϵ which modifies the value of t_0 for values of $\omega \leq \hat{\omega}$ in the following way:

$$t_0^\epsilon(\omega; \theta) \equiv \begin{cases} \epsilon \tilde{B}_\theta + (1 - \epsilon)t_0(\omega; \theta) & \omega \leq \hat{\omega} \\ t_0(\omega; \theta) & \omega > \hat{\omega}. \end{cases}$$

Let V_θ^ϵ be the value of V_θ which preserves the value of the LHS in (ii'). That is,

$$V_\theta^\epsilon \times \left(\int_0^{1-P} \frac{|b| \times F^\omega(d\omega)}{(P + t_0^\epsilon(\omega; \theta)) R} \right) = V_\theta \times \left(\int_0^{1-P} \frac{|b| \times F^\omega(d\omega)}{(P + t_0(\omega; \theta)) R} \right).$$

This perturbation relaxes (i') since $b < 0$, and increases the value of V_θ , which then relaxes (ii') since the RHS increases while the LHS remains constant (by construction). Constraint (v) is never affected by this perturbation, while (vi) is satisfied for small values of ϵ . As a result, the designer can increase $\pi(0|\omega; \theta)$ without violating any constraint. This is a contradiction, and hence we must have that $t_0(\omega; \theta) = \frac{z_\theta}{R}$ for all $\omega < \hat{\omega}$, and all θ .

Claim 2: $\exists \hat{\omega}_\theta \in [\hat{\omega}, 1 - P]$ so that $t_0(\omega; \theta) = \max\{1 - \omega - P, 1 - \hat{\omega}_\theta - P\}$ for all $\omega \in [\hat{\omega}_\theta, 1 - P]$.

Consider an arbitrary pricing policy \tilde{t}_0 . Construct the alternative policy $t_0(\omega) \equiv \max\{1 - \omega - P, 1 - \tilde{\omega} - P\}$ for all $\omega \in [\hat{\omega}, 1 - P]$, where $\tilde{\omega}$ is chosen so that:

$$\int_{\hat{\omega}_\theta}^{1-P} \frac{F^\omega(d\omega)}{(P + \tilde{t}_0(\omega; \theta)) R} = \int_{\hat{\omega}_\theta}^{1-P} \frac{f^\omega(\omega)}{\max\{1 - \omega, 1 - \tilde{\omega}\} R} d\omega.$$

I claim that t_0 dominates \tilde{t}_0 . To see this, note that constraints (i'), (ii'), (v) remain unchanged under the alternative policy, but constraint (vi) relaxes. In fact,

$$\sup_{\omega \in [\hat{\omega}_\theta, 1-P]} \left\{ \frac{V_\theta}{(P + t_0(\omega)) R} \right\} \leq \sup_{\omega \in [\hat{\omega}_\theta, 1-P]} \left\{ \frac{V_\theta}{(P + \tilde{t}_0(\omega)) R} \right\} \leq 1.$$

The first inequality is strict if $F^\omega(\{\omega \in [\hat{\omega}_\theta, 1 - P] : \tilde{t}_0(\omega; \theta) \neq t_0(\omega)\}) > 0$. \square

Claim 3: Constraint (i') must bind.

This constraint corresponds to obedience constraint (12), and requires that creditors have an incentive to follow the recommendation to keep rolling over the bank's debt. By contradiction, assume that this constraint does not bind. Then,

$$\mathbb{E}_\theta \left(\int_0^{\hat{\omega}_\theta} \frac{b \times F^\omega(d\omega)}{(P + t_0(\omega; \theta)) R} + \int_{\hat{\omega}_\theta}^{1-P} \frac{g \times F^\omega(d\omega)}{(P + t_0(\omega; \theta)) R} + g \times \frac{(1 - F^\omega(1 - P))}{(PR + z_\theta)} \right) > 0, \quad (33)$$

Observe that either (ii') or (vi) must be binding. Suppose first that (ii') is the binding constraint. Consider the following deviation from the optimal mechanism $\Upsilon^{\omega, \theta}$: We modify t_0 between $[\hat{\omega}_\theta, 1 - P]$ for some θ , so that the new price can be written as $\tilde{t}_0^\epsilon = \max\{1 - \omega - P, 1 - \tilde{\omega}_\theta^\epsilon - P\}$, where $\tilde{\omega}_\theta^\epsilon < \tilde{\omega}_\theta$ satisfies that

$$\int_{\hat{\omega}_\theta}^{1-P} \frac{F^\omega(d\omega)}{P + \tilde{t}_0^\epsilon(\omega; \theta)} = \int_{\hat{\omega}_\theta}^{1-P} \frac{F^\omega(d\omega)}{P + t_0(\omega; \theta)} - \epsilon,$$

for some $\epsilon > 0$ small enough so that the inequality above is respected²⁸. Next, let $\tilde{V}_\theta(\epsilon)$ be the maximal value that V_θ may take under the new policy so that (ii') remains unchanged. That is,

$$\tilde{V}_\theta(\epsilon) \times \left(\int_0^{\hat{\omega}_\theta} \frac{|b| \times F^\omega(d\omega)}{(P + t_0(\omega; \theta)) R} + \int_{\hat{\omega}_\theta}^{1-P} \frac{|b| \times F^\omega(d\omega)}{(P + \tilde{t}_0^\epsilon(\omega; \theta)) R} - g \times \frac{(1 - F^\omega(1 - P))}{PR + z_\theta} \right) = C_\theta, \quad (34)$$

where C_θ is a constant. Clearly, $C_\theta > 0$ for some $\theta_0 \in \Theta$. To see this, note that $\sum_\theta \mu_\theta C_\theta > 0$ since otherwise constraint (ii') cannot bind.

Next, differentiating (34) against ϵ and then taking the limit from the right as ϵ goes to 0, we get:

$$\lim_{\epsilon \downarrow 0} \tilde{V}'_{\theta_0}(\epsilon) = \frac{\tilde{V}_{\theta_0}(0) \times |b|}{\left(|b| \times \int_0^{1-P} \frac{f^\omega(\omega)}{(P + t_0(\omega; \theta_0)) R} d\omega - g \times \frac{(1 - F^\omega(1 - P))}{PR + z_{\theta_0}} \right)}.$$

This allows us to compute the effect of such a perturbation on the policy-maker's payoff

$$W = \sum_\theta \mu_\theta \times \tilde{V}_\theta \int_0^{1-P} \frac{F^\omega(d\omega)}{P + t_0(\omega; \theta)} + F^\omega(1 - P)$$

for small values of ϵ . In fact,

$$\begin{aligned} \lim_{\epsilon \rightarrow 0^+} \frac{dW}{d\epsilon} &\propto \left(\lim_{\epsilon \rightarrow 0^+} \tilde{V}'_{\theta_0}(\epsilon) \right) \cdot \frac{W(0)}{\tilde{V}_{\theta_0}(0)} - \tilde{V}_{\theta_0}(0) \\ &= \frac{V_{\theta_0}^2}{C} \times (g - b) \times \frac{(1 - F^\omega(1 - P))}{PR + z_{\theta_0}} \\ &> 0. \end{aligned}$$

which contradicts the optimality of $\Upsilon^{\omega, \theta}$.

Next, assume that (vi) is the binding constraint for some θ (which determines the value of V_θ). Consider the alternative policy

$$\hat{t}_0^\epsilon(\omega; \theta) \equiv \begin{cases} \tilde{b}_\theta & \omega \leq \hat{\omega}_\theta \\ \max\{1 - \omega - P, 1 - \tilde{\omega}_\theta^\epsilon - P\} & \omega \in (\hat{\omega}_\theta, 1 - P], \\ 0 & \omega > 1 - P \end{cases}$$

²⁸The existence of such ϵ comes from (33), since this inequality implies:

$$\int_{\hat{\omega}}^{1-P} \frac{f^\omega(\omega)}{P + t_0(\omega)} d\omega > \rho > \frac{F^\omega(1 - P) - F^\omega(\hat{\omega})}{P + z_L}.$$

with $\check{\omega}_\theta^\epsilon = \check{\omega}_\theta - \epsilon$ and ϵ small enough so that (ii') is still satisfied. Let $\check{V}_\theta^\epsilon$ be the maximal value that V may take under the new policy so that (vi) is still satisfied. That is,

$$\frac{\check{V}_\theta^\epsilon}{1 - \check{\omega}_\theta^\epsilon} = \frac{V_\theta}{1 - \check{\omega}_\theta}.$$

This implies that $\check{V}_\theta^\epsilon > V_\theta$ and hence $\hat{\pi}^\epsilon(0|\omega; \theta) \equiv \frac{\check{V}_\theta^\epsilon}{(P + t_0^\epsilon(\omega))R} > \pi(0|\omega; \theta)$ for all $\omega \leq \check{\omega}_\theta$ and $\hat{\pi}^\epsilon(0|\omega; \theta) = \pi(0|\omega; \theta)$ for all $\omega > \check{\omega}_\theta$, and hence the policy-maker's payoff must increase. This is a contradiction and hence (i') must be satisfied with equality. \square

This means that

$$\mathbb{E}_\theta \left(\int_{\hat{\omega}}^{1-P} \frac{F^\omega(d\omega)}{(P + t_0(\omega; \theta))R} \right) = \rho \in \left(\mathbb{E}_\theta \left(\frac{F^\omega(1-P) - F^\omega(\hat{\omega}_\theta)}{PR + \frac{z_\theta}{R}} \right), \mathbb{E}_\theta \left(\int_{\hat{\omega}_\theta}^{1-P} \frac{F^\omega(d\omega)}{(1-\omega)R} \right) \right),$$

which is feasible.

Therefore, we choose $t_0(\omega; \theta)$ in $[\hat{\omega}, 1-P]$ among all the policies satisfying (i') so that V_θ is largest. Let $\check{\omega}_\theta$ be implicitly defined by:

$$\int_{\hat{\omega}}^{\check{\omega}} \frac{F^\omega(d\omega)}{(1-\omega)R} + \frac{F^\omega(1-P) - F^\omega(\check{\omega})}{(1-\check{\omega})R} = \int_0^{\check{\omega}} \frac{|b| \times F^\omega(d\omega)}{g \times (P + \frac{z_\theta}{R})R} - \frac{1 - F^\omega(1-P)}{PR + z_\theta}.$$

That is, $\check{\omega}$ is the cutoff defining the price t_0 which maximizes $\min_{\omega \leq 1-P} (P + t_0(\omega; \theta))R$ while still respecting (i'). The optimal policy is thus given by:

$$t_0(\omega; \theta) = \begin{cases} \frac{z_\theta}{R} & \omega < \hat{\omega} \\ 1 - P - \omega & \omega \in [\hat{\omega}, \check{\omega}] \\ 1 - P - \check{\omega} & \omega \in (\check{\omega}, 1 - P) \\ 0 & \omega \geq 1 - P \end{cases}, \quad \pi(0|\omega; \theta) = \begin{cases} \frac{\bar{V}_\theta}{PR + \bar{b}} & \omega < \hat{\omega} \\ \frac{\check{V}_\theta}{(1-\omega)R} & \omega \in [\hat{\omega}, \check{\omega}] \\ \frac{\bar{V}_\theta}{(1-\check{\omega})R} & \omega \in (\check{\omega}, 1 - P) \\ \frac{\bar{V}_\theta}{PR + z_\theta} & \omega \geq 1 - P \end{cases}$$

where \bar{V}_θ is chosen so that (ii) and (vi) hold:

$$\bar{V}_\theta \equiv \min \left\{ (1 - \check{\omega})R, \frac{|\bar{U}_{LF}(P)|}{\int_0^{1-P} \frac{|b| \times f^\omega(\omega)}{(P + t_0(\omega; \theta))R} d\omega - g \times \frac{(1 - F^\omega(1-P))}{PR + z_\theta}} \right\}.$$

Finally, assume that

$$\rho \geq \int_{\hat{\omega}}^{1-P} \frac{F^\omega(d\omega)}{1-\omega}. \quad (35)$$

Then, the designer is unable to successfully dissuade creditors from running on the bank with positive probability. In other words, $\pi(0|\cdot) = \mathbf{0}$. To see this, rewrite the inequality 35 as:

$$\int_0^{\hat{\omega}} \frac{|b| \times F^\omega(d\omega)}{(PR + \bar{b})} - \frac{g \times (1 - F^\omega(1-P))}{PR + z_\theta} \geq g \times \int_{\hat{\omega}}^{1-P} \frac{f^\omega(\omega)}{(1-\omega)R} d\omega,$$

or equivalently,

$$\mathbb{E}(u(\omega, P, 1)|0) = \int_0^{\hat{\omega}} \frac{b \times F^\omega(d\omega)}{(PR + \bar{b})} + \int_{\hat{\omega}}^{1-P} \frac{g \times F^\omega(d\omega)}{(1-\omega)R} + \frac{g \times (1 - F^\omega(1-P))}{PR + z_\theta} \leq 0.$$

That is, creditors obtain a negative payoff if they pledge to the bank (and the rest does not), even if the designer were to offer enough funds so that every bank with $\omega > \hat{\omega}$ survives the liquidity shortage caused by all creditors refraining from rolling over the bank's debt. As a result, under the most adversarial equilibrium all creditors run on the bank. The policy-maker thus cannot engage in disclosing informative messages about the bank's liquidity buffer, and may only try to increase the likelihood of the bank's survival by purchasing claims on its asset. The optimal strategy for the policy-maker consists of purchasing the totality of the remaining claims on the asset at the largest price allowed by *fair price* constraint. Thus, the government purchases $y - s^*$ at price t^θ defined by:

$$t^\theta \equiv \sup \left\{ \tau \leq B : \frac{\sum_\theta \mu_\theta \mathbb{E}_\theta(y - s^*)}{R} \times \mathbb{P}\{\omega + P + \tau \geq 1\} \geq \tau \right\}.$$

□

Proof of Lemma 2

First, I claim that for any arbitrary mechanism $\Upsilon^{\omega, \theta}$, we have that $\Upsilon_{\text{OAK}}[\theta_L] \succeq_{\text{PM}} \Upsilon^{\omega, \theta}$. To see this, suppose we relax constraint (18) and assume instead that:

$$\pi_s \leq \frac{V_L}{PR + z_L}. \quad (36)$$

Clearly, the optimal screening mechanism of the relaxed problem dominates $\Upsilon^{\omega, \theta}$, which satisfies the original constraint (18). The optimal mechanism of the relaxed problem implements the mechanism $\Upsilon_{\text{OAK}}[\theta_L]$ for both types $\theta \in \{\theta_L, \theta_H\}$. In fact, constraint (20) requires that:

$$t(\omega, \theta_H) \leq (1 - \phi_H(\omega)) \times \frac{z_L}{R} \leq \frac{z_L}{R}, \quad \forall \omega.$$

As a consequence, the policy-maker may not pledge more than the value of the asset for a type-L bank. This, in turn, implies that there is no benefit associated with telling apart type-H banks from type-L ones. As a result, the optimal mechanism of the relaxed problem sets $\phi_h = 0$. The optimal mechanism of the relaxed problem then is given by $\Upsilon_{\text{OAK}}[\theta_L]$:

$$t(\omega, \theta_H) = t(\omega, \theta_L) = t_{\text{OAK}}(\omega; \theta_L), \quad \pi(\omega, \theta_H) = \pi(\omega, \theta_L) = \pi_{\text{OAK}}(\omega; \theta_L) \quad \forall \omega \in \Omega.$$

Finally, the conclusion obtains from the fact that $\Upsilon_{\text{OAK}}[\emptyset] \succeq_{\text{PM}} \Upsilon_{\text{OAK}}[\theta_L]$. That is, the optimal mechanism when the bank does not observe additional information (or, alternatively, the optimal mechanism when the policy-maker observes the private information of a bank that does not possess private information with respect to its asset), dominates the optimal mechanism that emerge when the bank possess pessimistic information about its asset, and this information is observed by the policy-maker. □

Proof of Theorem 2

Proof. Fix a message m^y disclosed with positive probability under Γ^y , and assume that the bank successfully raises P units of capital after the asset quality review Γ^y discloses m^y . Let $\pi_{\text{OAQ}}[\emptyset]$, $t_{\text{OAQ}}[\emptyset]$ be the disclosure and pricing policy associated with the screening mechanism $\Upsilon_{\text{OAQ}}^{\omega, \theta}[\emptyset]$, the optimal screening mechanism under the alternative setting wherein the bank does not possess private information regarding the quality of its asset. That is, $\Upsilon_{\text{OAQ}}^{\omega, \theta}[\emptyset]$ corresponds to the optimal screening mechanism characterized in proposition (6), when the bank has a unique, average, asset quality type $\bar{\theta} \equiv \mu_H \theta_H + \mu_L \theta_L$. This implies that there exist constants V_{\emptyset} and π_{\emptyset}^s so that the following constraints are satisfied:

$$\begin{aligned}
(i) \quad & \int_0^{1-P} \left((b-g) \cdot 1_{\{P+t_{\text{OAQ}}(\omega; \emptyset)+\omega < 1\}} + g \right) \pi_{\text{OAQ}}(0|\omega; \emptyset) F^\omega(d\omega) + \\
& + \pi_{\emptyset}^s \times g \times (1 - F^\omega(1-P)) \geq 0 \\
(ii) \quad & \left(\int_0^{1-P} |b| \times \pi_{\text{OAQ}}(0|\omega; \emptyset) (d\omega) \right) - g \times \pi_{\emptyset}^s \times (1 - F^\omega(1-P)) \leq |\bar{U}_{\text{LF}}(P)| \\
(iii) \quad & \pi_{\text{OAQ}}(0|\omega; \emptyset) \times ((P + t_{\text{OAQ}}(\omega; \emptyset)) R) = V_{\emptyset}, \quad \forall \omega \leq 1-P \\
(iv) \quad & \pi_{\emptyset}^s \leq \frac{V_{\emptyset}}{PR + \bar{z}} \\
(v) \quad & t_{\text{OAQ}}(\omega; \emptyset) \leq \frac{\bar{z}}{R}, \quad \forall \omega.
\end{aligned}$$

That securities purchased by the government are not penalized with a premium to compensate for rollover risk follows from the fact that the policy-maker only purchases when assigning the passing grade, in which case the probability that the bank fails equals 0. The proof shows that even if we assume that external investors pay the default-free price of the claims during the first period (which would be true if, for instance, there were not liquidity shock, i.e., $\lambda = 1$), the designer still prefers to minimize the claims that are sold to the asset market at $t = 1$. I show that when $\mathbb{E}(y|m^y)$ is high enough so that elicitation is in fact possible, the policy-maker prefers to minimize the amount raised by the bank during the fund-raising game in order to increase the value of \bar{z} , which provides her with more elicitation capacity during the second period. Assume that $\mathbb{E}(y|m^y) \geq \underline{E}$. This means, as it will become clear below, that there exists a non-empty set of screening policies. I characterize the optimal recapitalization and subsequent screening mechanism that follows the disclosure m^y .

Note that although $\Upsilon_{\text{OAQ}}^{\omega, \theta}[\emptyset]$ satisfies (i)-(v), it does not respect incentive compatibility under the original setting. In fact, $\Upsilon_{\text{OAQ}}^{\omega, \theta}[\emptyset]$ fails to satisfy constraint (19). That is, under this alternative mechanism safe banks (i.e., those with $\omega > 1 - P$), but with a low quality asset, have incentives to claim to be illiquid and receive a price for its asset above its fair value. I show that under the optimal persuasion mechanism $P^* = 0$ whenever $\underline{E} < \mathbb{E}(y) < K$. That is, the policy-maker minimizes the recapitalization rule in order to boost her elicitation capacity during the second period.

Claim 1: $\mathbb{E}(y|m^y) \geq \underline{E}$ implies that the set of potential policies satisfying (i)-(v) is non-empty.

By definition of \underline{E} , when $\mathbb{E}(y|m^y) \geq \underline{E}$ there exist transfers $t(\cdot)$ and probability π_s so that (i)

holds. Moreover, there always exist policies satisfying constraint (ii)-(iv), which can be seen by choosing V small enough, and then choosing $\pi(0|\cdot)$ consistently. \square

Following proposition (6), the optimal screening mechanism in the absence of bank's private information about asset quality, $\Upsilon_{\text{O AQ}}^{\omega, \theta}[\emptyset]$, can be characterized as a function of P and \bar{z} as follows:

$$(t_{\text{O AQ}}(\omega), \pi_{\text{O AQ}}(0|\omega)) = \begin{cases} \frac{\mathbb{E}(y|m^y)}{R}, \frac{V_\emptyset}{\mathbb{E}(y|m^y)} & \omega < \hat{\omega} \\ 1 - P - \omega, \frac{V_\emptyset}{(1-\omega)R} & \omega \in [\hat{\omega}, \check{\omega}] \\ 1 - P - \check{\omega}, \frac{V_\emptyset}{(1-\check{\omega})R} & \omega \in (\check{\omega}, 1 - P) \\ 0, \frac{V_\emptyset}{\mathbb{E}(y|m^y)} & \omega \geq 1 - P \end{cases}$$

with $\check{\omega}$ and V_\emptyset are chosen so that:

$$\int_{1-\frac{\mathbb{E}(y|m^y)}{R}}^{\check{\omega}} \frac{F^\omega(d\omega)}{(1-\omega)R} + \frac{F^\omega(1-P) - F^\omega(\check{\omega})}{(1-\check{\omega})R} = \int_0^{\frac{\mathbb{E}(y|m^y)}{R}} \frac{|b| F^\omega(d\omega)}{g \times \mathbb{E}(y|m^y)} - \frac{(1 - F^\omega(1-P))}{\mathbb{E}(y|m^y)}. \quad (37)$$

$$V_\emptyset \equiv \min \left\{ (1 - \check{\omega}) R, \frac{|\bar{U}_{\text{LF}}(P)|}{\int_0^{1-P} \frac{|b|F(d\omega)}{(P+t_{\text{O AQ}}(\omega))R} - g \times \frac{(1-F^\omega(1-P))}{\mathbb{E}(y|m^y)}} \right\}. \quad (38)$$

Claim 2: $V_\emptyset = (1 - \check{\omega}) R$.

To see this, note that constraint (ii) is satisfied with strict inequality. In fact, that $\bar{U}_{\text{LF}}(P) < 0$ for all $P < \frac{\mathbb{E}(y|m^y)}{R}$ implies that

$$\begin{aligned} & \int_0^{1-P} b \times (1 - \pi_{\text{O AQ}}(0|\omega; \emptyset)) F^\omega(d\omega) + g \times (1 - \pi_\emptyset^s) (1 - F^\omega(1-P)) \\ &= \int_0^{1-\frac{\mathbb{E}(y|m^y)}{R}} \left(1 - \frac{V_\emptyset}{\mathbb{E}(y|m^y)}\right) b F(d\omega) + \int_{1-\frac{\mathbb{E}(y|m^y)}{R}}^{\check{\omega}} \left(1 - \frac{V_\emptyset}{(1-\omega)R}\right) b F^\omega(d\omega) \\ & \quad + \int_{\check{\omega}}^{1-P} \left(1 - \frac{V_\emptyset}{(1-\check{\omega})R}\right) b F^\omega(d\omega) + g \times \left(1 - \frac{V_\emptyset}{\mathbb{E}(y|m^y)}\right) (1 - F^\omega(1-P)) \\ &< \left(1 - \frac{V_\emptyset}{\mathbb{E}(y|m^y)}\right) \times \left(\int_0^{1-\frac{\mathbb{E}(y|m^y)}{R}} b \times F(d\omega) + g \times \left(1 - F^\omega\left(1 - \frac{\mathbb{E}(y|m^y)}{R}\right)\right)\right) \\ &= \left(1 - \frac{V_\emptyset}{\mathbb{E}(y|m^y)}\right) \times \bar{U}_{\text{LF}}\left(\frac{\mathbb{E}(y|m^y)}{R}\right) \\ &\leq 0. \end{aligned}$$

As a consequence, the constraint defining the value of V_\emptyset is (iii). The result follows from the implicit restriction that $\pi_{\text{O AQ}}[\emptyset]$ is a probability measure:

$$(vi) \pi_{\text{O AQ}}(0|\omega; \emptyset) = \frac{V_\emptyset}{(P + t_{\text{O AQ}}(\omega; \emptyset)) R} \leq 1 \quad \forall \omega \leq 1 - P.$$

Thus, at the optimum:

$$V_\emptyset = \inf \{(P + t_{\text{O AQ}}(\omega; \emptyset)) R : \omega \leq 1 - P\} = (1 - \check{\omega}) R.$$

□

Next, I show that the designer can improve her payoff by decreasing P , and increasing \bar{z} accordingly, so that $PR + \bar{z} \leq \mathbb{E}(y|m^y)$.

Claim 3: The optimal persuasion mechanism sets either $P = 0$, or $P = \bar{P}(\mathbb{E}(y|m^y))$ (i.e., optimal interventions either involves the government, or the private sector, but not both) for any $\lambda \in [0, 1]$.

To see this, assume that $\lambda = 1$, so that the liquidity shock is a 0-probability event. This assumption exacerbates the incentives to let external investors (the private sector) purchase securities from the bank during the fund-raising game at $t = 1$, since the bank avoids discounts (haircuts) on its asset to compensate for default risk.

Consider the following function

$$\begin{aligned} \varphi^+(P, \tilde{\omega}) &\equiv \int_0^{1-P} \left((b-g) \cdot 1_{\{P+t_{\text{OAQ}}(\omega; \emptyset) + \omega < 1\}} + g \right) \pi_{\text{OAQ}}(0|\omega; \emptyset) F^\omega(d\omega) + \\ &\quad + \pi_\emptyset^s \times g \times (1 - F^\omega(1-P)) \\ &= \frac{\int_0^{1-\frac{\mathbb{E}(y|m^y)}{R}} b \left(\frac{\mathbb{E}(y|m^y)}{R}, 1 \right)}{\mathbb{E}(y|m^y)} F(d\omega) + \\ &\quad + g \times \left(\int_{1-\frac{\mathbb{E}(y|m^y)}{R}}^{\tilde{\omega}} \frac{F^\omega(d\omega)}{(1-\omega)R} + \frac{F^\omega(1-P) - F^\omega(\tilde{\omega})}{(1-\tilde{\omega})R} + \frac{(1 - F^\omega(1-P))}{\mathbb{E}(y|m^y)} \right). \end{aligned}$$

φ^+ corresponds to the expected payoff of creditors, at the optimal elicitation mechanism, under message '0' (pass). Function φ^+ decreases with P (or equivalently, increases with \bar{z}) if we keep the rest of variables (other than \bar{z}) constant, since $(1 - \tilde{\omega})R < \mathbb{E}(y|m^y)$. The case in which $(1 - \tilde{\omega})R = \mathbb{E}(y|m^y)$ corresponds to the situation in which the policy-maker can avoid default altogether (with certainty) and thus is not considered here. This implies that (i) is relaxed when we decrease the value of P , or equivalently, when we increase the value \bar{z} . Decreasing P (and therefore increasing \bar{z}) also relaxes (ii) and (v), and does not affect neither (iii), nor (iv). To see the first point, consider the following function:

$$\begin{aligned} \varphi^-(P, \tilde{\omega}, V_\emptyset) &\equiv \int_0^{1-P} b \times (1 - \pi_{\text{OAQ}}(0|\omega; \emptyset)) F^\omega(d\omega) + \\ &\quad + g \times (1 - F^\omega(1-P)) \times (1 - \pi_\emptyset^s) \\ &= \int_0^{1-\frac{\mathbb{E}(y|m^y)}{R}} b \times \left(1 - \frac{V_\emptyset}{\mathbb{E}(y|m^y)} \right) F(d\omega) + \int_{1-\frac{\mathbb{E}(y|m^y)}{R}}^{\tilde{\omega}} b \times \left(1 - \frac{V_\emptyset}{(1-\omega)R} \right) F^\omega(d\omega) \\ &\quad + \left(\int_{\tilde{\omega}}^{1-P} b \times \left(1 - \frac{V_\emptyset}{(1-\tilde{\omega})R} \right) F^\omega(d\omega) + g \times (1 - F^\omega(1-P)) \times \left(1 - \frac{V_\emptyset}{\mathbb{E}(y|m^y)} \right) \right). \end{aligned}$$

φ^- corresponds to the expected payoff of creditors, at the optimal elicitation mechanism, under message '1' (fail). Function φ^- increases with P if we keep the rest of variables (other than z) constant. As a result, reducing P relaxes constraint (ii). Finally to see that (iii) is not affected

by reductions of P , observe that for every reduction of P in the amount of Δ , the maximal price that may be pledged by the policy-maker (determined by constraint (v)) increases by Δ . Thus, the policy-maker can replicate the effect of P by increasing the price paid by the securities, t_{OAQ} , in the same amount.

Next, define $\tilde{\omega}(P)$ as the optimal cutoff associated with any price $P \in [0, \frac{E}{R}]$, as in (37). That is, $\tilde{\omega}(P)$ is chosen so that $\varphi^+(P, \tilde{\omega}(P)) = 0$. Consider the case where $P = 0$. The optimal elicitation mechanism is then given by:

$$(t_{\text{OAQ}}^{P=0}(\omega), \pi_{\text{OAQ}}^{P=0}(0|\omega)) = \begin{cases} \frac{\mathbb{E}(y|m^y)}{R}, \frac{(1-\tilde{\omega}(0))R}{\mathbb{E}(y|m^y)} & \omega < 1 - \frac{\mathbb{E}(y|m^y)}{R} \\ 1 - \omega, \frac{1-\tilde{\omega}(0)}{(1-\omega)} & \omega \in \left[1 - \frac{\mathbb{E}(y|m^y)}{R}, \tilde{\omega}(0)\right] \\ 1 - \tilde{\omega}(0), 1 & \omega \in (\tilde{\omega}(0), 1]. \end{cases}$$

Choose any alternative policy in which the bank raises a price $\tilde{P} \in (0, 1 - \tilde{\omega}(0))$ from external investors. That φ^+ decreases with P implies that $\tilde{\omega}(\tilde{P}) > \tilde{\omega}(0)$, since $\tilde{\omega}(\tilde{P})$ satisfies $\varphi^+(\tilde{P}, \tilde{\omega}(\tilde{P})) = 0$. This means that $\pi^{P=0}(0|\omega) > \pi^{\tilde{P}}(0|\omega)$ for all $\omega \leq \tilde{\omega}(P)$, and $\pi^{P=0}(0|\omega) = \pi^{\tilde{P}}(0|\omega) = 1$ for all $\omega > \tilde{\omega}(P)$. As a result, the policy-maker's payoff is strictly greater at $P = 0$. Finally, consider the case where $\tilde{P} \leq 1 - \tilde{\omega}(0)$. We note that:

$$\frac{\int_0^{1-\frac{\mathbb{E}(y|m^y)}{R}} b\left(\frac{\mathbb{E}(y|m^y)}{R}, 1\right) f^\omega(\omega) d\omega + g \cdot \left(\int_{1-\frac{\mathbb{E}(y|m^y)}{R}}^{1-P} \frac{f^\omega(\omega)}{(1-\omega)R} d\omega + \frac{(1-F^\omega(1-P))}{\mathbb{E}(y|m^y)} \right)}{\mathbb{E}(y|m^y)} < \varphi^+(0, \tilde{\omega}(0)) = 0,$$

which means that the policy-maker is unable to convince creditors to keep pledging to the bank, regardless of her chosen elicitation mechanism. Clearly, if best elicitation mechanism does not require long-term investors funding under $\lambda = 1$, it won't require it for $\lambda < 1$. Thus, the best liquidity provision program sets $P = 0$, which confirms that the optimal intervention will never involve the government, and the private sector at the same time. \square

Appendix D: General Mechanisms

In this section we consider general mechanisms and show that the solution found in section (4) is optimal in a broader sense than the one adopted in the main text. The mechanisms considered in this section differ from those considered in the main text in two aspects. First, I assume that the policy-maker may choose to observe θ , in addition to observing y , when conducting the asset quality stress test $\Gamma^{y,\theta}$.²⁹ Secondly, I assume that the policy-maker has commitment power and may choose in period 1 the set of recommendations and transfers she will conduct in the second period.

²⁹The leakage assumption made in the main text with respect to the information collected while conducting the asset quality review still applies. That is, any information about (y, θ) collected while conducting $\Gamma^{y,\theta}$ leaks.

Consider a mechanism $\Psi = \{\Gamma^{y,\theta}, s, P, \Gamma^\omega\}$, where the asset quality review $\Gamma^{y,\theta} \equiv \{\pi^{y,\theta}, M^{y,\theta}\}$ corresponds to a signal structure with disclosure rule $\pi^{y,\theta}$, and message space $M^{y,\theta}$:

$$\begin{aligned} \pi^{y,\theta} : \Theta \times \mathbb{R}_+ &\rightarrow \Delta M^{y,\theta} \\ (\theta, y) &\rightarrow \pi^{y,\theta}[\theta, y]; \end{aligned}$$

(s, P) corresponds to a public recommendation made to the bank and external investors, which are told to trade a security s at a price P , under the constraint that investors willingly accept the contract. I assume that (s, P) is measurable with respect to the public message disclosed by the designer, $m^{y,\theta}$, which rules out the possibility that the choice of (s, P) conveys information about (y, θ) , and therefore the only source of information about the quality of the asset will be given by message $m^{y,\theta}$;

The liquidity stress test $\Gamma^\omega = \{\pi^\omega, M^\omega\}$ conducted in period 2 is given by:

$$\begin{aligned} \pi^\omega : \Theta \times \mathbb{R}_+ \times M^{y,\theta} \times S \times \mathbb{R}_+ \times \Omega &\rightarrow \Delta M^\omega \\ (\theta, y, m^{y,\theta}, s, P, \omega) &\rightarrow \pi^\omega[s, P, \omega]. \end{aligned}$$

I rule out the possibility that Γ^ω depends on $(y, \theta, m^{y,\theta})$. This is without loss since at the moment of disclosing information about ω in period 2, the true value of (y, θ) is immaterial for short-term creditors. The message $m^{y,\theta}$, in turn, affects Γ^ω only through the price P . In fact, short-term creditors only care whether the (post-transfers) liquidity position $\omega + P$ is above the fraction of creditors running on the bank.

The contract (s, P) satisfies:

$$\begin{aligned} (s, P) : \quad \Theta \times \mathbb{R}_+ \times M^{y,\theta} &\rightarrow \Delta(S \times \mathbb{R}) \\ (\theta, y, m^{y,\theta}) &\rightarrow (s[m^{y,\theta}], P(m^{y,\theta})) \\ \text{s.t: } (a) \frac{1}{R} \mathbb{E}(1\{\omega + P > A(s, P, m^\omega)\} \times s | m^{y,\theta}, P, s) &\geq P. \end{aligned}$$

where $A(s, P, m^\omega)$ is the most aggressive fraction of creditors who run in period 2 after the public announcements (s, P, m^ω) . That is,

$$\begin{aligned} (b) \mathbb{E}(u(\omega + P, 1) | s, P, m^\omega) \leq 0 &\Rightarrow A(s, P, m^\omega) = 1, \\ (c) \mathbb{E}(u(\omega + P, 1) | s, P, m^\omega) > 0 &\Rightarrow A(s, P, m^\omega) = 0. \end{aligned}$$

Using an argument analogous to the one establishing lemma 5, we obtain that it is without loss to restrict attention to liquidity stress tests of the form $\Gamma^\omega = \{\{0, 1\}, \pi^\omega\}$ satisfying:

$$\begin{aligned} (b') \mathbb{E}(u(\omega + P, 1) | s, P, m^\omega = 1) &\leq 0 \\ (c') \mathbb{E}(u(\omega + P, 1) | s, P, m^\omega = 0) &> 0. \end{aligned}$$

Obedience of short-term creditors then requires that

$$(d) A(s, P, m^\omega) = m^\omega, \quad \forall m^\omega \in \{0, 1\}.$$

The designer thus maximizes:

$$\begin{aligned} & \max_{\Gamma^{y,\theta}, s, P, \Gamma^\omega, A} \mathbb{E}(W_0(A) \times 1\{\omega + P \geq A\}) \\ & \text{s.t.: } (a) - (d). \end{aligned}$$

I assume that the policy-maker has the authority to forbid the bank to net positive payoffs if it does not comply with the recommendation (s, P) . This is the reason why we do not specify incentive constraints for the bank. At the same time, this implies that it is without loss of optimality to set $s(y) \equiv y$, since this maximizes the amount of funds the bank may obtain from the market, which increases the likelihood of survival.³⁰ Next, we relax the problem by omitting (b'). This constraint is naturally satisfied at the optimum. Finally, we observe that under the obedience constraint (d), the policy-maker's objective can be reformulated as $W_0(0) \times \mathbb{E}(1 - m^\omega)$. The policy-maker's problem can then be written as:

$$\begin{aligned} & \max_{\Gamma^{y,\theta}, P, \Gamma^\omega} \mathbb{E}(1 - m^\omega) \\ & \text{s.t.: } (a') \quad \frac{1}{R} \mathbb{E}\left((1 - m^\omega) \times y | m^{y,\theta}, P\right) \geq P, \\ & \quad (c') \quad \mathbb{E}\left(u(\omega, P, 1) | m^{y,\theta}, P, m^\omega = 0\right) > 0 \end{aligned}$$

or equivalently as

$$\begin{aligned} & \max_{\Gamma^{y,\theta}, P, \Gamma^\omega} \int_{M^{y,\theta} \times \mathbb{R}_+ \times \Theta} \left(\int_{\Omega} \pi^\omega(0 | \omega, P(m^{y,\theta})) f^\omega(\omega) \right) \pi^{y,\theta}(m^{y,\theta} | y, \theta) f^y(y) \mu_\theta \\ & \text{s.t.: } (a'') \quad \frac{1}{R} \mathbb{E}(y | m^{y,\theta}) \times \int_{\Omega} \pi^\omega(0 | \omega, P(m^{y,\theta})) f^\omega(\omega) \geq P(m^{y,\theta}), \\ & \quad (c') \quad \mathbb{E}\left(u(\omega, P(m^{y,\theta}), 1) | P(m^{y,\theta}), m^\omega = 0\right) > 0 \end{aligned}$$

Fix an arbitrary message $m^{y,\theta} \in M^{y,\theta}$ and an arbitrary price $P(m^{y,\theta}) = P > 0$. Lemma 5 and Proposition 4 imply that the disclosure rule that maximizes $\int_{\Omega} \pi^\omega(0 | \omega, P) f^\omega(\omega)$ under constraint (c') is given by: $\{M^\omega = \{0, 1\}, \tilde{\pi}^\omega\}$ with $\tilde{\pi}^\omega(0 | P, \omega) = 1\{\omega \geq \bar{\omega}(P)\}$. As a result, the policy-maker's problem simplifies to:

$$\begin{aligned} & \max_{\{\pi^y, M^y\}, P} \mathbb{E}(1\{\omega \geq \bar{\omega}(P)\}) \\ & \text{s.t.: } (a'') \quad \frac{\mathbb{E}(y | m^{y,\theta})}{R} \times \mathbb{P}\{\omega \geq \bar{\omega}(P)\} \geq P, \end{aligned}$$

or equivalently,

$$\begin{aligned} & \max_{G^{\Gamma^y}} \int_0^\infty (1 - F^\omega(\bar{\omega}(\bar{P}(\tau)))) G^{\Gamma^y}(d\tau) \\ & \text{s.t.: } F^y \succeq_{\text{MPS}} G^{\Gamma^y}, \end{aligned}$$

³⁰Alternatively, we can assume that the policy-maker threatens the bank to conduct an adversarial liquidity stress test if she issues a security different from $s(\cdot) \equiv \cdot$.

which corresponds to the same problem considered in theorem 1, which proves that the solution found in section 4 is, in fact, optimal. \square

References

- Toni Ahnert, Kartik Anand, Prasanna Gai, and James Chapman. Asset encumbrance, bank funding, and fragility. *The Review of Financial Studies*, 32(6):2422–2455, 2018.
- Ricardo Alonso and Odilon Camara. Persuading voters. *American Economic Review*, 106(11):3590–3605, 2016a.
- Ricardo Alonso and Odilon Camara. Bayesian persuasion with heterogeneous priors. *Journal of Economic Theory*, 165:672–706, 2016b.
- Fernando Alvarez and Gadi Barlevy. Mandatory disclosure and financial contagion. *WP, NBER*, 2015.
- Ehsan Azarmsa and Lin William Cong. Persuasion in relationship finance. *Becker Friedman Institute for Research in Economics Working Paper*, (2017-16), 2018.
- Ana Babus and Maryam Farboodi. The hidden costs of strategic opacity. Technical report, mimeo. Washington University in St. Louis, 2018.
- Arjada Bardhi and Yingni Guo. Modes of persuasion toward unanimous consent. *Theoretical Economics*, forthcoming, 2017.
- Deepal Basak and Zhen Zhou. Diffusing coordination risk. *WP, NYU*, 2017.
- Dirk Bergemann and Stephen Morris. Bayes correlated equilibrium and the comparison of information structures in games. *Theoretical Economics*, 11(2):487–522, 2016a.
- Dirk Bergemann and Stephen Morris. Information design, bayesian persuasion and bayes correlated equilibrium. *The American Economic Review*, 106(5):586–591, 2016b.
- Ben S Bernanke. Stress testing banks: What have we learned? In *Maintaining Financial Stability: Holding a Tiger by the Tail (conference volume)*, 2013.
- Matthieu Bouvard, Pierre Chaigneau, and Adolfo de Motta. Transparency in the financial system: Rollover risk and crises. *The Journal of Finance*, 70(4):1805–1837, 2015.
- Charles W Calomiris and Charles M Kahn. The role of demandable debt in structuring optimal banking arrangements. *The American Economic Review*, pages 497–513, 1991.
- Giacomo Calzolari and Alessandro Pavan. On the optimality of privacy in sequential contracting. *Journal of Economic theory*, 130(1):168–204, 2006a.
- Ing-Haw Cheng and Konstantin Milbradt. The hazards of debt: Rollover freezes, incentives, and bailouts. *The Review of Financial Studies*, 25(4):1070–1110, 2012.

- Brendan Daley, Brett S Green, and Victoria Vanasco. Security design with ratings. *Available at SSRN 2940791*, 2016.
- Peter DeMarzo and Darrell Duffie. A liquidity-based model of security design. *Econometrica*, 67(1): 65–99, 1999.
- Peter M DeMarzo and Michael J Fishman. Optimal long-term financial contracting. *The Review of Financial Studies*, 20(6):2079–2128, 2007.
- Laura Doval and Jeffrey Ely. Sequential information design. *WP, Northwestern University*, 2017.
- Piotr Dworzak. Mechanism design with aftermarkets: Cutoff mechanisms. *WP, Stanford*, 2016.
- Thomas M Eisenbach. Rollover risk as market discipline: A two-sided inefficiency. *Journal of Financial Economics*, 126(2):252–269, 2017.
- Jeffrey C Ely. Beeps. *The American Economic Review*, 107(1):31–53, 2017.
- Miguel Faria-e Castro, Joseba Martinez, and Thomas Philippon. Runs versus lemons: information disclosure and fiscal capacity. *The Review of Economic Studies*, page 060, 2016.
- Mark Flannery, Beverl Hirtleb, and Anna Kovner. Evaluating the information in the federal reserve stress tests. *Journal of Financial Intermediation*, 29(29):1–18, 2017.
- Mark J Flannery, Simon H Kwan, and Mahendrarajah Nimalendran. The 2007–2009 financial crisis and bank opaqueness. *Journal of Financial Intermediation*, 22(1):55–84, 2013.
- William Fuchs and Andrzej Skrzypacz. Government interventions in a dynamic market with adverse selection. *Journal of Economic Theory*, 158:371–406, 2015.
- Matthew Gentzkow and Emir Kamenica. A rothschild-stiglitz approach to bayesian persuasion. *American Economic Review*, 106(5):597–601, 2016.
- Itay Goldstein and Yaron Leitner. Stress tests and information disclosure. *Journal of Economic Theory*, 177:34–69, 2018.
- Itay Goldstein and Haresh Sapra. Should banks’ stress test results be disclosed? an analysis of the costs and benefits. *Foundations and Trends (R) in Finance*, 8(1):1–54, 2014.
- Beverly Hirtle and Andreas Lehnert. Supervisory stress tests. *Annual Review of Financial Economics*, 7:339–355, 2015.
- Nicolas Inostroza and Alessandro Pavan. Persuasion in global games with application to stress testing, 2018.

- Emir Kamenica and Matthew Gentzkow. Bayesian persuasion. *American Economic Review*, 101: 2590–2615, 2011.
- Emir Kamenica and Matthew Gentzkow. A Rothschild-Stiglitz approach to bayesian persuasion. *American Economic Review*, 106(5):597–601, 2016.
- Anton Kolotilin, Tymofiy Mylovanov, Andriy Zapechelnuk, and Ming Li. Persuasion of a privately informed receiver. *Econometrica*, 85(6):1949–1964, 2017.
- Augustin Landier and Kenichi Ueda. *The economics of bank restructuring: Understanding the options*. Number 2009-2012. International Monetary Fund, 2009.
- Laurent Mathevet, Jacopo Perego, and Ina Taneva. Information design: The epistemic approach. *WP, NYU*, 2016.
- Donal Morgan, Stravos Persitani, and Savino Vanessa. The information value of the stress test. *Journal of Money, Credit and Banking*, Vol. 46, No. 7, 2014.
- Donald P Morgan. Rating banks: Risk and uncertainty in an opaque industry. *American Economic Review*, 92(4):874–888, 2002.
- Alfred Müller and Dietrich Stoyan. *Comparison methods for stochastic models and risks*, volume 389. Wiley New York, 2002.
- Stewart C Myers and Nicholas S Majluf. Corporate financing and investment decisions when firms have information that investors do not have. *Journal of financial economics*, 13(2):187–221, 1984.
- Roger B Myerson. Multistage games with communication. *Econometrica*, pages 323–358, 1986.
- David C Nachman and Thomas H Noe. Optimal design of securities under asymmetric information. *The Review of Financial Studies*, 7(1):1–44, 1994.
- Dmitry Orlov, Pavel Zryumov, and Andrzej Skrzypacz. Design of macro-prudential stress tests. *WP, Stanford University*, 2017.
- Giovanni Petrella and Andrea Resti. Supervisors as information producers: Do stress tests reduce bank opaqueness? *Journal of Banking and Finance*, 37(12):5406–5420, 2013.
- Thomas Philippon and Vasiliki Skreta. Optimal interventions in markets with adverse selection. *American Economic Review*, 102(1):1–28, 2012.
- Martin Szydlowski. Optimal financing and disclosure. *mimeo. University of Minnesota*, 2018.
- Ina A Taneva. Information design. *WP, University of Edinburgh*, 2016.

Jean Tirole. Overcoming adverse selection: How public intervention can restore market functioning. *American Economic Review*, 102(1):29–59, 2012.

Ming Yang. Coordination with flexible information acquisition. *Journal of Economic Theory*, 158: 721–738, 2015.