Lev eraging Exp erienced Consumers to Attract New Consumers: An Equilibrium Analysis of Displaying Deal Sales to Sell Daily Deals

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Abstract

Daily deal websites help small local merchants to attract new consumers. A strategy adopted by some websites is to continually track and display the number of deals sold by a merchant. We investigate the strategic implications of displaying deal sales and the website's incentive to implement this feature. We analyze a market in which a merchant offering an experience good is privately informed of its type. While daily deals cannibalize a merchant’s revenue from experienced consumers, we show that, by displaying deal sales, the website can transform this cannibalization into an advantage. Displaying deal sales can leverage discounted sales to experienced consumers to help a high-quality merchant signal its type and acquire new consumers at a higher margin. Signaling is supported through observational learning by new consumers from displayed deal sales. Nevertheless, the website may not implement this feature if signaling entails too much distortion in the merchant’s deal price. We also find that it can be optimal for the website to offer the merchant an upfront subsidy, but only if the website displays deal sales. Our analysis leads to managerial insights for daily deal websites.

(Keywords: Daily Deals, Observational Learning, Perfect Bayesian Equilibrium, Signaling)

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As customers, we like the counter because it indicates how popular deals are. - Director of Communications, Groupon (Groupon 2011)

We were concerned that the counter was having a negative impact on the consumers’ perception of the deal. - VP of Research, TroopSwap

1 Introduction

Daily deal websites have emerged as a popular means of conducting online promotions to attract new consumers for small local merchants. Some daily deal websites continually track and display the number of deals sold by a merchant through a ‘deal counter’. However, not all websites have adopted this strategy (e.g., AP Daily Deals, Restaurant.com, ValPak), and some websites that previously did no longer do (e.g., Dealsaver, KGB Deals, Tippr). As the opening comments (from websites Groupon and TroopSwap) indicate, while displaying deal sales can be useful to consumers, its impact on the website and the merchant are not immediately clear. Thus, the rationale for this website strategy requires further investigation. In this paper, we examine the strategic implications of displaying deal sales and the website’s incentive to do so. We analyze a model of strategic interaction between a daily deal website, merchant, and consumers to understand whether the website displaying deal sales is an equilibrium outcome. Our analysis leads to some managerial insights and recommendations for daily deal websites.

The daily deal website is one among many innovative business models that have emerged on the Internet. Daily deals are so called because new deals from different merchants are announced on the website every day. Each deal is available on the website for a specified period of time, ranging from a few days to a week or two. Most deals target consumers in a given city and are offered by merchants in that city, such as restaurants, spas, and gyms. In the U.S. alone, consumer spending on daily deals is estimated to have grown from $873 million in 2010 to $3.6 billion in 2012 and is expected to exceed $5 billion by 2015 (BIA/Kelsey 2011, 2012). Daily deal websites have also grown in their significance in many countries across the world. Thus, it is important to understand the strategies of daily deal websites.

At first glance, a daily deal website might appear to be simply an online counterpart to a traditional coupon mailer company, which distributes coupons from local merchants to consumers by mail. Similar to coupon mailers, daily deals serve to increase awareness of a merchant amongst potential consumers, and entice new consumers through discounts. For example, in a recent survey of small businesses, a majority identified daily deals as the most effective online tool to attract new consumers.
consumers (Clancy 2013). Similarly, other small business surveys have found that a majority of daily deal consumers are new to the business (Dholakia 2011, 2012; Edison Research 2012).

A closer look, however, brings forth important differences between the two. A coupon mailer company is not involved in the purchase transaction between the merchant and a consumer. To use a coupon, a consumer simply presents it to the merchant at the time of her purchase. In contrast, a daily deal website functions as a marketplace that also consummates the purchase transaction. This is made possible by the reduced cost of interactions on the Internet. To avail a daily deal, a consumer must purchase the deal upfront through the website, and redeem it later at the merchant. As a result, unlike a coupon mailer company, a daily deal website is able to monitor consumer purchases linked to a deal with great ease. How can the website use this capability? As noted earlier, an interesting strategy used by some daily deal websites is to continually track and display deal sales. Who benefits from this? Can this be an equilibrium outcome? If this strategy is profitable to the website, are there ways to make it more effective? We seek to address these questions in this paper. We believe that these questions are of managerial interest. For instance, the management at Groupon and TroopSwap were concerned about how displaying deal sales influenced consumers and whether displaying deal sales hurt their business (Groupon 2011; Vasilaky 2012).

A daily deal website also differs from a coupon mailer company in its business model. A daily deal website receives a share of the revenue from deal sales. Hence, the website is paid only if a sale occurs, i.e., it is paid for performance. Again, this is made possible by the website’s ability to monitor transactions. In contrast, a coupon mailer company cannot easily monitor transactions and is, therefore, paid an upfront fixed fee. In this paper, we also analyze the equilibrium revenue-sharing contract between the website and the merchant.

To shed light on these issues, we develop a theoretical model that explicitly considers the strategic interaction between a daily deal website, merchant, and consumers. We analyze a market in which a merchant offering an experience good is privately informed about its type (probability of meeting consumer needs), and can reach new consumers by offering a deal through a daily deal website. As in other forms of discount promotion, a daily deal cannibalizes the merchant’s regular sales, as the deal can be availed by experienced consumers who would have otherwise bought from the merchant at the regular price. We show, however, that displaying deal sales can help the merchant leverage the discounted sales to experienced consumers to attract new consumers. In particular, displaying deal sales may allow a high-type merchant to credibly signal its type to acquire new consumers at a higher margin. Signaling is supported through observational learning: new consumers can infer
the merchant’s type from displayed deal sales, which reflects how other (experienced) consumers responded to the deal. Thus, in contrast to a traditional coupon mailer company, a daily deal website can unlock the informational value of experienced consumers.

We find that displaying deal sales is, however, not a dominant strategy. The website implements this feature only if observational learning allows the merchant to signal without too much distortion in its deal price. It is important to note that the website has a role in determining whether the merchant can signal in equilibrium. Our model of a strategic website brings this out clearly. We also investigate the equilibrium contract between the website and the merchant. We find that it can be optimal for the website to offer the merchant an upfront subsidy, but only if the website displays sales. Our analysis leads to managerial insights that we discuss in the Conclusion.

1.1 Related Literature

To put our research in perspective, we briefly review several related streams of literature. We discuss in turn literature on the rationale for displaying deal sales for daily deals; empirical research on how consumers may infer quality through observational learning; firm strategies in the presence of observational learning; signaling quality especially through price and additional instruments; and how firms can leverage social interactions to make marketing more effective.

Early research on daily deals has focused on situations in which there is a minimum number of deals that must be sold before the deal is valid (Anand and Aron 2003; Jing and Xie 2011). Only Hu, Shi, and Wu (2013) have found a strategic role for displaying sales in the presence of minimum limits. They show that displaying deal sales informs consumers whether the minimum limit will be reached, thereby coordinating their buying decisions. They find that this always benefits the seller. We identify a different strategic role for displaying deal sales in the absence of minimum limits. This is relevant because daily deal websites such as Groupon, LivingSocial and AmazonLocal display deal sales even though they do not use minimum limits.

Empirical research has shown that consumers may engage in observational learning by drawing quality inferences by observing others’ choices. Zhang (2010) finds that, in the U.S. kidney market, patients draw negative quality inferences from earlier refusals by other patients to accept a kidney that is available for transplant. Through counterfactual simulations, she shows that while observational learning improves patient decisions, patients would benefit further if the reasons for kidney refusals could be shared. There is also evidence that a website can facilitate observational learning and influence consumer decisions by displaying popularity information (Chen and Xie, 2008; Tucker and Zhang, 2011). In particular, Luo, Andrews, Song, and Aspara (2014) find that deal sales infor-
mation facilitates observational learning on a daily deal website. Zhang and Liu (2012) find evidence of observational learning on a crowdfunding website, wherein lenders infer the creditworthiness of the borrower from the funding level. The question remains whether providing consumers such information is beneficial for the website. We address this question using a theoretical framework.

Past research has studied firm strategies assuming consumers can infer product quality by observing past sales. Caminal and Vives (1996) show that firms may compete more aggressively for market share in order to signal-jam consumer inferences. Bose, Orosel, Ottaviani, and Vesterlund (2006) show that the firm may distort its price to current buyers to facilitate information revelation to future buyers. Taylor (1999) shows that, in housing markets, an individual house seller may distort its price in order to minimize the negative inferences associated with her house remaining unsold. Miklós-Thal and Zhang (2013) show that a monopolist may visibly de-market its product to early adopters in order to improve the product's quality image amongst late adopters. We examine whether an intermediary, namely the daily deal website, should enable consumers to observe deal sales. We show that displaying deal sales can allow the high-type merchant to credibly charge a higher deal price, which in turn leads to higher website profit under some conditions.

Starting with the seminal works of Nelson (1974), Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986), past research has examined more broadly how a firm can signal its private information about quality to consumers. In our model, there is an intermediary, who is also strategic. We show that the website plays a crucial role in determining whether and how a merchant can convey private information to consumers. Indeed, we find that under some conditions the high-type merchant would prefer to signal its type, but is unable to do so because it is not in the interest of the website. Chu and Chu (1994) show that if a manufacturer is unable to signal its product quality, then the retailer carrying its product may be able to signal on the manufacturer’s behalf by making a non-salvageable investment. In our setting, the website determines whether consumers can engage in observational learning and, thereby, whether the merchant can signal to consumers. In this manner, we add to the extant literature.

Turning specifically to the role of price in revealing private information, researchers have examined whether price alone can signal a firm’s privately known quality.² Milgrom and Roberts (1986) show that price alone or price and non-informative advertising can signal a firm’s quality in a setting with repeat purchases. Desai (2000) shows that a manufacturer may use a combination of

²Researchers have also examined disclosure of verifiable quality information (e.g., Grossman 1981; Milgrom 1981; Guo and Zhao 2009; Kuksov and Lin 2010; Sun 2011). We study situations in which the merchant cannot disclose its type credibly and the website does not know the merchant's type.
wholesale price, slotting allowance, and advertising to signal demand for its product to a retailer. Moorthy and Srinivasan (1995) show that a combination of price and money-back guarantee may be necessary to signal product quality. Simester (1995) and Shin (2005) show that advertising prices of selected products can credibly signal the price image of a low-cost retailer. Our work has some similarity to Bagwell and Riordan (1991), who examine the role of informed consumers in enabling the high-quality firm to signal through price. We study situations in which only if the website displays deal sales do informed consumers play a role in enabling signaling, and that role is an indirect one. Stock and Balachander (2005) also find that a firm may not be able to signal its privately known quality unless consumers are aware of its product’s scarcity. We add to this literature by showing that for credible separation in deal price, consumers must both observe deal sales and engage in observational learning, thus requiring the website to implement a deal counter.

Our work is also broadly related to research on firm-level marketing strategies to leverage different forms of social interactions (e.g., Biyalogorsky, Gerstner, and Libai 2001; Amaldoss and Jain 2005; Godes et al. 2005; Mayzlin 2006; Chen and Xie 2008; Joshi et al. 2009; Kornish and Li 2010; Kuksov and Xie 2010; Jing 2011; Godes 2012). We study whether a daily deal website should allow a merchant to leverage one form of social influence, namely observational learning, to attract new consumers. Finally, we should note that researchers have also examined a website’s incentives in helping consumers make more informed decisions in various contexts. Wu, Zhang, and Padmanabhan (2013) show that a match-making website may have an incentive to deliberately reduce the effectiveness of it matching technology. Liu and Dukes (2014) show that an online shopping intermediary may design a search environment that limits search by consumers. We show that a website may not display deal sales in order to prevent observational learning by consumers.

2 Model
Our model consists of three strategic players, namely, a daily deal website, a merchant, and consumers. The daily deal website enables the merchant to reach new consumers by offering a deal through its website. We have in mind a setting in which the merchant’s product (or service) is an experience good, such as a restaurant, a spa or a gym. New consumers to the merchant are uncertain about whether the product can meet their needs. The merchant has private information about the probability that its product meets their needs. Our main interest is to examine the website’s strategic incentive to implement a certain website feature, namely, a deal counter that keeps track of and displays the number of deals sold. We model the following decisions. The website decides whether to display deal sales by implementing a deal counter. The merchant chooses whether to
offer a deal and the deal price. Consumers decide whether to buy the deal. Some consumers also
decide when to buy the deal. If the website implements a deal counter, then consumers can see the
information displayed by the counter before making their decisions. Later, in §4, we also examine
the website’s optimal revenue-sharing contract. We start by describing the merchant.

The merchant can be one of two types - \( H \) or \( L \). A type \( t \in \{ H, L \} \) merchant’s product meets
the needs of a proportion \( \alpha_t \in (0, 1) \) of consumers, where \( \alpha_H > \alpha_L \). A consumer derives a positive
utility \( r > 0 \) if the product meets her need, and zero utility otherwise. Thus, for a randomly chosen
consumer, the product meets the need with probability \( \alpha_t \). Hence, we refer to \( \alpha_t \) as the merchant’s
probability of fit, or simply fit. A merchant’s fit can be understood as how broadly its product will
appeal to consumers, and could be based on its ability to cater to the disparate needs of different
consumers. For example, restaurants differ in the flavors that they use. A restaurant that has
adapted its flavors to local tastes would appeal to a broad group of consumers, and correspond
to a high-fit merchant. A restaurant that is more authentic or more avant-garde may appeal to
a niche group of consumers, and correspond to a low-fit merchant. Or, in the case of hair salons,
consumers differ in their hair conditions and in their treatment preferences. A hair stylist that is
more experienced (less experienced) in different hair conditions and treatment procedures would
correspond to a high-fit (low-fit) merchant and can satisfy needs of a wide range (narrow range) of
consumers.

The merchant faces a mix of experienced consumers and new consumers on the daily deal
website. Because the merchant’s product is an experience good, consumers can know whether it
meets their needs only if they have tried its product in the past. Experienced consumers have tried
the merchant’s product in the past. Consequently, they are already aware of the merchant, know
the merchant’s type, and whether its product meets their needs. In contrast, new consumers become
aware of the merchant only if it offers a deal. Hence, they neither know the merchant’s type nor
whether its product will meet their needs. Let \( N \) denote the size of new consumers. Without loss
of generality, we normalize the size of experienced consumers to 1. Thus \( N \) captures the relative
proportion of new consumers on the website.

The difference between experienced consumers and new consumers becomes clear if we write
down their utility from buying the deal. We assume that a consumer may buy at most one unit of
the product and derives zero utility if she does not buy. Let \( d_t > 0 \) denote the deal price at which
a type \( t \) merchant offers the product. An experienced consumer’s utility from buying the deal is

\[ u_{EC} = i \cdot r - d_t, \]  

(1)
where $i \in \{0, 1\}$ is an indicator variable that equals 1 iff the product meets this consumer’s need. An experienced consumer knows both $i$ and $t$. We refer to an experienced consumer for whom $i = 1$ as a \textit{matched consumer} because the product meets her need. Only a matched consumer is willing to pay a positive price for the product. The number of matched consumers is equal to $\alpha_t$ for a type $t$ merchant. Thus, the type $H$ merchant has more matched consumers than a type $L$ merchant.

Unlike experienced consumers, a new consumer is uncertain about the merchant’s type and whether its product will meet her needs. Her expected utility from buying the deal is

$$u_{NC} = \theta r\alpha_H + (1 - \theta) r\alpha_L - d_t.$$  

(2)

where $\theta \in [0, 1]$ denotes her belief that the merchant’s type is $H$, and $r\alpha_t$ is her expected utility from the product conditional on the merchant’s type being $t$. A new consumer is, thus, willing to pay a positive price for the product. In particular, this is true even if she believes that the merchant is type $L$ ($\theta = 0$) because there is a positive probability ($= \alpha_L$) that the product meets her needs. A new consumer’s willingness to pay is increasing in her belief $\theta$. It is, however, always lower than that of matched consumers who are certain that the product meets their need. In general, a new consumer’s belief $\theta$ may depend on all observables including the deal price, and deal sales information (if displayed).

To study the implications of displaying deal sales, we will require that the deal is offered more than one period. We assume that a deal is available on the website for two periods, namely periods 1 and 2. We assume that some consumers visit the website in both periods and refer to them as \textit{frequent visitors}. Frequent visitors can buy the deal in either period. Other consumers are not able to visit the website frequently. They visit the website either only in period 1 or only in period 2, and we refer to them as \textit{early visitors} and \textit{late visitors}, respectively. The frequency of visits is an exogenous feature of our model. We note that consumers typically do not know a priori whether or when a particular merchant will offer a deal. Therefore, we model the visits of early and late visitors as being random relative to the deal timing (period 1) and assume that there is an equal number of either of them.\footnote{If some early visitors are able to return in period 2 for a specific deal, then this will be equivalent in our model to assuming that there are more frequent visitors.} Let $\beta \in [0, 1]$ denote the proportion of consumers who visit the website only once. Thus, a proportion $\frac{1}{2}\beta$ are early visitors, a proportion $\frac{1}{2}\beta$ are late visitors and a proportion $1 - \beta$ are frequent visitors. These proportions are the same for experienced consumers and new consumers. Thus, each consumer can be characterized along two dimensions - experienced vs. new, and frequent vs. early vs. late. For conciseness, we will say “early-new consumers” to refer to new
consumers who are early visitors, and so on.

Each consumer segment in our model will play an important role in determining the impact of displaying deal sales. Experienced consumers who can buy the deal early (early- and frequent-visitors) are important because their buying decisions can make deal sales informative. This is because the number of matched consumers depends on the merchant’s type. New consumers who can buy the deal later (frequent- and late-visitors) are important because they can potentially infer the merchant’s type through observational learning from displayed deal sales. Together, these consumer segments will determine the impact of displaying deal sales. Our analysis will determine whether and how firms leverage this interaction between consumer segments in equilibrium.

We now describe the situation if the merchant does not offer a deal. In this case, we assume that the experienced consumers from the website can still buy from the merchant at its regular price. Let $p$ denote the merchant’s regular price, where $p \leq r$. We take the regular price $p$ to be exogenous. Because daily deals are of relatively short duration and offered infrequently (to attract new consumers rather than to promote to existing ones; see, for example, Dholakia 2011), we do not expect a daily deal promotion to affect the merchant’s regular price. Thus, we assume that there are significant “menu costs” to changing the regular price. Hence, the merchant decides its daily deal strategy taking its regular price as given. We then take the regular price to be exogenous in order to keep our analysis focused on the interactions on the daily deal website. What will be important for our main insights is that the regular price is higher than the price at which new consumers are willing to buy from the merchant. To ensure that this occurs in equilibrium, we assume that the following sufficient condition holds: $p > r\alpha_H$, where $r\alpha_H$ is the upper bound for the maximum price that new consumers will pay (as seen from equation (2)).

Let $R_t^O$ denote the revenue for a type $t$ merchant from selling to experienced consumers at the regular price.

Next, we describe the revenue-sharing arrangement between the website and the merchant. Let $R_t^D$ denote the deal revenue for a type $t$ merchant if it offers a deal. Let $\lambda \in (0, 1)$ denote the merchant’s share of deal revenue. Let $\Pi^W$ denote the website’s expected profit. We assume that the merchant has zero marginal cost. The revenue-sharing arrangement can be profitable for both the merchant and the website as long as offering the deal generates incremental revenue, i.e., $R_t^D > R_t^O$. To begin with, we will assume that $\lambda$ is exogenous and sufficiently high such that if a deal can generate incremental revenue, then it will also be profitable for the merchant to offer.

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4 One might expect that allowing $p$ to be determined endogenously will result in $p = r$, which is the reservation price of matched consumers. This is indeed the case, and is allowed by our restriction $p > r\alpha_H$. Thus, endogenizing $p$ will not affect our results. We thank two anonymous reviewers for this suggestion.
a deal. Assuming $\lambda$ to be exogenous in this manner allows us to bring out the essential effects of displaying deal sales on consumers and the merchant, and thereby the website’s incentives to display deal sales. Having established these effects, we later let $\lambda$ be endogenous and examine the website’s optimal equilibrium contract in §4.

We should note that our assumption of zero marginal cost is not without loss of generality. With non-zero marginal cost, the merchant’s profit from deal sales will equal its share of deal revenue $\lambda R^D_t$. In particular, if the marginal cost is a high enough fraction of the deal price (higher than $\lambda$), then offering a deal can never be profitable. Nevertheless, as long as offering the deal generates incremental profit (after accounting for the merchant’s marginal cost), it is still the case that the revenue-sharing arrangement will be mutually profitable if $\lambda$ is sufficiently high. Consequently, while a non-zero marginal cost imposes additional restrictions on $\lambda$, it does not qualitatively affect our main insights. Hence, for clarity of exposition, we assume zero marginal cost.\footnote{We thank the AE for this suggestion.}

Having described the merchant and consumer decisions, we turn to the website’s decision. The website can choose between two possible website regimes: (i) displaying deal sales by implementing a deal counter, and (ii) not displaying deal sales by not implementing a deal counter.\footnote{We remark that the common practice on websites such as Groupon, Living Social and AmazonLocal is to display the counter for all deals, i.e., it is website feature, and not specific to a deal.} We refer to these website regimes as transparent and opaque respectively. In the transparent website regime, the deal counter is displayed every period, and updated at the start of each period. In other words, the counter displays zero at the start of period 1, and the number of deals sold in period 1 at the start of period 2. The website regime is known to the merchant and to consumers.

It is important to note that we assume that it is too costly for the website to misreport sales. The implication of this assumption, and an essential condition in our analysis, is that consumers believe that the website will not deceive them by misreporting sales. Thus, we identify how displaying deal sales can benefit or hurt the website in this boundary case.\footnote{We thank an anonymous reviewer for suggesting this interpretation.} Misreporting sales can be costly for two reasons. First, laws against false or deceptive advertising can act as a constraint.\footnote{In our discussion with website managers, we found that laws against false advertising do act as a constraint.} For example, the website would have engaged in deceptive advertising if misreporting sales caused consumers to buy a deal they otherwise would not have. Second, misreporting sales in our model can be detected by experienced consumers. Experienced consumers on one occasion (in one merchant or product category), can be new consumers on a future occasion. Consequently, misreporting sales can hurt the website in future interactions with the same consumers, as they would no longer believe that
the website reports sales truthfully. In other words, it can be in the website’s interest to report sales truthfully in order to maintain its reputation.\footnote{See Chu and Chu (1994) for a formal development of the idea that reputational concerns prevent a firm from deceiving consumers in a signaling framework with repeated interactions. We thank an anonymous reviewer for this suggestion.}

We also assume that it is not feasible for the merchant to significantly manipulate deal sales by buying its own deal. Daily deal websites typically do not allow bulk purchases, presumably to prevent reselling of these deals.\footnote{For instance, most deals on Groupon have a limit of two purchases per person (including one as a gift).} Therefore, to significantly manipulate deal sales, a merchant would have to create several user accounts as well as use different credit cards to make purchases.

We assume that doing so is too cumbersome and, hence, not feasible. Before we proceed to the analysis, it is useful to make clear the sequence of the game:

**Stage 1 (Period 0):** Website decides whether or not to implement a deal counter.

**Stage 2 (Period 0):** Merchant decides whether or not to offer a deal, and the deal price $d_t$ if it will offer a deal.

**Period 1 (if deal is offered):** Deal sales counter, if implemented, displays 0. Early and frequent visitors visit the website and decide whether to buy the deal. Frequent visitors can also decide to wait till period 2.

**Period 2 (if deal is offered):** Deal sales counter, if implemented, displays number of deals sold in period 1. Frequent and late visitors visit the website and decide whether to buy the deal.

We assume that the merchant knows its type $t$ and this is private information. The website and new consumers have a belief about the type of the merchant and this is common knowledge. This belief can be conditioned on the regular price $p$ and other information about the attributes of the merchant. Nevertheless, it will be useful to think of this belief as the initial or prior belief about the merchant’s type. Denote this belief by $\theta^0 \in (0, 1)$. The prior belief $\theta^0$ can be contrasted with the belief $\theta$ used in equation (2), which is a new consumer’s posterior belief at the time of making her buying decision. This posterior belief will depend on the website regime, the merchant’s decision to offer a deal, the deal price, and the deal sales information if it is displayed.

It is useful to think about the information that the website has about the merchant’s type. Daily deal websites utilize a relatively low-skilled sales force to market their services to a large number of small merchants. As a result, qualifying the merchants is often impractical as it makes the selling process time-consuming and effortful.\footnote{In our discussion with Groupon, we found that their sales team does not spend time qualifying merchants.} In particular, it would be especially difficult for a low-skilled sales person, without any specialized expertise about a merchant’s product category, to determine
the merchant’s fit. In other words, it is too costly for the website to verify the merchant’s type.

We assume that firms maximize their expected profits and consumers maximize their expected utility. We solve for a perfect Bayesian equilibrium (PBE). We restrict our attention to pure-strategy equilibria. It is well known that in games of incomplete information, multiple PBE can be supported by specifying sufficiently pessimistic off-equilibrium beliefs so as to make any deviation unattractive. In our context, this pertains to specifying pessimistic beliefs for new consumers (i.e., $\theta = 0$) at off-equilibrium deal prices, leading to multiple equilibria for merchant strategies. As in Miklós-Thal and Zhang (2013), because the intuitive criterion refinement (Cho and Kreps 1987) is not sufficiently strong to rule out unreasonable off-equilibrium beliefs in our model, we use the strongly-undefeated equilibrium (SUE) refinement (Mailath, Okuno-Fujiwara, and Postlewaite 1993; Spiegel and Spulber 1997; Taylor 1999; Mezzetti and Tsoulouhas 2000; Gomes 2000; Gill and Sgroi 2012) to obtain a unique equilibrium. As noted in Miklós-Thal and Zhang (2013), the SUE refinement is equivalent to selecting the PBE that yields the type $H$ merchant the highest profit (amongst all PBEs). This property has intuitive appeal because it is the type $L$ merchant that will have an incentive to mimic the type $H$ merchant and not vice versa. Said differently, in an SUE, the type $H$ merchant follows its sequentially optimal strategy given that the type $L$ merchant can mimic its strategy. It is important to note that, in our model, the SUE is also the unique PBE surviving the intuitive criterion that yields the highest profit for both merchant types. We provide a more formal description of the SUE refinement in Appendix A. Without loss of generality, we normalize $r = 1$ in the remainder of our analysis. Appendix E summarizes our model notation.

### 3 Strategic Implications of Displaying Deal Sales

We develop our insights regarding the strategic implications of displaying deal sales by identifying sufficient conditions for each website regime to occur in equilibrium. We proceed as follows. First, in §3.1, we establish an upper bound for the website’s equilibrium profit and show that this can be attained only in a separating equilibrium. Then, in §3.2, we show that the separating equilibrium in which the website attains the upper bound occurs under certain conditions, but only if the website displays deal sales and not otherwise. We thus obtain a sufficient condition for the website to display deal sales in equilibrium. Next, in §3.3, we obtain a sufficient condition for the website not to display deal sales in equilibrium. This occurs in a pooling equilibrium. Finally, in §3.4, we complete the analysis by examining the remaining situations.

Three observations help simplify our analysis. First, the merchant’s profit from the deal is $\lambda R^D_t$, where $\lambda \in (0, 1)$. Therefore, the merchant’s equilibrium deal price must be set to maximize
the deal revenue $R^D_t$. Second, if no new consumers buy the deal, then the deal does not generate incremental revenue ($R^D_t \leq R^O_t$), and, hence, cannot be mutually profitable for the merchant and the website. Therefore, at least some new consumers must buy the deal in equilibrium, if a deal is offered. It follows that it is sufficient to focus on equilibrium deal price $d_t \in [\alpha_L, \alpha_H]$, since the reservation price of new consumers is in this price range. Finally, the regular price $p > \alpha_H$ is higher than the equilibrium deal price $d_t \in [\alpha_L, \alpha_H]$. Hence, all matched consumers will buy the deal in equilibrium, if a deal is offered. In other words, deal sales cannibalizes the merchant’s regular sales to experienced consumers.

3.1 Upper Bound for Website’s Equilibrium Profit

We obtain the upper bound for the website’s equilibrium profit from a benchmark setting in which new consumers are assumed to know the merchant’s type $t$. We refer to this setting as the *symmetric information benchmark*, noting that the source of asymmetric information in our model is about the merchant’s type. If new consumers know the merchant’s type $t$, then they will be willing to pay $\alpha_t$. Recall that offering a deal can be profitable only if at least some new consumers buy the deal. Therefore, conditional on offering a deal, the optimal deal price for a type $t$ merchant is $d_t = \alpha_t$. All new consumers and matched consumers will buy the deal, resulting in deal revenue of

$$R^D_t = \alpha_t (\alpha_t + N),$$

(3)

If the type $t$ merchant does not offer a deal, then it sells to matched consumers at its regular price, and obtains revenue of

$$R^O_t = p\alpha_t.$$  

(4)

Offering the deal is profitable for the merchant if it generates incremental revenue, i.e., if $R^D_t > R^O_t$ so that $\lambda R^D_t > R^O_t$ for sufficiently high $\lambda \in (0, 1)$. Hence, the type $t$ merchant offers a deal if

$$\alpha_t (\alpha_t + N) > p\alpha_t.$$ 

(5)

The foregoing inequality will hold if $N$ is sufficiently large. In other words, offering a deal is profitable if there are sufficient number of new consumers on the website (relative to the number of experienced consumers). To keep our analysis straightforward, we will assume that $N > p$, which is sufficient to ensure that condition (5) holds for both merchant types. In other words, offering a deal is efficient in the absence of asymmetric information.

To summarize, in the symmetric information benchmark, the deal price reflects the merchant’s fit, i.e., $d_t = \alpha_t$. The type $H$ merchant charges a higher deal price due to its higher fit. All matched consumers buy the deal and obtain a surplus of $p - \alpha_t$ (relative to buying at the regular price). All
new consumers also buy the deal and their ex-ante surplus is zero. The website’s expected profit is

$$\Pi_{si}^W = (1 - \lambda) (\bar{\theta} \alpha_H (\alpha_H + N) + (1 - \bar{\theta}) \alpha_L (\alpha_L + N))$$  \hspace{1cm} (6)$$

We now show in Lemma 1 that the website’s equilibrium expected profit cannot exceed $$\Pi_{si}^W$$. Moreover, the website can attain this upper bound only in a separating equilibrium that replicates the benchmark outcome, i.e., an equilibrium in which the type $$t$$ merchant sets $$d_t = \alpha_t$$ and all matched consumers and new consumers buy the deal.

**Lemma 1.** The website’s expected profit in the symmetric information benchmark ($$\Pi_{si}^W$$) is an upper bound for the website’s equilibrium expected profit, where $$\Pi_{si}^W$$ is given in equation (6). This upper bound can be attained only in a separating equilibrium in which the merchant types offer a deal at a price $$d_t = \alpha_t$$ for $$t \in \{H, L\}$$, and all matched consumers and new consumers buy the deal.

**Proof.** See Appendix B.

We emphasize that we obtain the upper bound for the website’s expected profit without explicitly solving for the equilibrium. To see the intuition behind Lemma 1, consider a separating equilibrium in which the merchant types set different prices, i.e., $$d_H \neq d_L$$. We require $$d_t \leq \alpha_t$$ in equilibrium, such that new consumers buy the deal. Therefore, the equilibrium deal revenue cannot exceed that in the symmetric information benchmark for either merchant type. Consequently, the website’s equilibrium expected profit cannot exceed $$\Pi_{si}^W$$. Next, consider any pooling equilibrium in which the merchant types set the same deal price, i.e., $$d_H = d_L$$.\footnote{By pooling equilibrium, we mean an equilibrium in which both merchant types follow the same equilibrium strategy. Consumers may still learn the merchant type in equilibrium through the deal counter.} The equilibrium deal price can be at most be $$\bar{\alpha} = \bar{\theta} \alpha_H + (1 - \bar{\theta}) \alpha_L$$, which is the reservation price of new consumers based on their prior belief $$\bar{\theta}$$. Otherwise, one or both types will not sell to any new consumers in equilibrium. Therefore, the website’s expected profit is highest in a pooling equilibrium if the deal price is $$\bar{\alpha}$$, and all matched and new consumers buy the deal. In such a pooling equilibrium, the website’s expected profit from new consumers is the same as in the symmetric information benchmark. But its expected profit from experienced consumers is lower than in the benchmark. This is because the type $$H$$ merchant, which has more matched consumers than the type $$L$$ merchant, sets a higher price in the benchmark than in the pooling equilibrium. Said differently, there is higher cannibalization of revenue from experienced consumers in a pooling equilibrium than in the symmetric information benchmark. Therefore, the website’s expected profit is strictly lower than in the benchmark. It follows that the website can attain the benchmark profit level only in a separating equilibrium that replicates
the benchmark outcome. Such an equilibrium maximizes the revenue from new consumers, and minimizes the cannibalization of revenue from experienced consumers.

### 3.2 Can Displaying Deal Sales Help the Website?

Lemma 1 provides us a convenient way to establish a sufficient condition under which the website will display deal sales in equilibrium. First, in Lemma 2, we show that a separating equilibrium in which the merchant types set different deal prices cannot occur in the opaque website regime. Therefore, the website’s equilibrium expected profit in this regime is strictly lower than $\Pi_{si}^{W}$. Next, in Lemma 3, we show that a separating equilibrium that replicates the symmetric information benchmark outcome can occur in the transparent website regime. We, thus, obtain our main result in Proposition 1, that the website can attain the upper bound $\Pi_{si}^{W}$ for its expected profit only if it displays deal sales and not otherwise. Thus, displaying deal sales can help the website maximize its expected profit.

**Lemma 2.** In the opaque website regime, there is no separating equilibrium in which both merchant types offer a deal.

*Proof (by contradiction).* Suppose that there exists a separating equilibrium in the opaque website regime in which both merchant types offer a deal. Then, $d_H \neq d_L$. As shown in the proof of Lemma 1, it must be that $d_L = \alpha_L$ and $d_H \leq \alpha_H$. Further, $d_H > \alpha_L$ since the reservation price of new consumers cannot be below $\alpha_L$ irrespective of their beliefs. In this equilibrium, new consumers must believe that the merchant’s type is $t$ if the deal price is $d_t$, and be willing to pay $\alpha_t$. Consequently, new consumers will buy the deal at either equilibrium deal price. Matched consumers will also buy the deal at either equilibrium deal price regardless of the merchant’s type as the deal price is below the regular price. But this implies that the type $L$ merchant can increase its margin by deviating to $d_H > \alpha_L$ without affecting its demand. New consumers will still buy its deal as they mistakenly believe it to be a type $H$ merchant. Matched consumers will also buy the deal since the deal price is below the regular price. Therefore, the type $L$ merchant can profitably mimic the type $H$ merchant, which contradicts the existence of a separating equilibrium.

Lemma 2 shows that the type $H$ merchant cannot credibly separate and set a higher price than the type $L$ merchant in the opaque website regime. In particular, the type $H$ merchant cannot attract new consumers at the price $\alpha_H$ (that it charges in the symmetric information benchmark). This is because the type $L$ merchant always has an incentive to mimic the type $H$ merchant by also

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13 It is understood from hereon that, when we refer to an equilibrium in a given website regime, we mean the subgame equilibrium in the subgame that follows the website’s choice of that regime.
charging the deal price \( \alpha_H \). New consumers rationally anticipate this mimicking. Hence, they are not willing to pay \( \alpha_H \) for the deal. We next show that displaying deal sales can help avoid mimicking in a separating equilibrium, thereby enabling the type \( H \) merchant to acquire new consumers at a deal price \( \alpha_H \), and allowing the website to attain the upper bound for its equilibrium profit.

**Lemma 3.** In the transparent website regime, the website attains the upper bound \( \Pi^W_{\text{si}} \) for its equilibrium expected profit iff \( \alpha_L \geq \alpha_1 \), where \( \alpha_1 = \frac{1}{2} \left[ \sqrt{N^2 - 2N(1 - \beta)\alpha_H + \alpha_H^2} - (N - \alpha_H) \right] \) and \( \alpha_1 \in (0, \alpha_H) \). This occurs in a separating equilibrium in which new consumers in period 2 condition their buying decisions on the displayed deal sales even though the merchant types separate on deal prices.

**Proof (by construction).** From Lemma 1, the website can attain the upper bound profit level only in a separating equilibrium in which \( d_t = \alpha_t \) for \( t \in \{H, L\} \) and all matched consumers and new consumers buy the deal. If new consumers buying the deal in period 2 do not condition their buying decisions on period 1 deal sales, then the situation is identical to that in the opaque regime; from Lemma 2 a separating equilibrium cannot occur in this case. We now propose a candidate separating equilibrium in which new consumers buying the deal in period 2 condition their buying decisions on deal sales, and the website attains the upper bound profit level. We show that this candidate equilibrium exists iff \( \alpha_L \geq \alpha_1 \), survives the SUE (strongly-undefeated equilibrium) refinement, and that it is sufficient to focus on this candidate equilibrium.

Consider a candidate equilibrium in which the type \( t \) merchant sets a deal price \( d_t = \alpha_t \), and consumers adopt the following strategies:

- Matched consumers buy the deal at either deal price \( (\alpha_H \text{ and } \alpha_L) \) regardless of the merchant’s type, with frequent-matched consumers buying the deal in period 1;
- At the deal price \( \alpha_L \), all new consumers buy the deal, with frequent visitors buying in period 1.
- At the deal price \( \alpha_H \), early-new consumers buy in period 1, and frequent-new consumers wait till period 2. In period 2, frequent- and late-new consumers buy iff period 1 deal sales is not less than a threshold \( \tau \), where \( \tau \in (\tau_L, \tau_H) \) and \( \tau_t = \alpha_t \left( 1 - \frac{1}{2} \beta \right) + \frac{1}{2} \beta N \) for \( t \in \{H, L\} \).

By conditioning their buying decisions on deal sales, frequent- and late-new consumers ensure that they buy only from the type \( H \) merchant at a deal price \( \alpha_H \). This is because deal sales will be below their threshold \( \tau \) only if the merchant is type \( L \). We note that \( \tau_t \) is the number of consumers who will buy the deal in period 1 if a type \( t \) merchant offered the deal at a price \( \alpha_H \). Specifically, \( \alpha_t \left( 1 - \frac{1}{2} \beta \right) \), is the number of matched consumers who will buy in period 1 and \( \frac{1}{2} N \beta \) is the number of new consumers who buy in period 1. We have \( \tau_H > \tau_L \), since more experienced consumers buy
from the type $H$ merchant owing to its higher fit.

Clearly, consumer strategies are optimal given the merchant strategies. All experienced consumers buy the deal and obtain a positive surplus $p - d_t$. All new consumer also buy and their ex-ante surplus is zero. Consumers cannot obtain a higher utility by deviating from their strategies. Further, the type $H$ merchant can also not do any better. It cannot sell to new consumers at any price higher than $\alpha_H$, and offering a deal is profitable since $N > p$ and condition (5) holds. Similarly, it is also profitable for the type $L$ merchant to offer a deal. It remains to be seen whether the type $L$ merchant has an incentive to mimic the type $H$ merchant.

Suppose that the type $L$ merchant deviated to $d_L = \alpha_H$. Deal sales in period 1 is then $\tau_L$. Consequently, frequent- and late-new consumers will not buy in period 2. Therefore, the total deal sales for the type $L$ merchant will be $\alpha_L + \frac{1}{2} N \beta$. The corresponding deal revenue is

$$ R'_L = \alpha_H \left( \alpha_L + \frac{1}{2} \beta N \right). \quad (7) $$

The equilibrium deal revenue for the type $L$ merchant is

$$ R^D_L = \alpha_L (\alpha_L + N). \quad (8) $$

For mimicking to be unattractive in equilibrium, we require $R^D_L \geq R'_L$, which yields

$$ \alpha_L^2 + (N - \alpha_H) \alpha_L - \frac{1}{2} \alpha_H \beta N \geq 0. \quad (9) $$

We note that the LHS of the above no-mimicking constraint is continuous and convex in $\alpha_L$, is negative if $\alpha_L \to 0$, and positive if $\alpha_L \to \alpha_H$. It follows that the LHS has a unique positive root $\alpha_L = \alpha_1 \in (0, \alpha_H)$, and the no-mimicking constraint holds iff $\alpha_L \geq \alpha_1$, where $\alpha_1$ is as defined in the statement of the lemma.

We now show that the candidate equilibrium survives the SUE refinement. Moreover, it is sufficient to focus on this candidate equilibrium. A PBE (perfect Bayesian equilibrium) survives the SUE refinement if there is no other PBE in which the type $H$ merchant derives higher profit. As shown in the proof of Lemma 1, $d_H \leq \alpha_H$ and $d_L = \alpha_L$ in any separating PBE, and $d_t \leq \bar{\alpha}$ in any pooling PBE. Three observations follow immediately. First, the type $H$ merchant cannot derive higher profit in any other PBE than in the candidate equilibrium. Therefore, the candidate equilibrium is a SUE. Second, any other (separating) PBE that yields the same profit for the type $H$ merchant as the candidate equilibrium (and hence survives the SUE refinement) cannot lead to a different outcome.\footnote{For instance, it may be possible to construct a separating PBE in which the no-mimicking constraint is enforced only by the late-new consumers conditioning their buying decisions on deal sales.} Hence, the equilibrium outcome is unique. Lastly, any such PBE that leads
to the same outcome cannot exist for a wider range of parameters (e.g., for $\alpha_L < \alpha_1$) than the candidate equilibrium. This is because the no-mimicking constraint cannot be made stronger than equation (9). Therefore, it sufficient to focus on the candidate equilibrium.

Lemma 3 establishes how a separating equilibrium that resembles the symmetric information benchmark can occur in the transparent website regime. Intuitively, mimicking is prevented in this case because frequent- and late-new consumers can learn the merchant’s type from period 1 deal sales, and avoid buying from the type $L$ merchant if it deviates to mimic the deal price of the type $H$ merchant. Mimicking still allows the type $L$ merchant to sell to early-new consumers, who infer the merchant’s type from the deal price. Lemma 3 shows that the loss in demand from frequent- and late-new consumers makes mimicking unattractive iff $\alpha_L \geq \alpha_1$. In this way, displaying deal sales allows the type $H$ merchant to credibly signal its type to early-new consumers by charging a higher deal price than the type $L$ merchant. Consequently, the type $H$ merchant acquires new consumers at a higher price than in the opaque website regime (in which a separating equilibrium cannot occur).

Period 1 deal sales is informative to new consumers because the type $H$ merchant has more matched consumers than the type $L$ merchant, resulting in higher period 1 sales for the former. In equilibrium, the type $H$ merchant charges a deal price $\alpha_H$. In period 1, early- and frequent-matched consumers buy the deal as they obtain a positive surplus $p - \alpha_H$; all early-new consumers also buy the deal because they believe that the merchant is type $H$ and their expected surplus is non-negative. Therefore, the equilibrium period 1 sales is $\tau_H = \alpha_H \left(1 - \frac{1}{2} \beta\right) + \frac{1}{2} \beta N$, where $\alpha_H \left(1 - \frac{1}{2} \beta\right)$ is the number of early- and frequent-matched consumers for a type $H$ merchant, and $\frac{1}{2} \beta N$ is the number of early-new consumers. If the type $L$ merchant deviates to set a deal price $\alpha_H$, then the number of early- and frequent-matched consumers who will buy the deal is $\alpha_L \left(1 - \frac{1}{2} \beta\right)$ (as they obtain a positive surplus $p - \alpha_H$); all $\frac{1}{2} \beta N$ early-new consumers will buy the deal as they (incorrectly) believe that the merchant’s type is $H$. Hence, the period 1 sales following this deviation is $\tau_L = \alpha_L \left(1 - \frac{1}{2} \beta\right) + \frac{1}{2} \beta N$. We note that $\tau_L < \tau_H$ as the type $L$ merchant has fewer matched consumers.

Thus, displaying deal sales enables observational learning by new consumers. That is to say, frequent- and late-new consumers are able to infer their utility of buying the deal by observing the buying decisions of other consumers which reflects their private information. Observational learning influences the buying decisions of frequent- and late-new consumers at the deal price $\alpha_H$. If they observe deal sales of $\tau_L$, which is possible only if the merchant is type $L$, they update their belief
to \( \theta = 0 \) and their expected surplus is negative. Therefore, they do not buy. In contrast, they buy on the equilibrium path, because they observe deal sales of \( \tau_H \), which indicates that the merchant is type \( H \), and their expected surplus is non-negative. It is important to note that experienced consumers are necessary for observational learning to occur. If there are no experienced consumers, then period 1 sales is not informative.

Moreover, observational learning has force in the separating equilibrium. Even though the merchant types set different deal prices in equilibrium, new consumers buying the deal in period 2 must condition their buying decisions on period 1 sales. While frequent- and late-new consumers buy on the equilibrium path, they do not buy off the equilibrium path. This is necessary to ensure that if the type \( L \) merchant mimics the type \( H \) merchant, which is an off-equilibrium occurrence, then they do not buy the deal. In other words, mimicking is avoided and separation occurs only through observational learning. We remark that the off-equilibrium strategy of frequent- and late-new consumers is both sequentially rational and credible in that it is robust to a small tremble by the type \( L \) merchant. Specifically, given any arbitrarily small probability that the type \( L \) merchant deviates to \( d_L = \alpha_H \), it is the strictly dominant strategy for frequent-new consumers to wait till period 2 and for frequent- and late-new consumers to buy only if deal sales is not below a threshold \( \tau \), where \( \tau \in (\tau_L, \tau_H] \).

It is useful to understand how the model parameters determine whether mimicking can be avoided in this equilibrium. Mimicking involves a trade-off for the type \( L \) merchant between gaining a higher margin and losing demand from frequent- and late-new consumers. The no-mimicking constraint in equation (9) captures this trade-off. All else equal, it is easier to avoid mimicking if the type \( H \) merchant’s fit (\( \alpha_H \)) is lower or if type \( L \) merchant’s fit (\( \alpha_L \)) is higher, because the gain in margin from mimicking is then lower. It is also easier to prevent mimicking if there are fewer early visitors (\( \beta \) is lower) or the number of new consumers (\( N \)) is higher, since more consumers then condition their purchase on period 1 deal sales and the loss in demand from mimicking is higher. The no-mimicking constraint holds iff \( \alpha_L \geq \alpha_1 \). We note that \( \alpha_1 \) is decreasing in \( \beta \), and \( \alpha_1 \to 0 \) if \( \beta \to 0 \). Hence, mimicking is easier to prevent if more new consumers engage in observational learning. Comparing Lemmas 2 and 3 provides a sufficient condition for the website to display deal

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15 The equilibrium strategy of frequent- and late-new consumers can be made robust to trembles by an arbitrarily small measure of consumers in period 1 by specifying a \( \tau \) suitably in the interior of \( (\tau_L, \tau_H] \). We thank the AE and an anonymous reviewer for this suggestion.

16 The alert reader will note that for the equilibrium to be trembling-hand perfect, the type \( H \) merchant should set a deal price \( \alpha_H - \epsilon \), where \( \epsilon > 0 \) is arbitrarily small. This ensures that, for a sufficiently small tremble by the type \( L \) merchant, early-new consumers still buy from the type \( H \) merchant in period 1. Thus, the equilibrium that we have constructed in Lemma 3 is the limit of the trembling-hand perfect equilibrium as \( \epsilon \to 0 \).
sales in equilibrium. The following proposition describes this result.

**Proposition 1.** The website can attain the upper bound ($\Pi_{W}^{1}$) for its equilibrium expected profit iff it displays deal sales and $\alpha_L \geq \alpha_1$, where $\alpha_1$ is defined in Lemma 3. The website displays deal sales in equilibrium if $\alpha_L \geq \alpha_1$.

**Proof.** Follows from Lemmas 1, 2 and 3.

We have thus identified a role for the website to display deal sales under asymmetric information. Displaying deal sales can enable the type $H$ merchant to credibly separate and charge a deal price equal to new consumers’ willingness to pay for its product. Therefore, it acquires new consumers at a higher deal price than would be possible if deal sales are not displayed. In turn, this allows the website to attain its maximum possible expected profit. This role of displaying deal sales is different from the role it can play in coordinating consumer choices in the presence of a minimum limit in Hu, Shi, and Wu (2013). An interesting point is that though the two merchant types charge different deal prices, they can do so credibly only if deal sales are displayed by the website. In other words, what sustains a separating equilibrium is not the merchant types choosing different prices, but it is the website’s decision to display deal sales. This separating equilibrium has an additional interesting feature, namely, there is no distortion in the deal price or demand compared to the symmetric information benchmark. This differs from what we commonly encounter in signaling models, where signaling entails a distortion to avoid mimicking.\(^\text{17}\) This can be understood better in our model, by noting that the frequent- and late-new consumers, who are responsible for preventing mimicking, learn the merchant’s types not from prices but through observational learning from deal sales. The distortionless separation also enables the website to attain its maximum possible profit, which is not possible without displaying deal sales.

Intuitively, displaying deal sales leverages sales to experienced consumers to attract new consumers at a higher price, thereby benefiting the type $H$ merchant and the website.\(^\text{18}\) Offering a deal cannibalizes revenue from experienced consumers, who would have otherwise bought the product at the regular price. Such cannibalization is common in many forms of discount promotions. What is different in this case is that sales to experienced consumers is leveraged by the type $H$ merchant to credibly communicate its type to new consumers and charge a higher price. In other words, experienced consumers have informational value that is unlocked by displaying deal sales. By using

\(^{17}\)Separation without distortion in prices through the use of multiple instruments also occurs in Moorthy and Srinivasan (1995) and Stock and Balachander (2005).

\(^{18}\)We thank an anonymous reviewer for suggesting this intuition.
experienced consumers to attract new consumers at a higher price, the costs due to cannibalization are minimized. This mechanism is made possible by the fact that consumers purchase the deal upfront, and this can be monitored by the website. The idea of paying existing consumers (through a discount) to help acquire new consumers has some resemblance to selling mechanisms examined before by Biyalogorsky et al. (2001) and Jing and Xie (2011). In our setting, however, there are no direct interactions between experienced consumers and new consumers. Instead, new consumers are influenced indirectly by experienced consumers through observational learning.

3.3 Can Displaying Deal Sales Hurt the Website?

We now show that the website will not display deal sales in equilibrium if the merchant types will pool and set the same deal price as each other. First, in Lemmas 4 and 5, we obtain the outcome in the unique candidate pooling equilibrium in each website regime. Comparing these outcomes leads to our main result in Proposition 2. We show that if the equilibrium in the transparent website regime is a pooling equilibrium, then the equilibrium in the opaque website regime must also be a pooling equilibrium. Moreover, the website’s expected profit is higher in the latter pooling equilibrium. Therefore, the conditions under which a pooling equilibrium occurs in the transparent website regime are sufficient for the website not to display deal sales in equilibrium. We derive these conditions in Lemma 6, thus establishing that displaying deal sales can also hurt the website.

Let $R^I_{D|pooling}$ and $R^O_{D|pooling}$ denote the deal revenues for a type $t$ merchant in a pooling equilibrium in the transparent and opaque website regimes, respectively. Let $\Pi^I_{W|pooling}$ and $\Pi^O_{W|pooling}$, respectively, denote the corresponding expected website profits. Consider first a pooling equilibrium in the opaque website regime. At the equilibrium deal price, new consumers must maintain their prior belief $\bar{\theta}$ and be willing to pay $\bar{\alpha}$. It is possible, however, to construct pooling PBE (perfect Bayesian equilibrium) in which both merchant types set $d_t = d < \bar{\alpha}$ by specifying pessimistic beliefs ($\theta = 0$) at off-equilibrium deal prices. In Lemma 4, we show that only a pooling PBE in which $d_t = \bar{\alpha}$ for $t \in \{H, L\}$ can survive the SUE (strongly-undefeated equilibrium) refinement. Thus, there is a unique candidate for a pooling equilibrium. In this equilibrium, all matched consumers and new consumers buy the deal. Lemma 4 also describes the equilibrium buying behavior, merchant revenue, and the website’s expected profit. The conditions under which this pooling equilibrium occurs will be established through subsequent results.

**Lemma 4.** Consider a pooling equilibrium in the opaque website regime. In this equilibrium:

(i) The merchant’s pricing strategy is $d_t = \bar{\alpha}$ for $t \in \{H, L\}$;

(ii) All matched consumers and new consumers buy the deal;
(iii) Deal revenue is $R_{t}^{D}_{pooling} = \alpha (\alpha t + N)$ for $t \in \{H, L\}$;

(iv) Website’s expected profit is $\Pi_{W}^{0}_{pooling} = (1 - \lambda) \bar{\alpha} (\bar{\alpha} + N)$.

Proof. Let $d$ denote the deal price in a pooling PBE in the opaque website regime, i.e., $d_t = d$ for $t \in \{H, L\}$. In the proof of Lemma 1, we showed that $d \leq \bar{\alpha}$ in a pooling PBE as otherwise new consumers will not buy. Also $d \geq \alpha_L$ since the reservation price of new consumers cannot be below $\alpha_L$ irrespective of their beliefs. Therefore, $d \in [\alpha_L, \bar{\alpha}]$. All matched consumers and new consumers will buy the deal in equilibrium, as they obtain an expected surplus $\bar{\alpha} - d \geq 0$. Deal revenue is then

$$R_{t}^{D} = d (\alpha t + N) \quad \forall t \in \{H, L\}$$  \hspace{1cm} (10)

The pooling PBE exists iff offering a deal can be profitable for the merchant. Therefore,

$$R_{t}^{D} > R_{t}^{O} \quad \Rightarrow \quad d (\alpha t + N) > p\alpha t,$$  \hspace{1cm} (11)

for $t \in \{H, L\}$. Since $R_{t}^{D}$ is increasing in $d$, if condition (11) holds for $d < \bar{\alpha}$, then it also holds for $d = \bar{\alpha}$. Therefore, if a pooling PBE in which $d < \bar{\alpha}$ exists, then a pooling PBE in which $d = \bar{\alpha}$ must also exist. Moreover, the latter PBE leads to higher revenue and profit for the type $H$ merchant. Hence, a pooling PBE in which $d < \bar{\alpha}$ cannot be a SUE. Thus, the pooling PBE in which $d = \bar{\alpha}$ is the unique candidate for a pooling equilibrium. As noted above, all matched consumers and new consumers will buy the deal in this equilibrium. Therefore, $R_{t}^{D}_{pooling}$ and $\Pi_{W}^{0}_{pooling}$ are as in the statement of the lemma. \hfill \Box

Next, consider the transparent website regime. We show in Lemma 5 that there is unique candidate pooling equilibrium. Again, $d_t = d = \bar{\alpha}$ for $t \in \{H, L\}$ in this equilibrium. However, frequent- and late-new consumers do not buy from the type $L$ merchant. These consumers infer the merchant’s type from period 1 deal sales. If they determine that the merchant is type $L$, which is the case if the realized deal sales is $\tau_L$, then they do not buy since $\bar{\alpha} > \alpha_L$. We note that, while frequent-new consumers will obtain non-negative surplus if they buy in period 1, it is a dominant strategy for them to wait till period 2. By waiting, they can observe deal sales and avoid buying from the type $L$ merchant. Consequently, the type $L$ merchant sells only to matched consumers and early-new consumers. Lemma 5 also describes the equilibrium buying behavior and outcomes.

**Lemma 5.** Consider a pooling equilibrium in the transparent website regime. In this equilibrium:

(i) The merchant’s pricing strategy is $d_t = \bar{\alpha}$ for $t \in \{H, L\}$;

(ii) All matched consumers and early-new consumers buy the deal.
(iii) Frequent and late-new consumers buy if period 1 sales is $\tau_H$ and do not buy if period 1 sales is $\tau_L$, where $\tau_t = \alpha_t \left(1 - \frac{1}{2}\beta\right) + \frac{1}{2}\beta N$ for $t \in \{H, L\}$;

(iv) Deal revenues are $R_{D|pooling}^H = \bar{\alpha} (\alpha_H + N)$ and $R_{D|pooling}^L = \bar{\alpha} (\alpha_L + \frac{1}{2}\beta N)$;

(v) Website’s expected profit is $\Pi_{W|pooling}^1 = (1 - \lambda) \bar{\alpha} (\bar{\alpha} + N (\bar{\theta} + \frac{1}{2}\beta (1 - \bar{\theta})))$.

Proof. See Appendix B.

We now show that if pooling occurs in the transparent website regime, then the website will not display deal sales in equilibrium. Comparing the candidate pooling equilibrium outcomes in the two website regimes from Lemmas 4 and 5, we observe that the type $H$ merchant equilibrium deal revenue and, therefore, profit are the same in both regimes; the type $L$ merchant equilibrium deal revenue and profit are lower in the transparent regime. Therefore, the conditions for a pooling equilibrium are more stringent in the transparent website regime. Hence, if pooling occurs in the transparent website regime, then pooling must also occur in the opaque website regime. Moreover, the website’s expected profit will be higher in the latter regime. Therefore, the website will not display deal sales in equilibrium.

Proposition 2. If both merchant types set the same deal price as each other in equilibrium in the transparent website regime, then they also set the same deal price as each other in equilibrium in the opaque website regime. Moreover, the website will not display deal sales in equilibrium.

Proof. Given that the equilibrium in the transparent website regime is a pooling equilibrium, we first show that the equilibrium in the opaque regime is also a pooling equilibrium. In the pooling equilibrium in the transparent website regime, it must be profitable for both merchant types to offer a deal. We, therefore, have

$$R_{D|pooling}^t > R_O^t \quad \forall t \in \{H, L\}$$

(12)

From Lemmas 4 and 5, we also have

$$R_{D|pooling}^t \geq R_{D|pooling}^{t+1} \quad \forall t \in \{H, L\}$$

(13)

Conditions (12) and (13) imply that

$$R_{D|pooling}^t > R_O^t \quad \forall t \in \{H, L\}$$

(14)

Consider the opaque website regime. Condition (14) ensures that the candidate pooling PBE described in Lemma 4 exists. From Lemma 4, no other pooling PBE can be a SUE. From Lemma 2, there is no separating PBE in which both merchant types offer a deal. Also, there is no separating
PBE in which only the type $H$ merchant offers a deal. This is because, from condition (5), it is always profitable for the type $L$ merchant to offer a deal at a deal price $\alpha_L$ even if new consumers knew its type. Thus, the candidate pooling PBE of Lemma 4 is the unique SUE, and the equilibrium in the opaque regime is also a pooling equilibrium. Moreover, the website’s expected profit is higher in the opaque regime, i.e., $\Pi_W|_{pooling}^0 > \Pi_W|_{pooling}^1$. It follows that the website will not display deal sales in equilibrium.

Proposition 2, thus, shows that observational learning can be a double-edged sword: if it does not prevent mimicking, then it causes a loss of demand and hurts the website. Therefore, the website will not display deal sales in equilibrium if pooling occurs in the transparent website regime. We identify the conditions for this to occur in the following lemma.

**Lemma 6.** Both merchant types will set the same deal price as each other in equilibrium in the transparent website regime iff $\alpha_L < \alpha_1$ and $\bar{\theta} > \max \{\theta_1, \theta_2\}$, where $\alpha_1$ is defined in Lemma 3, $\theta_1 = 1 - \frac{\alpha_H(N+\alpha_H-p)}{(N+\alpha_H)(\alpha_H-\alpha_L)}$ and $\theta_2 = \frac{N\alpha_L(1-\frac{1}{2}\beta)}{(\alpha_L+\frac{1}{2}\beta N)(\alpha_H-\alpha_L)} \in (0,1)$.

**Proof.** See Appendix B.

### 3.4 Should the Website’s Strategy Always Favor the Better Merchant?

In the equilibrium situations considered thus far, the website’s equilibrium strategy has been aligned with the interests of the “better” merchant, namely, the type $H$ merchant. Specifically, if the transparent website regime supports distortionless separation, then the website displays deal sales in equilibrium, which benefits the type $H$ merchant and hurts the type $L$ merchant. If the transparent website regime does not support separation, then the website does not display deal sales in equilibrium, which does not affect the type $H$ merchant and benefits the type $L$ merchant. We now ask whether the website’s strategy is aligned with the interests of the type $H$ merchant in the remaining situation as well. We show that this is not the case. Specifically, we examine the situation in which the transparent website regime supports a separating equilibrium, but with distortion in the deal price. We derive this (subgame) equilibrium in Lemma 7. In this equilibrium, the type $H$ merchant prefers to separate than to pool as it can acquire new consumers at a higher margin. Nevertheless, the website does not always display deal sales in equilibrium. We establish this result in Proposition 3. Thus, the website’s strategy does not always favor the “better” merchant.

Consider the transparent website regime. From Lemma 3, a distortionless separating equilibrium cannot occur if $\alpha_L < \alpha_1$. It may be possible, however, to support a separating equilibrium in which the type $H$ merchant distorts its deal price downwards from $\alpha_H$, i.e., $d_H \neq d_L$, and $d_H = d_H^* < \alpha_H$.  

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This separating equilibrium still requires observational learning: frequent- and late-new consumers buy the deal at the deal price $d^*_H$ only if period 1 sales exceeds $\tau_L$. But, observational learning alone is not sufficient to prevent mimicking. The type $H$ merchant must also distort its price downwards. In a SUE, the type $H$ merchant will set the highest deal price $d^*_H < \alpha_H$ such that the type $L$ merchant will not mimic it. We also require that $d^*_H \geq \bar{\alpha}$ for a SUE. Otherwise, the type $H$ merchant is better off pooling than separating. That is to say, it prefers to set a deal price $\bar{\alpha}$ and allow the type $L$ merchant to mimic it than to separate through a deal price $d^*_H < \bar{\alpha}$. We derive this equilibrium and the conditions under which it occurs in Lemma 7. Let $R^D_1$ denote the deal revenue for a type $t$ merchant in this separating equilibrium, and let $\Pi^W_1$ denote the corresponding expected website profit.

Lemma 7. Consider a separating equilibrium in the transparent website regime. If $\alpha_L < \alpha_1$, then in this equilibrium:

(i) The merchant’s pricing strategy is $d_L = \alpha_L$ and $d_H = d^*_H = \frac{\alpha_L(\alpha_L+N)}{\alpha_L+\frac{1}{2}\beta N} < \alpha_H$;

(ii) Deal revenues are $R^D_{1, \text{separation}} = d^*_H (\alpha_H + N)$ and $R^D_{L, \text{separation}} = \alpha_L (\alpha_L + N)$.

(iii) Website’s expected profit is $\Pi^W_{1, \text{separation}} = (1 - \lambda) (\bar{\theta} d^*_H (\alpha_H + N) + (1 - \bar{\theta}) \alpha_L (\alpha_L + N))$

This separating equilibrium occurs iff $\alpha_L \in (\alpha_2, \alpha_1)$ and $\bar{\theta} \leq \theta_2$, where $\alpha_1$ and $\theta_2$ are defined in Lemmas 3 and 6, and $\alpha_2 = \frac{\sqrt{(N^2+N\alpha_H-\alpha_H)^2+2N\alpha_H(N+\alpha_H)+\alpha_H-N(\alpha_H)}}{2(N+\alpha_H)} \in (0, \alpha_1)$.

Proof. See Appendix B. \qed

Even if the transparent website regime supports separation, this does not mean that it is profitable for the website to display deal sales. From Lemma 7, the website’s expected profit is increasing in $d^*_H$. The level of $d^*_H$ reflects the relative attractiveness of mimicking for the type $L$ merchant. Higher the relative attractiveness of mimicking, lower is $d^*_H$, and lower is the website’s expected profit. In particular if $d^*_H = \bar{\alpha}$, then the equilibrium in the opaque website regime is a pooling equilibrium, and the website’s expected profit is strictly higher than in the transparent website regime, i.e., $\Pi^W_{0, \text{pooling}} > \Pi^W_{1, \text{separation}}$. This is because the type $H$ merchant’s deal revenue is the same in either case, while that of the type $L$ merchant is higher in the opaque website regime. Consequently, for $d^*_H$ sufficiently close to $\bar{\alpha}$, it is not profitable for the website to display deal sales, even though the type $H$ merchant prefers to separate. The following proposition describes this finding and the conditions under which the website displays deal sales in equilibrium.

Proposition 3. The website may not display deal sales in equilibrium even if doing so benefits the type $H$ merchant. Specifically, under the conditions for the separating equilibrium described in
Lemma 7, the type $H$ merchant’s equilibrium profit is higher in the transparent website regime than in the opaque website regime. However, the website displays deal sales in equilibrium iff one of the following conditions hold: $\alpha_L \in (\alpha_2, \alpha_3)$ and $\bar{\theta} \leq \theta_1$, or $\alpha_L \in (\alpha_4, \alpha_1)$ and $\bar{\theta} \leq \theta_3$, where $\alpha_1$, $\alpha_2$, $\theta_1$ and $\theta_2$ are defined in Lemmas 3, 7, and 6, $\alpha_3 = \frac{\rho H}{\bar{N} + \alpha_H}$, $\alpha_4 \in (\alpha_2, \alpha_1)$ is the unique positive root for $\alpha_L$ in equation (31) in Appendix C, and $\theta_3 = \frac{d_H(N+\alpha_H)-N\alpha_H-(2\alpha_H-\alpha)(\alpha_L)}{(\alpha_H-\alpha_L)^2} < \theta_2$.

Proof. See Appendix C. \qed

Proposition 3 shows that the website’s strategy does not always favor the better merchant. It may not display deal sales even if doing so would allow the type $H$ merchant to acquire new consumers at a higher deal price. While the type $H$ merchant prefers the separating equilibrium in the transparent website regime, the type $L$ merchant prefers the pooling equilibrium in the opaque website regime. The website’s strategy maximizes its profit from both merchant types. Consequently, it displays deal sales only if there is not much (downward) distortion in the type $H$ merchant’s deal price. It is important to note that the website has a part in determining whether the merchant types can separate in equilibrium. Moreover, its incentives are distinct from that of the merchant. Our analysis of a strategic website brings this out clearly.

An interesting point to note is that the type $L$ merchant revenue is lower in the transparent website regime, and the website’s expected profit is more dependent on type $L$ merchant revenue if $\bar{\theta}$ is low. However, the website displays deal sales only if $\bar{\theta}$ is sufficiently low ($\bar{\theta} \leq \theta_1$ or $\bar{\theta} \leq \theta_3$ depending on $\alpha_L$). To understand why, it is useful to consider the website’s trade-off. On the one hand, in the transparent website regime, the website cannot attain its highest profit because the type $H$ merchant distorts its deal price downwards to credibly separate, i.e., there is a cost of separation. On the other hand, in the opaque website regime, the website cannot attain its highest profit because of an adverse selection problem. That is to say, because of asymmetric information, the type $H$ merchant is forced to either set the same deal price as the type $L$ merchant or not offer a deal (if offering a deal at $d_H = \bar{\alpha}$ is not profitable). Thus, the website faces a trade-off between the cost of separation and the cost of adverse selection. If $\bar{\theta}$ is low, then the pooling price $\bar{\alpha}$ is low, and the cost of adverse selection dominates: there is a steep drop in the type $H$ merchant’s price if it must pool with the type $L$ merchant, whereas the type $L$ merchant is not hurt as much if it must separate. Consequently, the website displays deal sales. Whereas if $\bar{\theta}$ is high, then the cost of separation dominates, and the website prefers not to display deal sales.

In summary, the website displays deal sales if $\alpha_L \geq \alpha_1$ (Proposition 1), or $\alpha_L \in (\alpha_2, \alpha_3)$ and
\[\bar{\theta} \leq \theta_1, \text{ or } \alpha_L \in (\alpha_4, \alpha_1) \text{ and } \bar{\theta} \leq \theta_3 \text{ (Proposition 3).} \] It does not display deal sales otherwise.\(^{19}\)

### 4 Should the Website Offer the Merchant an Upfront Subsidy?

Some daily deal websites such as Groupon and LivingSocial offer considerable support to a merchant in designing the promotional material for the deal and employ a substantial team of copywriters and editorial staff for this purpose (e.g., Streitfeld 2011, LivingSocial 2013). But they do not charge the merchant for this service. Hence, it could be thought of as a subsidy. This raises the question whether it can be optimal for the website to offer a subsidy. To examine this question, we endogenize the revenue-sharing contract. We assume that the contract consists of the revenue-sharing rate \(\lambda \in [0, 1]\), which is the merchant’s share of revenue, and a fixed-fee \(F\) that the merchant must pay the website to offer a deal, with \(F < 0\) denoting a subsidy. In stage 1 of period 0, the website decides the website regime and the revenue-sharing contract. In stage 2, the merchant must accept the contract to be able to offer a deal. The rest of the game proceeds as before.

Given the contract terms \((\lambda, F)\), the type \(t\) merchant will accept the contract iff

\[\lambda R_t^D - F \geq R_t^O. \tag{15}\]

We refer to the incremental revenue \(R_t^D - R_t^O\) as the surplus generated by the type \(t\) merchant. If the merchant accepts the contract, then it retains a portion \(\lambda R_t^D - F - R_t^O\) of the surplus, while the website captures a portion \((1 - \lambda) R_t^D + F\). If only one of the merchant types accepts the contract in equilibrium, then the equilibrium contract is not uniquely determined, i.e., there is a range of \((\lambda, F)\) that will lead to the same outcome. Therefore, for our analysis to be meaningful, we focus on situations in which the website offers a contract that both merchant types accept in equilibrium.\(^{20}\)

As in our main analysis, the contract terms do not affect the merchant’s pricing decision conditional on offering a deal. Therefore, the equilibrium deal price and revenue are the same as in our main analysis. We now examine whether the equilibrium contract will have a subsidy built in. First, in Lemma 8, we obtain a necessary and sufficient condition for this to occur in terms of the equilibrium deal revenue. Then, in Proposition 4, we examine when this condition can hold in equilibrium.

**Lemma 8.** The equilibrium revenue sharing contract will involve a subsidy iff \(\frac{R_H^D}{R_H^O} > \frac{R_L^D}{R_L^O}\).

**Proof.** See Appendix B. \(\square\)

\(^{19}\)If \(\alpha_L \in (0, \alpha_3)\) and \(\bar{\theta} < \theta_1\), then only the type \(L\) merchant offers a deal in equilibrium in either website regime (since offering a deal at a price \(d_H = \bar{\alpha}\) is not profitable for the type \(H\) merchant). Hence, the website is indifferent between either website regime. In this case, we assume that it does not display deal sales.

\(^{20}\)We note in passing that it will be optimal for the website to offer a contract that both merchant types accept if \(\alpha_L \geq \alpha_5 \in (0, \alpha_4)\), where \(\alpha_4\) is defined in Proposition 4. Otherwise, the website offers a contract that only the type \(L\) merchant accepts. Essentially, the type \(H\) merchant is “driven” out of the market because of adverse selection. We do not include an analysis of this outcome as it does not provide any additional insights.
Intuitively, the revenue-sharing component allows the website to capture more surplus from the type $H$ merchant than from the type $L$ merchant. In any equilibrium, the type $H$ merchant realizes higher deal revenue ($R^D_H > R^D_L$) since it sets a (weakly) higher deal price and realizes (strictly) higher deal sales. Therefore, revenue-sharing extracts higher surplus from the type $H$ merchant. Consequently, we find that if the type $H$ merchant’s deal generates considerably more surplus than the type $L$ merchant’s, then it is optimal for the website to offer a subsidy in conjunction with taking a larger share of deal revenue (low $\lambda$). The low $\lambda$ extracts the higher surplus of the type $H$ merchant’s deal, and the subsidy ensures that the type $L$ merchant offers a deal despite a low $\lambda$. Without the subsidy, the website cannot fully capture the higher surplus of the type $H$ merchant while also serving the type $L$ merchant. Lemma 8 establishes the necessary and sufficient condition for a subsidy. It is useful to note that this condition can hold only if the type $H$ merchant’s deal generates higher surplus, i.e., only if $R^D_H - R^O_H > R^D_L - R^O_L$.

We find that offering a subsidy is not optimal if the website does not display deal sales in equilibrium because the type $L$ merchant’s deal generates higher surplus. In the pooling equilibrium in the opaque website regime, both merchant types obtain the same revenue from new consumers, while the type $H$ merchant faces higher cannibalization as it has more matched consumers. Consequently, the type $H$ merchant’s deal generates lower surplus than the type $L$ merchant’s. In contrast, offering a subsidy can be optimal if the website displays deal sales in equilibrium. Displaying deal sales allows the type $H$ merchant to attract new consumers at a higher margin than the type $L$ merchant. If the type $H$ merchant is able to charge a sufficiently higher deal price than the type $L$ merchant, then its deal can generate higher surplus than the type $L$ merchant, and offering a subsidy can be optimal. In particular, a subsidy always occurs in situations in which the website attains the upper bound on its expected profit. Proposition 4 describes the conditions under which the website should provide a subsidy.

**Proposition 4.** The equilibrium revenue sharing contract will involve a subsidy iff the website displays deal sales in equilibrium and $\alpha_L > \alpha_4$, where $\alpha_4 = \frac{1}{2} \beta \alpha_H$.

**Proof.** See Appendix C. \hfill $\square$

A daily deal website’s ability to monitor deal purchases does not preclude it from charging the merchant an upfront fee for the service it provides. Proposition 4 shows that it can, nevertheless, be in the interest of the website not to charge an upfront fee and, in fact, provide a subsidy. This is because it enables the website to better capture the surplus generated by the high-type merchant.
In practice, there may be other reasons why the website does not charge the merchant an upfront fee. For instance, this could be a means for the website to differentiate itself from traditional forms of promotional advertising, such as coupon mailers or newspapers that charge the merchant upfront for their service. In other words, it can be a cost of doing business. Setting aside such considerations, we find that it can actually be a means for the website to extract more surplus from the merchant, but only if it displays deal sales.

5 Discussion and Conclusion

Daily deal websites have become a popular means for small merchants to attract new consumers. Our work contributes to the understanding of this emerging business model. Unlike traditional coupon mailer companies, a daily deal website functions as a marketplace enabling transactions between a merchant and consumers. Consequently, it can monitor consumer purchases linked to a deal. We show how the website can capitalize on this capability. By tracking and displaying deal sales, the website can enable a merchant to leverage its sales to experienced consumers to attract new consumers. Thus, while a daily deal promotion cannibalizes the merchant’s revenue from experienced consumers, displaying deal sales can unlock the informational value of these consumers and minimize the costs from cannibalization. Displaying deal sales is, however, not a dominant strategy. In particular, the website may not implement this feature even if doing so will enable the high-type merchant to credibly signal its type to attract new consumers. Thus, the incentives of the website are distinct from that of the merchant.

5.1 Managerial Implications

Managers of daily deal websites have been concerned about the implication of displaying deal sales, and how to improve its effectiveness. For instance, Groupon and TroopSwap conducted field experiments to determine how displaying sales influenced consumer behavior (Groupon 2011; Vasilaky 2012). Our work examines the implications while explicitly considering the strategic behavior of the merchant as well as consumers within an equilibrium framework. Our findings lead to some managerial implications.

First, daily deal websites have been criticized both for the high share of revenue that they take and the deep discounts that merchants offer (Mulpuru 2011; Bice 2012; Kumar and Rajan 2012). These criticisms essentially question the viability of the business model. Our results provide guidance to daily deal websites on how the depth of discounts offered on the website can be managed so as to maximize the profitability of daily deals. Essentially, this occurs in a separating equilibrium. We show that it can be necessary to display deal sales to obtain a separating equilibrium. One might
conjecture that a daily deal website could instead sort the merchants by individually verifying their characteristics or by offering a menu of contracts. However, given that daily deal websites typically market their services to a large number of small merchants through a relatively low-skilled sales force, these approaches can be impractical as they can make the selling task more effortful and complex. In this context, reporting deal sales can play an important role in inducing the merchant to provide the right level of discount and increasing industry profitability.

A second implication is the important role of experienced consumers buying the deal. In the case of traditional promotions, there is no benefit if experienced consumers who would have bought at the regular price buy the deal, because this only results in cannibalization and lowers profit. Based on this logic, industry experts recommend that, when possible, daily deal offers should include restrictions to ensure that they are availed only by new consumers (Mulpuru 2011; Bice 2012; Kumar and Rajan 2012). But observational learning cannot occur if experienced consumers do not buy the deal. Thus, based on our analysis, we can conclude that such restrictions can hurt in the case of a daily deal website that displays deal sales and lead to lower industry profitability.

Third, a website can benefit from offering the merchant an upfront subsidy. Not charging the merchant an upfront fee or providing costly services free of charge are typically thought of as the website’s “costs of doing business” to attract merchants. Our analysis, however, shows that providing a subsidy can help the website capture more of the merchant’s profit. By offering a subsidy, the website may be able to retain a higher share of deal revenues. But this is the case only if the website displays deal sales such that the high-type merchant earns sufficiently higher margin and, therefore, higher profit than the low-type merchant.

Finally, a daily deal website should also explore ways to promote observational learning. As our model suggests, if more consumers visit the website frequently, then it is beneficial. One way to attract consumers to the website frequently is by choosing the appropriate assortment of goods and services. Another way would be to use additional communication methods such as targeted emails and advertisements. For instance, Groupon found that including deal sales information in their emails significantly increased website traffic. Keeping the duration of the deal longer can also enhance the opportunities for observational learning.

5.2 Caveats and Directions for Future Work

We showed that experienced consumers can be leveraged to acquire new consumers. One reason to acquire new consumers is that they may make repeat purchases after the initial purchase. Our analysis did not explicitly consider repeat purchases. In Appendix D, we analyze a setting in which
new consumers can make a repeat purchase at the regular price. It turns out that incorporating repeat purchases strengthens the signaling role of displaying deal sales, i.e., it increases the range of model parameters for which displaying deal sales enables the website to attain the upper bound for its expected profit level. Repeat purchases make mimicking less attractive for the type \( L \) merchant in a separating equilibrium (but only if deal sales are displayed). This is because, in the presence of observational learning, the type \( L \) merchant loses not only current sales, but also future sales from frequent- and late-new consumers, if it mimics the type \( H \) merchant. Thus, our results are robust to allowing repeat sales.

We assumed that the merchant’s deal price is fixed during the duration of the deal, which is the common practice on daily deal websites such as Groupon, Living Social and AmazonLocal. We suspect that this is because allowing prices to vary dynamically over the relatively short duration of a daily deal may cause consumer confusion, and may discourage consumers from using the website. There may also be significant “menu costs” associated with keeping consumers updated about dynamically changing prices over the short duration of the deal. Dynamic prices may however be feasible in other online e-retailing setting. We leave it for future research to examine the implications of displaying sales in other online contexts.

We focused on implications of private information about the merchant’s type. Since, merchants are small players, many in number, and not known a priori to new consumers, we assumed that the merchant cannot undertake significant advertising or brand-building initiatives that are noticed by consumers. We then showed that deal sales can convey private information about the merchant to consumers through observational learning, which is consistent with prior empirical research (e.g., Zhang and Liu 2012; Luo et al. 2014). Nevertheless, displaying deal sales may also serve other purposes such as convey private information about the website (e.g., its marketing ability). However, unlike merchants, the website has more visible market presence and can also use advertising and brand building activities to signal its private information. Future research can examine whether and how a daily deal website can jointly employ these different means to convey its private information.

Competition between daily deal websites may provide an additional motive for a daily deal website to display deal sales. Displaying deal sales may allow a website to differentiate itself from a rival website that does not display deal sales, by enabling consumers make better-informed decisions. It may also enable the website to attract high-type merchants away from the rival website, since they benefit from signaling their type. We expect that observational learning would still be crucial even if only one of the websites displays deal sales, i.e., consumers must still condition their buying
decisions on the displayed sales level and not rely on the website’s decision to display deal sales alone to infer the merchant’s type. This would be necessary to ensure that only merchants that are truly high-types join the website displaying deal sales. An interesting question for future research is whether this can result in asymmetric outcomes, wherein only one of the website displays deal sales in equilibrium.

Lastly, our analysis provides market conditions under which displaying deal sales is profitable for the website. As mentioned in the Introduction, some daily deal websites display deal sales while others do not. Future empirical research could determine whether the market conditions faced by these websites are consistent with our model results.

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### Appendix A  Formal Description of the SUE Refinement

The SUE refinement constrains off-equilibrium beliefs in a candidate PBE using the on-equilibrium beliefs in an alternative PBE. If the candidate PBE does not survive such a belief restriction, then it is said to be defeated by the alternative PBE. A SUE is a PBE that is not defeated by any alternative PBE.

Consider a candidate PBE in which the deal price $d$ is not used in equilibrium. Suppose there exists an alternative PBE in which the deal price $d$ is used in equilibrium and at least one of the merchant types that sets this deal price realizes higher profit than in the candidate PBE. Then, the refinement requires that the beliefs following the deal price $d$ in the candidate PBE do not assign lower probability than the alternative PBE to the merchant types that are strictly better off in the alternative PBE. Formally, let $T$ denote the set of merchant types that set the deal price $d$ in the alternative PBE. Let $T_1 \subseteq T$ denote the set of types that realize strictly higher profit in the alternative PBE than in the candidate PBE. Let $\mu(t \mid d)$ denote the belief in the candidate equilibrium that the merchant’s type is $t$ at some information set following a deal price $d$. Let
\(\mu'(t | d)\) denote the corresponding belief in the alternative equilibrium. The candidate PBE is not defeated by the alternative PBE iff \(\mu(t | d) \geq \mu'(t | d)\) for any \(t \in T_1\).

Two implications follow from such a belief restriction. First, a candidate PBE will be defeated if the type \(H\) merchant is better off in the alternative PBE. This is because consumer beliefs at the deal price \(d\) must be at least as optimistic as in the alternative PBE, i.e., we require that \(\mu(H | d) \geq \mu'(H | d)\). Given any beliefs that satisfy this restriction, consumer demand for the type \(H\) merchant at the deal price \(d\) cannot be lower than that in the alternative PBE, because consumers derive higher utility from the type \(H\) merchant and their decisions must be sequentially rational. It follows that it will be profitable for the type \(H\) merchant to deviate to \(d_H = d\) in the candidate PBE, because it can earn at least as much profit as in the alternative PBE. Hence, the candidate PBE is defeated by the alternative PBE.

A second implication is that a candidate PBE will not be defeated if only the type \(L\) merchant is better off in the alternative PBE. In this case, we require that consumer beliefs are not more optimistic than in the alternative PBE, i.e., \(\mu(L | d) \geq \mu'(L | d)\). But this allows for the beliefs to be pessimistic, i.e., \(\mu(L | d) = 1\). Since to be a PBE, the candidate PBE must have survived under such pessimistic beliefs, it follows that it is not defeated by the alternative PBE.

It follows from these two implications that a PBE that yields the highest profit for the type \(H\) merchant: (a) will defeat any other PBE that yields lower profit for the type \(H\) merchant, and (b) is itself not defeated by any other PBE. Thus, only the PBE that yields the highest profit for the type \(H\) merchant can be a SUE.

Appendix B Proofs of Lemmas

Proof of Lemma 1: We show that \(\Pi^W \leq \Pi^W_{sl}\) in any potential PBE (perfect Bayesian equilibrium) in either website regime (subgame). First, consider a potential separating PBE in which both merchant types offer a deal and \(d_H \neq d_L\). In this PBE, new consumers must believe that the merchant’s type is \(t\) if the deal price is \(d_t\), and be willing to pay \(\alpha_t\). If \(d_t > \alpha_t\), then new consumers will not buy the deal and the deal cannot generate incremental revenue. Hence, it must be that \(d_t \leq \alpha_t\) for \(t \in \{H, L\}\) in any separating PBE. Further, it must be that \(d_L = \alpha_L\), since all new consumers will buy the deal at this price even if they hold pessimistic beliefs \(\theta = 0\). Now, if in a separating PBE, both merchant types set \(d_t = \alpha_t\) and all matched consumers and new consumers buy the deal, then \(R^D_t = \alpha_t(\alpha_t + N)\) and \(\Pi^W = \Pi^W_{sl}\). It follows that website’s expected profit cannot exceed \(\Pi^W_{sl}\) in any other potential separating PBE since neither the deal price nor the demand can be higher than in the benchmark, i.e., \(R^D_t \leq \alpha_t(\alpha_t + N)\) for \(t \in \{H, L\}\). Following
similar arguments, it can also be shown that if only one of the merchant types offers a deal in a PBE, then \( R^D_t \leq \alpha_t (\alpha_t + N) \) for that merchant type and \( R^D_t = 0 \) for the other merchant type. Therefore, \( \Pi^W < \Pi_{si}^W \) in any potential PBE in which only one of the merchant types offers a deal.

Next, consider a potential pooling PBE in which both merchant types offer a deal and \( d_H = d_L = d \). At the deal price \( d \), new consumers must maintain their prior belief that \( \theta = \bar{\theta} \) in period 1. Consider a potential pooling PBE in which \( d > \bar{\alpha} \). In period 1, frequent- and early-new consumers will not buy the deal because their expected surplus is \( \bar{\alpha} - d < 0 \). Only frequent- and early-matched consumers will buy the deal. The resulting period 1 sales of \( (1 - \frac{1}{2} \beta) \alpha_t \) for \( t \in \{H, L\} \) depends on the merchant’s type. In period 2, if deal sales are not displayed, frequent- and late-new consumers will maintain their prior belief \( \bar{\theta} \) and will not buy the deal. If deal sales are displayed, then frequent- and late-new consumers will correctly identify the merchant’s type from period 1 sales. They will not buy the deal if the merchant’s type is \( L \), since their expected surplus will be \( \alpha_L - d < 0 \). Thus, whether or not deal sales are displayed, new consumers will not buy the deal from the type \( L \) merchant if \( d > \bar{\alpha} \), and its deal cannot generate any incremental revenue. Therefore, it must be that \( d \leq \bar{\alpha} \) in any potential pooling PBE. If \( d = \bar{\alpha} \) in a pooling PBE and all matched consumers and new consumers buy, then the website’s expected profit will be

\[
\Pi^W = (1 - \lambda) \bar{\alpha} (\bar{\theta} \alpha_H + (1 - \bar{\theta}) \alpha_L + N).
\]

The above profit is strictly lower than \( \Pi_{si}^W \) since

\[
\Pi_{si}^W - \Pi^W = (1 - \lambda) \bar{\theta} (1 - \bar{\theta}) (\alpha_H - \alpha_L)^2.
\]

It follows that in any other potential pooling PBE \( \Pi^W < \Pi_{si}^W \) since \( d \leq \bar{\alpha} \). Therefore, \( \Pi^W = \Pi_{si}^W \) only in a separating PBE in which \( d_t = \alpha_t \) and all matched consumers and new consumers buy the deal, and \( \Pi^W < \Pi_{si}^W \) in any other PBE.

**Proof of Lemma 5:** Let \( d \) denote the deal price in a pooling PBE in the transparent regime, i.e., \( d_t = d \) for \( t \in \{H, L\} \). Following the same arguments as in the proof of Lemma 4, we have \( d \in [\alpha_L, \bar{\alpha}] \) in any pooling PBE. We show that only a pooling PBE in which \( d = \bar{\alpha} \) can survive the SUE refinement. First, consider a pooling PBE in which \( d \in (\alpha_L, \bar{\alpha}] \), i.e., \( d > \alpha_L \). In this PBE, early-new consumers will maintain their prior belief \( \bar{\theta} \) and buy the deal as they obtain an expected surplus \( \bar{\alpha} - d \geq 0 \). All matched consumers will also buy the deal as they obtain a positive surplus, with frequent-matched consumers buying in period 1. Therefore, period 1 sales is informative about the merchant’s type. Consequently, it is a strictly dominant strategy for frequent-new consumers to wait till period 2 to learn the merchant’s type by observing deal sales. This is because their
expected surplus from buying from a type $L$ merchant is $\alpha_L - d < 0$, and they can avoid buying from the type $L$ merchant by waiting till period 2. In period 2, frequent- and late-new consumers will buy if period 1 sales equals $\tau_H$, which would indicate that the merchant’s type is $H$, and will not buy if it equals $\tau_L$, which would indicate that the merchant’s type is $L$. Therefore, the deal revenues for a type $H$ and type $L$ merchant in this PBE are

$$R^D_H = d(\alpha_H + N), \quad R^D_L = d(\alpha_L + \frac{1}{2}\beta N) \quad (18)$$

This pooling PBE exists iff the following conditions hold: (i) $R^D_t > R^O_t = p\alpha_t$ for $t \in \{H, L\}$, such that offering a deal can be profitable for the merchant, and (ii) $R^D_L \geq \alpha_L (\alpha_L + N)$, such that the type $L$ merchant does not have an incentive to deviate to $d_L = \alpha_L$ to sell to all new consumers.

We note that the deal revenues in equation (18) are increasing in $d$. Therefore, if a pooling PBE in which $d \in (\alpha_L, \bar{\alpha})$ exists, then a pooling PBE in which $d = \bar{\alpha}$ must also exist. Furthermore, the latter PBE leads to higher revenue and profit for the type $H$ merchant. Hence, a pooling PBE in which $d < \bar{\alpha}$ cannot be a SUE.

Next, consider a pooling PBE in which $d = \alpha_L$. In this case, the expected surplus of new consumers is non-negative even if they believe that the merchant’s type is $L$. Therefore, all matched consumers and new consumers will buy in equilibrium. Deal revenue is then

$$R^D_t = \alpha_L (\alpha_t + N) \quad \forall t \in \{H, L\} \quad (19)$$

This pooling PBE exists iff $R^D_t > R^O_t$ for $t \in \{H, L\}$. We show that this candidate PBE cannot be a SUE because, whenever it exists, one of the following alternative PBE that yields strictly higher profit for the type $H$ merchant also exists: (i) the pooling PBE in which $d = \bar{\alpha}$ described previously, or (ii) a separating PBE in which $d_H = \bar{\alpha}$ and $d_L = \alpha_L$. Consider first the alternative pooling PBE. Let $R^D_t'$ denote the deal revenue for a type $t$ merchant in this PBE. We have $R^D_H' = \bar{\alpha} (\alpha_H + N)$ and $R^D_L' = \bar{\alpha} (\alpha_L + \frac{1}{2}\beta N)$ (from equation (18) for $d = \bar{\alpha}$). We note $R^D_H' > R^D_H$. Given that the candidate pooling PBE exists, i.e., $R^D_t > R^O_t$ for $t \in \{H, L\}$, the alternative pooling PBE exists iff $R^D_L' \geq R^D_L$. This is because $R^D_H' > R^D_H$ and $R^D_L' \geq R^D_L$ are sufficient to ensure that it is profitable for either merchant type to offer a deal in the alternative pooling PBE, and $R^D_L' \geq R^D_L$ is necessary and sufficient to ensure that it is not profitable for the type $L$ merchant to deviate from $d_L = \bar{\alpha}$ to $d_L = \alpha_L$ in the alternative pooling PBE. Therefore, if the candidate pooling PBE exists and $\bar{\alpha} (\alpha_L + \frac{1}{2}\beta N) \geq \alpha_L (\alpha_L + N)$, then the alternative pooling PBE in which $d = \bar{\alpha}$ also exists and yields higher profit for the type $H$ merchant.

If instead $\bar{\alpha} (\alpha_L + \frac{1}{2}\beta N) < \alpha_L (\alpha_L + N)$, then we can construct the alternative separating PBE.
that yields higher profit for the type $H$ merchant as follows. If the deal price is $\alpha_L$, then all matched consumers and new consumers buy. If the deal price is $\bar{\alpha}$, then all matched consumers and early-new consumers buy, with frequent-new consumers buying in period 1. Frequent- and late-new consumers buy in period 2 if deal sales is $\tau_H$ and do not buy if deal sales is $\tau_L$. Let $R_{t}^{D'}$ denote the deal revenue for a type $t$ merchant in this separating PBE. We have $R_{H}^{D'} = \bar{\alpha} (\alpha_H + N)$ and $R_{L}^{D'} = \alpha_L (\alpha_L + N)$.

Since $\bar{\alpha} (\alpha_L + \frac{1}{2} \beta N) < \alpha_L (\alpha_L + N)$, the type $L$ merchant does not have an incentive to mimic the type $H$ merchant. Moreover, $R_{H}^{D'} > R_{H}^{O}$ and $R_{L}^{D'} = R_{L}^{O}$. Therefore, if the candidate pooling PBE exists and $\bar{\alpha} (\alpha_L + \frac{1}{2} \beta N) < \alpha_L (\alpha_L + N)$, then the alternative separating PBE exists and yields higher profit for the type $H$ merchant. Thus, the pooling PBE in which $d = \bar{\alpha}$ is the unique candidate for a pooling equilibrium, and $R_{t}^{O,1}_{pooling}$ and $\Pi_{W}^{1}_{pooling}$ are as in the statement of the lemma.

**Proof of Lemma 6:** We first show that if the pooling PBE described in Lemma 5 exists, then there is no other separating PBE in which the type $H$ merchant earns higher profit. Hence, the pooling PBE is the unique SUE. The pooling PBE exists iff:

(i) Offering a deal is profitable for both merchant types. Therefore, we have,

\[
R_{H}^{D,1}_{pooling} \geq R_{H}^{O} \implies \bar{\alpha} (\alpha_H + N) \geq p\alpha_H; \quad (20)
\]

\[
R_{L}^{D,1}_{pooling} \geq R_{L}^{O} \implies \bar{\alpha} (\alpha_L + \frac{1}{2} \beta N) \geq p\alpha_L; \quad (21)
\]

(ii) The type $L$ merchant does not deviate to $d_L = \alpha_L$ to sell to all new consumers, which yields

\[
\bar{\alpha} (\alpha_L + \frac{1}{2} \beta N) > \alpha_L (\alpha_L + N); \quad (22)
\]

We note that there is no separating PBE in which only the type $H$ merchant offers a deal. This is because, from condition (5), it is always profitable for the type $L$ merchant to offer a deal at a deal price $\alpha_L$ even if new consumers knew its type. Consider instead a separating PBE in which both merchant types offer a deal. As shown in the proof of Lemma 1, in this PBE we require that $d_L = \alpha_L$ and $d_H \leq \alpha_H$. On the equilibrium path, all matched consumers and new consumers will buy the deal as they obtain non-negative surplus. Hence, $R_{L}^{D} = \alpha_L (\alpha_L + N)$. If the type $L$ merchant mimics the type $H$ merchant, early-new consumers will still buy the deal as they (incorrectly) believe that the merchant is of type $H$, while frequent- and late-new consumers can avoid buying the deal by observing deal sales. Therefore, a necessary condition for mimicking to be unprofitable in a separating PBE is

\[
\alpha_L (\alpha_L + N) \geq d_H (\alpha_L + \frac{1}{2} \beta N). \quad (23)
\]
But conditions (22) and (23) cannot both hold for \( d_H \in [\bar{\alpha}, \alpha_H] \). Therefore, if the pooling PBE exists, then a separating PBE in which the type \( H \) merchant realizes a higher profit does not exist.

We now derive the parametric conditions under which the pooling equilibrium occurs. We note that condition (22) is sufficient for condition (21) because of condition (5). Conditions (20) and (22) are linear in \( \bar{\theta} \). Condition (20) holds iff \( \bar{\theta} \geq \theta_1 = 1 - \frac{\alpha_H(N + \alpha_H - p)}{(N + \alpha_H)(\alpha_H - \alpha_L)} \), where \( \theta_1 < 1 \). Condition (22) holds iff \( \bar{\theta} > \theta_2 = \frac{N(1 - \frac{1}{2} \beta)}{(\alpha_L + \frac{1}{2} \beta N)(\alpha_H - \alpha_L)} \). We note that if \( \bar{\theta} \to 0 \), then \( \bar{\alpha} \to \alpha_L \) and condition (22) must hold. Also if \( \bar{\theta} \to 1 \), then \( \bar{\alpha} \to \alpha_H \) and condition (22) will hold only if \( \alpha_L < \alpha_1 \) (as shown in the proof of Proposition 1, \( \alpha_L (\alpha_L + N) < \alpha_H (\alpha_L + \frac{1}{2} \beta N) \) iff \( \alpha_L < \alpha_1 \) because the no-mimicking constraint (9) will not hold). Therefore, \( \theta_2 \in (0, 1) \) if \( \alpha_L < \alpha_1 \). Thus, the pooling equilibrium occurs in the transparent regime iff \( \alpha_L < \alpha_1 \) and \( \bar{\theta} > \max \{ \theta_1, \theta_2 \} \).

**Proof of Lemma 7:** We construct a candidate separating PBE that can occur if \( \alpha_L < \alpha_1 \). We show that this candidate PBE exists iff \( \alpha_L \in (\alpha_2, \alpha_1) \), it is the unique SUE iff \( \bar{\theta} \leq \theta_2 \), and no other separating PBE can be a SUE. As shown in the proof of Proposition 1, \( \alpha_L (\alpha_L + N) < \alpha_H (\alpha_L + \frac{1}{2} \beta N) \) if \( \alpha_L < \alpha_1 \), because the no-mimicking constraint (9) will not hold. Therefore, there exists a deal price \( d \in (\alpha_L, \alpha_H) \) such that \( \alpha_L (\alpha_L + N) = d (\alpha_L + \frac{1}{2} \beta N) \). This deal price \( d \) equals \( d_H^* \) defined in the statement of the Lemma. Consider a separating PBE in which \( d_H = d_H^* \) and \( d_L = \alpha_L \) and consumers adopted the following strategies:

- Matched consumers buy the deal at either deal price regardless of the merchant’s type, with frequent-matched consumers buying the deal in period 1.
- At the deal price \( \alpha_L \), all new consumers buy the deal with frequent visitors buying in period 1.
- At the deal price is \( d_H^* \), early-new consumers buy in period 1, and frequent-new consumers wait till period 2. In period 2, frequent- and late-new consumers buy iff period 1 deal sales is not less than a threshold \( \tau \), where \( \tau \in (\tau_L, \tau_H) \).

In equilibrium, all matched consumers and new consumers buy the deal. It is straightforward to verify that given the merchant strategies, consumers do not have an incentive to deviate. By construction, the type \( L \) merchant will not have an incentive to mimic the type \( H \) merchant. Merchant revenue and website expected profit are as given in the statement of the lemma. This separating PBE exists iff \( R_t^1 \left| \text{separation} > R_t^0 \right. \) so that it is profitable to offer a deal. Therefore

\[
d_H^r (\alpha_H + N) > p\alpha_H, \tag{24}
\]

\[
\alpha_L (\alpha_L + N) > p\alpha_L. \tag{25}
\]

Condition (25) holds because of condition (5). If \( \alpha_L \to \alpha_1 \), then \( d_H^* \to \alpha_H \) and condition (24) holds because of condition (5). If \( \alpha_L \to 0 \), then \( d_H^* \to 0 \) and condition (24) cannot hold. Also \( d_H^* \) is
strictly increasing in $\alpha_L$. Therefore, by continuity there exists $\alpha_2 \in (0, \alpha_H)$ such that condition (24) holds iff $\alpha_L \in (\alpha_2, \alpha_1)$. Further condition (24) must hold as an equality if $\alpha_L = \alpha_2$, from which we obtain $\alpha_2$ as defined in the statement of the Lemma.

We next show that no other separating PBE can be a SUE. As shown in the proof of Lemma 1, in any separating PBE we require that $d_L = \alpha_L$ and $d_H \leq \alpha_H$. By our construction of the candidate PBE, a separating PBE in which the type $H$ merchant charges a higher price than $d_H^*$ cannot exist (since the corresponding no-mimicking constraint will not hold). Also, if a separating PBE in which $d_H < d_H^*$ exists, then the separating PBE in which $d_H = d_H^*$ will also exist and yields higher profit for the type $H$ merchant. Therefore, a separating PBE in which $d_H < d_H^*$ cannot be the SUE. Lastly, there is no separating PBE in which only the type $H$ merchant offers a deal. This is because, from condition (5), it is always profitable for the type $L$ merchant to offer a deal at a deal price $\alpha_L$ even if new consumers knew its type.

Finally, we show that the candidate PBE is an SUE iff $d_H^* \geq \bar{\alpha}$. As shown in the proof of Lemma 6, if $d_H^* \geq \bar{\alpha}$, then the pooling PBE in which $d_t = \bar{\alpha}$ for $t \in \{H,L\}$ cannot exist. This is because the mimicking condition (22) will not hold and the type $L$ merchant will prefer to separate than to pool with the type $H$ merchant. Conversely, if $d_H^* < \bar{\alpha}$, then the pooling PBE will exist whenever the separating PBE exists. This is because the mimicking condition (22) will be satisfied, and condition (20) holds if condition (24) holds. Moreover, the type $H$ merchant’s profit is higher in the pooling PBE since $d_H^* < \bar{\alpha}$. Therefore, the separating PBE we constructed is an SUE iff $d_H^* \geq \bar{\alpha}$. Moreover, no other PBE that leads to a different equilibrium outcome can be a SUE if $d_H^* \geq \bar{\alpha}$. It is straightforward that $d_H^* \geq \bar{\alpha}$ if $\bar{\theta} \leq \theta_2 = \frac{N\alpha_L(1-\frac{1}{2}\beta)}{(\alpha_L+\frac{1}{4}\beta N)(\alpha_H-\alpha_L)}$, where $\theta_2 \in (0,1)$ was derived in the Lemma 6.

**Proof for Lemma 8:** The equilibrium contract is one that maximizes the website’s expected profit subject to the IR constraints (15) for both merchant types and the feasibility constraint $\lambda \in [0,1]$. There are two possible equilibrium scenarios depending on whether the IR constraint (15) binds for one or both merchant types. We will show that the equilibrium contract involves a subsidy in the latter case under certain conditions. Two observations will be useful in our analysis. First, $R_H^Q > R_L^Q$, since the type $H$ merchant has more matched consumers. Second, in any equilibrium, $R_H^D > R_L^D$ since the type $H$ merchant sets a (weakly) higher deal price and realizes (strictly) higher deal sales in any equilibrium.

First, consider the case in which the IR constraint (15) binds for both merchant types. By solving
the resulting pair of simultaneous equations, we obtain

$$\lambda R^D_t - F = R^O_t \quad \forall t \in \{H, L\} \implies \lambda = \frac{R^O_t - R^O_H}{R^D_t - R^D_H}, \quad F = \frac{R^D_t R^O_t - R^D_H R^O_H}{R^D_t - R^D_H}; \tag{26}$$

The contract in equation (26) is feasible if $\lambda \in [0, 1]$. Since $R^O_H > R^O_L$ and $R^D_H > R^D_L$, we have $\lambda > 0$. We have $\lambda \leq 1$ if $R^D_H - R^O_H \geq R^D_L - R^O_L$. The contract is optimal, since it fully extracts the surplus of both merchant types. The contract involves a subsidy ($F < 0$) if $\frac{R^D_H}{R^D_L} > \frac{R^O_H}{R^O_L}$. In fact, we also have that $\lambda < 1$ if $\frac{R^D_H}{R^D_L} > \frac{R^D_H}{R^D_L}$ since,

$$\frac{R^D_H}{R^D_L} > \frac{R^D_L}{R^D_L} \implies \frac{R^D_H - R^O_H}{R^D_H} > \frac{R^D_L - R^O_L}{R^D_L} \implies R^H_H - R^H_O > R^D_L - R^D_L \tag{27}$$

where the last step follows because $R^O_H > R^O_L$. Therefore, if $\frac{R^D_H}{R^D_L} > \frac{R^D_H}{R^D_L}$, then the optimal contract is given by equation (26) and involves a subsidy.

Next, consider the case in which the IR constraint (15) binds for only one of the merchant types. From our analysis above, this can occur iff $R^D_H - R^O_H < R^D_L - R^O_L$. We note that

$$\lambda R^D_H - F \geq R^O_H \implies (R^D_H - R^O_H) - (1 - \lambda) R^D_H - F \geq 0,$$

$$\implies (R^D_L - R^O_L) - (1 - \lambda) R^D_L - F > 0,$$

$$\implies \lambda R^D_L - F > R^O_L, \tag{28}$$

where the penultimate step follows because $R^O_H > R^O_L$ and $R^D_H - R^O_H < R^D_L - R^O_L$. Therefore, the IR constraint can be binding only for the type $H$ merchant. Consequently, the optimal contract in this case can fully extract the surplus of only the type $H$ merchant. Now consider a revenue-sharing contract that fully extracts the surplus of the type $H$ merchant in which $\lambda \in (0, 1)$. We can construct an alternative contract with a higher revenue-sharing rate $\lambda' > \lambda$ and a higher fixed-fee $F' = F + (\lambda' - \lambda) R^O_H$. This contract also fully extracts the surplus of the type $H$ merchant. Moreover, it will extract a higher portion of the type $L$ merchant’s surplus since $R^O_H > R^D_L$. Therefore, website’s expected profit is higher under the alternative contract. It follows that the optimal contract is one in which $\lambda = 1$ and $F = R^D_H - R^O_H > 0$, which does not involve a subsidy. Hence, a subsidy occurs iff $\frac{R^O_H}{R^D_H} > \frac{R^O_L}{R^D_L}$.

**Appendix C  Proofs of Propositions**

**Proof of Proposition 3:** Lemma 7 provides the conditions under which the transparent regime supports a separating equilibrium with price distortion. We first show that if $d^*_H$ is sufficiently close to $\bar{a}$ in this separating equilibrium, then the website will not display deal sales in equilibrium, even though doing so will benefit the type $H$ merchant. Next, we obtain the parametric conditions under
which the website will display deal sales in equilibrium.

We compare the outcomes of the separating equilibrium with price distortion in the transparent regime and the pooling equilibrium in the opaque regime described in Lemmas 7 and 5, respectively. We have $R_L^D|_{\text{separation}} \leq R_L^D|_{\text{pooling}}^0$. If $d_H^* = \bar{\alpha}$, then we have $R_H^D|_{\text{separation}} = R_H^D|_{\text{pooling}}^0$ and, therefore, $\Pi_W^1|_{\text{separation}} \leq \Pi_W^0|_{\text{pooling}}$. If the separating equilibrium exists, then $R_H^D|_{\text{separation}} > R_H^D$ since it is profitable for either merchant type to offer a deal. Therefore, the pooling PBE must also exist since $R_L^D|_{\text{pooling}} \geq R_L^D|_{\text{separation}}$. Moreover, the pooling PBE must be the SUE in the opaque regime as there is no other equilibrium in which the type $H$ merchant can earn higher profit. Thus, if $d_H^* = \bar{\alpha}$, then the website will not display deal sales in equilibrium. Now, as shown in the proof of Lemma 7, $d_H^* = \bar{\alpha}$ iff $\bar{\theta} = \theta_2$ and $d_H^* > \bar{\alpha}$ if $\bar{\theta} < \theta_2$. Hence, by continuity, for $\bar{\theta} < \theta_2$ and sufficiently close to $\theta_2$, displaying deal sales results in lower expected website profit but higher type $H$ merchant revenue and profit (since $d_H^* > \bar{\alpha}$). Thus, there are situations in which displaying deal sales will benefit the type $H$ merchant, but the website will not display deal sales in equilibrium.

To determine the conditions under which the website will display deal sales, we note that there are two candidates for an equilibrium in the opaque regime: (i) a separating equilibrium in which only the type $L$ merchant offers a deal at $d_L = \alpha_L$; and (ii) the pooling equilibrium described in Lemma 4.\textsuperscript{21} The website’s expected profit in the first candidate equilibrium is strictly lower than $\Pi_W^1|_{\text{separation}}$, because $R_H^D = 0$ and $R_L^D = \alpha_L (\alpha_L + N) = R_L^D|_{\text{separation}}$. Therefore, a necessary and sufficient condition for the website to display deal sales is that either the first candidate equilibrium occurs in the opaque regime, or the expected website profit in the transparent regime is higher than that in the pooling equilibrium in the opaque regime.

The candidate equilibrium in which only the type $L$ merchant offers a deal is always a PBE (as it can be supported through pessimistic beliefs). It is also the SUE in the opaque regime if the pooling equilibrium (in which $d_H = d_L = \bar{\alpha}$) cannot be supported. We, therefore, require $R_H^D|_{\text{pooling}} = \bar{\alpha} (\alpha_H + N) \leq p\alpha_H$. It is straightforward to verify that this holds iff $\alpha_L < \alpha_3 = \frac{p\alpha_H}{N + \alpha_H}$ and $\bar{\theta} \leq \theta_1$. Next, for the expected website profit in the transparent regime to be higher than that in the pooling equilibrium in the opaque regime, we require $\Pi_W^1|_{\text{separation}} \geq \Pi_W^0|_{\text{pooling}}$, which yields

$$
(1 - \lambda) \bar{\theta} \left( d_H^* (N + \alpha_H) - N\alpha_H - (2\alpha_H - \alpha_L) \alpha_L - (\alpha_H - \alpha_L)^2 \bar{\theta} \right) \geq 0,
$$

(29)

The above condition holds iff $\bar{\theta} \leq \theta_3$, where $\theta_3 = \frac{d_H^* (N + \alpha_H) - N\alpha_H - (2\alpha_H - \alpha_L) \alpha_L}{(\alpha_H - \alpha_L)^2} < \theta_2$. We still need

\textsuperscript{21}From Lemma 2, there is no separating PBE in which both merchant types offer a deal. There is also no separating PBE in which only the type $H$ merchant offers a deal. From condition (5), it is always profitable for the type $L$ merchant to offer a deal at a deal price $\alpha_L$ even if new consumers knew its type.
to determine if $\theta_3 > 0$. Substituting for $d_H^*$ in the expression for $\theta_3$, we have

$$\theta_3 = \frac{\alpha_3 L + \left( (N - \alpha_H) + \frac{1}{2} N \beta \right) \alpha^2_3 + N (N - \alpha_H) \alpha_L - \frac{1}{2} N^2 \alpha_H \beta}{(\alpha_H - \alpha_L)^2 (\alpha_L + \frac{1}{2} N \beta)}.$$  \hspace{1cm} (30)

By Descartes sign-change rule, the numerator in the RHS of equation (30) has at most one positive root for $\alpha_L$. We note that if $\alpha_L \to \alpha_1$, then $d_H^* \to \alpha_H$ (see condition (9) in proof of Lemma 3) and $\theta_3 > 0$. If $\alpha_L \to \alpha_2$, then $d_H^* \to \frac{p \alpha_H}{N + \alpha_H}$ (see condition (24) in proof of Lemma (7)) and $\theta_3 < 0$. Therefore, by continuity, there exists $\alpha_3 \in (\alpha_2, \alpha_1)$ such that $\theta_3 > 0$ iff $\alpha_L > \alpha_4$, where $\alpha_4$ is the unique positive root of the equation

$$\alpha_3^2 + \left( (N - \alpha_H) + \frac{1}{2} N \beta \right) \alpha^2_3 + N (N - \alpha_H) \alpha_L - \frac{1}{2} N^2 \alpha_H \beta = 0$$  \hspace{1cm} (31)

**Proof of Proposition 4:** Consider the opaque regime. In this case, the only equilibrium in which both merchant types offer a deal is the pooling equilibrium that is described in Lemma 4. In this equilibrium, we have

$$\frac{R^D}{R^O_H} - \frac{R^D}{R^O_L} = - \frac{N \bar{\alpha} (\alpha_H - \alpha_L)}{p \alpha_H \alpha_L} < 0.$$  \hspace{1cm} (32)

Therefore, from Lemma 8, the contract will not involve a subsidy. Next, consider the transparent regime. The website displays deal sales only in a separating equilibrium. From Proposition 1, if $\alpha_L \geq \alpha_1$, then the equilibrium is a separating equilibrium in which $d_t = \alpha_t$. We have,

$$\frac{R^D}{R^O_H} - \frac{R^D}{R^O_L} = \frac{\alpha_H - \alpha_L}{p} > 0.$$  \hspace{1cm} (33)

Therefore, the contract always involves a subsidy. If $\alpha_L < \alpha_1$ and the equilibrium is a separating equilibrium as described in Lemma 7, we have

$$\frac{R^D}{R^O_H} - \frac{R^D}{R^O_L} = \frac{N (N + \alpha_L) \left( \alpha_L - \frac{1}{2} \beta \alpha_H \right)}{p \alpha_H \left( \alpha_L + \frac{1}{2} \beta N \right)},$$  \hspace{1cm} (34)

which is positive iff $\alpha_L > \alpha_4 = \frac{1}{2} \beta \alpha_H$.

**Appendix D  Repeat Purchase**

If new consumers can make a repeat purchase at the regular price $p$, then they will do so if they had bought the deal and found that the product meets their needs. We assume that future profit and future utility are not discounted. New consumers whose needs are met will buy again and obtain a surplus $r - p \geq 0$. The probability that a type $t$ merchant’s product meets a new consumer’s need is $\alpha_t$. Hence, a new consumer’s utility from the deal is

$$u_{NC} = \theta \left( r \alpha_H + (r - p) \alpha_H \right) + (1 - \theta) \left( \alpha_L r + \alpha_L (r - p) \right) - d_t,$$

$$= \left( \theta r \alpha_H + (1 - \theta) r \alpha_L \right) \left( 2 - \frac{p}{r} \right) - d_t.$$  \hspace{1cm} (35)
Comparing equation (35) with equation (2), we observe that repeat purchases increase new consumers’ willingness to pay for the deal by a factor \((2 - \frac{p}{r})\). It will be sufficient to consider repeat purchases by new consumers. Experienced consumers already know the merchant and whether its product can meet their needs. Hence, the deal will neither affect their repeat purchase decision nor the associated merchant revenue. As before, we normalize \(r = 1\). A sufficient condition for the deal price to be lower than the regular price is \(p > (2 - p) \alpha_H\).

We now proceed as in the main model. We show that a separating equilibrium that resembles the symmetric information benchmark can be supported only if the website displays deal sales. Moreover, it can be supported for a larger range of parameters than in the main model. In the symmetric information benchmark, new consumers are willing to pay \((2 - p) \alpha_t\) for a type \(t\) merchant (from equation (35)). Therefore, a type \(t\) merchant charges \(d_t = (2 - p) \alpha_t\), and all new consumers and matched consumers buy the deal. Offering a deal will be attractive for a type \(t\) merchant iff

\[
\alpha_t (N + \alpha_t) (2 - p) + pN \alpha_t > p \alpha_t, \tag{36}
\]

Comparing equation (36) with equation (5), we note that offering a deal is more attractive because of repeat purchases. The website’s expected profit is

\[
\Pi_W = (1 - \lambda) (2 - p) ( \theta \alpha_H (\alpha_H + N) + (1 - \theta) \alpha_L (\alpha_L + N)), \tag{37}
\]

which is higher than before by a factor \(2 - p\). As in Lemma 1, the website’s expected profit cannot be higher than that in the symmetric information benchmark, and can be equal to it only in a separating equilibrium that replicates the symmetric information benchmark outcome. As in Lemma 2, a separating equilibrium does not exist in the opaque regime, because the type \(L\) merchant will always mimic the type \(H\) merchant in such an equilibrium to sell to all new consumers and matched consumers at a higher price.

Consider a separating equilibrium in the transparent website regime in which \(d_t = (2 - p) \alpha_t\) for \(t \in \{H, L\}\). At the deal price \((2 - p) \alpha_H\), frequent- and late-new consumers buy the deal iff period 1 sales is not below \(\tau \in (\tau_L, \tau_H]\), where \(\tau_t\) is as in Lemma 3. They always buy at the deal price \((2 - p) \alpha_L\). All matched consumers buy the deal at either equilibrium deal price from either merchant type. Early-new consumers buy the deal at either equilibrium deal price. The type \(L\) merchant’s expected profit is

\[
\Pi_L = \lambda \alpha_L (\alpha_L + N) (2 - p) + pN \alpha_L. \tag{38}
\]

If it mimics the type \(H\) merchant, then it cannot sell to frequent- and late-new consumers. Its
expected profit is
\[ \Pi'_L = \lambda \alpha_H (\alpha_L + \frac{1}{2} \beta N) (2 - p) + \frac{1}{2} p \beta N \alpha_L. \]  
(39)

For mimicking to be unattractive, we require \( \Pi_L \geq \Pi'_L \), which yields
\[ \alpha_L^2 + (N - \alpha_H) \alpha_L - \frac{1}{2} \alpha_H \beta N + \frac{p}{\lambda (2 - p)} N \alpha_L (1 - \frac{1}{2} \beta) \geq 0. \]  
(40)

Comparing equation (40) with the non-mimicking constraint in equation (9), we note that the former is less stringent as there is an additional positive term \( \frac{p}{\lambda (2 - p)} N \alpha_L (1 - \frac{1}{2} \beta) \) on the LHS. Thus, with repeat purchases, displaying deal sales supports a distortionless separating equilibrium for a larger range of parameters.

**Appendix E  Notation**

\[ t : \text{Merchant’s type, } t \in \{H, L\} \]
\[ r : \text{Consumer utility if product fits consumer need, normalized to 1} \]
\[ \alpha_t : \text{Probability of fit for type } t \text{ merchant} \]
\[ \bar{\theta} : \text{Prior belief that merchant’s type is } H \]
\[ N : \text{Number of new consumers on the website} \]
\[ \beta : \text{Proportion of infrequent visitors on the website} \]
\[ p : \text{Merchant’s regular price, } p \in (r \alpha_H, r] \]
\[ j : \text{Website regime, } j = 1 \text{ for transparent regime, } j = 0 \text{ otherwise} \]
\[ \theta : \text{New consumers’ posterior belief that merchant’s type is } H \]
\[ d_t : \text{Deal price of type } t \text{ merchant} \]
\[ R_t^D : \text{Deal revenue for type } t \text{ merchant} \]
\[ R_t^Q : \text{Revenue from regular sales for type } t \text{ merchant} \]
\[ \lambda : \text{Merchant’s share of deal revenue} \]
\[ F : \text{Fixed fee paid by merchant to website; } F = 0 \text{ in main model} \]
\[ \Pi^W : \text{Website’s expected profit} \]
\[ \Pi_{W_{si}}^W : \text{Website’s expected profit in the symmetric information benchmark} \]
\[ X_{\text{separating}}^j, X_{\text{pooling}}^j : \text{Variable } X \text{ in separating and pooling equilibrium in website regime } j \]