Price Discrimination in a Market with Uninformed Consumer Preferences

June 23, 2016

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Abstract

We study the price discrimination problem when consumers have uninformed preferences – imperfect prior knowledge about their match values with the firm's product. Each consumer receives a private noisy signal that is correlated with her value. If the firm can collect and aggregate these signals, it is able to learn beyond the consumer's private knowledge. This confounds the monopolist's ability to extract differential surpluses, because consumers may under-estimate their value. We show that the firm can effectively price discriminate, but it requires the firm to signal to consumers through personalized prices and a list price. Several novel results arise from this setting: (i) under certain conditions, uniform pricing is more profitable than price discrimination; (ii) the optimal uniform price can decrease with the portion of consumers whose value is higher than others; (iii) relative to uniform pricing, price discrimination may be a strong Pareto improvement, strictly increasing the payoff for the firm and every consumer.

Keywords: Price discrimination, uninformed consumer preference, price signaling, data collection, privacy, strong Pareto improvement

1. Introduction

In typical models of price discrimination, each consumer is fully aware of her own willingness-to-pay while the firm is not. However, in many situations, the informational circumstances are reversed. A consumer often has only her own experience to assess her willingness-to-pay, which may be uncertain for products that are difficult to value, but a firm that deals with many consumers can observe their experiences in aggregate. Consider, for instance, an agent selling automobile insurance. A potential client estimates her accident risk through her own experience with driving (e.g., the number of traffic tickets), which is also a function of environmental factors, such as law enforcement efforts or traffic conditions. The client, therefore, may be unable to perfectly assess how much she should value insurance without observing all the factors that affect her driving experience. The agent, by contrast, has a wealth of actuarial data from the other drivers in the risk pool. Based on that data, the agent can control for environmental factors and obtain a reliable estimate of his client's actuarial accident risk and match value for insurance.

But then, how can the insurance agent quote personalized prices to clients who may be, a priori, uninformed of their match value for the insurance? More generally, how can a firm price discriminate uninformed consumers? Can personalized pricing ever benefit uninformed consumers? If so, under what conditions?

We examine a price discriminating monopolist who faces two types of consumers, each of whom may be uninformed of her *match value* (H or L) for a firm's product. Nature provides each consumer a noisy signal, referred to as her *transient value*, which is correlated with her match value. Though potentially informative, the transient value is subject to uncertainties about the state of the market. Thus, for an individual consumer, the transient value is only an imperfect measure and may not reveal her match value. By contrast, the firm that observes a large sample of consumers' transient values can deduce the state of the market and identify each consumer's type. Therefore, the firm's choice of personalized prices can be informative and facilitate the objectives of price discrimination.

We illustrate our setting using again the case of car insurance. Consider a driver who has been cited for many moving violations. Such a driving record indicates a high transient value and she can be relatively certain of her high risk status. Instead, a record with a few moving violations may not perfectly reveal her type because of unobserved market factors that might have contributed to this outcome. For example, it may not be common knowledge that police have reduced traffic controls. An insurance company, however, has access to actuarial data, including the driving records of many clients and the conditions of their citations. Thus, by observing his client's individual driving record (i.e., her transient value) and comparing it with the risk pool, the insurance agent can better estimate the driver's actuarial risk (i.e. her match value).

The key feature of this setting is that transient values, which may be acquired by the firm and viewed in aggregate, give the firm an informational advantage over consumers when designing personalized pricing schemes. This feature captures the notion that a firm has access to a broader set of observations than any particular consumer does. This is not limited to the case of an insurance agent discussed above. Consider, for instance, an online service that tracks consumers' usage patterns, or a firm that acquires consumers' behavioral data through third-party brokers. By aggregating consumers' data and comparing an individual consumer with others, the firm can rank the consumer and deduce her type even if the consumer cannot.

If consumers are uninformed of their type, then the firm would like consumers all to believe they have a high match value with the product (they are H-types) and charge them accordingly. But rational consumers who know the firm has superior information should be suspicious when the firm tries to overcharge them. The monopolist, therefore, faces a signaling problem in choosing personalized prices that may either inform consumers of their type or manipulate their believes. We show that there exists a separating equilibrium, which is the only Perfect Bayesian equilibrium surviving the D1 criterion (Banks and Sobel, 1987; Cho and Kreps, 1987). The firm is able to effectively price discriminate by identifying consumers and deducing their types. It is, however, unable to capture the entire consumer surplus, because to convince H-types of their type the firm has to leave them positive surplus, which serves as the firm's signaling cost.

The monopolist in our model has the option of using a *list price*, which is an upper bound on all prices in the market and is observable by all consumers. We show that the optional list price is an essential feature of the separating equilibrium. A list price enables the firm to credibly communicate to uninformed H-types that they are not being tricked into overestimating their value when it is below a threshold, and it helps an uninformed L-type to correctly update her belief by observing a personalized price discount. This suggests a novel rationale for the practice of setting a list price with personalized discounts.[1]

[1] While list prices have been explored in prior literature (Horowitz 1992 and Knight et al. 1994), their role is restricted to single-unit sales, such as in the housing market. However, list prices are often used when a firm sells multiple units to differentiated consumers. For example, furniture, insurance, and industrial products typically state a list price though selected buyers are quoted lower prices.

The signaling cost needed for effective price discrimination can be substantial when there is a large portion of uninformed *H*-types. To avoid this cost, the firm may refrain from initially collecting data on consumers' transient values and publically commit itself to being uninformed. This further binds the firm to setting a uniform price. When the firm does not observe consumers' transient values, the firm's price conveys no information and an uninformed consumer estimates her match value to be the weighted market average. If there is a larger portion of *H*-types, uninformed *L*-types increasingly over-estimate their value, which can be a valuable source of profits for the firm. Therefore, we show that the monopolist may actually find it unprofitable to collect consumer information for price discrimination.

Under the conventional assumption that consumers are perfectly informed, the optimal uniform price from the uninformed firm is an increasing function of *H*-type' proportion. This need not be the case with uninformed consumer preferences, because *L*-types raise their estimates of match value when the portion of *H*-types increases. Suppose, for illustration, that it is optimal for the firm to skim the top of the market with a high uniform price. To exploit *L*-types' over-estimation, the firm may lower its uniform price to that estimated value when there are more *H*-types.

Another standard result under the conventional assumption is that *H*-types will be worse-off with price discrimination, even if price discrimination increases average consumer surplus. Again, this need not be the case when consumers have uninformed preferences. In our model, due to the signaling cost, price discrimination prevents *L*-types from overpaying and may reduce *H*-types' personalized price. As a result, it can simultaneously improve the expected surplus for both *H*-types and *L*-types. Price discrimination is nevertheless profitable because it helps the firm serve additional consumers: the informed *L*-types who may not buy under uniform pricing. That is, price discrimination can be a strong Pareto improvement, strictly increasing the payoff for the firm and every consumer.

Our paper is most related to the literature on the economics of price discrimination and firm learning. Existing literature emphasizes welfare loss when firms collect private information from consumers (Taylor, 2004; Acquisti and Varian, 2005; Calzolari and Pavan, 2005).[2] Varian (2002) suggests that consumers may want to hide their willingness-to-pay from the firm, because they will be worse-off if the firm price discriminates (Pigou, 1924; Robinson, 1933; Maskin and Riley, 1984; Varian, 1985; Aguirre et al., 2010). Bergemann et al. (2015) show that price

[2] For a review of economic perspectives of consumer privacy, see Hui and Png (2006).

discrimination may raise average consumer surplus without reducing the firm's profit, but it always harms at least some consumers. By contrast, our results indicate that price discrimination using the data from uninformed consumers is not only economically efficient in raising total social welfare, but it may also enable a strong Pareto improvement. Thus, helping uninformed consumers learn their match value is a source of incremental consumer surplus achieved through price discrimination, which is not seen in earlier literature.

This research also provokes several issues related to consumer privacy. First, our model suggests that price discrimination by a firm acting in self-interest may willingly inform the consumers, even without regulation. This is because by aggregating consumers' data the firm can learn additional information (uninformed consumers' type), of which it prefers to share with at least some consumers (uninformed *H*-types). Therefore, despite its purpose of price discrimination, data collection is a process of not only one-way information transmission but also knowledge creation and bilateral communication. Second, the data-collecting firm identifies whether a consumer is informed of her value and uses the information for price discrimination. Our model, therefore, suggests that the definition of private consumer information may be broader than typically assumed. Specifically, consumers' private information should include not only their private value but also their private knowledge (prior belief). Finally, our findings have implications for the current debate on the welfare consequence of mandatory disclosure of data. Because the data disclosure is enforced and monitored by a third-party, the firm may credibly inform the consumers without incurring signaling costs. We show that market forces alone may enable uninformed consumers to correctly infer their type, therefore the regulation only harms consumers by allowing the firm to fully exploit their willingness to pay. This stands in contrast to policies, such as RACAP, that advocate firms to reveal the private information they collected to the consumers (Kamenica et al. 2011).

Because prices in our setting serve as signals to uninformed consumers, our paper also relates to the classic literature on price signaling and contract design by an informed principal. That work emphasizes a one-way communication of the firm's private information to consumers (Milgrom & Roberts 1984, Bagwell & Riordan 1991; Maskin and Tirole, 1990; Beaudry, 1994). Our work departs in two ways. First, in our model, uninformed consumers' match value becomes private information for the data-collecting firm, and the pricing scheme serves as a signaling message. But given the message (prices) and the consumers' purchasing actions, the firm's private knowledge affects neither the consumers' utility nor the firm's profit. By contrast, in previous price signaling models, the payoffs of both the sender and the receiver depend directly on the sender's private information (e.g. product quality or efficiency level). This distinction implies an iterative process of consumer belief updates by assessing firm incentives under all possible consumer beliefs. Second, classic price signaling models typically assume one receiver. By contrast, the signaling process in our model involves different receivers. The increase in the number of receivers is not a trivial extension because an uninformed consumer has to infer from the message by role playing across the different types she could be.

Finally, our model relates to the literature on contextual inferences and deliberation by uninformed consumers. Wernerfelt (1995) and Kamenica (2008) demonstrate that uninformed consumers may infer about their product utility from the informational content of the product line. They assume that consumers' utility consists of two parts: consumers' type, which is common knowledge, and a global preference parameter, which is known by only a fraction of consumers. Therefore uninformed consumers exogenously know the relative ranking but not the absolute value of their utility. In contrast to their setting, we assume that consumers learn only noisy information on their utility, thus uninformed consumers may not even know their type. This distinction extends the Kamenica's model such that consumers also need to infer the other type's belief. It also allows us to construct a scenario in which the firm may learn beyond uninformed consumers only by observing their data. Other papers examine uninformed consumers' choice of deliberation effort. For example, Guo and Zhang (2012) assumes an exogenous cost in consumer preference retrieval. Because such cost reduces total social welfare, it may be inefficient for the firm to accommodate the effort of consumer deliberation. By contrast, we show that it can be Pareto efficient for consumers to infer their match value, even when the firm is able to manipulate their belief using personalized pricing schemes.

We provide a general modeling framework in the next section and then derive the equilibrium pricing strategy of the firm who has access to consumer data. In Section 3, we study the case in which the firm does not collect data as a benchmark to understand the firm's incentive for price discrimination. In Section 4, we study the implication of price discrimination on consumer surplus and overall economic welfare. We close the analysis by considering the case of naïve consumers who do not make inferences from the firm's behavior in Section 5. Section 6 concludes.

2. The Model

In this section, we develop a model in which the firm designs personalized pricing schemes for consumers who are uninformed of their match value. We first define the relevant notions regarding uninformed consumers and their transient value. Then we analyze the equilibrium pricing when the firm observes consumer transient values.

2.1 Model Setup

A monopolist markets to a set of consumers (normalized to unitary measure), each of which has imperfect knowledge of her *match value* with the firm's product. There are two types,[3] indexed by $i \in \{H, L\}$. A fraction $\lambda \in (0,1)$ of consumers are H-types with a match value α_H and the others have a match value α_L, where $\alpha_L < \alpha_H$. Consumer $i's$ utility from purchase is $U_i = \alpha_i - p_i$, where p_i is the purchase price. Consumer i does not directly learn α_i, instead she observes a private signal that is correlated with α_i. We denote this signal by $\theta_i(M)$ and refer to it as a *transient value*, which is a function of an unobserved *market state*, M, distributed on $[-m, m]$ with density $f(M)$. As will become clear in the argument for Proposition 1, the boundedness property of this state space ensures existence of an equilibrium. Furthermore, we assume that consumers cannot observe others' transient value.

The transient value θ is the key component of the model. To interpret θ, recall the example of car insurance: the number of traffic tickets (θ) serves as a noisy signal for the client's risk type (α). For simplicity of the analysis we assume that transient values preserve the same order as that of match values. In other words, H-types observe greater transient values than L-types under any market state. We also assume that the transient value function is monotonic and differentiable. The assumed properties of the transient values are reflected in the following assumption:

Assumption 1: $\theta_H > \theta_L$ and $\partial\theta_i/\partial M > 0$ for all $M \in [-m, m]$

As will become clear in Section 2.2, Assumption 1 allows transient values to be uninformative. A monopoly firm observes all consumers' transient values.[4] The firm chooses whether to charge consumer i a personalized price, p_i, which only she observes, and whether

[3] The restriction to only two types is made to keep the exposition clear, but is not essential for the results.

[4] We show that by aggregating θ_i, the firm can learn the market state, and correspondingly, identify each consumer's type α_i and perform price discrimination.

to use a *list price*, $\bar{p}$, for everyone in the market to observe. Because consumers can purchase at any price they observe, the list price can be considered as a price ceiling on all personalized prices: $\overline{p} \geq p_i$ for each i. In principal we allow the firm to set $\overline{p} = \infty$ to represent the case in which the firm does not set a list price. Formally, we denote the firm's pricing scheme as a triple $S \equiv (p_L, p_H, \overline{p}) \in \mathbb{R}^2 \times \overline{\mathbb{R}}$. Since a list price is the maximum price that the firm charges for any consumer, it may become a credible signal of the firm's profit.

The timing of the game proceeds as follows. In period 1, nature randomly draws M, each consumer i observes her own transient value θ_i, and the firm observes $\{\theta_L, \theta_H\}$. In period 2, the firm chooses a pricing scheme S and each consumer i decides whether to purchase at the best price available to her: $\min\{\bar{p},\, p_i\}$.

2.2 Uninformed Consumers

Since some values of θ_i may correspond to both types of consumers, it is possible that consumers do not learn their types from transient values in period 1. We call such consumers uninformed.

> **Definition 1**: Consumer $i \in \{L, H\}$ is *uninformed* after observing her transient value $\theta_i(M)$, if and only if there exists another possible market state $M' \in [-m, m] \backslash M,$ such that $\theta_i(M) = \theta_j(M')$ for $i \neq j$. Otherwise, consumer i is *informed*.

Definition 1 implies that a consumer is uninformed whenever her transient value does not reveal her match value. Denote $M_k^i(M) \equiv \theta_k^{-1}[\theta_i(M)]$ as the uninformed consumer i's inferred market state, given her belief that she is a *k*-type (with match value α_k) when observing the transient value $\theta_i(M)$. $M_k^i(.)$ is well defined for all market states, since the monotonicity properties of Assumption 1 assure an inverse relationship between market states and transient values, such that $\theta_k^{-1}(.)$ exists for all transient values. From Definition 1, an *H*-type could believe she is possibly an *L*-type under any market state M, if and only if $M_L^H(M) \in [-m, m]$. Conversely, an *L*-type could believe that she is possibly an *H*-type under any market state M, if and only if $M_H^L(M) \in [-m, m]$.

Lemma 1 specifies conditions under which there may exist a market with uninformed consumer preferences.

Lemma 1: The following statements are equivalent.

(i) $M_H^L(m) \geq -m$;

(ii) $M_L^H(-m) \leq m$;

(iii) H-types are uninformed if and only if under market state $M \in [-m, M_H^L(m)]$;

(iv) L-types are uninformed if and only if under market state $M \in [M_L^H(-m), m]$.

The proof of Lemma 1 is straightforward and relegated to the Appendix. Note that our framework can incorporate the classic case when consumers ax-ante know their match value. By Lemma 1, when $M_H^L(m) < -m$, then the intervals $[-m, M_H^L(m)]$ and $[M_L^H(-m), m]$ are empty, and consumers are always informed by observing their transient value. Since our focus is on uninformed consumer preferences, we assume that uninformed consumers exist with a positive probability for the remainder of the analysis:

Assumption 2: $M_H^L(m) \geq -m$.

Under Assumption 2, Figure 1 illustrates a region of market states under which consumers may be uninformed.

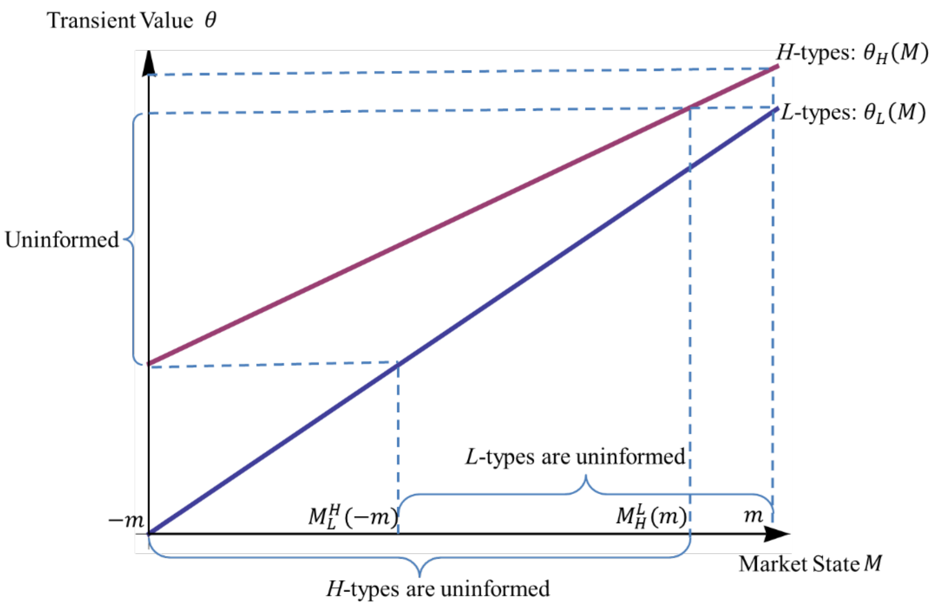

Figure 1: Uninformed Consumers in All Market States

2.3 Informed Firm

An informed firm is one that observes consumers' transient values $\{\theta_L, \theta_H\}$, which we call *consumer data*. Because the firm observes $\{\theta_L, \theta_H\}$, it can, by Assumption 1, identify each consumer's type based on the ranking of the consumer's transient value. Furthermore, the firm can also infer the market state M, which informs the firm about the distribution of uninformed consumers' type and their prior belief. Therefore, the firm may design its personalized pricing scheme as a function of the market state, i.e., $S = \{s_H(M), s_L(M)\}$, where $s_i(M) \equiv (p_i, \bar{p}): [-m, m] \to \mathbb{R} \times \overline{\mathbb{R}}$, is the pricing scheme that consumer i observes.

By contrast, an uninformed consumer i, who observes only her own transient value, does not learn the market state M and infers it from two possible values, i.e., $M \in \{M_H^i, M_L^i\}$. Because the pricing scheme is a function of the market state, the consumers may infer the market state from the pricing scheme whenever possible. In particular, consumer i may update her belief about the probability that she is an H-type based on both the pricing scheme s_i and her transient value θ_i. For convenience we denote the updated belief of being an H-type as $\mu(s_i)$. For instance, if consumer i is uninformed, then upon observing s_i, she assigns probability $\mu(s_i)$ to the market state M_H^i, under which she is an H-type, and probability $1 - \mu(s_i)$ that the market state is M_L^i, under which she is an L-type. As a result, she estimates her match value as $E[\alpha|\mu(s_i)] = \mu(s_i)\alpha_H + [1 - \mu(s_i)]\alpha_L$.

We examine the equilibrium pricing scheme and uninformed consumers' rational beliefs using Perfect Bayesian Equilibrium (PBE). As in many signaling games, there are a multitude of PBE in our model. Most of these equilibria involve unreasonable out-of-equilibrium consumer beliefs. To eliminate these beliefs, we employ the D1 criterion to refine any equilibrium (Banks and Sobel 1987; Cho and Kreps 1987; Cho and Sobel, 1990). By applying D1, we specify "reasonable" beliefs $\mu(s_i')$ for uninformed consumers when the firm chooses an out-of-equilibrium pricing scheme s_i'. Particularly, upon observing a deviation, an uninformed consumer assigns zero probability to a given market state, if and only if she believes that whenever the firm has weak incentives to deviate in that state, it has strong incentives to deviate in a different state.[5]

Formally, consider a PBE $\{s_i^*, r_i^*, \mu(s_i^*)\}_{i=H,L}$, where $r_i \in \{0,1\}$ is consumer i's purchase decision, and $\{0,1\}$ corresponds to "No Purchase" and "Purchase", respectively. When consumer i is uninformed, she infers the market states to be either M_L^i or M_H^i. Deonte

[5] Formal details of the equilibrium refinement process are relegated to the Appendix.

the firm's equilibrium profit as $\Pi^*(M)$. For an out-of-equilibrium pricing scheme $s_i' \equiv (p_i', \bar{p}') \neq s_i^*$, denote $\mu(s_i')$ as the corresponding consumer belief and $\widehat{\Pi}_i(M, s_i', r_i)$ as the deviation profit that consumer i attributes to the firm in the market state M, where $r_i = BR(s_i', \mu)$ is the consumer's best response to that deviation s_i' given belief μ. For convenience of presentation, we establish the existence of an unique PBE surviving D1 under the restriction that $M_H^L(m) < M_L^H(-m)$, which implies informed consumers exist under any market state. We relax this restriction in the Appendix and show that our results are robust under general conditions.[6] Under this restriction, denote $\mathcal{M}^0 \equiv (M_H^L(m), m]$ and $\mathcal{M}^1 \equiv [-m, M_H^L(m)]$ as the set of market states in which *H*-types are informed and uninformed, respectively. Proposition 1 shows the firm's equilibrium pricing strategy is separating between $\mathcal{M}^0$ and $\mathcal{M}^1$.

Proposition 1: There exists a PBE, $\{s_i^*, r_i^*, \mu(s_i^*)\}_{i=H,L}$, with the following properties:

(i) The firm sets prices $s_i^* = (p_i^*, \bar{p}^*)$ according to

$$p_H^*(M) = \bar{p}^*(M) = \begin{cases} \alpha_H & \text{if } M \in \mathcal{M}^0 \\ \lambda\alpha_H + (1-\lambda)\alpha_L & \text{if } M \in \mathcal{M}^1, \end{cases}$$

$p_L^*(M) = \alpha_L$, for all $M \in [-m, m]$.

(ii) All consumers purchase: $r_i^* = 1,\ i = L, H$.

(iii) Uninformed consumer *i* updates her belief for any pricing strategy s_i' as follows:

$\mu(s_i') = 1$, if $\widehat{\Pi}_i\left(M_L^i, s_i', r_i\right) \leq \Pi^*\left(M_L^i\right)$, where $r_i = BR\left(s_i', M_H^i\right)$; and

$\mu(s_i') = 0$, if otherwise.

(iv) This is the unique PBE surviving D1.

Proof: Here we establish existence of a PBE satisfying properties (i)-(iii). The claim (iv) is relegated to the Appendix.

When $M \in \mathcal{M}^0$, *H*-types are informed and, therefore, purchase at any price at or below α_H. Since $M_H^L(m) < M_L^H(-m)$, $\mathcal{M}^0 = \left(M_H^L(m), M_L^H(-m)\right) \cup [M_L^H(-m), m]$. If $M \in$

[6] This restriction simplifies our analysis without changing any key results, so we use it to present our results in the remaining sections of the paper. In the Appendix we relax it and establish our result under general conditions. Specifically, $M_H^L(m) < M_L^H(-m)$ is equivalent to $M_H^L[M_H^L(m)] < -m$. Thus Proposition 1 is a special case for $-m \in \left(M_H^{L\,(N+1)}(m), M_H^{L\,(N)}(m)\right]$ when $N = 1$. The models that assume all consumers are informed are a special case when $N = 0$, i.e., $M_H^L(m) < -m \leq m$. The generalization of Proposition 1 for any integer $N > 1$ is proved in the Appendix.

$\left(M_H^L(m), M_L^H(-m)\right)$, then L-types are also informed, thus they are willing to pay at most α_L. Therefore, the firm's optimal pricing strategy is $p_H^* = \alpha_H$, and $p_L^* = \alpha_L$. Otherwise if $M \in [M_L^H(-m), m]$, then L-types are uninformed. To show s_L^* is optimal for the firm, consider an arbitrary deviation $s_L' = (p_L', \bar{p}') \neq s_L^*$. A deviation s_L' is profitable if and only it convinces L-types to pay more, which requires a belief $\mu(s_L') > 0$. Therefore by (iii), $\widehat{\Pi}_L(M_L^L, s_L', r_L) \leq \Pi^*(M_L^L)$, where $r_L = BR(s_L', M_H^L) = 1$ if $p_L' \leq \alpha_H$. Since the belief $M = M_H^L$ is most favorable for the firm, i.e., $\mu_L = 1 | M = M_H^L$, we have $\widehat{\Pi}_L(M_L^L, s_L', r_L) \geq \widehat{\Pi}_L\left(M_L^L, s_L', BR(s_L', \mu)\right)$ for any $\mu \leq 1$, which then implies $\widehat{\Pi}_L\left(M_L^L, s_L', BR(s_L', \mu)\right) \leq \Pi^*(M_L^L)$. Thus, no deviation s_L' is more firm-profitable than s_L^*. In addition, $r_L^* = 1$ is a best response, since $p_L^* = \alpha_L$. Therefore (i) and (ii) satisfy the sequential rationality conditions. Finally, since by (i) s_L^* is offered only when $M \in \mathcal{M}^0$, the updated belief $\mu(s_L^*) = 0$ satisfies Bayes' rule.

Now suppose $M \in \mathcal{M}^1$. Since $M_H^L(m) < M_L^H(-m)$, L-types are informed. H-types are uninformed and infer from two possible states M_L^H or M_H^H. Since $M \in \mathcal{M}^1$ and $M_H^L(m) < M_L^H(-m)$ imply $M_L^H \in \mathcal{M}^0$, by (i) the firm's equilibrium profit under M_L^H is $\Pi^*(M_L^H) = \lambda\alpha_H + (1-\lambda)\alpha_L = p_H^*$. Next, we examine H-types' estimate of the firm's profit for an arbitrary deviation pricing scheme $s_H' = (p_H', \bar{p}') \neq s_H^*$. Clearly, s_H' is never profitable unless H-types held the belief $\mu(s_H') > 0$. Then, by (iii), we must have $\mu(s_H') = 1$ with $\widehat{\Pi}_H(M_L^H, s_H', r_H) \leq \Pi^*(M_L^H)$. When H-types consider that the market state is M_L^H, then they assume the following: (1) they are L-types (i.e. $\mu(s_H') = 0$); (2) the other consumers are informed H-types and pay at $\min\{\bar{p}', \alpha_H\}$. Thus, H-types assess the firm's profit in M_L^H when they are tricked into overpaying as $\widehat{\Pi}_H(M_L^H, s_H', r_H) = \lambda \min\{\bar{p}', \alpha_H\} + (1-\lambda) p_H' r_H$. Since $r_H = BR(s_H', M_H^H) = 1$ when H-types observe $p_H' \leq \alpha_H$, and since it is a dominant strategy to set $\bar{p}' \geq p_H'$, we have $\widehat{\Pi}_H(M_L^H, s_H', r_H) \geq p_H'$. Because $\Pi^*(M_L^H) = p_H^*$, the inequality $\widehat{\Pi}_H(M_L^H, s_H', r_H) \leq \Pi^*(M_L^H)$ implies $p_H' \leq p_H^*$. In addition, L-types are informed and pay no more than α_L, thus $p_L' \leq \alpha_L = p_L^*$, so the firm could do no better with s_H' than the equilibrium s_H^*. Further, $r_H^* = 1$ is a best response, since $p_H^* < \alpha_H$. Thus the equilibrium satisfies the sequential rationally conditions. Finally, since, by (i), $s_H^*(M \in \mathcal{M}^1)$ is offered to only uninformed H-types, thus $\mu(s_H^*) = 1$ as specified in (iii) satisfy Bayes' rule. ■

First note from Proposition 1 that this PBE is a separating equilibrium since $s_L^*(M) \neq s_H^*(M)$ for all $M \in [-m, m]$. Therefore, any uninformed consumer can infer her type from her personalized pricing scheme in equilibrium. Such learning is sustained by the belief updating rule in (iii). For example, suppose L-types are uninformed (i.e. $M \in \mathcal{M}^0$), then their belief can be intuitively expressed in the following speech: “If the firm has a strong incentive to deviate by tricking me into believing that I am an H-type with the pricing scheme that I observe, then I shall assess that I am an L-type by observing that pricing scheme.” This belief is rational and survives D1, because we show in the Appendix that the firm may deviate more often under M_L^i. In this way, equilibrium beliefs ensure an uninformed consumer maintains a level of suspicion that prevents the firm from misleading her. Since $p_L^* = \alpha_L$, we see that such beliefs prevent the firm from overcharging uninformed L-types.

In contrast to L-types, uninformed H-types can obtain positive surplus from their personalized pricing scheme, even though it allows H-types to correctly infer their match value. Specifically, to convince uninformed H-types that the market state is low ($M \in \mathcal{M}^1$), the firm sets the list price, $\overline{p}^*$, no higher than what it could earn, even under most favorable beliefs, when the market state is high ($M \in \mathcal{M}^0$). Furthermore, by setting the list price $\overline{p}^* \leq \Pi^*(M \in \mathcal{M}^0)$, the firm expresses to any uninformed H-type “I am not charging anyone more than I am charging you, so that you know I would never use this pricing scheme if you were not an H-type.” Therefore, in equilibrium, H-types receive positive expected surplus and L-types avoid paying more than their match value. The inability of the firm to either extract the full surplus of H-types or overcharge L-types in equilibrium constitute the cost of signaling the firm’s private information. As we see in the next section, such signaling costs can be, in some circumstances, a significant burden to the price discriminating monopolist.

3. Firm’s Incentive for Price Discrimination

In this section, we examine the impact of price discrimination on firm profit. In Section 3.1, we analyze the benchmark case when the firm does not observe consumers’ transient values. In Section 3.2, we compare the benchmark case with the equilibrium in Proposition 1 to assess when the firm has incentives to collect data for price discrimination. In Section 4, we assess the impact of price discrimination on consumer welfare.

Without loss of generality, we employ the following restrictions on the functional form of θ_i and the distribution of M to simplify the analysis for the benchmark case on uninformed firm.

Assumption 3: The transient value of consumer i with market state M is $\theta_i(M) \equiv \alpha_i + \beta_i M$, where $M \sim U[-m, m]$, $\alpha_H > \alpha_L$, and $\beta_i > 0$.

Assumption 3 is a stronger form of Assumption 1. We also add another period to the game, period 0, in which the firm decides whether or not to collect consumer data.

3.1 Uninformed Firm

An uninformed firm is a firm that commits in period 0 not to observe the transient value θ_i from any consumer i. Each consumer, as before, observes her own transient value and updates her belief accordingly. By Lemma 1, since H-types are uninformed if and only if $M \leq M_H^L(m)$, and L-types are uninformed if and only if $M \geq M_L^H(-m)$, we denote ρ_i as the initial probability that consumer i is uninformed, $i = H, L$:

$$\rho_H \equiv \int_{-m}^{M_H^L(m)} f(M)dM, \text{ and } \rho_L \equiv \int_{M_L^H(-m)}^{m} f(M)dM.$$

Next we derive uninformed consumers' updated beliefs. Denote Θ as the random variable with transient value realizations θ_i for uninformed consumer i, i.e., $\Theta \in [\theta_H(-m), \theta_L(m)]$, and $\Theta|\alpha$ as the conditional random variable with conditional transient value realizations $\theta_i|\alpha_k$. From Assumption 3, the conditional probability density function is

$$f_{\Theta|\alpha}(\theta_i|\alpha_k) = \left|\frac{d}{d\theta_i}\theta_k^{-1}(\theta_i)\right| f\left(M_k^i\right).$$

Therefore, uninformed consumers have the same posterior Bayesian estimate:

$$e \equiv E[(\alpha|\theta_i)] = \alpha_L + (\alpha_H - \alpha_L)\Pr(\alpha_H|\theta_i),$$

where $\Pr(\alpha_H|\theta_i) = \frac{Pr(\alpha_H)f_{\Theta|\alpha}(\theta_i|\,\alpha_H)}{\sum_k[Pr(\alpha_k)f_{\Theta|\alpha}(\theta_i|\,\alpha_k)]} = \frac{\lambda}{\lambda+(1-\lambda)w}$, $w \equiv \frac{\beta_H}{\beta_L}$. Note that $\frac{\rho_L}{\rho_H} = \frac{\beta_H}{\beta_L}$, so that w can also be interpreted as the ratio of the probability for L-types to be uninformed over that for H-types. We summarize the consumers' updated estimate of their match value in Table 1.[7]

Table 1: Distribution of Uninformed Consumers' Beliefs

Market State	**Consumer's Estimates**	
	***L*-Type**	***H*-Type**
$M \in [-m, M_H^L(m)]$	α_L	e
$M \in (M_H^L(m), M_L^H(-m))$	α_L	α_H
$M \in [M_L^H(-m), m]$	e	α_H

[7] Since $M_H^L(m) < M_L^H(-m)$, there are only three scenarios: (1) when $-m \leq M \leq M_H^L(m)$, only H-types are uninformed; (2) when $M_H^L(m) < M < M_L^H(-m)$, both consumers are informed; (3) when $M_L^H(-m) \leq M \leq m$, only L-types are uninformed.

Without collecting data on consumers, the firm cannot identify the consumers and use personalized pricing. That is, an uninformed firm posts only a uniform price, p_U, to all consumers. The firm, however, knows the distribution of consumers' updated estimates in Table 1. Thus the optimal uniform price must be one of these updated estimates listed in Table 1: $p_U^* \in \{\alpha_L, e, \alpha_H\}$. Denote the expected profit for the firm conditional on price p as $\Pi_U(p;\lambda)$, we express the firm's problem as:

$$\underset{p\in\{\alpha_L,e,\alpha_H\}}{\text{Max}} \Pi_U(p;\lambda)$$

where

$$\Pi_U(\alpha_H,\lambda) = \lambda(1-\rho_H)\alpha_H,$$
$$\Pi_U(e,\lambda) = [\lambda + (1-\lambda)\rho_L]e,$$
$$\Pi_U(\alpha_L,\lambda) = \alpha_L.$$

Proposition 2: The unique equilibrium list price for the uninformed firm is characterized as follows:

(i) $p_U^* = \alpha_H$, if and only if $\rho_H \leq \frac{1}{2}\left(1-\sqrt{\frac{\alpha_L}{\alpha_H}}\right)$ and $\lambda \in [\max\{\lambda_{HE1},\lambda_{HL}\},\lambda_{HE2}]$, where $\lambda_{HE1},\lambda_{HE2}$ solve $\Pi_U(\alpha_H,\lambda) = \Pi_U(e,\lambda)$,[8] $\lambda_{HE1} < \lambda_{HE2} \in (0,1)$, and $\lambda_{HL} \equiv \frac{\alpha_L}{\alpha_H(1-\rho_H)} \in (0,1)$.[9]

(ii) Otherwise, there exists a unique $\lambda_{LE} \in (0,1)$ that solves $\Pi_U(e,\lambda) = \Pi_U(\alpha_L,\lambda)$, and

$$p_U^* = \begin{cases} e, & \text{if } \lambda \geq \lambda_{LE} \\ \alpha_L, & \text{if } \lambda < \lambda_{LE} \end{cases}.$$

Proposition 2 provides the benchmark in Section 4 to assess the impact of price discrimination on welfare. But it also directly illustrates a novel consequence of uninformed consumer preferences. When the probability that *H*-types are uninformed is below the threshold, $\frac{1}{2}\left(1-\sqrt{\alpha_L/\alpha_H}\right)$, the equilibrium uniform price is non-monotonic in the portion of *H*-types, λ. The conventional wisdom, under the assumption that consumers are fully informed about their type, suggests that a non-discriminating monopoly will (weakly) raise the uniform price as λ increases. However, in our model the uninformed firm may prefer to lower the uniform price when there are more *H*-types. To understand this result, note first that the prior probability they

[8] In Lemma A2 of the Appendix we show that $\rho_H \leq \frac{1}{2}\left(1-\sqrt{\alpha_L/\alpha_H}\right)$ implies $\lambda_{HE1},\lambda_{HE2} \in (0,1)$ exist.

[9] In the proof of Proposition 2 we show that $\rho_H \leq \frac{1}{2}\left(1-\sqrt{\alpha_L/\alpha_H}\right)$ implies that $\lambda_{HL} \in (0,1)$.

are uninformed, ρ_H, is independent of λ. However, uninformed consumers' estimate e is an increasing function of λ, since the expectation of match value is higher if there are more H-types. Therefore, the firm may prefer to lower the uniform price from α_H to e to exploit this increased over-estimation from uninformed L-types. Figure 2 depicts the optimal list price as a function of λ.[10]

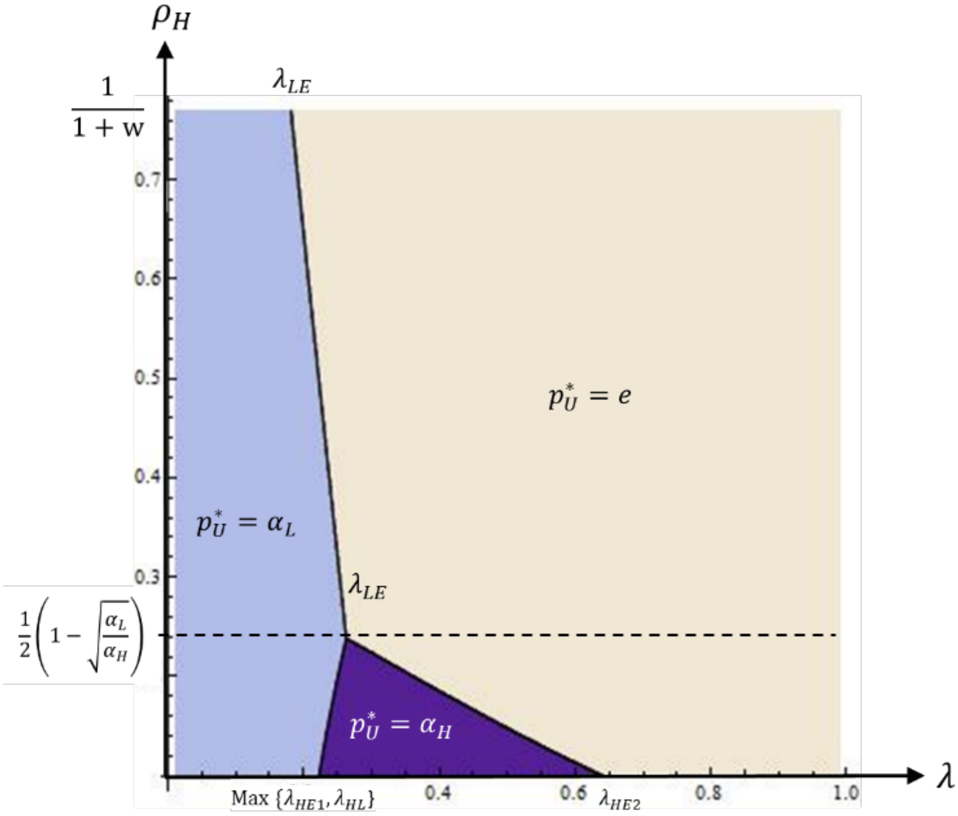

Figure 2: Prices without Price Discrimination ($\alpha_H = 5\alpha_L, w = 0.3$)

Note: The boundary curves in Figure 2 are solved in Proposition 2. $w \equiv \frac{\beta_H}{\beta_L} = \frac{\rho_L}{\rho_H}$. *The curve between the light and darker region is* λ_{LE}, *which is the iso-profit curve along which* $\Pi_U(e,\lambda) = \Pi_U(\alpha_L,\lambda)$ *. Similarly,* λ_{HL} *is the iso-profit curve along which* $\Pi_U(\alpha_H,\lambda) = \Pi_U(\alpha_L,\lambda)$*; and* λ_{HE1} *and* λ_{HE2} *are the two iso-profit curves along which* $\Pi_U(\alpha_H,\lambda) = \Pi_U(e,\lambda)$*, which exist if and only if* $\rho_H \leq \frac{1}{2}\left(1-\sqrt{\alpha_L/\alpha_H}\right)$*. The upper bound for* ρ_H *is* $\frac{1}{1+w}$ *by the restriction that* $M_H^L(m) < M_L^H(-m)$*.*

3.2 When Is Price Discrimination Profitable?

We compare the firm's profits from the equilibrium in Proposition 1 and Proposition 2. That is, we examine whether data collection improves the firm's profit by facilitating price

[10] The numerical example is $\rho_H = \rho_L = 0.1$; $\alpha_H = 2\alpha_L = 2$. We can solve that $e = 1 + \lambda$; $\Pi_U(\alpha_H;\lambda) = 1.8\lambda$; $\Pi_U(e;\lambda) = (0.9\lambda + 0.1)(1+\lambda)$; $\Pi_U(\alpha_L;\lambda) = 1$. Note: The numerical values are $\rho_H = \rho_L = 0.1$; $\alpha_H = 2\alpha_L = 2$. Note that the list price is reduced when $\lambda > \lambda_{HE2} \approx 0.75$.

discrimination relative to no data collection and uniform pricing. Conventional wisdom suggests that acquiring the ability to discriminate in price always improves the firm's profit. Therefore a monopoly always prefers personalized pricing under the standard assumption. This need not be the case with uninformed consumer preference. Specifically, the firm prefers not to price discriminate at least under some conditions.

Before the firm decides whether to collect data on consumers, it solves the expected profit given the distribution of the market states. From the analysis above, the expected profit for the informed firm is

$$\Pi_I^* = [\lambda^2\alpha_H + (1-\lambda^2)\alpha_L]\rho_H + [\lambda\alpha_H + (1-\lambda)\alpha_L](1-\rho_H)$$
$$= \alpha_L + \lambda(1-\rho_H+\lambda\rho_H)(\alpha_H-\alpha_L).$$

From Proposition 2, the expected profit for the uninformed firm is

$$\Pi_U^* = \max\{\Pi_U(\alpha_L), \Pi_U(e), \Pi_U(\alpha_H)\},$$

where we drop the argument of λ. Note that, for all λ, $\Pi_I^* > \lambda(1-\rho_H)\alpha_H = \Pi_U(\alpha_H)$. That is, acquiring consumer data on transient values and price discriminating is always more profitable than skimming the high end of the market with a uniform price, α_H. Furthermore, since $\rho_H \in (0,1)$, we have $\Pi_I^* > \alpha_L = \Pi_U(\alpha_L)$. Again, observing transient values and price discriminating is more profitable for the firm than serving the entire market with a low uniform price, α_L. Thus, the firm prefers uniform pricing over price discrimination only if it can exploit *L*-types' over-estimation by charging $p_U^* = e$. To assess the precise condition, we only need to compare Π_I^* with $\Pi_U(e)$. Proposition 3 suggests that the firm prefers no price discrimination if and only if the probability of *H*-types being uninformed, ρ_H, exceeds a threshold.

Proposition 3 Let $\underline{\rho}_1 \equiv \frac{w[(\alpha_H-\alpha_L)\lambda+(1-\lambda)\alpha_L]+\lambda\alpha_L}{(\alpha_H-\alpha_L)\lambda^2+\lambda w[(1-\lambda)(\alpha_H-\alpha_L)+\alpha_H]+w^2\alpha_L(1-\lambda)} > 0.$

(i) If $\rho_H \geq \underline{\rho}_1$, then it is profitable for the firm to commit not collecting data on consumers and setting a uniform price $p_U^* = e$.

(ii) Otherwise, if $\rho_H < \underline{\rho}_1$, then it is profitable for the firm to price discriminate according to Proposition 1.

When consumers are uninformed of their match value, they use their personalized prices to infer their type. The set of personalized prices is, therefore, a signal to consumers and, as in most signaling models, reveals the incurrence of signaling cost. This signaling cost is increasing in ρ_H: As the probability of *H*-types being uninformed increases, the firm must compromise larger margins on H-types to convince them of their high match value. But, while the informed firm's profit is decreasing in ρ_H, the uninformed firm's profit $\Pi_U(e)$ is increasing in ρ_H, because uninformed consumers raise their estimates of match value when ρ_H increases. As such, the firm prefers not observing transient values when $\rho_H > \underline{\rho}_1$ to avoid the signaling cost. The reverse intuition holds when $\rho_H < \underline{\rho}_1$.

Finally, it is instructive to consider the comparative statics of profits with respect to λ. Both the uninformed and informed firms benefit from a larger portion of H-types. The rate of increase, however, is different. For the price discriminating firm, increasing λ raises only the uninformed H-types' personalized price. By contrast, for the uninformed firm, increasing λ raises all uninformed consumers' price, e. Therefore, increasing λ increases $\Pi_U(e)$ at a faster rate than Π_I^*. Figure 3 depicts the region in which price discrimination is profitable for the firm, as implied by Proposition 3. The impact just described is reflected by the fact that the boundary line is downward sloping.

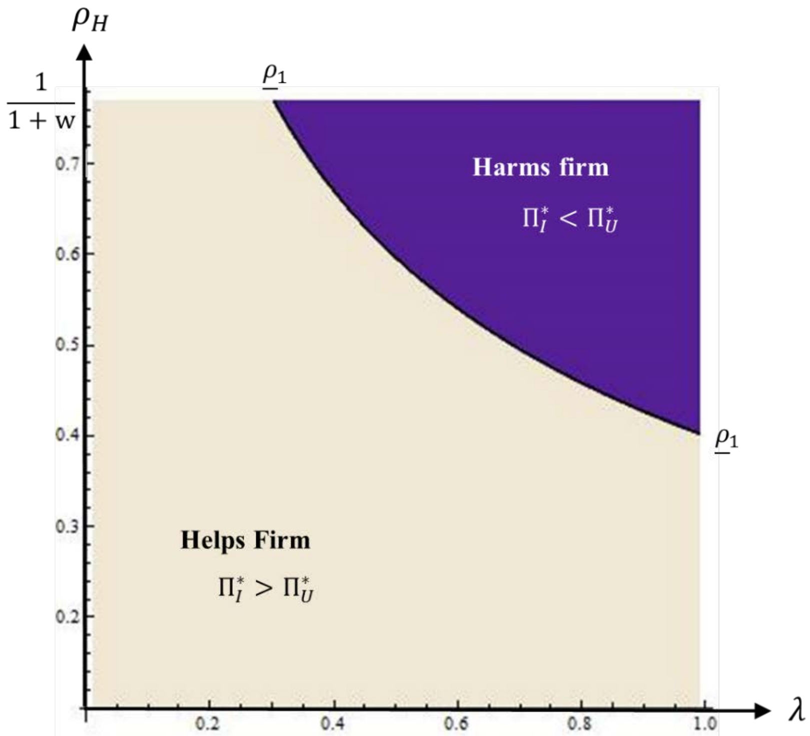

Figure 3: Equilibrium Region for Price Discrimination ($\alpha_H = 5\alpha_L, w = 0.3$)

Note: The boundary curve in Figure 3 is solved in Proposition 3 as $\rho_H = \underline{\rho}_1$, *which is the iso-profit curve along which* $\Pi_I = \Pi_U(e)$. $w \equiv \frac{\beta_H}{\beta_L} = \frac{\rho_L}{\rho_H}$. *The upper bound for* ρ_H *is* $\frac{1}{1+w}$ *by the restriction that* $M_H^L(m) < M_L^H(-m)$.

4. Impact of Price Discrimination on Consumer and Total Welfare

In this section, we examine the effect of price discrimination on consumer surplus and total social welfare. Proposition 1 quickly establishes that price discrimination never reduces the total social welfare because the informed firm serves every consumer in equilibrium. It is also immediate that the firm collects data if and only if it is profitable to do so. Therefore, we assess whether equilibrium price discrimination is a Pareto improvement by examining its impact on consumer surplus, and whether the improvement is strong or weak.

When the firm is informed and price discriminates, the average consumer surplus is $CS_I = \lambda\rho_H(1-\lambda)(\alpha_H - \alpha_L)$. When the uninformed firm charges the list price p_U^*, we denote the average consumer surplus as $CS_U(p_U^*)$:

$$CS_U(\alpha_H) = 0;$$

$$CS_U(e) = \frac{\lambda(1-\lambda)(\alpha_H-\alpha_L)w}{\lambda+w(1-\lambda)}(1-\rho_H); \text{ and}$$

$$CS_U(\alpha_L) = \lambda(\alpha_H - \alpha_L).$$

From Proposition 3, the firm always prefers personalized pricing to uniform pricing when $p_U^* = \alpha_H$ or $p_U^* = \alpha_L$. In addition, direct comparison of the $CS_U(p_U^*)$ suggests that $CS_U(\alpha_H) < CS_I < CS_U(\alpha_L)$. Therefore price discrimination improves both average consumer surplus and the firm's profit when $p_U^* = \alpha_H$. In this case, price discrimination is a weak Pareto improvement, because L-types' surplus remains zero as they are either unserved by the uninformed firm or fully-exploited by the discriminating firm. This intuition is consistent with the conventional wisdom that price discrimination may help consumers on average only if the total demand increases (Varian 1985, Bergemann et al. 2015).

When $p_U^* = e$, price discrimination raises the firm's profit if and only if $\rho_H < \underline{\rho}_1$ by Proposition 3. In addition, price discrimination raises the average consumer surplus if and only if $CS_I > CS_U(e)$, which is equivalent to $\rho_H > \frac{w}{\lambda+w(2-\lambda)} \equiv \underline{\rho}_2$. Therefore when $p_U^* = e$ and $\rho_H \in (\underline{\rho}_2, \underline{\rho}_1)$, price discrimination also benefits both the consumers on average and the firm.

Further, when $p_U^* = e$, uninformed L-types always benefit from the price discrimination, because they can avoid over-paying when they over-estimate their match value ($e > \alpha_L$). Therefore L-types have strictly higher expected surplus under price discrimination. Uninformed H-types may also be better off if $p_H^* = E[\alpha] < e$, or equivalently, $w < 1$. But since informed H-types are worse off by paying a higher price α_H, we need to examine whether price discrimination may improve H-types' expected surplus. Denote each H-type's expected surplus with price discrimination as CS_{IH}, and that with the uninformed firm who charges the list price $p_U^* = e$ as $CS_{UH}(e)$. We have

$$CS_{IH} = \rho_H(1-\lambda)(\alpha_H - \alpha_L);$$
$$CS_{UH}(e) = \alpha_H - e.$$

The condition $CS_{IH} > CS_{UH}(e)$ is equivalent to $\rho_H > \frac{w}{\lambda + w(1-\lambda)} \equiv \underline{\rho}_3 > \underline{\rho}_2$. Therefore, if $\rho_H \in (\underline{\rho}_3, \underline{\rho}_1)$, price discrimination strictly raises the expected surpluses of both types as well as the firm's profit, making it a strong Pareto improvement. Otherwise if $\rho_H \in (\underline{\rho}_2, \underline{\rho}_3]$, price discrimination harms *H*-types but still benefits both the consumers on average and the firm.[11] Proposition 4 formalizes the above results:

Proposition 4 Price discrimination

(i) is a weak Pareto improvement, if $p_U^* = \alpha_H$;

(ii) is a strong Pareto improvement, if $p_U^* = e$ and $\rho_H \in (\underline{\rho}_3, \underline{\rho}_1)$;

(iii) harms *H*-types but benefits consumers on average, if $p_U^* = e$ and $\rho_H \in (\underline{\rho}_2, \underline{\rho}_3]$,

where $\underline{\rho}_1$ is given in Proposition 3, $\underline{\rho}_2 \equiv \frac{w}{\lambda + w(2-\lambda)}$ and $\underline{\rho}_3 \equiv \frac{w}{\lambda + w(1-\lambda)}$.

Figure 4 illustrates the conditions described in Proposition 4. The conditions in part (i) of the proposition correspond to the triangular section in the lower-center portion of Figure 4 (Region V) and that of Figure 2. The conditions in part (ii) of Proposition 4 correspond to the second upper region in Figure 4 (Region II). In this region, the uninformed firm is charging the uniform price $p_U^* = e$. On the consumer-side, there is an increase in consumer surplus stemming from the benefit to uninformed *L*-types who no longer overpay. Price discrimination also imposes a strict gain in states where uninformed *H*-types pay the informed firm a price lower than e. The intuition is that when ρ_H is sufficiently high, i.e., $\rho_H > \underline{\rho}_3$, then the signaling cost is so high (but not unprofitably so) that *H*-types need a low-enough price to be convinced of their type. In this case, the gain of *H*-types when they are uninformed more than compensates the loss when they are informed, thus price discrimination raises *H*-types' expected surplus.

[11] It remains to show that there exists a feasible parameter space in which $\underline{\rho}_3 < \underline{\rho}_1$, which can be verified by a numerical example: $\alpha_H/\alpha_L = 5$, $w = 0.4$, $\lambda = 0.5$, then $\underline{\rho}_3 = 4/7 \leq \underline{\rho}_1 = 75/124$.

Otherwise if $\rho_H \in (\underline{\rho}_2, \underline{\rho}_3]$, H-types are worse-off under price discrimination, but their loss is less than L-types' gain, thus price discrimination still raises consumer surplus on average.

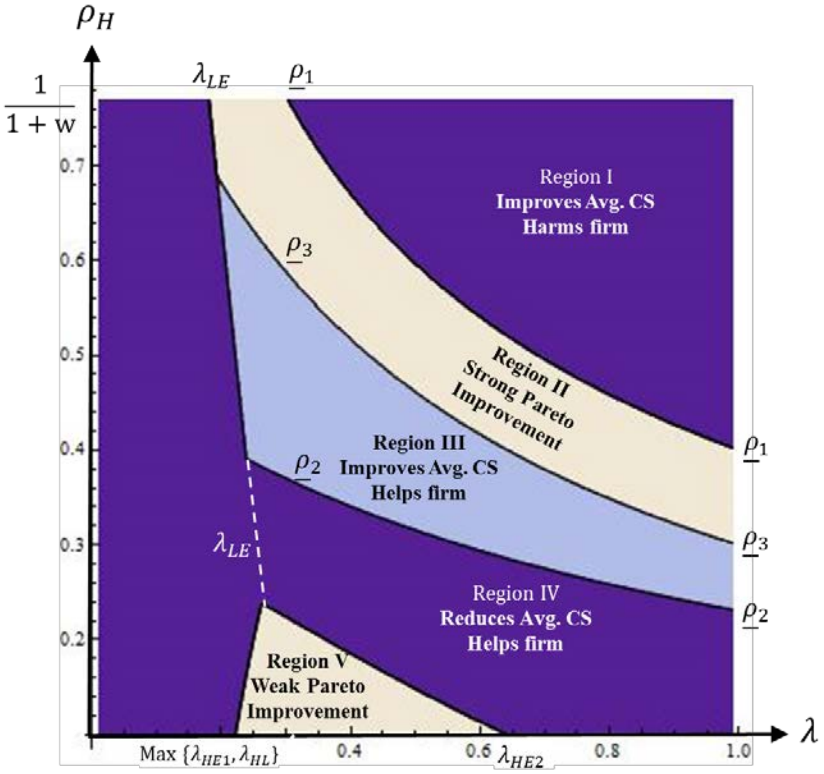

Figure 4: Welfare Impacts of Price discrimination ($\boldsymbol{\alpha_H = 5\alpha_L}, w = \mathbf{0.3}$)

Note: The boundary curves in Figure 4 are solved in Proposition 4. $w \equiv \frac{\beta_H}{\beta_L} = \frac{\rho_L}{\rho_H}$. *Price discrimination helps both the firm and consumers on average in Region II, III and V. Region II (V) is the region in which price discrimination is a strong (weak) Pareto improvement. The upper boundary curve of Region II is* $\rho_H = \underline{\rho}_1$, *which is the iso-profit curve under which* $E(\Pi^c) = \Pi^E$ *in Figure 3, the left boundary curve is* $\lambda = \lambda_{LE}$ *in Figure 2, and the lower boundary curve on Region II is* $\rho_H = \underline{\rho}_3$, *which is the iso-surplus curve under which* $CS_{IH} = CS_{UH}(e)$*; The lower boundary curve on Region III is* $\rho_H = \underline{\rho}_2$, *which is the iso-surplus curve under which* $CS_U(e) = CS_I$. *The boundary curves on Region V are* $\lambda = \max\{\lambda_{HE1}, \lambda_{HL}\}$ *and* $\lambda = \lambda_{HE2}$ *from Figure 2. The upper bound for* ρ_H *is* $\frac{1}{1+w}$ *by the restriction that* $M_H^L(m) < M_L^H(-m)$.

In summary, Proposition 4 implies three distinct economic gains from price discrimination. First, uninformed H-types who may undervalue their reservation value are served under price discrimination at a lower price, due to the firm's signaling cost. Second, uninformed L-types, who overpay and obtain negative surplus with uniform pricing, obtain strictly more surplus (zero) with price discrimination. Third, informed L-types, who may be unserved with uniform pricing, will be profitably served due to price discrimination. These gains are obtainable only when the discriminating firm (1) collects data from uninformed consumer and (2) and credibly signals through the pricing scheme. Correspondingly, Proposition 4 implies that consumers can be worse-off when third-party regulates that either (1) the firm is prohibited from collecting data on consumers or (2) the firm must disclose the collected data to the consumers. Because market forces alone may enable uninformed consumers to correctly infer their type, mandatory disclosure of data only harms consumers by allowing the firm to avoid the signaling cost and fully exploit their willingness to pay. This stands in contrast to policies, such as RACAP, that advocate firms to reveal the private information they collected to the consumers for the sake of consumer welfare (Kamenica et al., 2011).

5. Naïve Consumers

We wrap up the analysis by studying the impact of consumer naïveté. We will call consumers naïve, if they ignore the firm's data collection decision in period 0, or if belief updating of the sort described earlier is too difficult or costly for them. In either case naïve consumers do not utilize the information contained in their personalized prices. Our objective is to understand whether or not consumers can be unambiguously exploited when they are naïve.

Intuitively, if consumers are naïve, then the firm always prefers to collect data on consumers and price discriminate because there are not losses for the firm associated with price discrimination. Recall, for instance, that rational (non-naïve) H-types may not be convinced of their type unless the firm incurs some signaling cost. If consumers are naïve, their inferred match value is independent from the firm's updated pricing strategy. In this case, the firm uses the consumer updates in Table 1 to determine the personalized prices. Note that uninformed L-types may overestimate their match value and be overcharged by the informed firm. This establishes the following result.

Lemma 2

When consumers are naïve and the firm is informed, the firm's equilibrium pricing strategy utilizes personalized pricing without the list price ($\overline{p} = \infty$) and

$$\hat{p}_H = \begin{cases} \alpha_H & \text{if } M \in \mathcal{M}^0 \\ e & \text{if } M \in \mathcal{M}^1 \end{cases} \quad \text{and} \quad \hat{p}_L = \begin{cases} \alpha_L & \text{if } M \in [-m, M_L^H(-m)) \\ e & \text{if } M \in [M_L^H(-m), m] \end{cases}.$$

Consistent with the suggestion from Kamenica et al. (2011), Lemma 2 states that the firm always has incentive to collect data from naïve consumers, because it can charge higher prices without losing consumers in any market state. As a result, price discrimination strictly lowers naïve consumers' surplus on average. This finding is also consistent with the price discrimination literature (Varian 1985; Taylor 2012; Bergemann et al, 2015). However, when consumers are fully informed, personalized pricing benefits L-types but harms H-types. When consumers are uninformed and naïve, the reverse effect can occur. Specifically, personalized pricing harms L-types but benefits H-types when $p_U^* = \alpha_H$. Since L-types may be overcharged by the informed firm at the price e and obtain a negative surplus, they are strictly worse-off by price discrimination. Uninformed H-types, on the other hand, have their surplus improved when the optimal uniform price is α_H, since they obtain a positive surplus when the firm identifies them and charge them the price e. This result is formally summarized in Proposition 5.

Proposition 5

When consumers are naïve, the firm always prefers to collect data. Relative to uniform pricing when $p_U^* = \alpha_H$ (under the condition in Proposition 2(i)), personalized pricing simultaneously reduces L-types' and raises H-types' surplus; otherwise it strictly reduces either type's surplus.

While price discrimination always benefits the firm, there may be detrimental impact on overall consumer welfare. In contrast to the analysis of the sophisticated and rational consumers of Section 4, naïveté (i) prevents consumers from making better purchases decisions because they do not update their beliefs about their match values; and (ii) eliminates the firm's need to reduce its price in order to signal to consumers. Thus, Proposition 5 provides another perspective on the signaling role of price discrimination plays in our welfare results.

6. Conclusion

This work sought to understand the implication of data collection on the firm's ability to price discriminate uninformed consumers. While it is often assumed that consumers have perfect knowledge about their willingness to pay, there are many situations in which they have some degree of uncertainty about their match value for a product or service (e.g. insurance). Furthermore, in the digital era, firms have access to a wealth of consumer data, which, through aggregated analysis, can enable the firm to learn about individual consumers better than consumers themselves do. As we demonstrated, this situation creates a challenge for a price discriminating monopolist: How to price discriminate when consumers' are uninformed of their match value? In answering this question, we found that the presence of uniformed consumers provokes the collected wisdom about monopolistic price discrimination.

First, we showed that even if the firm has perfect knowledge about consumers' valuations, it may not be able to extract the entire surplus. In particular, if consumers know that the firm has superior information, then consumers anticipate the possible exploitation by the firm. Therefore, the firm must signal to consumers their match value through its pricing scheme to facilitate exploitation. As is the case in signaling models, the sender must expend costs to convince the receivers.

Second, we showed that the ability to price discriminate, through the collection of consumer data, may not always benefit the firm. We compared equilibrium pricing with and without price discrimination. Without price discrimination, the firm is restricted to uniform pricing and consumers know that the firm has no informational advantage. However, the firm is able to exploit consumers' (rational) over-estimation of their match value. Indeed, we show that the firm's uniform price may actually decrease when the distribution of consumers shifts toward higher valuation. If the firm can exploit this form of overpaying with a uniform price, then it may be better off by committing not to collect data for price discrimination.

Third, we demonstrated that both consumer surplus and economic welfare can increase with price discrimination. A firm collects data only if it is profitable to do so. What is not so clear is how that can also benefit consumers. We found two mutually exclusive conditions under which this happens. First, if the firm can better exploit the high end of the market through price discrimination, then it may be willing to give up the opportunity to over-charge the low end of the market with a uniform price. Second, the firm can expand the market by serving the low-end of the market via discriminatory prices, as per the usual benefit of price discrimination. However, because of uninformed preferences, the firm incurs a signaling cost to the high-end of the market, which generates positive consumer surplus.

These results may have implications for the debate about consumer privacy and the collection of consumer's personal data. As we showed, through the collection of consumer data, the firm actually acquires more information than the individual sum of consumer's private data. That is, it is through aggregation that the firm assembles its own informational advantage, which then becomes private to the firm. As a result, despite its purpose of price discrimination, data collection is a process of not only one-way information transmission but also knowledge creation and bilateral communication. The additional knowledge from data collection helps the firm, not only learn the consumer types, but also whether consumers are informed of their types.

Therefore, the definition of private consumer information may be broader than typically assumed: consumers' private information should include not only their private value but also their private knowledge (prior belief). If consumers are rational, then the firm can inform them of their type. And, as we showed, this information can be communicated through personalized pricing schemes.

Appendix

This appendix contains the technical details omitted from the main text, including the proof to all lemmas and propositions.

Proof of Lemma 1

First, we show that (i) and (ii) are equivalent. Suppose $-m \leq M_H^L(m)$. Then, by Assumption 1, $M_L^H(\cdot)$ is increasing, thus $M_L^H(-m) \leq M_L^H[M_H^L(m)] = m$. Now suppose $M_L^H(-m) \leq m$. Again, because $M_H^L(\cdot)$ is increasing, $-m = M_H^L[M_L^H(m)] \leq M_H^L(m)$.

Next, we show that (i) and (iii) are equivalent. By (i), $[-m, M_H^L(m)]$ is non-empty. Thus, for any M with $-m \leq M \leq M_H^L(m)$, we have (1) $M_L^H(M) \leq m$, since $M_L^H(\cdot)$ is increasing, and (2) $-m \leq M_L^H(M)$, since $M \leq M_L^H(M)$ by Assumption 1. Therefore $M_L^H(M) \in [-m, m]$. Set $M' \equiv M_L^H(M) = \theta_L^{-1}[\theta_H(M)]$. Re-writing gives $\theta_L(M') = \theta_H(M)$, which, by Definition 1, means that H-types are uninformed under any state $M \in [-m, M_H^L(m)]$. By contrast, for any M with $M_H^L(m) < M$, then $m < M_L^H(M)$, since $M_L^H(\cdot)$ is increasing. Thus H-types are informed by Definition 1 for any $M > M_H^L(m)$. Conversely, when (iii) holds, $[-m, M_H^L(m)]$ is non-empty, thus $-m \leq M_H^L(m)$.

Finally, we show that (ii) and (iv) are equivalent. By (ii), $[M_L^H(-m), m]$ is non-empty. Thus, we can take any M with $M_L^H(-m) \leq M \leq m$, such that (1) $M_H^L(M) \geq -m$, since $M_H^L(\cdot)$ is increasing, and (2) $M_H^L(M) \leq m$, since $M_H^L(M) \leq M$ by Assumption 1. Therefore $M_H^L(M) \in [-m, m]$. Set $M' \equiv M_H^L(M) = \theta_H^{-1}[\theta_L(M)]$. Re-writing gives $\theta_H(M') = \theta_L(M)$, which, by Definition 1, means that L-types are uninformed under any state $M \in [M_L^H(-m), m]$. By contrast, for any M with $M < M_L^H(-m)$, then $M_L^H(M) < -m$. Thus L-types are informed by Definition 1 for any $M < M_L^H(-m)$. Conversely, when (iv) holds, $[M_L^H(-m), m]$ is non-empty, thus $M_L^H(-m) \leq m$. ∎

Equilibrium Concept and Refinement

Definition A (Perfect Bayesian Equilibrium): Any triple $\{s_i^*, r_i^*, \mu(s_i^*)\}_{i=H,L}$ is a Perfect Bayesian Equilibrium (PBE) if it satisfies both of the conditions below.

(A.1) Sequential Rationality:

(i) $r_i^*(s_i, \mu(s_i)) = 1$, if and only if $E[\alpha|\mu(s_i)] \geq \min\{\overline{p}, p_i\}$.

(ii) $S^*(M) = (s_L^*(M), s_H^*(M))$ maximizes the firm's profit:

$$\Pi(S) = \lambda \ \min\{\overline{p}, p_H\}\, r_H^* \ (s_H, \mu(s_H)) + (1-\lambda) \min\{\overline{p}, p_L\}\, r_L^*(s_L, \mu(s_L)).$$

(A.2) Consistency (Bayes' Rule): If an uninformed consumer i observes $s_i = (p_i, \overline{p})$, then her updated belief $\mu(s_i)$ follows Bayes' Rule whenever possible.

As noted in the main text, we apply the D1 Criterion to eliminate unreasonable equilibria arising in the model with the uninformed firm. In this model D1 is equivalent to D2 (Cho and Kreps, 1987; Banks and Sobel, 1987; Cho and Sobel, 1990), since there are only two possible market states, either M_L^i or M_H^i, for any uninformed consumer i who observes her transient value. D1 can interpret this criterion as a two-step check: The first step is to check whether there exists a market state under which firm deviates less often under arbitrary consumer belief. For instance, if the firm has a strong incentive to deviate in market state M_L^i whenever it has a weak incentive to deviate in M_H^i, then we say that the firm *less often* deviates under M_H^i and *more often* deviates under M_L^i. The second step is to check whether the consumer's updated belief assigns zero weight to the market state under which the firm less often deviates. For instance, in the previous case if the uninformed consumer assigns positive probability that she is an *H*-type, then the equilibrium fails D1.

Definition B (D1 Criterion): A perfect Bayesian equilibrium $\{s_i^*, r_i^*, \mu(s_i^*)\}_{i=H,L}$ survives D1, if and only if for any out-of-equilibrium pricing strategy $s_i' \neq s_i^*$, $i \in \{H, L\}$, such that whenever the following condition holds for $j \neq k \in \{H, L\}$:

$$\cup_\mu\{r_i | \Pi^*(M_j^i) \leq \widehat{\Pi}_i(M_j^i, s_i', r_i)\} \subsetneq \cup_\mu\{r_i | \Pi^*(M_k^i) < \widehat{\Pi}_i(M_k^i, s_i', r_i)\},$$

where r_i is a best response to s_i' given an arbitrary belief $\mu \in [0,1]$, i.e., $r_i = BR(s_i', \mu \in [0,1])$, consumer i updates her belief as

$$\mu(s_i') = \begin{cases} 0, & j = H \\ 1, & j = L \end{cases}.$$

Proof of Proposition 1(iv)

We first establish that the PBE described in (i)-(iii) survives D1. Suppose that $j = H$ and $k = L$, the strict inclusion condition in Definition A implies that for any s_i', the set $\cup_\mu\{r_i|\Pi^*(M_L^i) < \widehat{\Pi}_i(M_L^i, s_i', r_i)\} \neq \emptyset$, which then implies $\mu(s_i') = 0$ by (iii). Next, we show the case for $j = L$ and $k = H$. Specially we want to show that $\mu(s_i') = 1$ for any s_i' such that $\cup_\mu\{r_L|\Pi^*(M_L^i) \leq \widehat{\Pi}_i(M_L^i, s_i', r_i)\} \subsetneq \cup_\mu\{r_L|\Pi^*(M_H^i) < \widehat{\Pi}_i(M_H^i, s_i', r_i)\}$. Suppose $\mu(s_i') \neq 1$ by contradiction, then by (iii), $\mu(s_i') = 0$ and $\Pi^*(M_L^i) < \widehat{\Pi}_i(M_L^i, s_i', r_i)$, $\cup_\mu\{r_L|\Pi^*(M_L^i) > \widehat{\Pi}_i(M_L^i, s_i', r_i)\} = \emptyset$, violating the strict inclusion condition. Therefore $\mu(s_i') = 1$. By Definition A, the PBE described in (i)-(iii) survives D1.We prove by contradiction. Suppose there exist another PBE, i.e., $(\tilde{s}_i, \tilde{r}_i, \tilde{\mu})_{i=H,L} \neq (s_i^*, r_i^*, \mu)_{i=H,L}$ that survives D1, where $\tilde{r}_i = BR(\tilde{s}_i, \tilde{\mu})$. Denote $\widetilde{\Pi}(M)$ as the equilibrium profit under $(\tilde{s}_i, \tilde{r}_i, \tilde{\mu})_{i=H,L}$. By Definition A, we need to show that for $j, k \in \{H, L\}$, $j \neq k$, there exists an off-equilibrium pricing scheme $s_i' \neq \tilde{s}_i$, such that for all possible beliefs $\tilde{\mu} \in (0,1)$ off the equilibrium path,

$$\cup_{\tilde{\mu}}\{r_i|\widetilde{\Pi}(M_j^i) \leq \widehat{\Pi}_i(M_j^i, s_i', r_i)\} \subsetneq \cup_{\tilde{\mu}}\{r_i\, |\widetilde{\Pi}(M_k^i) < \widehat{\Pi}_i(M_k^i, s_i', r_i)\},$$

where $r_i = BR(s_i', \tilde{\mu})$, but $\mu(M_j^i|s_i') > 0$.

Next, we show the uniqueness claim of (iv). This is done in two steps.

Step 1: Any separating PBE that is not specified in (i)-(iii) fails D1

For convenience of the notation, we denote $\mathrm{E}[\alpha] \equiv \lambda\alpha_H + (1-\lambda)\alpha_L$ and $\mathrm{E}^2[\alpha] \equiv \lambda^2\alpha_H + (1-\lambda^2)\alpha_L$. First we derive the equilibrium profit $\widetilde{\Pi}(M)$: When $M \in \mathcal{M}^0$, H-types are informed. Since $(\tilde{s}_i, \tilde{r}_i, \tilde{\mu})_{i=H,L}$ is separating, $\tilde{\mu}(\tilde{s}_L) = 0$, L-types pay at most α_L. Therefore $\widetilde{\Pi}(M \in \mathcal{M}^0) = \mathrm{E}[\alpha]$. When $M \in \mathcal{M}^1$, L-types are informed. Since $\tilde{\mu}(\tilde{s}_H) = 1$, $\widetilde{\Pi}(M \in \mathcal{M}^1) = \lambda\tilde{p}_H + (1-\lambda)\alpha_L$. Since $(\tilde{s}_i, \tilde{r}_i, \tilde{\mu})_{i=H,L}$ is a PBE and $\tilde{s}_H \neq s_H^*$, we must have $\tilde{p}_H < \mathrm{E}[\alpha]$. To see this, suppose the contrary that either $\tilde{p}_H > \mathrm{E}[\alpha]$, or $\tilde{p} > \tilde{p}_H = \mathrm{E}[\alpha]$, then the firm always deviate to $\tilde{s}_H$ in $M \in \mathcal{M}^0$ under the belief $\tilde{\mu}(\tilde{s}_H) = 1$, because $\widehat{\Pi}_L\Big(M \in \mathcal{M}^0, \tilde{s}_H, BR\big(\tilde{s}_H, \tilde{\mu}(\tilde{s}_H)\big)\Big) = \lambda\tilde{p} + (1-\lambda)\tilde{p}_H > \mathrm{E}[\alpha] = \widetilde{\Pi}(M \in \mathcal{M}^0)$, contradicting with $\tilde{\mu}(\tilde{s}_H) = 1$. As a result, $\tilde{p}_H < \mathrm{E}[\alpha]$, thus $\widetilde{\Pi}(M \in \mathcal{M}^1) < \mathrm{E}^2[\alpha]$.

Let $s_i' = \{p_i', \overline{p}'\}$ and $p_i' = \overline{p}' \in (\tilde{p}_H, \mathrm{E}[\alpha])$. We show that $\tilde{\mu}(s_i') < 1$: Suppose the contrary that $\tilde{\mu}(s_i') = 1$, then the firm always prefers deviating s_H' to $\tilde{s}_H$ in $M \in \mathcal{M}^1$ because $p_H' > \tilde{p}_H$, therefore $\tilde{s}_H$ cannot be on a PBE. Next we want to show that for an arbitrary $\tilde{\mu}$ and $r_i = BR(s_i', \tilde{\mu})$, $\widetilde{\Pi}(M_H^i) < \widehat{\Pi}_i(M_H^i, s_i', r_i)$ whenever $\widetilde{\Pi}(M_L^i) \leq \widehat{\Pi}_i(M_L^i, s_i', r_i)$. When consumer i is uninformed, $M = M_L^i$ is equivalent to $M \in \mathcal{M}^0$, thus $\widetilde{\Pi}\left(M_L^i\right) = \widetilde{\Pi}(M \in \mathcal{M}^0) = \mathrm{E}[\alpha]$. To substitute in the profit function, consider $r_i = BR(s_i', \tilde{\mu})$ as that consumer i accepts p_i' in probability $\tilde{\mu}$, thus $\widehat{\Pi}_i\left(M_L^i, s_i', r_i\right) = (1-\lambda)\tilde{\mu}\overline{p}' + \lambda\overline{p}'$. Since $\overline{p}' < \mathrm{E}[\alpha]$ and $\widetilde{\Pi}\left(M_L^i\right) \leq \widehat{\Pi}_i\left(M_L^i, s_i', r_i\right)$ imply that $\tilde{\mu}\overline{p}' \geq \frac{\mathrm{E}[\alpha]-\lambda\overline{p}'}{1-\lambda} > \mathrm{E}[\alpha]$, thus $\widehat{\Pi}_i\left(M_H^i, s_i', r_i\right) = \lambda\tilde{\mu}\overline{p}' + (1-\lambda)\alpha_L \geq \mathrm{E}^2[\alpha]$. From previous discussion, $\widetilde{\Pi}\left(M_H^i\right) = \widetilde{\Pi}(M \in \mathcal{M}^1) < \mathrm{E}^2[\alpha]$, thus $\widetilde{\Pi}\left(M_H^i\right) < \widehat{\Pi}_i\left(M_H^i, s_i', r_i\right)$. Therefore by Definition A, we must have $\tilde{\mu}(s_i') = 1$, which contradicts with the assumption that $\tilde{\mu}(s_i') < 1$. As a result, the separating equilibrium with s_i' fails D1.

Step 2: Any pooling PBE fails D1

Since $(\tilde{s}_i, \tilde{r}_i, \tilde{\mu})_{i=H,L}$ is pooling, then $\tilde{\mu}(\tilde{s}_L) = \tilde{\mu}(\tilde{s}_H) > 0$, and since uninformed consumers cannot update their belief from the pricing scheme, they estimate their match value as $\mathrm{E}[\alpha|\tilde{s}_i] = e$, therefore it is optimal for the firm to set $\tilde{p} \leq e$, and $\widetilde{\Pi}(M \in \mathcal{M}^0) \leq e$. Comparing to the separating equilibrium in Proposition 1, $\Pi^*(M \in \mathcal{M}^0) = \mathrm{E}[\alpha]$, thus when $e \leq \mathrm{E}[\alpha]$, there is no pooling because the firm prefers to use s_L^* to inform L-types their true type.

Now consider the case when $e > \mathrm{E}[\alpha]$. We show that the belief $\tilde{\mu}(\tilde{s}_i) > 0$ violates D1. Particularly, we show that for any $\tilde{\mu}$ and $r_i = BR(\tilde{s}_i, \tilde{\mu})$, $\Pi^*(M \in \mathcal{M}^0) < \widehat{\Pi}_i\left(M_L^i, \tilde{s}_i, r_i\right)$, whenever $\Pi^*(M \in \mathcal{M}^1) \leq \widehat{\Pi}_i\left(M_H^i, \tilde{s}_i, r_i\right)$. Consider the pricing scheme in pooling, $\tilde{p} = \tilde{p}_i = e$. Since $\widehat{\Pi}_i\left(M_H^i, \tilde{s}_i, r_i\right) = \lambda\tilde{\mu}e + (1-\lambda)\alpha_L$, $\Pi^*(M \in \mathcal{M}^1) \leq \widehat{\Pi}_i\left(M_H^i, \tilde{s}_i, r_i\right)$ implies that $\tilde{\mu}e \geq \mathrm{E}[\alpha]$. Since $e > \mathrm{E}[\alpha]$, $\widehat{\Pi}_i\left(M_L^i, s_i', r_i\right) = (1-\lambda)\tilde{\mu}e + \lambda e \geq (1-\lambda)\mathrm{E}[\alpha] + \lambda e > \mathrm{E}[\alpha] = \Pi^*(M \in \mathcal{M}^0)$. Therefore by Definition A, $\tilde{\mu}(\tilde{s}_i) = 0$, which contradicts with $\tilde{\mu}(\tilde{s}_i) > 0$ in the pooling equilibrium. Thus consumer i should never assign positive probability that she is an H-type when she observes $\tilde{p} = \tilde{p}_i = e$. As a result, the pooling equilibrium is not optimal.

Steps 1 and 2 establish the uniqueness claim in (iv). ■

The main equilibrium result presented Proposition 1 is a special case of a more general result, which we establish in the two lemmas below. Here we relax the restriction made in the main text that $M_H^L[M_H^L(m)] < -m < M_H^L(m)$. Let $N \in \mathbb{N}$ be the unique exponent satisfying

$$M_H^{L^{(N+1)}}(m) < -m < M_H^{L^{(N)}}(m),$$

the existence of which is guaranteed by Assumption 1: since $\theta_H > \theta_L$, $M_H^L(M) < M$, for all M, thus $M_H^{L^{(N+1)}}(m) < M_H^{L^{(N)}}(m)$ for any $M_H^{L^{(N)}}(m) \in [-m, m]$. In the main text we restricted

$N = 1$, which implied a simple partition of the state space in which either $M \in [-m, M_H^L(m)]$ in which L-types are informed, or $M \in [M_H^L(m), m]$ in which H-types are informed. For the more general case in which $N > 1$, we define a partition of the state space consisting of $N + 1$ intervals of the form, from right to left,

$$\mathcal{M}^0 = (M_H^L(m), m],$$

$$\mathcal{M}^n \equiv \left(M_H^{L^{(n+1)}}(m), M_H^{L^{(n)}}(m)\right],\ n = 1, \dots, N-1, \text{ and}$$

$$\mathcal{M}^N = \left[-m, M_H^{L^{(N)}}(m)\right].$$

H-types are informed whenever $M \in \mathcal{M}^0$, otherwise, they are uninformed. Conversely, L-types are informed only if $M \in \mathcal{M}^N$, but otherwise uninformed. To better understand the meaning of this partition, consider an arbitrary $M \in \mathcal{M}^0$. In this case, L-types observe the transient value $\theta_L(M)$, which is the same transient value for an H-type under state $M_H^L(M) \in \mathcal{M}^1$: $\theta_H[M_H^L(M)]$. For $M \in \mathcal{M}^1$, L-types are unsure over the states M, as an L-type, or $M_H^L(M) \in \mathcal{M}^2$ as an H-type, while H-types are unsure over the states M, as an H-type, or $M_L^H(M) \in \mathcal{M}^0$, as an L-type. And so on.

With this basic structure of consumers' information sets, we can establish the existence of an equilibrium in this game.

Lemma A1: **(General Result)**

Let $n \in \{0,1, \dots, N\}$ and consider any $M \in \mathcal{M}^n$. Then there exists a unique perfect Bayesian equilibrium $(s_i^*(M), r^*, \mu_i)_{i=H,L}$ that survives D1:

$p_H^*(M) = \bar{p}^*(M) = \lambda^n \alpha_H + (1 - \lambda^n)\alpha_L,\ p_L^*(M) = \alpha_L;$
r_i^*: purchase at p_i^*;
$\mu(s_i^*) = 1$, if $\widehat{\Pi}_i\left(M_L^i, s_i^*, r_i^*\left(s_i^*, M_H^i\right)\right) \leq \Pi^*\left(M_L^i\right)$, and $\mu(s_i^*) = 0$, if otherwise.

Proof of Lemma A1:

In Proposition 1, we have shown the case when $N = 1$. To show that the claim holds for an arbitrary $N > 1$, we use the mathematical induction method: Suppose that the claim holds for

$N = k,$ where $k > 1$ is arbitrary, we show that the claim is true for $N = k + 1$. For convenience of notation, denote $\mathrm{E}^n[\alpha] \equiv \lambda^n \alpha_H + (1 - \lambda^n)\alpha_L$ for any n.

First, we establish existence of a PBE satisfying properties (i)-(iii).

Since the claim holds for $N = k$, the equilibrium in (i)-(iii) is a PBE for any $M \in \mathcal{M}^n$, $n = 1, \dots, k$. Now suppose $N = k + 1$. When $M \in \mathcal{M}^{k+1}$, L-types are informed. H-types are uninformed and consider two possible states M_L^H or M_H^H. Since $M_L^H \in \mathcal{M}^k$, we have the equilibrium price $p_H^*(M_L^H) = \mathrm{E}^k[\alpha]$ and the firm's profit $\Pi^*(M_L^H) = \mathrm{E}^{k+1}[\alpha]$. The estimated profit in deviation to $s_i' = \{p_i', \bar{p}'\}$ is $\widehat{\Pi}_i\left(M_L^i, s_i', BR(s_i', M_H^i)\right) = \lambda\, \bar{p}' + (1 - \lambda)p_i'$ for any $p_i' \leq \bar{p}' \leq \alpha_H$.

By the consumer belief (iii), $\mu(s_i') = 1$ if and only if $\Pi^*(M_L^i) \geq \widehat{\Pi}_i\left(M_L^i, s_i', BR(s_i', M_H^i)\right)$. If $\bar{p}' > \Pi^*(M_L^H)$, then $\Pi^*(M_L^H) \geq \widehat{\Pi}_H\left(M_L^H, s_H', BR(s_H', M_H^H)\right)$ implies $p_H' < \Pi^*(M_L^H)$. Because s_i' is arbitrary, the firm could do no better than with the equilibrium pricing scheme s_H^* defined in (i), therefore (i) is firm-optimal given the belief in (iii). Additionally, since by (i) s_H^* is offered only in $M \in \mathcal{M}^{k+1}$ when the uniformed consumers are H-types, thus (iii): $\mu(s_H^*) = 1$ satisfies Bayes' rule, and (ii): H-types purchase, $r_H^* = 1$ is a best response, since $p_H^* < \alpha_H$.

Second, we establish that the PBE described in (i)-(iii) satisfies D1. The argument is the same as in the proof to proposition 1: $\mu(s_i') = \{0,1\}$ in (iii) implies that the PBE cannot fail D1.

Third, we establish the uniqueness claim by using the same notations in the proof to proposition 1. The proof is similar so we only show step 1: Any separating PBE that is not specified in (i)-(iii) fails D1. Denote an arbitrary separating equilibrium $(\tilde{s}_i, \tilde{r}_i, \tilde{\mu})_{i=H,L} \neq (s_i^*, r_i^*, \mu^*)_{i=H,L}$ and $\widetilde{\Pi}(M)$ as the equilibrium profit. Suppose that for the uninformed consumer i, $M_L^i \in \mathcal{M}^k$ and $M_H^i \in \mathcal{M}^{k+1}$, then since $(\tilde{s}_i, \tilde{r}_i, \tilde{\mu})_{i=H,L}$ is separating, we have $\tilde{s}_L(M_L^i) \neq \tilde{s}_H(M_H^i)$.

We derive the equilibrium profit $\widetilde{\Pi}(M)$: When $M \in \mathcal{M}^k$, since there is no other separating equilibrium for $N = k$, we have $\widetilde{\Pi}(M \in \mathcal{M}^k) = \Pi^*(M \in \mathcal{M}^k) = \mathrm{E}^{k+1}[\alpha]$. When $M \in \mathcal{M}^{k+1}$, L-types are informed. Since $\tilde{\mu}(\tilde{s}_H) = 1$, $\widetilde{\Pi}(M \in \mathcal{M}^{k+1}) = \lambda\tilde{p}_H + (1 - \lambda)\alpha_L$. Since $(\tilde{s}_i, \tilde{r}_i, \tilde{\mu})_{i=H,L}$ is a PBE and $\tilde{s}_H \neq s_H^*$, we must have $\tilde{p}_H < \mathrm{E}^{k+1}[\alpha]$. To see this, suppose the contrary that either $\tilde{p}_H > \mathrm{E}^{k+1}[\alpha]$, or $\tilde{p} > \tilde{p}_H$, then the firm always deviate to $\tilde{s}_H$ in $M \in$

$\mathcal{M}^k$ under the belief $\tilde{\mu}(\tilde{s}_H) = 1$, contradicting with $\tilde{\mu}(\tilde{s}_H) = 1$ is on a PBE. As a result, we must have $\widetilde{\Pi}(M \in \mathcal{M}^{k+1}) < \mathrm{E}^{k+2}[\alpha]$.

Let $s_i' = \{p_i', \overline{p}'\}$ and $p_i' = \overline{p}' \in (\tilde{p}_H, \mathrm{E}^{k+1}[\alpha])$. First we show that $\tilde{\mu}(s_i') < 1$, otherwise the firm always prefers deviating s_i' to $\tilde{s}_H$ in $M \in \mathcal{M}^1$, therefore $\tilde{s}_H$ cannot be on a PBE. Next we want to show that $\widetilde{\Pi}(M_H^i) < \widehat{\Pi}_i(M_H^i, s_i', r_i)$ for an arbitrary $\tilde{\mu}$ and $r_i = BR(s_i', \tilde{\mu})$, whenever $\widetilde{\Pi}(M_L^i) \leq \widehat{\Pi}_i(M_L^i, s_i', r_i)$. When consumer i is uninformed, $M = M_L^i$ is equivalent to $M \in \mathcal{M}^k$, thus $\widetilde{\Pi}(M_L^i) = \widetilde{\Pi}(M \in \mathcal{M}^k) = \mathrm{E}^{k+1}[\alpha]$. To substitute in the profit function, consider $r_i = BR(s_i', \tilde{\mu})$ as that consumer i accepts p_i' in probability $\tilde{\mu}$, thus $\widehat{\Pi}_i(M_L^i, s_i', r_i) = [(1-\lambda)\tilde{\mu} + \lambda]\overline{p}'$. Since $\overline{p}' < \mathrm{E}^{k+1}[\alpha]$ and $\widetilde{\Pi}(M_L^i) \leq \widehat{\Pi}_i(M_L^i, s_i', r_i)$ imply that $\tilde{\mu}\overline{p}' \geq \frac{\mathrm{E}^{k+1}[\alpha] - \lambda\overline{p}'}{1-\lambda} > \mathrm{E}^{k+1}[\alpha]$, thus $\widehat{\Pi}_i(M_H^i, s_i', r_i) = \lambda\tilde{\mu}\overline{p}' + (1-\lambda)\alpha_L > \lambda\mathrm{E}^{k+1}[\alpha] + (1-\lambda)\alpha_L = \mathrm{E}^{k+2}[\alpha]$. Because $\widetilde{\Pi}(M_H^i) = \widetilde{\Pi}(M \in \mathcal{M}^{k+1}) < \mathrm{E}^{k+2}[\alpha]$, we must have $\widetilde{\Pi}(M_H^i) < \widehat{\Pi}_i(M_H^i, s_i', r_i)$. Thus by Definition A, $\tilde{\mu}(s_i') = 1$, contradicting with the assumption that $\tilde{\mu}(s_i') < 1$. Therefore any separating equilibrium with s_i' fails D1. ■

Lemma A2 There exist two real values of λ, $\lambda_{HE1} < \lambda_{HE2} \in (0,1)$, such that $\Pi_U(\alpha_H; \lambda) - \Pi_U(e; \lambda) = 0$, if and only if $(1 - 2\rho_H) \geq \sqrt{\alpha_L/\alpha_H}$.

Proof of Lemma A2

We start by writing $\Pi_U(\alpha_H; \lambda) - \Pi_U(e; \lambda) = \frac{G_1(\lambda)}{\rho_L(1-\lambda)+\rho_H\lambda}$, where $G_1(\lambda) = \{\rho_L^2\alpha_L - \rho_H^2\alpha_H - \rho_L[(\alpha_H - \alpha_L) - 2(\rho_H\alpha_H - \rho_L\alpha_L)]\}\lambda^2 + \rho_L[(\alpha_H - \alpha_L) - 2(\rho_H\alpha_H - \rho_L\alpha_L)]\lambda - \rho_L^2\alpha_L$. Since $G_1(\lambda)$ is a quadratic function with the following properties: $G_1(0) = -\rho_L^2\alpha_L < 0$, $G_1(1) = -\rho_H{}^2\alpha_H < 0$. Therefore, $\Pi_U(\alpha_H; \lambda) \geq \Pi_U(e; \lambda)$ for some $\lambda \in (0,1)$ only if there exists two roots $\lambda_{HE1}, \lambda_{HE2} \in (0,1)$ for $G_1(\lambda)$. Because $G_1(\lambda)$ is quadratic, it suffices to examine three conditions to check whether such roots may exist:

(1) The discriminant of $G_1(\lambda) = 0$ is positive.

(2) $\left.\frac{\partial G_1}{\partial \lambda}\right|_{\lambda=0} > 0$;

(3) $\left.\frac{\partial G_1}{\partial \lambda}\right|_{\lambda=1} < 0$;

Below we show that the intersection of the above three conditions (1) through (3) is equivalent to $(1 - 2\rho_H) \geq \sqrt{\frac{\alpha_L}{\alpha_H}}$.

Note that the above condition (1) is equivalent to $(1-2\rho_H)^2 > \frac{\alpha_L}{\alpha_H}$. Condition (2) is equivalent to $(1-2\rho_H) > (1-2\rho_L)\frac{\alpha_L}{\alpha_H}$, which then implies $\rho_H \le \frac{1}{2}$, otherwise if $\rho_H > \frac{1}{2}$, then $\rho_L > \frac{1}{2}$ and $\rho_H + \rho_L > 1$, contradicting with the restriction that $M_H^L(m) < M_L^H(-m)$. Condition (3) is equivalent to $\frac{(2\rho_H^2)}{\rho_L} - 2\rho_H + 1 \ge \frac{\alpha_L}{\alpha_H}$. If $\rho_H \le \frac{1}{2}$, then $\frac{(2\rho_H^2)}{\rho_L} - 2\rho_H + 1 > (1-2\rho_H)^2$, thus condition (1) and (2) imply condition (3). Further, condition (1) and $\rho_H \le \frac{1}{2}$ are equivalent to $(1-2\rho_H) \ge \sqrt{\frac{\alpha_L}{\alpha_H}}$. It remains to show that $\sqrt{\frac{\alpha_L}{\alpha_H}} > (1-2\rho_L)\frac{\alpha_L}{\alpha_H}$, so the previous inequality is also sufficient for Condition (2). Since $\rho_L, \frac{\alpha_L}{\alpha_H} \in (0,1)$, we have $|1-2\rho_L| < 1$ and $\sqrt{\frac{\alpha_L}{\alpha_H}} > \frac{\alpha_L}{\alpha_H}$, thus $\sqrt{\frac{\alpha_L}{\alpha_H}} > \frac{\alpha_L}{\alpha_H} > |1-2\rho_L|\frac{\alpha_L}{\alpha_H} \ge (1-2\rho_L)\frac{\alpha_L}{\alpha_H}$. ■

Proof of Proposition 2

(i) Suppose $(1-2\rho_H) \ge \sqrt{\alpha_L/\alpha_H}$. Then, from Lemma A2, $\Pi_U(\alpha_H;\lambda) > \Pi_U(e;\lambda)$, if and only if $\lambda_{HE1} \le \lambda \le \lambda_{HE2}$. And, by definition of λ_{HL}, $\Pi_U(\alpha_H;\lambda) > \Pi_U(\alpha_L;\lambda)$ if and only if $\lambda > \lambda_{HL}$. Since $\lambda \in (0,1)$, this last condition requires $\lambda_{HL} < 1$, which we establish next: The condition $1-2\rho_H \ge \sqrt{\alpha_L/\alpha_H} > 0$ implies $\rho_H < \frac{1}{2}$, which further implies $1-\rho_H > (1-2\rho_H)^2$. Thus, $(1-2\rho_H)^2 \ge \alpha_L/\alpha_H$ implies $(1-\rho_H) > \alpha_L/\alpha_H$, which is equivalent to $\lambda_{HL} < 1$. Therefore, $\Pi_U(\alpha_H;\lambda) > \max\{\Pi_U(e;\lambda), \Pi_U(\alpha_L;\lambda)\}$ if and only if $\max\{\lambda_{HE1}, \lambda_{HL}\} \le \lambda \le \lambda_{HE2}$, and $(1-2\rho_H) \ge \sqrt{\alpha_L/\alpha_H}$.

(ii) Suppose the condition of (i) do not hold. Then the uniform price of α_H is not optimal and we need only to compare profits $\Pi_U(\alpha_L;\lambda)$ and $\Pi_U(e;\lambda)$. We show that $\Pi_U(e;\lambda) > \Pi_U(\alpha_L;\lambda) \Leftrightarrow \lambda > \lambda_{LE}$, where $\lambda_{LE} \in (0,1)$ and is unique. This ordering of profits is equivalent to $G_2(\lambda) < 0$, where $G_2(\lambda) \equiv -[\rho_H\alpha_H - \rho_L\rho_H\alpha_H - \rho_L\alpha_L + \rho_L^2\alpha_L]\lambda^2 - [\rho_L\rho_H\alpha_H + 2\rho_L\alpha_L - 2\rho_L^2\alpha_L - \rho_H\alpha_L]\lambda + \rho_L(1-\rho_L)\alpha_L$. Since $G_2(.)$ is continuous, $G_2(0) = \rho_L(1-\rho_L)\alpha_L > 0$ and $G_2(1) = -\rho_H(\alpha_H - \alpha_L) < 0$, by the intermediate value theorem, there must exist at least a $\lambda_{LE} \in (0,1)$, such that $G_2(\lambda_{LE}) = 0$. Suppose that there exists another $\lambda'_{LE} \in (0,1)$, such that $\lambda'_{LE} \ne \lambda_{LE}$, and $G_2(\lambda_{LE}') = 0$. Then because $G_2(\lambda_{LE}) = G_2(\lambda'_{LE}) = 0$, G_2 is quadratic, and $G_2(0) > 0$, we must have $G_2(1) > 0$, which contradicts with $G_2(1) < 0$. Therefore there exists a unique $\lambda_{LE} \in (0,1)$.■

Figure A1 illustrates a numerical example for Proposition 2.

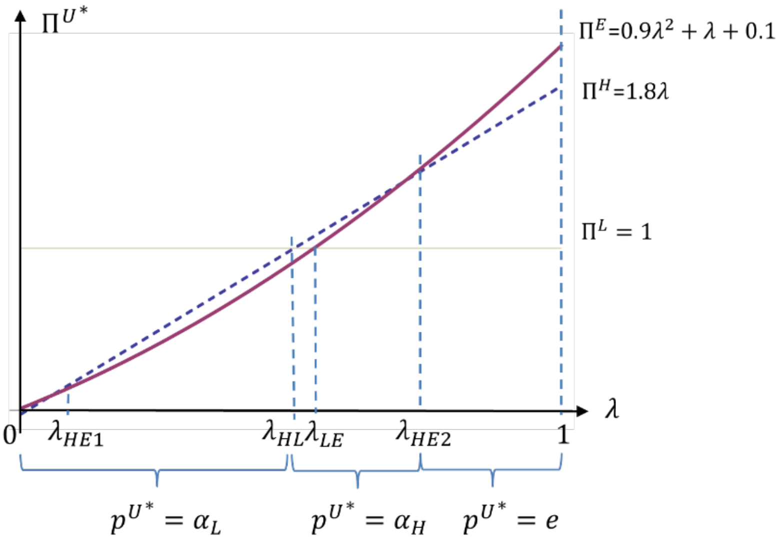

Figure A1: Firm's Profit with Uniform Prices

Proof of Proposition 3

Since $\frac{\partial[\Pi_I - \Pi_U(e)]}{\partial \rho_H} = \frac{-(1-\lambda)}{w(1-\lambda)+\lambda}[(\alpha_H - \alpha_L)\lambda(w+\lambda) + w(w+\lambda)\alpha_H(1-\lambda) + w\lambda^2\alpha_L] < 0$, the ordering of Π_I and $\Pi_U(e)$ holds as claimed. The relative profitability of price discrimination is determined by the ordering of ρ_H and $\underline{\rho_1}$. We only need to establish one numerical example to show that a feasible parameter space exist for $\rho_H > \underline{\rho_1}$. Suppose $\alpha_H = 2$, $\alpha_L = 1$, $\beta_H = 1, \beta_L = 2$, $\lambda = 0.8$, and $m = 0.55$, then $\rho_H \approx 0.59$, and $\underline{\rho_1} \approx 0.57$. Note that $\rho_L \approx 0.30$, $\rho_H, \rho_L, \rho_H + \rho_L \in (0,1)$, satisfying all restrictions on the parameter space. Since $\rho_H > \underline{\rho_1}$ on this point, it is feasible $\Pi_I < \Pi_U(e)$. Moreover, given the continuity of $\Pi_I - \Pi_U(e)$ in all parameter values, there is a feasible range of parameters for which $\rho_H > \underline{\rho_1}$. ■

Proof of Proposition 4

Since $\underline{\rho_3} > \underline{\rho_2}$, thus the existence of the feasible parameters that $\underline{\rho_1} > \underline{\rho_3}$ implies $\underline{\rho_1} > \underline{\rho_3}$ is a feasible condition. Moreover, since $\rho_H > \underline{\rho_2}$ is equivalent to $CS_I > CS_U(e)$, it remains to show that such condition is feasible in the parameter space. Since Assumption 3 implies that $\rho_H < \frac{1}{1+w}$, feasibility requires $\frac{1}{1+w} > \frac{w}{\lambda + w(2-\lambda)}$. Because $w > 0$, the necessary condition is equivalent to $w < 1$, which is a feasible condition. Therefore $\rho_H > \underline{\rho_3}$ is also feasible. ■

Proof of Proposition 5

When $p_U^* = \alpha_H$, all consumers obtain zero surplus from the uninformed firm. But when the firm uses personalized pricing, *L*-types pay at $e > \alpha_L$ whenever they are uninformed, and thus they obtain a negative expected surplus from a discriminating firm. However, naïve *H*-types obtain positive surplus by paying $e < \alpha_H$ with a discriminating firm, when they are uninformed. ■

References

Acquisti, A., & Varian, H. R. 2005. Conditioning prices on purchase history. *Marketing Science* 24(3), 367–381.

Aguirre, I., Cowan, S., & Vickers, J. 2010. Monopoly price discrimination and demand curvature. *The American Economic Review*, 100(4), 1601-1615.

Banks, J. S., & Sobel, J. 1987. Equilibrium selection in signaling games. *Econometrica*: *Journal of the Econometric Society*, 647-661.

Bagwell, K., & Riordan, M. H. 1991. High and declining prices signal product quality. *The American Economic Review, 81*(1), 224–239.

Beaudry, P. 1994. Why an informed principal may leave rents to an agent. *International Economic Review*, 821-832.

Bergemann, D., Brooks, B., & Morris, S. 2015. The limits of price discrimination. *American Economic Review*, 105(3), 921-957

Calzolari, G., & Pavan, A. 2005. On the optimality of privacy in sequential contracting. *Journal of Economic Theory*, 30 (1), 168–204.

Cho, I. K. & Kreps, D. M. 1987. Signaling games and stable equilibria. *Quarterly Journal of Economics*, 102, 179-221.

Cho, I. K. & Sobel, J. 1990. Strategic stability and uniqueness in signaling games. *Journal of Economic Theory*, 50 (2), 381-413.

Guo, L. & Zhang, J., 2012. Consumer deliberation and product line design. *Marketing Science*, 31(6), pp.995-1007.

Horowitz, Joel. 1992. The role of the list price in housing markets: Theory and an Econometric Model. *Journal of Applied Econometrics*, 7, 115-129.

Hui, K. L., & Png, I. 2003. Piracy and the legitimate demand for recorded music. *Contributions in Economic Analysis & Policy*, 2 (1).

Kamenica, Emir. 2008. Contextual inference in markets: on the informational content of product lines. *American Economic Review*. 98(5), 2127–2149.

Kamenica, Emir, Sendhil Mullainathan, & Richard Thaler. 2011. Helping consumers know themselves. *American Economic Review: Papers & Proceedings*. 101(3), 417-422.

Knight, John, C.F. Sirmans, and Geoffrey Turnbull. 1994. List price signaling and buyer behavior in the housing market. *Journal of Real Estate Finance and Economics*, 9, 177-192.

Larréché, J. C. 2008. The momentum effect: How to ignite exceptional growth. Wharton School Publishing, New Jersey, Pearson Education.

Maskin, E., & Riley, J. 1984. Monopoly with incomplete information. *The RAND Journal of Economics*, 15(2), 171-196.

Maskin, E., & Tirole, J. 1990. The principal-agent relationship with an informed principal: The case of private values. *Econometrica*, 379-409.

Milgrom, P. & Roberts, J. 1986, Price and advertising as a signal of quality, *Journal of Political Economy*, 94, 796-812**.**

Pigou, A. C. 1924. The economics of welfare. Transaction Publishers.

Robinson, J. 1933. The economics of imperfect competition, Macmillan, London.

Samuelson, P. 1999. Privacy as intellectual property. *Stanford Law Review*, 52, 1125.

Solove, D. J. 2007. I've got nothing to hide and other misunderstandings of privacy. *San Diego Law Review*, 44, 745.

Taylor, C. R. 2004. Consumer privacy and the market for customer information. *The RAND Journal of Economics*, 35(4), 631–650.

Varian, H. R. 1985. Price discrimination and social welfare. *The American Economic Review*, 75(4), 870-875.

Varian, H. R. 2002. Economic aspects of personal privacy. In Cyber Policy and Economics in an Internet Age, 127-137. Springer.

Wernerfelt, B. 1995. A rational reconstruction of the compromise effect: using market data to infer utilities. *Journal of Consumer Research*, 21(4): 627-33