What Triggers Default?
A Study of the Default Boundary

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Abstract

In structural models of risky debt default is triggered when the market value of the firm's assets falls below a certain solvency boundary. Based on market values of defaulting firms, I estimate the default boundary to be 66% of the face value of debt, and find support for models in which the default timing is chosen endogenously to maximize the value of equity. Although default predictions based entirely on solvency can match observed average default frequencies, they misclassify a substantial number of firms in cross-section, affecting the accuracy of boundary-based models. In particular, cash shortages play an important independent role in triggering default, but only when access to external financing is restricted.

Keywords: Credit risk; Default boundary; Insolvency; Cash shortage; Default

JEL Classification Numbers: G21, G30, G33

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1. Introduction

One of the central assumptions of virtually any model involving risky debt concerns the conditions that result in default. Different theories about what causes firms to default result in dramatically different predictions regarding default probabilities, security prices, and corporate financing decisions. This paper is an empirical study of the conditions that trigger default in structural models of corporate debt, originating with Black and Scholes (1973) and Merton (1974). Since Black and Cox (1976), Leland (1994), and Longstaff and Schwartz (1995),[1] the de facto standard approach in such models has been to assume that there is a solvency threshold, known as the default boundary, which separates firms' defaulting and nondefaulting states. Formally, default is commonly described as the first time the market value of the firm's assets, which summarizes the firm's economic distress or prosperity, falls below the boundary. For instance, if the boundary is set equal to the face value of debt, the firm defaults as soon as it becomes economically insolvent, that is, when the value of its assets falls below its debt obligations. The boundary-based representation of default is simple and intuitively appealing, but, despite its nearly-universal use in structural modeling, there is little evidence regarding how well it describes actual defaults.

This paper documents the empirical properties of the default boundary and studies their implications for models of risky debt. It addresses the following questions. First, where is the default-triggering solvency threshold located and what does it depend on? The level of the default boundary is one of the most important parameters in structural models (Leland (2004)), which hitherto has not been measured directly. Second, is the default trigger exogenous and determined by covenants, or is it chosen endogenously to maximize the value of equity? The endogeneity of default has profound implications for optimal financing decisions and hedging (Leland (1994), Acharya and Carpenter (2002)). Third, structural models have difficulty explaining the cross-section of empirically observed spreads (Eom, Helwege, and Huang (2004)). What are the implications of the default boundary assumption for models' accuracy? Fourth, the assumption that default is driven by low asset values (economic distress) ignores the role of cash shortages (financial distress). Is the role of illiquidity in triggering default distinct from that of insolvency, and what affects the relative importance of the two factors for different firms?

[1]Structural models also include Geske (1977), Kim, Ramaswamy, and Sundaresan (1993), Anderson and Sundaresan (1996), Leland and Toft (1996), Collin-Dufresne and Goldstein (2001), Duffie and Lando (2001), Goldstein, Ju, and Leland (2001), Acharya and Carpenter (2002), and Hsu, Saá-Requejo, and Santa-Clara (2010), among others. Huang and Huang (2003), Eom, Helwege, and Huang (2004), Leland (2004), Ericsson, Reneby, and Wang (2005), and Huang and Zhou (2008) evaluate the models' ability to explain observed credit spreads and default probabilities. Structural modeling has also become increasingly popular in studies of corporate financing decisions in a dynamic setting, such as Fischer, Heinkel, and Zechner (1989), Leland (1994), Goldstein, Ju, and Leland (2001), Anderson and Carverhill (2007), and Strebulaev (2007).

Understanding the role of insolvency (assets falling short of liabilities) and its interaction with illiquidity (liquid assets falling short of current obligations) is instrumental both to credit risk modeling and to the analysis of financial reorganization and corporate financing and hedging decisions. Yet assessing the firm's economic solvency is challenging, because the market value of assets, though used as a state variable in most risky debt models, is unobservable and rarely measured empirically. To my knowledge, this paper is the first to measure the value-based default boundary directly from market values of defaulting firms,[2] to document its empirical properties, and to evaluate the relative importance of illiquidity versus insolvency in triggering default.

Prior evidence on the role of different factors in default comes primarily from empirical bankruptcy-predicting models, such as Altman (1968). Such studies do not measure the market value of assets and hence do not control for economic solvency, which is central to structural credit risk modeling. Shumway (2001) shows that a low value of equity is a strong predictor of bankruptcy. Yet this result does not necessarily imply that default is driven by a low value of total assets. Indeed, regardless of whether default is triggered by insolvency, illiquidity, or any other factor, equity prices should fall endogenously whenever the probability of default increases, as long as equity is likely to be wiped out in bankruptcy. In contrast to existing studies, the main goal of this paper is not to build a better empirical forecasting model, but rather to document stylized facts about the structural default boundary. To this end, I estimate firms' unobserved unlevered asset values uncontaminated by investors' expectations of default, which allows me to separate the effect of economic and financial distress on asset prices.

I overcome the asset measurement problem by using a sample of firms with observed market prices of bonds, bank loans, and equity, which renegotiated with their bondholders, missed a bond payment, or filed for bankruptcy. I estimate the value of unlevered assets in the run-up to default as the total market value of the firm's debt and equity, adjusted for the effect that expected default costs have on market prices when default is partially anticipated by investors. Empirically, I find that the mean (median) market value of assets at default is only 66% (61.6%) of the face value of debt. Thus, contrary to the view that firms default as soon as they become economically insolvent, on average one-third of creditors' claim already has been destroyed by the time default is declared. To illustrate the importance of knowing the location of the default boundary, using formulae and parameter values from Leland (2004), the 10-

[2] Brockman and Turtle (2003) and Reisz and Perlich (2007) estimate the default boundary implied by a structural model calibrated to match equity values of nondefaulting firms. The idea is similar in spirit to estimating the option-implied equity volatility by inverting the Black-Scholes formula. Such model-based estimates of the boundary are conditional on all the assumptions of the particular model used, including the assumption that the asset value is a sufficient statistic for default.

year probability of default for an average BBB–rated firm when the boundary is fixed at 100% of face debt is 13.8%. Adjusting the boundary to equal its sample median of 61.6% of the debt claim and keeping all other parameters constant, the predicted default probability drops by three-quarters to 3.6%, which is much closer to historical default rates for BBB firms. The default boundary also has a dramatic effect on credit spreads and optimal capital structure (Leland (1994)).

How well does the assumption of a solvency boundary separating defaulting and nondefaulting firms describe actual defaults? Consistent with the standard structural approach, the ratio of the market value of assets to the face value of debt is the most powerful variable explaining the timing of default, outperforming most popular accounting– and equity–based variables used in empirical studies put together. Yet using it as a sufficient statistic for default risk results in sizable cross-sectional errors. Two factors contribute to its lack of accuracy.

First, observed asset values at default are highly heterogenous, ranging from 30% to 122% of face debt at the 5^{th} and 95^{th} percentiles, respectively. Some of this variation can be explained by endogenous-default models, which assume that firms' managers choose the timing of default to maximize the value of equity (Leland (1994); Leland and Toft (1996); Goldstein, Ju, and Leland (2001)). I find that in the cross-section, the empirically observed boundary is decreasing in asset volatility and increasing in default costs in a manner consistent with equityholders' endogenous default. By contrast, there is only weak evidence that the boundary depends on debt covenants, as in models in which it is exogenously specified. These results suggest that endogenous-default models can explain important patterns in the data and are likely to be essential to any analysis of credit risk and optimal financing decisions. Nonetheless, the proportion of the observed variation in asset value at default that extant models can explain quantitatively is modest.

Second, I find many severely distressed firms that are able to avoid default or delay it for years, even when their asset values become very low. In fact, *most* economically insolvent firms with market assets below debt obligations do not default for at least a year. This makes it difficult to separate defaulting and nondefaulting firms on the basis of the asset value alone. The threshold value of assets that balances type-I and type-II errors is 64.8% of the face value of debt. This boundary correctly "predicts" the observed default frequency in the sample, in the sense that the total number of firms below it equals the total number of defaulted firms. Yet as many as 42% of all defaults occur at asset values above this level, and an equal number of firms below it do not default on their next bond payment. Hence, even though boundary-based models can be calibrated to explain the probability of default on average (Leland

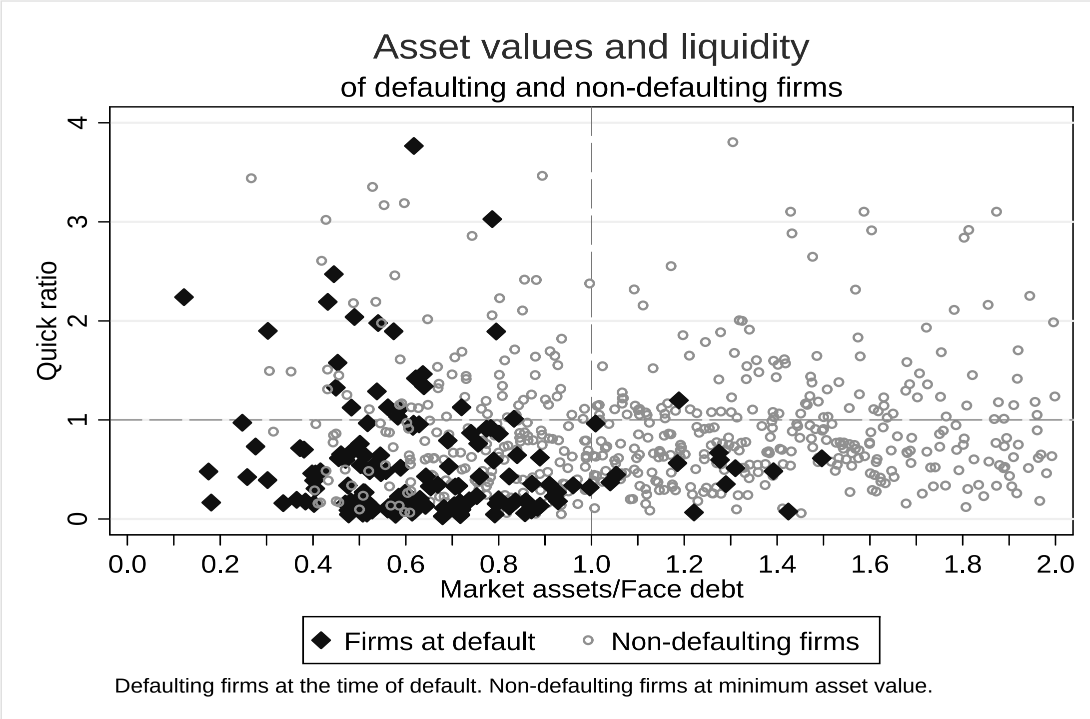

Fig. 1. This graph shows observed combinations of asset values and quick ratios for firms at default and firms that do not default on their next bond payment. Each nondefaulting firm is represented by one point, corresponding to the month-end when the ratio of its market asset value to the face value of debt is at its sample period minimum.

(2004)), the cross-sectional error implied by the observed lack of a sharp boundary may be substantial.[3]

One potential reason for the limited accuracy of predictions based entirely on solvency is that they ignore the role of liquid assets, such as cash. Structural models assume that if the cash flow falls short of required debt payments, shareholders will supply liquidity for as long as the firm's economic prospects remain good enough to keep it alive, which makes temporary cash shortages irrelevant. Yet I find that, controlling for solvency, cash shortages are the second most important factor explaining the timing of default. Figure 1 displays asset values (relative to the face value of debt) and balance sheet liquidity (measured by the quick ratio, equal to cash and receivables divided by current liabilities) for firms at default, as well as for distressed firms that do not default. Firms with assets-to-debt ratios below 1 are economically insolvent, whereas those with a quick ratio below 1 have current liabilities in excess of liquid assets. Figure 1 shows that most firms at default are both insolvent *and* illiquid, as continued losses eventually deplete the firm's cash reserves. However, in general, liquidity and value are distinct potential

[3]This conclusion is based on the time-independent boundary specification, which is the most common by far, but not universal. In Merton (1974), Geske (1977), Black and Cox (1976), Collin-Dufresne and Goldstein (2001) and Hsu et al. (2010) the default boundary is either time-dependent or stochastic. Empirically, I find that firms tend to default close to scheduled bond payment dates even when they do not coincide with maximum insolvency, as predicted, for example, by Geske (1977). This evidence suggests that models with variable default triggers may potentially perform better.

default triggers. In the sample, 14.9% of defaulting firms have enough liquid assets to cover their current liabilities (sometimes several times over) but have low asset values. At the same time, 11.4% of firms are economically solvent at default but have low liquidity ratios. Figure 1 suggests that neither liquidity nor value can alone explain all observed defaults.

I hypothesize that, for a given asset value, a cash shortage is more likely to trigger default when the firm is more constrained in accessing external financing. In the absence of market frictions firms can raise new financing as long as the business remains sufficiently valuable. In this case, cash shortages are irrelevant, and default is triggered by low values of productive assets, as in traditional value-based models. In contrast, if the required cash cannot be raised at any cost, then any temporary cash shortage can push the firm into default, regardless of its economic fundamentals. In between these two extreme cases, firms for which external financing is neither costless nor infinitely costly should be able to overcome some but not all liquidity shortages. Using firm-specific and economy-wide proxies for financing constraints, I find cash shortages to be highly significant for constrained firms, but insignificant for unconstrained firms.

The role of cash in triggering default has important implications for corporate financing policies. In the presence of financing frictions and costly default, firms may find it optimal not only to rely less on debt financing ex ante, but also to maintain a cash reserve to avoid default driven by illiquidity (Acharya, Davydenko, and Strebulaev (2008)). This precautionary motive for hoarding cash may contribute to the importance that managers attach to financial flexibility and financial slack (Graham and Harvey (2001)), and is likely to be particularly prominent when market disruptions, such as the credit freeze that followed the collapse of Lehman Brothers in 2008, severely restrict firms' access to external financing.

Taken together, this study's findings suggest that the reliance of existing structural models on the default boundary assumption may be an important factors that adversely affects their accuracy, particularly in cross-sectional predictions. This is consistent with Eom, Helwege, and Huang (2004), who find that existing models cannot explain the cross-section of bond spreads even when they perform reasonably well on average. Leland (2004) shows that, difficulties with spreads nothwithstanding, default probabilities predicted by structural models for an average firm are in line with observed historical default frequencies, assuming that the default boundary is close to 73% of the face value of debt. Yet I find that in the cross-section, almost a third of all defaults in my sample occur above this boundary level, while a similar number of firms below it do not default for at least a year.

More accurate explicit modeling of the default trigger while maintaining analytical tractability is likely to be challenging, not only because both value and liquidity play a role, but also because their relative importance depends on firm-specific and economy-wide financing constraints. Some recent models, such as Acharya et al. (2006) and Anderson and Carverhill (2007), incorporate financial slack in the dynamic structural framework and provide a multitude of new insights, but their complexity necessitates numerical analysis. Given the wide variation of asset values at default documented in this paper, which cannot be explained by common factors such as maturity and asset volatility, structural models may need to incorporate the uncertainty associated with the default trigger explicitly. Duffie and Lando (2001) show that imperfect information can make the timing of default unpredictable to investors. Advancing this line of research to explore the nature and the implications of information imperfections in credit markets may be key for the next generation of credit risk models.

The remainder of this paper is organized as follows. Section 2 outlines assumptions regarding the default trigger used in the literature, as well as the procedure that I employ to estimate the market value of assets at default. Section 3 describes the data set. Section 4 reports estimated levels of the default boundary, and Section 5 presents their cross-sectional regressions. Section 6 documents classification errors that arise when the value of assets is assumed to be a sufficient statistic for default. Section 7 studies the role of liquidity and financing frictions. Section 8 documents the performance of popular formulae for firm-specific boundaries, and Section 9 concludes. Details of the procedures used to estimate the value of the firm and the market value of assets appear in the appendices.

2. Estimating the default boundary

2.1. Default triggers in the existing literature

Conditions that result in default are discussed in many different areas of financial research, including theoretical corporate finance models, studies of distressed reorganizations, and empirical bankruptcy-predicting models.[4] Yet it is in structural models of credit risk that explicit formal assumptions regarding the nature of the default trigger play the most prominent role. Structural models start by specifying the dynamics of a state variable that describes the firm's economic conditions. Since Merton (1974), a large majority of models have assumed that the state variable is the value of the firm's productive assets,

[4]Theoretical models of the default decision include Bulow and Shoven (1978) and White (1989). Empirical bankruptcy-predicting ("credit scoring") models include Altman (1968), Zmijewski (1984), and Shumway (2001), among others. In empirical studies of distressed reorganizations, Asquith, Gertner, and Scharfstein (1994) define distress based on low interest coverage ratios; Gilson, John, and Lang (1990) relate it to low equity returns; and Pulvino (1998) assumes that distressed firms are those which have high leverage and low current ratios. In general, different definitions of distress can result in different conclusions about the effect that "distress" has on the firm.

i.e., the value of the unlevered firm.[5] In most models the value of assets follows a geometric Brownian motion, and hence is independent of the firm's capital structure. The market value of the levered firm (i.e., the aggregate value of the firm's debt and equity) is equal to the value of assets, plus the value of tax shields, minus the expected cost of financial distress. Thus, in contrast to the unobservable value of assets, the value of the firm does depend on its capital structure. In addition to the dynamics of the state variable, structural models specify the firm's financial structure, the way the assets are divided in default (e.g., according to the absolute priority rule), and the economic conditions that give rise to default. The assumption describing the default trigger has a particularly large effect on predicted default probabilities, recovery rates, credit spreads, and optimal financing decisions (Leland (1994)).

The Merton (1974) model assumes that debt consists of one zero-coupon bond, and that default occurs if at debt maturity the value of assets is below the face value of the bond. Geske (1977) generalizes the Merton (1974) model to allow for coupon-paying debt. In his model, default can only happen at the time of a scheduled debt payment, whereas in between two payments the value of assets can fall to almost nothing without triggering default. Black and Cox (1976) introduce intermediate default, which occurs if at any time before maturity the value of assets falls below a certain threshold level, known as the default boundary. Extended by Leland (1994), Longstaff and Schwartz (1995), Leland and Toft (1996), and others, the boundary-based description of the default event is now de facto standard in the literature.[6] The level of the default boundary may be specified exogenously, typically in relation to the face value of debt. For example, the negative economic net-worth boundary postulates that default happens when the market value of assets falls below the face value of debt. An alternative approach is to assume that the boundary is chosen endogenously to maximize the value of equity (Leland (1994); Leland and Toft (1996)).[7] In models with endogenous default, required debt payments may at times exceed operating cash flow without triggering default. In such cases, the firm will continue to meet debt obligations by selling newly issued equity, as long as the equity option value is higher than the required debt payment. Once the asset value falls below the boundary, the firm defaults, because the hopes of recovery become so remote that new funds cannot be raised no matter how much dilution shareholders are prepared to

[5] Although models like Goldstein, Ju, and Leland (2001) use cash flow as the state variable, under standard assumptions the value of assets is a constant multiple of the cash flow, which makes such models virtually identical to value-based models.

[6] Kim, Ramaswamy, and Sundaresan (1993) and Ross (2005) postulate that default happens when cash flows fall short of current obligations. However, because the asset value in their models is proportional to cash flow, their default trigger is equivalent to a value-based boundary. Recent models by Acharya et al. (2006), Anderson and Carverhill (2007), and Asvanunt, Broadie, and Sundaresan (2007) separate the role of value (solvency) from that of liquidity.

[7] François and Morellec (2004) and Galai, Raviv, and Wiener (2007) modify the boundary to distinguish between default and liquidation. Carey and Gordy (2009) assume that the default boundary is chosen optimally by bank lenders. In Hsu, Saá-Requejo, and Santa-Clara (2010), the default boundary follows a stochastic process. In CreditGrades, a model popular with practitioners, it is a random variable drawn from a given distribution (Finger et al. (2002)).

suffer. As Leland (2004) notes, in models with exogenous default, the default boundary can be set at arbitrary levels, including the level implied by endogenous boundary models, which is the lowest boundary compatible with a non-negative value of equity.

Despite their central role in credit risk studies, standard boundary assumptions have not been evaluated empirically. One difficulty is that the market value of assets is unobservable. As a result, extant studies provide no direct, model-free estimates of the default boundary. This often presents a challenge to practical applications of structural models. To implement an exogenous-boundary model such as Longstaff and Schwartz (1995), it is necessary to choose a particular level of the default boundary. Although the 'zero net-worth' boundary equal to the face value of debt is often considered a natural choice, Huang and Huang (2003) argue that a boundary of 60% of face debt is more consistent with typical debt recovery rates. As another example, Vassalou and Xing (2004) calibrate the Merton (1974) model using an approach similar to that employed by MKMV (Crosbie and Bohn (2002)), but, instead of Merton's default trigger, their boundary equals short-term debt plus 50% of long-term debt. It is unclear to what extent such adjustments to the default boundary are justified on empirical grounds. Applying models with endogenous default, such as Leland (1994) or Leland and Toft (1996), is more straightforward, because they derive the boundary as a function of other parameters; yet there is no empirical evidence regarding how well formula-based default boundaries describe actual conditions that result in default.

Accurate calibration of the default boundary is crucial to quantitative analysis of credit risk, because predictions of structural models are highly sensitive to its choice. To illustrate, using formulae and base-case parameter values from Leland (2004), changing the level of the boundary from 60% (used in Huang and Huang (2003)) to 73% (used in Leland (2004)) to 100% of face debt, and keeping everything else constant, changes the predicted ten-year probability of default for an average BBB-rated firm from 2.5% to 5.1% to as much as 13.8%. The default boundary also has a dramatic effect on credit spreads and optimal capital structure decisions (Leland (1994)). Because of this sensitivity, the lack of empirical evidence regarding levels and properties of the boundary is a prominent gap in existing research.

2.2. Estimating the market value of assets

In most models, the default boundary is the threshold market value of the firm's productive assets (equivalently, the unlevered value of the firm) that separates defaults from nondefaulting states. Therefore, the location of the boundary can in principle be measured as the market value of assets immediately prior to default. In practice, the market value of productive assets is not observable and must be estimated.

One common approach to estimating the value of assets is to infer it from observed equity prices using a particular structural model, such as Merton (1974) (e.g., Vassalou and Xing (2004); Hillegeist et al. (2004)). However, this approach relies on the model's specification for the default boundary (including its level) and therefore cannot be used when the boundary is the object of study. Another disadvantage of model-based estimators is their reliance on a number of other assumptions, such as the dynamics of the asset value, the firm's financial structure, and how assets are split between different claimants in default. For most firms that are close to default equity is only a small fraction of the total asset value (in my sample, 4.3% for the median firm; see Table 3). In these conditions, estimates of the value of assets based on equity values alone are likely to be particularly sensitive to model misspecification. To isolate the effect of one particular assumption of structural models – that of the default boundary – it is desirable to estimate the boundary, to the extent possible, directly from the data.

To measure the market value of assets, I use a sample of risky firms with observed market prices of bonds, bank loans, and equity. I collect detailed information on the debt structure of these firms, and use it in conjunction with the pricing data to compute the total value of each firm's debt and equity. Unfortunately, when default is costly, the value of the distressed firm is not an unbiased proxy for the value of its unlevered assets, because expected default costs may affect debt and equity prices long before default is declared.[8] To undo the effect of expected default costs on debt and equity prices, I use the procedure suggested by Davydenko, Strebulaev, and Zhao (2010). The approach is based on the idea, first introduced in the structural credit risk literature by Duffie and Lando (2001), that investors do not have the full information necessary to conclude with certainty whether or not the firm is about to default in the next instant. As a result, the observed market value of the firm just prior to default depends on the "recovery" value of assets in default (which can be observed ex post), on the value that the assets would have if default were never possible (i.e., the unlevered asset value), and on the probability that investors attach to default conditional on the information available to them (which can be parameterized and estimated from the data). Based on these observations, one can solve for the unlevered asset value implied by the value of the firm just prior to default and its value immediately after. This unlevered asset value at default corresponds to the value-based default boundary of structural models. The specifics of the estimation procedure are outlined in Appendix B.

Although the resulting estimates are model based, the assumptions behind the model are significantly

[8]In addition to default costs, another factor that makes the value of the firm differ from the value of its assets is the tax shield. However, as 93% of firms at default are making losses, taxes are not a major factor for them. As shown below in Table 2, tax rates for firms close to default are much lower than those for nondefaulting firms.

less restrictive than those of structural models. For example, I need not assume that a value-based boundary exists, nor that absolute priority is enforced upon default. The estimation procedure is structured with a view of maximizing transparency while limiting the effect of potential model misspecification.[9] Most importantly, the role of the model is limited to adjusting the observed value of the firm for the part of the cost of default that is embedded in pre-default prices. Because the expected cost of default is a modest fraction of total assets (in my sample, 8% for the median firm; see Table 3), the specifics of the estimation procedure have a relatively small effect on the resulting estimates of the default boundary.

3. Data description

3.1. Data sources and sample selection

The empirical analysis is based on a sample of speculative-grade bond issuers for which the market value of the firm can be estimated from observed prices of debt and equity, a subset of which defaulted on their bonds between 1997 and 2005. To make the control sample of nondefaulting firms comparable to defaulting firms, I limit it to junk firms (rated BB+ and below), because defaults by investment-grade firms are very rare indeed.[10] Default events include bond contract renegotiations via a distressed exchange, payment omissions, and bankruptcy filings.

To estimate the market value of firms' bonds, I use monthly quotes from Merrill Lynch's bond trading desks for bonds included in the Merrill Lynch U.S. High Yield Master II Index (MLI) between December 1996 (the month the index was created) and March 2004. The MLI consists of speculative-grade bonds with par amounts of at least $100 million and remaining maturity of one year or more. Bank loan prices come from the LSTA/LPC Mark-to-Market Pricing Database, which includes monthly secondary market loan quotes, each obtained from several dealers, starting from May 1998. Mergent's Fixed Income Securities Database (FISD) provides descriptive information on bonds, and Loan Pricing Corporation's DealScan is used for information on bank loans and covenants, as well as aggregate statistics for the distressed loan market. Information on types of outstanding debt, including the use of credit lines, is manually collected from 10-K and 10-Q filings. Using bond, loan, and equity prices in conjunction with the debt structure data, I estimate monthly market values of firms' total debt and equity, as described in Appendix A, and then adjust them for the effect of expected default costs, as described in Appendix B.

[9] Davydenko, Strebulaev, and Zhao (2010) discuss the assumptions of the model in detail.

[10] Collin-Dufresne, Goldstein, and Helwege (2010) note that since 1937, only four firms with an investment-grade rating from Moody's have defaulted on their bonds. Including high-quality firms in the control sample would have little effect besides strengthening my conclusions regarding the explanatory power of the market value of assets.

From the set of firms whose bond prices are included in the MLI, I first exclude all financial and non-U.S. firms, and then manually merge the remaining firms with Compustat, CRSP, FISD, and DealScan, taking into account mergers, acquisitions, and name changes and excluding all firms that cannot be merged reliably. Accounting data are from quarterly Compustat,[11] and monthly equity prices are from CRSP. Firm-month observations following default are removed from the sample.

3.2. The sample of defaults

"Cashflow" or "payment" defaults are failures to supply creditors with the cash flows stipulated in the debt contract. I adopt the definition of default used by the rating agency Moody's, which encompasses bankruptcy filings and out-of-court renegotiations with bondholders through either a distressed bond exchange or payment delays or omissions (see Emery and Ou (2010)). Driven primarily by data availability, this focus on bond defaults as defined by rating agencies implies that certain distress reorganizations do not appear in the analysis. First, this study is limited to firms that have outstanding public bonds. Second, neither firms that restructure their private debt nor those that attempt to renegotiate their bonds *unsuccessfully* are considered to have defaulted as long as they continue uninterrupted bond service, as per the bond contract. Third, covenant violations per se do not change creditors' payoffs and therefore do not constitute an event of default. Nevertheless, rating agencies' definition of default includes all payment defaults on public bonds, which are the main focus of most credit risk studies.

The master list of defaults is based on the Default Risk Service (DRS) from Moody's, which purports to include all defaults on public bonds since 1970.[12] Not all defaults in DRS are independent events. First, firms often default together with their subsidiaries, which can result in multiple entries in DRS. For such cases, I use the parent's consolidated financial information to study the default decision for the whole family of firms. Second, for some companies DRS reports multiple defaults. Because not all of them are independent, I study only the first default event for each firm to ensure comparability. For distressed exchanges, DRS reports the date of successful completion as the date of default. Because the date of the announcement of the bond exchange is more relevant for studying the timing of the default decision, I collect information on announcement dates from news reports in Factiva.

[11] The use of quarterly financial data is now commonplace in default research (e.g., Bharath and Shumway (2008) and Campbell, Hilscher, and Szilagyi (2008)), and allows me to account for quick declines in liquid assets in the months leading to default. Acharya, Davydenko, and Strebulaev (2008) show that because of the endogeneity of cash, cash reserves are *positively* associated with the probability of default at horizons of one year and longer, but not at short horizons.

[12] To identify possible omissions in the DRS database, I cross-check it against Standard & Poor's LossStats and CreditPro databases and default records in FISD. On the few occasions that default events reported in these databases are missing from DRS and could not be confirmed independently, I exclude the firm from the sample.

In addition to defaulted firms, the sample includes 593 junk firms that did not default during the sample period. These firms, plus firm-month observations for defaulted firms that precede the date of default by more than a year, form the control sample. Thus, the overall sample consists of 806 high-yield firms, including 213 firms that defaulted between 1997 and 2005, comprising about 36% of all bonds defaults in the US during this period. The main reason for exclusion from the sample is the lack of market bond prices. Another important factor is that many original-issue junk firms, which constitute a large majority of all defaulters, are privately held firms without traded equity or accounting data necessary to compute their market value. In particular, many defaulting high-yield firms have undergone a levered buy-out, and as such are privately held.

Panel A of Table 1 shows the number of defaults in the sample by year and the first default event. There were few defaults in the late 1990s, but after the dot-com crash, the default rate increased dramatically. It peaked in 2001, when 10.3% of all speculative-grade firms defaulted, which at the time was the highest default rate since 1933 (Emery and Ou (2010)). Not surprisingly, Panel B shows that telecom firms are particularly common in the sample. Overall, the sample includes both relatively calm periods and years with extremely high failure rates, which makes it possible to relate the default boundary to the state of the economy. In Panel A, 37.1% of firms default by filing for bankruptcy directly, 54.0% miss or delay a payment, and 8.9% complete a distressed bond exchange. Further analysis (not tabulated) reveals that 87.8% of payment omissions and 31.6% of successful distressed exchanges lead to a bankruptcy filing within two years of the first default event. Although the proportion of defaults that result in bankruptcy varies from year to year, the incidence of bankruptcy in the sample is close to the DRS average.[13]

[TABLE 1 HERE]

Do firms default at debt maturity, at dates when interest is due, or in between scheduled debt payments? Interest on most corporate bonds is paid semiannually, and even when there are several bonds outstanding for the same issuer, they often pay interest in the same calendar month. As a result, for the median firm in my sample, all bond payments occur in only two calendar months each year (most frequently, June and December). If the timing of default were random and unrelated to debt payments, only 2 out of 12 defaults would be expected to occur in the month preceding a scheduled interest payment. Yet in my sample 59% of all defaults occur within the 30 days preceding a scheduled

[13] At the same time, there is evidence that the *practice* of Chapter 11 bankruptcy has changed dramatically since reorganizations of the 1980s, for which well-known empirical studies like Gilson, John, and Lang (1990) and Asquith, Gertner, and Scharfstein (1994) document prolonged bargaining and large violations of absolute priority. For example, Baird and Rasmussen (2003) state that “traditional reorganizations [in Chapter 11] have largely disappeared. Put concretely, in 84% of all large Chapter 11s from 2002, the investors entered bankruptcy with a deal in hand or used it to sell the assets of the business. In the remaining cases, going-concern value was small or nonexistent” (p. 674).

bond payment, including 34% that happen on a scheduled payment date. Although this analysis does not include payments on bank debt, it clearly indicates that most firms do not default until a debt payment is due. Most of these defaults are on bond interest payments, with only 4 firms (1.9%) in my sample defaulting close to a bond maturity date.

3.3. Descriptive statistics

Table 2 reports descriptive statistics for firms at default and for nondefaulting junk firms. The "nondefaulting" control sample consists of 593 firms that did not default throughout the sample period, as well as the firm-month observations for defaulted firms that precede the default date by at least a year. For each firm in the control sample, I calculate the sample period average of each variable and report descriptive statistics for this set of firm averages. [TABLE 2 HERE]

Panel A of Table 2 shows that defaulting and nondefaulting junk firms are large and do not differ much in terms of book assets or sales-to-assets ratios. The volatility of assets, estimated as the industry median standard deviation of monthly firm returns, is marginally higher for defaulted firms, with a median of 27.5%, compared with 26.7% for the control sample. For comparison, Schaefer and Strebulaev (2008) estimate asset volatilities of 28% for B-rated firms. Other statistics in Panel A indicate that firms at default are more distressed than the control sample. Their market-to-book ratios are noticeably lower. The median marginal corporate tax rate, estimated by John Graham using the simulation method described in Graham (1996),[14] is 32.4% for the control sample, but only 13.1% for the defaulting sample, reflecting its distressed state and the lack of taxable profits expected in the foreseeable future. These statistics suggest that tax shields are unlikely to be an important consideration for firms close to default. The median nominal share price at default is only \$0.84 compared with \$19.30 for nondefaulting junk firms. Thus, it is unsurprising that the firm's market capitalization can be used as a predictor of bankruptcy (Shumway (2001)). The weighed average price of debt is close to 50 cents on the dollar for firms at default, but close to par for nondefaulting firms. This suggests that approximating the market value of debt by its face value, as if often done in academic studies, may be acceptable for healthy junk firms, but not for distressed ones. In contrast with the control sample, most firms at default have negative book equity. Finally, as many as 85% of defaulting firms are original-issue high-yield (or junk) bond issuers, meaning that none of their outstanding bonds have ever had an investment-grade rating. This proportion is only 69% for nondefaulting junk firms, while 31% of them are "fallen angels."

[14] I thank John for making these data available through his website (`http://faculty.fuqua.duke.edu/~jgraham/`).

Panel B reports various measures of leverage. The median market leverage, defined as the market value of debt over the market value of the firm, equals 91.7% for defaulting firms, but only 47.5% for the control sample. Thus, for nondefaulting junk firms, equity accounts for slightly more than half of the total value, but at default, it is only 8.3% of the median firm. The other two measures of leverage in Panel B use book rather than market values of debt. The quasi-market leverage ratio, which uses market equity values in the denominator, is close to the market leverage for firms away from default, but at default, it understates the equity's share in the capital structure significantly. Book leverage is also close to the other two measures for nondefaulting firms, but is far off at default. These statistics underscore the importance of using market rather than book values of debt for very distressed firms.

Panel C of Table 2 shows various statistics describing the debt structure of sample firms. Almost two-thirds of their debt is in bonds. Bond maturity is slightly lower and the proportion of short-debt higher for defaulting firms.[15] This finding is consistent with firms defaulting due to difficulties in refinancing short-term debt. An alternative possibility is that riskier firms endogenously issue debt with shorter initial maturity. Debt interest rates and average numbers of loan covenants are higher for firms at default, with differences significant at the 5% level. Taken together, shorter maturities, higher interest rates, and more onerous covenants suggest that firms in the defaulted sample were perceived as riskier at the time when the debt was issued and its characteristics were negotiated.

4. The location of the solvency boundary

I estimate the default boundary as the total market value of the firm's debt, equity, and expected costs of default, measured at the end of the calendar month preceding default. The default boundary is commonly expressed as a fraction of the face value of debt. One frequently used assumption is that default is triggered when the firm's economic net worth becomes negative, i.e., when the market value of its assets falls below the face value of debt. Huang and Huang (2003) argue that, based on historical debt recovery rates and plausible estimates of financial distress costs, the default boundary is likely to be closer to 60% than 100% of debt. Leland (2004) uses the formula derived by Leland and Toft (1996) and finds that for an average BBB-rated firm, it implies a boundary of 73.1%. Reisz and Perlich (2007) calibrate a barrier option model to observed equity prices and estimate the boundary at 66.2% of debt.

[15] Balance sheet data on short-term debt for distressed firms should be interpreted with caution. Should the firm be in violation of a debt covenant, all its debts must be classified for accounting purposes as due immediately, regardless of actual maturity. This requirement results in a sharp increase in reported short-term debt for firms close to default compared with the previous quarter. To preserve the information on the maturity structure, for firms at default, I use the second most recent balance sheet for information on debt due in one year, rather than the most recent.

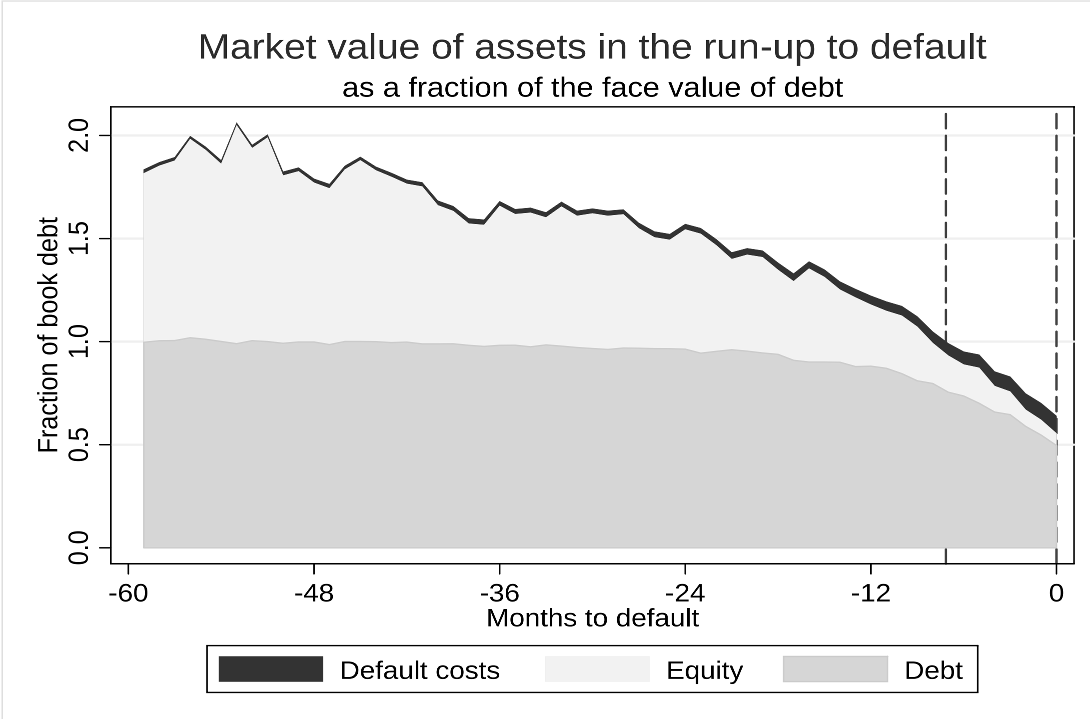

Fig. 2. This graph illustrates the evolution of the market value of debt, equity, and expected default costs for the hypothetical median defaulting firm in the run-up to default. All market values are normalized by the face value of debt. For each month before default, I first find the median firm value (expressed as a fraction of the debt claim) and the median market leverage ratio, then calculate the value of debt as the product of the two. The value of equity is the value of the firm minus the value of debt. Expected default costs are estimated for each month based on the median firm value, assuming the total cost of default equals its sample median, $\alpha = 19.4\%$.

Panel A of Table 3 shows that empirically, the average market value of assets at default is 66.0% of the face value of debt, and the median is 61.6%. Thus, contrary to the zero net worth assumption, by the time the average firm defaults, more than one-third of the creditors' value is already destroyed. The default boundary is highly dispersed, ranging from 30% to 122% of face debt at the 5^{th} and 95^{th} centiles, respectively. Thus, some firms default while their economic net worth is still positive, and others do not default until they are deep in distress. To my knowledge, these are the first estimates of the levels and the variation of the value-based default boundary obtained directly from market values of defaulted firms.

In panel A, for junk firms that do not default for at least a year the mean (median) value of assets is 311% (225%) of the face value of debt. The difference between assets values of nondefaulting and defaulting firms is economically large and highly statistically significant. It would be even bigger if the control sample were not limited to junk firms, some of which are also very distressed. The median asset value for all firm months in the sample, including defaulting and nondefaulting firms, is 201% of debt. [TABLE 3 HERE]

Figure 2 illustrates the evolution of the market value of assets and its components for the median defaulting firm during the five years preceding default. The value of assets initially fluctuates at about twice the face value of debt, which is its sample median for all defaulting and nondefaulting firm-months. About four years before default the asset value starts to decline visibly. Initially, the decline is absorbed by falling equity values. About two years before default, debt prices also start to fall. For the median firm, the total value of assets falls below the face value of debt approximately eight months prior to default. Contrary to the zero net worth boundary assumption, the firm continues to operate with negative net worth, and does not default until its asset value falls to 61.6% of face debt. In untabulated analysis, I find that in the month of default, the value of assets of the median firm falls by an additional 8.4%. The debt recovery rate, calculated as the ratio of the market value of debt to its face value at the end of the calendar month of default, is 41.8% for the median defaulting firm. This recovery rate is very close to the 42% documented by Acharya, Bharath, and Srinivasan (2007), suggesting that asset values at default for my firms are likely to be similar to those in other defaulted samples.

When investors realize that costly default may be possible, the value of the firm will differ from the market value of assets due to expected cost of default, which depend both on the total default cost for the firm and on the probability of default. Table 3 shows that for the average (median) firm at default, the value of the firm is 88.7% (92.0%) of the market value of assets, whereas expected default costs incorporated in observed debt and equity prices are 11.3% (8.0%). For the median firm just prior to default, expected default costs are substantially larger than the value of equity, which is only 4.3% of the asset value. In other words, the error from failing to adjust the firm value for expected default costs at default is larger than that from ignoring the equity claim. The median value of the firm at default is 55.5% of face debt, compared with 61.6% for the total assets. Nonetheless, as long as the market value of assets is still substantial, investors perceive only a small probability of default. Consequently, the mean expected cost of default for nondefaulting junk firms is only 1% of the value of assets, and the median is as low as 0.4%, consistent with estimates of ex ante costs of financial distress in Elkamhi, Ericsson, and Parsons (2010). As can be seen from Figure 2, expected default costs become a noticeable fraction of the asset value only at advanced stages of distress. Thus, the market value of the firm is a good approximation of the asset value in most situations, except for firms that are very close to default.

Estimated default costs are discussed in detail in Davydenko, Strebulaev, and Zhao (2010). The mean (median) cost of default in the sample is 20.4% (19.3%) of the pre-default market value of assets. For comparison, using a sample of 31 highly levered firms that became distressed, Andrade and Kaplan (1998) estimate that the costs of financial distress are likely to be in the 10%–20% range. I find that by the time the firm defaults, roughly half of the total default costs are already incorporated in debt and equity prices, while the remaining half is realized as the drop in the value of the firm upon default.

5. Determinants of the default boundary

This section reports cross-sectional regressions of the solvency boundary. Because the boundary is only observed for a nonrandom sample of firms that default, I use the Heckman (1979) two-step procedure to control for self-selection. The first-stage regressions estimate the probability of being in the defaulted sample. The inverse of the Mills' ratio is then included in the second-stage regressions, in which the dependent variable is the asset value at default normalized by the face value of debt. I use the quick ratio, which measures the firm's liquid assets relative to its current liabilities, as an instrument in first-stage regressions. The role of liquidity in triggering default is discussed in detail in Section 7. For the purposes of the current analysis, what is important is that the quick ratio is highly correlated with the short-term probability of default but almost uncorrelated with the value of assets at default.

What factors can be expected to affect the level of the default boundary? As Leland (2004) points out, in models with exogenous default, such as Longstaff and Schwartz (1995), the boundary is independent of firm characteristics. In contrast, models with endogenous default assume that the default boundary is chosen to maximize the value of equity, and derive the boundary as a function of firm characteristics. Leland (1994) and Goldstein, Ju, and Leland (2001) provide explicit formulae for the default boundary when debt is a console bond promising a continuous coupon payment. The boundary in these models depends on the asset volatility, the coupon rate, the asset payout ratio, the risk-free rate, and the corporate tax rate.[16] Leland and Toft (1996) extend the time-independent setting to allow for finite-maturity debt. The boundary in their model depends additionally on debt maturity and default costs. Most variables enter their boundary formula nonlinearly and interact in a complex way. To determine the direction of the predicted effect, I compute the Leland-Toft boundary when all the inputs are set to their sample averages for defaulted firms, and then evaluate the formula with each factor varying between its sample minimum and maximum. The predicted signs and the regression results are reported in Table 4a. [TABLE 4a HERE]

A distinctive prediction of endogenous-default models is that the default boundary is lower when the volatility of assets is high. Intuitively, high volatility means that shareholders' limited liability option is more valuable, and therefore, it is worth keeping the option alive by making required debt payments

[16] In Leland (1994), the tax rate enters the formula for the boundary explicitly, whereas in Goldstein, Ju, and Leland (2001), taxes are part of the asset payout ratio. In the tests of this section, the payout ratio is estimated as in Eom, Helwege, and Huang (2004) and does not include taxes, corresponding to Leland (1994) and Leland and Toft (1996).

until more advanced stages of distress. Regression (1) tests this prediction, and shows that the coefficient for the asset volatility is negative and strongly statistically and economically significant, as predicted by endogenous-default models.[17] Also, in column (2), the boundary is increasing in the cost of default, as predicted by Leland and Toft (1996), although not by models with infinite-maturity debt. The strong correlation of observed solvency levels at default with asset volatility and default costs is encouraging for models that assume that the default timing is chosen optimally to maximize the value of equity.

At the same time, the evidence regarding the effect of other variables in Table 4a is mixed. Although the coefficients for the asset payout ratio and the risk-free rate are negative as predicted, they are only marginally significant. Moreover, the signs of the coefficients for the coupon ratio and the tax rate are the opposite of what is expected.[18] These variables are insignificant in most multivariate regressions. Importantly, the effect of debt maturity is also not clear-cut. Based on the Leland-Toft formula, the default boundary is predicted to be negatively correlated with the weighted average debt maturity. A negative correlation with maturity is also expected in some exogenous-default models, such as Black and Cox (1977). Yet in regression (3), the coefficient for the average maturity is positive an insignificant.[19] However, the average maturity may not be a good proxy in this context. In the Leland-Toft model, debt is offered and retired continuously, the debt structure is time-independent, and the average maturity is one-half of the initial maturity for all debt issues. In reality, the debt structure changes with time as debt ages and the remaining maturity of outstanding debt issues shortens. In such a setting, the firm's ability to repay or refinance the debt issue that is coming due first is likely to affect its default boundary more than longer-maturity issues. Therefore, as another proxy for maturity, I use the time left before the first of the firm's outstanding bonds is due. As predicted, in regression (4) this variable is negatively and significantly associated with the default boundary.

However, an empirical concern with these tests is that debt maturity, coupon rate, and the asset payout ratio are potentially endogenous variables that are likely to depend on firm characteristics, especially its

[17] I measure asset volatility as the industry median standard deviation of monthly firm returns, excluding firms with fewer than 10 monthly return observations. The use of industry rather than firm-specific volatility estimates increases the number of usable observations and reduces noise. Moreover, because the median firm in the industry is typically not distressed, its firm and asset values are very close. As a result, the volatility of assets can be approximated by the volatility of the firm computed as in Choi and Richardson (2008), without the need to adjust for expected default costs. For robustness, I also use firm-specific firm volatility estimates, with similar results.

[18] Of note, for the subsample of firms for which simulation-based estimates of the corporate tax rate are available, the coefficient for the tax rate is positive and significant at the 6% level. This likely reflects the fact that firms with higher asset values are generally expected to be more profitable and hence pay taxes at higher marginal rates in the future. A regression of the estimated tax rate on the value of assets (normalized by the debt claim) for all defaulting and nondefaulting observations yields a positive coefficient significant at the 1% level.

[19] I measure the average maturity as the weighted average of maturities of all debt instruments, assuming that all bank debt is due in one year. I also use the weighted-average bond maturity as an alternative proxy, with similar results.

default risk. Regressions (9) and (10) show that the statistical significance of these factors declines or disappears in the presence of controls for volatility and default costs. To investigate this issue further, I estimate instrumental variable regressions using Treasury rates at debt origination as an instrument for the potentially endogenous debt characteristics. The idea is that, for a firm of a given credit quality, the coupon rate and the overall asset payouts will be higher, and the initial debt maturity lower, if interest rates are high at the time when the firm's debt is issued. I compute the weighted average of ten-year Treasury rates at bond origination, and find it to be strongly related to maturity, coupons, and asset payout ratios. However, columns (11) through (14) show that when I use this variable as an instrument in regressions of the default boundary, the coefficients for the four variables, including the shortest bond maturity, are not statistically significant (although three of the four have the predicted sign).

Overall, the value of assets at default is associated with the volatility of assets and the cost of default in a manner consistent with endogenous-boundary models, while the evidence regarding the effect of other variables is more mixed. The relatively low R^2s of the regressions in Table 4a imply that these variables can explain only a modest fraction of the observed variation in the asset value at default. Table 4b reports the effect of additional factors, including covenants, macroeconomic conditions, and balance sheet liquidity. These regressions also control for all of the variables that enter the Leland-Toft formula, with the exception of the tax rate, which is only available for a fraction of firms.

Debt covenants give creditors the right to demand debt repayment upon covenant violation (technical default). If covenants are so strict that violations occur even in the absence of severe distress, and if creditors routinely accelerate repayment upon violation, ignoring covenants may cause the default boundary to be underestimated. Dichev and Skinner (2002) find that bank covenants are set so tightly that they appear violated for about one-quarter to one-third of all loans, and most apparent violations do not indicate distress. Instead, covenants serve as tripwires, allowing banks to renegotiate loan terms and reduce their risk exposure if the firm's financial position deteriorates (Smith (1993)). Therefore, whether the covenant structure affects the actual timing of bond defaults is an empirical question.

I use the DealScan database to compute the number of covenants specified in each firm's loan contracts. DealScan provides information on 16 financial covenants. I look at those 11 that are present for at least 5% of firms, and classify them as either balance sheet–based or cash flow–based. The first group includes covenants restricting the firm's net worth and leverage. The second group specifies various coverage ratios, such as the interest coverage ratio and the ratio of debt to cash flow. I calculate the total number of covenant types for these two groups. The number of networth covenant types varies between zero and six and the number of coverage covenants varying between zero and five. Columns (1) and (2) of Table 4b report regressions of the default boundary on these variables. Regression (1) shows that the presence of networth–based covenants is associated with higher asset values at default, though the effect is marginal. This evidence, although not strong, is consistent with exogenous-default models that assume that firms default when they violate a networth covenant. In contrast, cash flow–based covenants in regression (2) are not statistically related to the default boundary. [TABLE 4b HERE]

To gain further insight into the role of covenants, I collect media reports on covenant violations and waivers from Factiva. In my sample of 213 defaults, at least 8.0% are explicitly said to be forced by senior creditors as a direct result of technical default. In addition, at least 7.0% of defaults happen around the time of expiration of a previously granted covenant waiver; it is likely that at least some of these defaults are covenant-related. For 76.5% of covenant-triggered defaults, banks waive or loosen covenants before eventually pulling the plug. Regressions (3) and (4) show that reports of a covenant violation and waiver are associated with lower solvency at default, but neither factor is statistically significant.[20]

Carey and Gordy (2009) assume that upon covenant violation banks optimally decide whether to ignore it or force default. The default boundary in their model maximizes the value of bank debt, and is predicted to be higher when bank debt is a larger fraction of total debt. Regression (5) shows that, though the correlation between the boundary and the proportion of bank debt is positive, it is not statistically significant. Overall, regressions (1) through (5) suggest that the effect of bank covenants on the level of solvency at which firms default on their bonds is at best marginal for my sample period. Banks may be using covenants to improve their own position by reducing their exposure and requiring additional collateral in informal negotiations, without triggering default on public bonds that can push the firm into bankruptcy (Smith (1993)). Put differently, the evidence on widespread covenant waivers and renegotiations, coupled with the low incidence of covenant-induced bond defaults, suggests that covenants mostly affect the boundary that triggers bank debt renegotiations, rather than the default boundary for bonds that these regressions focus on.

Recent models by Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010) incorporate macroeconomic risk in the structural framework, and predict that the value-based default boundary is lower at times of economic downturns. To proxy for macro conditions, I use the seasonally adjusted quarterly growth rate of the GDP, as well as the proportion of bond issuers that default in each year. Regressions (6) and (7) of Table 4b report the results. Using both proxies, firms at default are less valuable in

[20] Further results on covenant violations, renegotiations, and covenant-triggered defaults are available upon request.

economic downturns, consistent with the above models in which the value-based boundary is procyclical.

Finally, columns (8) and (9) report OLS regressions on two proxies for balance sheet liquidity, the quick ratio and the current ratio. The correlation between liquidity and solvency at default is not statistically significant, and its sign is not consistent across the two proxies. I investigate the role of liquidity further in Section 7, and document that the *probability* of default is strongly correlated with liquidity. Taken together, these findings suggest that the effect of cash shortages on the timing of default cannot be captured adequately by simply modeling the value-based boundary as a function of liquidity.

6. Classification errors

The assumption of a threshold value of assets that separates defaulting and nondefaulting firms is a convenient simplification that is widely used in structural models. This section documents the errors it implies, focusing on the simple case of a constant boundary common to all firms. Firm-specific boundaries, such as those predicted by models with endogenous default, are studied in Section 8.

In order to quantify the accuracy of boundary-based default predictions, it is necessary to define what it means that a certain level of insolvency "triggers" or "fails to trigger" default. Suppose, for example, that in a certain month the value of assets falls to the face value of debt. According to the boundary-based view of default, if the firm defaults immediately after, it means that default has been triggered by the firm's economic net worth falling to zero. But what if the firm defaults not immediately, but shortly thereafter? Classifying such observations as nondefaulting and counting them as failures of the zero net-worth boundary to detect the subsequent default may be too harsh on boundary-based models. If the model is used to price bonds and determine the probability of default for a healthy firm over a period of several years, the fact that default occurred one or two months after hitting the boundary can arguably be considered a success of the boundary-based prediction, whereas the failure of the boundary to pinpoint the timing of default precisely is likely to be less important. Thus, to calibrate the boundary for practical purposes, situations in which the firm defaults after crossing the boundary, but only with some delay, should not be automatically considered type-I errors (i.e., a false positives).

Empirically, contrary to the first-passage time models with a constant boundary, only 52% of defaults in my sample coincide with the lowest historical value of assets (normalized by debt claim) for the firm. However, for another 28% the minimum asset value is followed by default before the next scheduled bond payment is due. Such behavior is consistent with models like Merton (1974) and Geske (1977), in which the firm does not need to default as long as its assets are high enough at times of scheduled debt payments, but not necessarily in between. Based on these observations, for the base case I assume that the observed asset value has failed to trigger default if it is not followed by default at least until after the next scheduled bond payment.

Figure 1 on p. 4 suggests two stylized facts that contradict the assumption of a pronounced boundary that separates defaulting and nondefaulting states. First, asset values at default are highly dispersed. This implies that for moderate boundary levels, there will always be firms that default above the boundary, resulting in a significant probability of a type-II error (wrongly classifying a firm that is about to default as nondefaulting). Second, many low-value firms do not default for years. Such firms are likely to be incorrectly classified as defaulting by the boundary rule, resulting in a considerable type-I error.

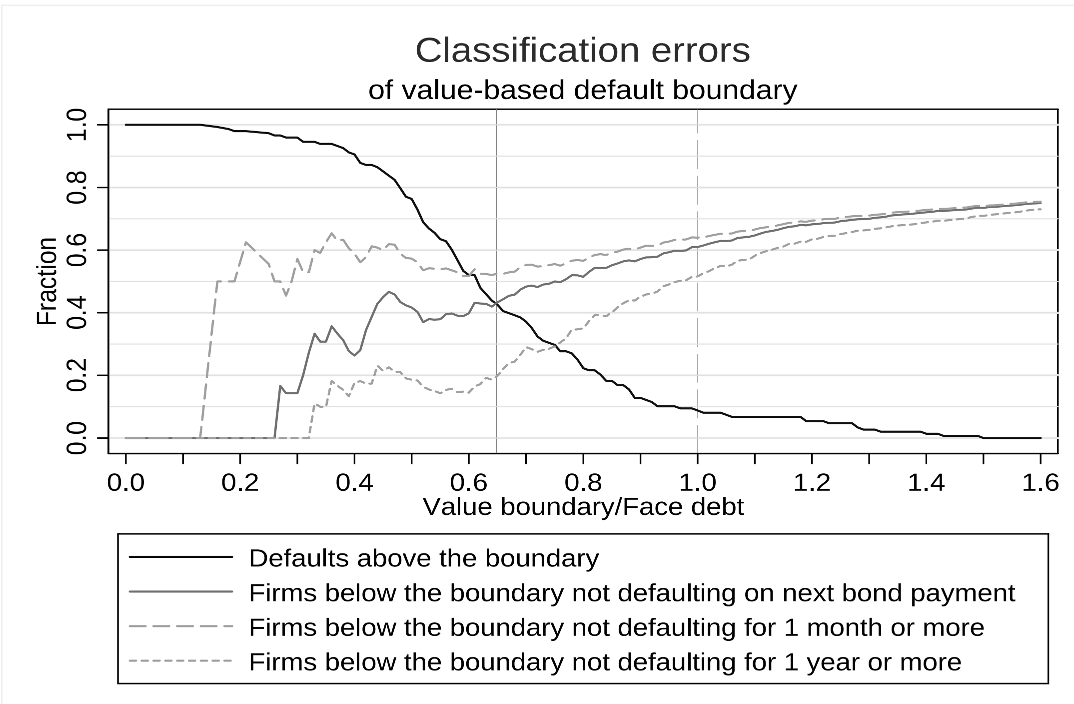

Fig. 3. For each assumed boundary value of assets, this graph plots the fraction of observed defaults that happen at asset values above the boundary and the fraction of firms with asset values below it that do not default.

Figure 3 plots classification errors as functions of the assumed boundary level. The zero net-worth boundary misses 8.8% of defaults, but produces a large number of false positives: *Most* economically insolvent firms (51.6%) do not default for at least a year, and 61.0% of them do not default on the next bond payment.[21] These statistics gauge Huang and Huang's (2003) observation that many firms continue to operate with negative net worth, and provide a justification for adjusting the default boundary

[21] Longer term, the fraction of negative networth firms that avoid default for three years is 44.1%; and for five years, 25.0%.

downward in implementations of exogenous-default models, such as Longstaff and Schwartz (1995). For lower levels of the default boundary, the proportion of nodefaulting firms below the boundary is lower, but the number of defaults above it is higher. Over the one-month horizon, false positives typically amount to 60% or more of all firms predicted to default. Because more firms default over time, false positives are less frequent for longer prediction horizons, but generally their presence remains a significant problem.

The level of the boundary that equalizes the number of defaults above the boundary with the number of firms below it that do not default on their next debt payment is 64.8% of face debt. This boundary level correctly "predicts" the probability of default in the sample, in the sense that the total number of firms with asset values below it equals the observed number of defaults. However, the cross-sectional errors that it implies are substantial: As many as 42.6% of defaults happen at asset values above this boundary level, and an equal number of firms below it do not default at least until after the next scheduled debt payment. Even assuming, perhaps contrary to the spirit of the first-passage time view of default, that an excursion below the boundary can trigger default for up to a year afterwards, the boundary that equates the number of type-I and type-II errors, which equals 75% of face debt in Figure 3, misclassifies 30% of defaults and an equal number of nondefaulting firms. These results quantify the cross-sectional error that arises when firms are classified as defaulting and nondefaulting based on their solvency alone.

Eom, Helwege, and Huang (2004) find that existing structural models cannot explain the cross-section of bond spreads even when they perform reasonably well on average. My findings suggest that an important reason for this lack of accuracy may be their reliance on the boundary assumption. With the boundary at 73.1%, Leland (2004) finds that first-passage time structural models predict default probabilities for an average firm that accord well with historical default frequencies over horizons of five years and beyond. Yet in my sample, 31% of firms default above this boundary and a similar number of firms do not default for at least a year. Thus, even though models with a constant boundary can match the observed default probability for an *average* firm, the *cross-sectional* errors arising from the boundary assumption alone can be substantial. Developing specifications of the default trigger that could account for the stylized facts documented in this study could be an important step towards improving the accuracy of future models.

7. Value versus liquidity as default triggers

A potentially important factor missing from traditional structural models is the availability of cash and other liquid assets, which allow firms to continue uninterrupted debt service when their cash flow declines temporarily. Cash shortages are irrelevant in value-based models, because these models assume that firms can always raise external financing against future cash flow as long as the value of assets is high enough. However, in reality market frictions may restrict firms' access to external financing in distress, for instance, due to information asymmetry or agency problems (Myers (1977), Myers and Majluf (1984)). If external financing is unavailable, temporary cash shortages can push the firm into default even if the business is still valuable. Corporate finance models, such as Bulow and Showen (1978) and White (1989), argue that the firm's ability to access external financing depends on its net worth, implying that both asset values and liquidity can affect the timing of default. This section shows that, although the value of assets is by far the most powerful factor explaining default, liquidity also plays an independent role, but only for financially constrained firms.

7.1. Summary statistics on cash flows and liquidity

Table 5 shows that there are major differences in cash flows and liquid asset holdings between defaulting and nondefaulting firms. At default, the median profit margin is –21.4%, and the mean is –239%. Accounting income is negative for 93.1% of defaulting firms, and as many as 81.1% of them have negative operating cash flow. Nondefaulting firms in my sample also have negative profitability on average (–20.3%), but it is driven by a much smaller number of loss-making firms (33.8%), and the proportion of negative cash flow firms in the control sample is only 17.1%. The interest coverage ratio (EBITDA over interest payments) provides additional evidence that defaulting firms do not generate sufficient cash flow to cover their obligations. While the median firm in the control sample can cover its interest payments out of its EBITDA 3.3 times, the median firm at default is making *losses* equal to 17% its interest payments. [TABLE 5 HERE]

Panel B of Table 5 reports various measures of balance sheet liquidity. The primary proxy for liquidity used in this paper is the quick ratio, or the ratio of cash and near-cash plus accounts receivable to current liabilities. The mean (median) quick ratio at default is 0.55 (0.35), compared with 1.27 (0.92) for the control sample. For 85% of firms at default, the quick ratio is less than 1, compared with only 59% for nondefaulting firms in the sample and 47.1% for all firm quarters in Compustat. While low liquidity ratios may be the norm rather than a sign of distress in some industries, 80% of firms at default have a quick ratio below their industry median, whereas nondefaulting firms are fairly similar to the industry norm. Other measures of liquidity, such as the current ratio (current assets divided by current liabilities) and cash ratios (cash over current liabilities) are also much lower at default.[22]

[22] The current ratio is similar to the quick ratio but includes inventories and other current assets in the numerator. Because

With insufficient liquid asset reserves, distressed firms must rely on either the operating cash flow or external financing to pay their creditors and suppliers. However, as many as 66.5% of firms at default have *both* a quick ratio below the industry median and negative cash flow, compared with only 7.2% for nondefaulting junk firms. Loss-making firms with insufficient liquid assets need access to external financing to avoid default.

7.2. Value and liquidity at default

Figure 1 on p. 4 displays asset values (relative to the face value of debt) and balance sheet liquidity (measured by the quick ratio) for firms at default, as well as for the control sample of firms that do not default for at least a year. In the graph, diamonds represent firms at default, and circles show nondefaulting firms in the month when their asset value is at its historical minimum. Moving left (down) in the graph corresponds to increasing insolvency (illiquidity).

Several important insights emerge from Figure 1. Nearly all sample firms at default have negative economic net worth, a quick ratio below 1, or, in most cases, both. This finding is consistent with the intuition that economic distress (falling cash flows) eventually causes financial distress (the inability to honor financial obligations) when the firm's continued losses deplete its cash reserves. In general though, liquidity and value are distinct potential default triggers. Of special interest are defaults in the upper left quadrant of Figure 1, representing companies with low asset values but liquidity sufficient to meet their current liabilities, sometimes several times over. In the sample of firms at default, the quick ratio is above the industry median for 20% of firms, above 1 for 14.9%, and above 2 for 3.6%. For comparison, the quick ratio is above 2 for 26% of all firm-quarters in Compustat.

I use Factiva to gain insight into the motives that drive firms with substantial liquid assets into default, looking into the details of all such defaults on a case-by-case basis. News releases and press statements for these firms emphasize continuing losses, difficulties in obtaining additional financing, and insufficient resources for vital investment expenditures as frequent reasons for default. Some distressed firms file for bankruptcy in recognition of their inability to generate sufficient cash flow to support their obligations over the long term; others carry out recapitalizations involving a bond exchange, which may seriously dilute existing shareholders' stake but improve the firm's balance sheet.[23] Observed defaults by

a firm in decline often cannot convert its inventories into cash quickly, this ratio may be less informative about the firm's liquidity in distress than the quick ratio.

[23] As an example, Focal Communications' 10-Q filing dated June 30, 2001, indicates a quick ratio of 2.19. On August 9, 2001, Focal announced a distressed bond exchange, completed in October: "The [recapitalization] plan dilutes the stake of existing shareholders to 20%, but steers Focal away from a potentially debilitating cash crunch. The $80.8 million in cash

high-liquidity, low-value firms are consistent with models that use a value-based default boundary but not with the popular perception that firms default when they run out of cash.

At the same time, Figure 1 indicates that some defaults happen while the market asset value is still substantial. In the sample, 8.8% of firms at default have positive economic net worth but low liquidity ratios.[24] A case-by-case Factiva analysis indicates that such defaults may be driven by litigation, covenant violations, and, commonly, insufficient liquid reserves, coupled with the inability to obtain additional liquidity from external sources. For firms that are unable to pay their suppliers because of a cash shortage, Chapter 11 of the U.S. bankruptcy code relaxes financing constraints by providing access to Debtor-In-Possession (DIP) financing. Overall, Figure 1 suggests that though most defaults happen when both value and liquidity are low, neither factor alone can fully explain observed default decisions.

Finally, the large number of circles in Figure 1 indicates the presence of nondefaulting firms with low asset values and/or low liquidity. Even though nondefaulting firms are more valuable and liquid on average, many of them at times become very distressed. Defaulting and nondefaulting firms often appear in similarly poor conditions, as measured by the value of assets and the quick ratio, so liquidity and value alone cannot separate them perfectly. Below, I explore the role of financing constraints in moderating the effect of cash shortages in triggering default.

7.3. Hazard regressions of default

To study the role of various factors in triggering default, I estimate hazard models of default using all defaulting and nondefaulting firm-month observations. Hazard analysis has become the instrument of choice in empirical studies predicting default and bankruptcy (e.g., Shumway (2001); Bharath and Shumway (2008); and Campbell, Hilscher, and Szilagyi (2008)). In contrast to predictive studies, the main goal is not to build a better forecasting model, but rather to evaluate the role of two specific factors, value and liquidity, at the time when default is triggered.[25] Following Bharath and Shumway (2008), I avoid the need to specify the baseline hazard by using the Cox (1972) proportional-hazard model.

that sat on Focal's balance sheet before the recapitalization was expected to run out early in 2002. [...] Analysts say that without the massively dilutive recapitalization, shareholders may have ended up with nothing" (*Focal Closes Recapitalization For Shot At Survival*, Dow Jones Newswires, 24 October 2001).

[24]Although in models with endogenous default, such as Leland and Toft (1996), shareholders' optimal boundary can sometimes exceed the face value of debt, this does not explain defaults by solvent firms in my sample. The Leland-Toft boundary is above 1 for 5.7% of defaulted firms, but the empirical boundaries for these firms are all below one, with the mean and median close to those of the overall defaulted sample. Conversely, for the 8.8% of firms observed to default while solvent, estimated Leland-Toft boundaries are all below one and similar to those of other defaulted firms.

[25]Prominent contributions include Altman (1968), Zmijevski (1984), Shumway (2001), and Hillegeist et al. (2004), among others. Predictive studies typically look at horizons of one year or more. Acharya et al. (2008) show that, because cash reserves are endogenous and depend on credit risk, the role of liquidity is sharply different for shorter horizons.

7.3.1. Value, liquidity, and traditional predictors of default

Table 6 compares the market value of assets with other accounting-based and market-based variables commonly used in empirical default-predicting studies. To facilitate comparisons of different models, all regressions in this table use the same subsample of 25,018 firm-months for which all variables are available. Following Shumway (2001) and Bharath and Shumway (2008), all independent variables are winsorized at their 1% and 99% levels. The winsorization boosts the predictive power of accounting and equity-based variables significantly, but has almost no effect on the asset value. [TABLE 6 HERE]

Column (1) shows that the ratio of the market value of assets to the face value of debt is a very powerful variable affecting the timing of default, with a z-statistic of –14.1 and pseudo-R^2 of 56%. Remarkably, its explanatory power exceeds that of all other factors in Table 6 put together, including predictors based on the market value of equity. Regression (2) includes the five ratios used in Altman's (1968) z-score model, namely, working capital over total assets (WC/TA), retained earnings over total assets (RE/TA), EBIT over total assets ($EBIT/TA$), market equity over total liabilities (ME/TL), and sales over total assets (S/TA). Three of the ratios are statistically significant, but all five together explain less of the variation of the timing of default than does the asset-value alone in regression (1). Moreover, column (3) shows that once the value of assets is added to the specification, the only variable that remains significant is WC/TA, which is Altman's proxy for balance sheet liquidity.

Regression (4) includes three accounting ratios suggested by Zmijewski (1984), namely, the current ratio (CA/CL), net income over total assets (NI/TA), and total liabilities over total assets (TL/TA). As in Shumway (2001), these variables are strongly statistically significant. However, when they are combined with the value of assets in regression (5), the coefficient for TL/TA changes sign and becomes insignificant, and that for NI/TA falls by two thirds, with the z-statistic dropping in magnitude from –12.2 to –2.9.[26] At the same time, the current ratio CA/CL, which measures balance sheet liquidity, remains strongly significant, similar to WC/TA in (3).

Columns (6) to (8) of Table 6 use the distance to default (DD), as well as the Expected Default Frequency (EDF) provided by Moody's/KMV (MKMV). These are two related measures of distress based on the Merton (1974) model, calibrated to observed market values of equity. The use of these variables was pioneered by KMV LLC and later introduced into academic research by Vassalou and Xing (2004).

[26] Bharath and Shumway (2008) find that NI/TA remains strongly significant after controlling for distance-to-default proxies, which is also the case in my sample. However, as shown in columns (10) to (13), in the presence of additional controls NI/TA loses much of its predictive power and becomes insignificant in most specifications.

The distance to default is a sufficient statistic for calculating the probability of default in the Merton model. The estimation procedure developed by KMV involves solving a series of nonlinear equations in order to estimate the unobserved market asset value and volatility implied by observed equity prices and leverage ratios (Crosbie and Bohn (2002)). MKMV applies proprietary modifications to the estimation algorithm, and transforms the distance to default into predicted expected default frequencies (*EDFs*) using a proprietary sample of defaulted firms, rather than Merton's formula. Bharath and Shumway (2008) find that the distance to default estimated using a nonproprietary version of the KMV algorithm is, if anything, a slightly weaker predictor of default than a "naïve" measure that uses the same inputs and functional form, but is much easier to estimate. Consequently, I compute the naïve distance to default (DD) as suggested by Bharath and Shumway (2008).[27]

Regressions (6) and (7) show that both the naïve distance to default and the EDF are strongly associated with default. These regressions produce pseudo-R^2s of 31% and 44%, which are high compared to most other univariate predictors, but still substantially lower than the 56% produced by the market value of assets in regression (1). Interestingly, in column (8) both DD and EDF remain significant in each other's presence (the correlation between the two is –0.65), though MKMV's proprietary adjustments seem to give the EDF an edge compared to the naïve distance to default measure. Importantly, even though both variables incorporate information embedded in the market value of equity, they lose much of their power in the presence of the market value of total assets in column (9). The coefficient for the EDF drops from 0.31 to 0.09, and its z-statistic from 9.2 to 2.2, while the naïve distance to default becomes insignificant. By contrast, the coefficient for the value of assets changes relatively little in the presence of these and other controls in the table.

Overall, in these tests the market value of assets is the most powerful variable by far, explaining a higher proportion of the variation in the timing of default than all other default predictors put together, most of which become insignificant in its presence. In addition to MKMV's EDF, the only consistent exception to this rule comes from proxies for balance sheet liquidity, such as WC/TA and CA/CL.

7.3.2. Financing constraints and the relative importance of value versus liquidity

I hypothesize that whether or not shortages of liquid assets cause the firm to default depends on the availability of external financing. If there are no financing constraints, then any cash shortage can be met

[27] Bharath and Shumway (2008) use implied default probabilities, computed as $N(-DD)$, where $N(\cdot)$ is the cumulative normal distribution function. I find that for firms very close to default this nonlinear transformation reduces predictive power in hazard regressions compared to DD. Therefore, I use DD rather than implied default probabilities in the tests.

by raising new financing, rendering liquidity irrelevant. If, in contrast, external financing is completely unavailable, then any temporary cash shortage can push the firm into default regardless of its solvency. In between these two extreme cases, firms for which external financing is neither costless nor infinitely costly can overcome some, but not all, liquidity shortages. Thus, the relative importance of value versus liquidity as default triggers should depend on how difficult it is to raise external financing in distress.

The existence and the real effects of financing constraints are a subject of extensive debate in corporate finance (e.g., Fazzari, Hubbard, and Petersen (1988); Kaplan and Zingales (1997)). Unfortunately, most standard proxies for constraints suggested in the literature, such as whether or not the firm is rated by a major credit rating agency and whether it pays dividends, are inapplicable for my sample of bond issuers on the brink of default. Instead, because banks are the most likely source of external cash for very distressed firms, I construct three firm-specific and two economy-wide proxies related primarily to how difficult it is to raise new bank debt in distress. The first proxy is the number of classes of bond covenants that restrict the firm's ability to raise cash, including restrictions on senior and subordinated debt, equity issuance, and asset sales, as well as the "negative pledge" covenant that prohibits secured borrowing unless the bonds are also secured on a pari passu basis. This covenant index varies between 0 and 5. The second proxy is the ratio of secured debt (loans and bonds) to fixed assets. The idea behind this variable is that banks are less likely to extend new credit if the firm has few unincumbered assets that can be pledged as collateral. The third proxy equals one minus the amount of undrawn credit lines, normalized by current liabilities. The presence of authorized but unused credit lines may indicate better prospects for obtaining additional cash, even though they may be curtailed in distressed.

In addition to firm-specific proxies for constraints, I use two variables that characterize the overall state of the distressed loan market. Cash shortages are more likely to result in default when junk loan markets are "cold" and few new risky loans are extended, as was the case after the failure of Lehman Brothers in 2008 (Ivashina and Scharfstein (2010)). I compute the quarterly par amount of all new junk loans in DealScan, subtract the linear trend, and normalize the resulting variable by its maximum value during the sample period. The fourth proxy for financing frictions equals one minus this measure of the junk loan market activity. Finally, the fifth proxy is the spread between Baa and Aaa corporate bonds reported by Moody's, expected to be negatively associated with the market activity and the availability of financing for distressed firms. These five proxies for financing constraints are used in regressions (4) to (8) of Table 7, respectively. [TABLE 7 HERE]

To test the hypothesis that cash shortages are positively associated with default for constrained firms, but not necessarily for unconstrained firms, I define *Cash shortage* as a dummy variable that equals one if the firm's quick ratio is below the industry median, and zero otherwise. For each of the five proxies for constraints, observations with the value of the proxy above its sample median are classified as constrained, and those below the median as unconstrained. Table 7 reports the results. As previously, the ratio of the market value of assets to the face value debt is a very powerful variable explaining the timing of default, and cash shortages are significant over and above the asset value. Moreover, regressions (4) to (8) show that liquidity is only important when financing constraints are high, whereas for unconstrained firms, cash shortages are insignificant.

The results of this section have a number of important implications. First, the superior power of the market value of assets in explaining the timing of default suggests that the asset value is a good candidate for the state variable in traditional single-factor structural models. Second, such models cannot explain all observed defaults, in particular because cash shortages play an independent role in triggering default. The effect of cash shortages cannot be captured simply by allowing the value-based default boundary to depend on liquidity. Third, the relative importance of value versus liquidity depends on the availability of external financing, which can vary across firms and over time. To improve the accuracy of structural models, it may be necessary to take into account the firm's endogenous cash holdings in the presence of financing frictions, as in Acharya et al. (2006) and Anderson and Carverhill (2007).

8. Firm-specific boundary levels

A number of studies depart from the assumption that the default boundary is an exogenously specified fixed proportion of the face value of debt, and introduce firm-specific boundaries that are functions of firm characteristics. Leland (1994), Duffie and Lando (2001), and Goldstein, Ju, and Leland (2001) all derive explicit formulae for the default boundary when debt consists of a console bond promising a continuous coupon payment. The formulae in these models are very similar, except for differences in asset growth rate specifications and the treatment of taxes. I compute the boundary derived by the Goldstein, Ju, and Leland (2001) model (V_b^{GJL}), because Leland (1994) does not treat tax payments as cash outflows, and Duffie and Lando (2001) assume that investors are risk-neutral. I also estimate the boundary predicted by the Leland and Toft (1996) model, in which finite-maturity debt is continuously rolled over, resulting in a time-independent boundary (V_b^{LT}). Finally, I compute the so-called KMV boundary (V_b^{KMV}), equal to the face value of the short-term debt plus one-half of the long-term debt. Denoting face values of short-term debt as S and of total debt as B, the KMV boundary can be expressed as $V_b^{KMV} = (S + 0.5(B - S))/B = 0.5(1 + S/B)$, which is always between 0.5 and 1.[28]

Estimated boundaries are reported in Panel A of Table 8.[29] None of the three formulae predict average observed asset values at default accurately. Both V_b^{GJL} and V_b^{LT} overestimate the boundary, with means of 73.5% and 74.3% of face debt, compared with the empirically observed level of 66%. For comparison, Leland (2004) calculates V_b^{LT} for different ratings between 72.4% and 73.7% of face debt, with the base-case value at 73.1%. The variation in the boundary predicted by the Goldstein, Ju, and Leland (2001) model matches the variability of the observed boundary better, although V_b^{GJL} appears sensitive to noise in inputs such as the applicable coupon rate. In contrast to the endogenous-boundary models, V_b^{KMV} is on average too low, with the median only slightly above its lower bound of 50%. [TABLE 8 HERE]

Panel B reports regressions of the observed asset value at default (i.e., the empirical default boundary) on the predicted boundary value. Both V_b^{GJL} and V_b^{LT} are strongly associated with the observed boundary, primarily because all three are negatively related to the volatility of assets. Thus, endogenous-boundary formulae do capture some of the variation in the observed solvency levels at default. At the same time, if their predictions were correct, the regression coefficients would be close to 1.0 and the regression constants would be insignificant, neither of which holds in the sample. The regressions produce R^2 coefficients of only 5.3% and 4.4%. Although the low explanatory power can be due partly to mismeasured inputs, the formulae appear unlikely to be able to explain most of the cross-sectional variation in observed asset values at default. Of note, the explanatory power of these regressions is less than half of what the inputs to the Leland-Toft formula yield in a linear specification (see regressions (9) and (10) of Table 4a), suggesting that the functional form of of the boundary in endogenous-default models is less valuable than their prediction that the boundary depends on the volatility of assets.

As to the KMV boundary, its values for defaulting firms are mostly clustered around its lower and upper bounds of 50% and 100%. Moreover, the correlation between V_b^{KMV} and the empirical boundary is *negative* and significant at the 10% level. This is consistent with the empirical tendency of riskier firms both to be financed with shorter-maturity debt (Helwege and Turner (1999)) and to default at more advanced stages of insolvency. Thus, V_b^{KMV} does not approximate empirical boundary levels well.

[28] The use of this specification for the default boundary in academic implementations of the KMV algorithm originates with Vassalou and Xing (2004), but the details of the boundary actually used by MKMV are proprietary.

[29] Proxies for the inputs into the endogenous-boundary formulae are computed as in Table 4a, with the following exceptions. For V_b^{LT}, the initial debt maturity is twice the average remaining maturity of all outstanding debt issues. When firm-specific estimates of the cost of default and/or the marginal tax rate are unavailable, their sample averages of 20.4% (17.0%, respectively) are used instead. The asset payout ratio used to calculate V_b^{GJL} includes reported tax payments in addition to equity and debt payouts in the numerator. To calculate V_b^{KMV}, short-term debt is defined as debt in current liabilities, lagged one quarter for defaulting firms to reduce the impact of the accounting convention that requires firms in technical default to classify as current liabilities all their debts regardless of maturity.

Does the use of firm-specific boundary levels improve the power of boundary-based predictions, compared with a simple model in which the boundary is a constant fraction of the face value of debt? A standard measure of classification quality is the area under the ROC (receiver operating characteristic) curve, which plots the test's sensitivity (the "true positive" rate, or the proportion of defaults correctly classified as such) against 1–specificity (the "false positive" rate, or the proportion of nondefaulting observations wrongly classified as defaulting).[30] Constructing the ROC curve involves calculating the number of firms classified as defaulting and nondefaulting for each assumed threshold value of the classification variable. The area under the curve equals the probability that a randomly selected firm at default has a lower value of the classification variable than a randomly selected nondefaulting observation, and varies from a minimum of 0.5 for a random predictor with no power to a maximum of 1 for a predictor that makes no classification errors.

In structural models, the market value of assets V_t and the level of the default boundary V_b affect the (risk-neutral) probability of default through their ratio, $X_t = V_t/V_b$ (see, for example, Eq. (2.6) in Leland (2004)). Therefore, I study the classification properties of the market value of assets when it is normalized by the face value of debt (as in models in which the boundary is a fixed proportion of the face value of debt) versus when it is normalized by the predicted firm-specific boundary levels. Panel C of Table 8 reports that the area under the ROC curve for the ratio of the market value of assets to the face value of debt 0.976. For comparison, for Altman's (1968) z-score, it is only 0.893, and for the equity-based naïve distance to default (Bharath and Shumway (2008)), it is 0.930. These statistics once again confirm that the value of assets is a very powerful variable relative to the available alternatives.

At the same time, the table also shows that the use of firm-specific boundary levels, if anything, reduces the classification accuracy marginally, compared with a simpler rule that specifies the boundary value as a fixed proportion of the face value of debt. The differences are minor and not statistically significant, except for V_b^{GJL}, whose power appears to be slightly lower. Curiously, the ranking of the power of the three firm-specific boundaries is the opposite of their ranking in explaining the cross-section of the empirically observed solvency level at default. For practical purposes, all four predictors have essentially the same power to discriminate between defaulting and nondefaulting states. This suggests that for quantitative predictions, the fixed-boundary specification used in models with exogenous default may be preferable due to its flexibility. Leland (2004) notes that the exogenous boundary can be set at different levels, including those suggested by endogenous-boundary models and those that allow it

[30]For a discussion of ROC curves and other metrics of classification quality for default prediction models, see Stein (2002).

to match model-predicted probabilities of default with observed default frequencies. Overall, the main advantage of existing models with endogenous default is not the superior accuracy of their quantitative predictions, but rather their ability to capture some of the aspects of the data, such as the dependence of the boundary on the volatility of assets, that are important in qualitative analysis of credit risk and financing and hedging decisions (Acharya and Carpenter (2002)).

9. Summary and conclusions

This paper studies the properties of the value-based default boundary and evaluates the relative importance of insolvency (low asset value relative to debt) and illiquidity (low liquid assets relative to current liabilities) in triggering default. Consistent with the core assumption of value-based models, the market value of assets is the most powerful variable explaining the timing of default, outperforming most available alternatives put together. The average level of the solvency boundary is 66% of the face value of debt, and the calibrated boundary that balances type-I and type-II errors is 65% of debt. Empirically observed asset values at default for a typical firm are reliably below the popular "zero net worth" boundary. Endogenous-default models such as Leland and Toft (1996) correctly identify some of the factors that affect the boundary, but at present they explain only a fraction of its observed variation. Cross-sectionally, boundary-based predictions misclassify a significant proportion of defaulting firms, both because empirically observed boundary levels are widely dispersed, and also because there are many deeply insolvent firms that are able to avoid default or delay it for years.

Difficulties in specifying the default boundary likely contribute to the limited cross-sectional accuracy of debt pricing models (Eom, Helwege, and Huang (2004)) and their limited success in explaining short-term default probabilities (Leland (2004)). One factor missing from traditional value-based models is balance sheet liquidity. I find that, controlling for the value of assets, cash shortages are the second most important factor explaining the timing of default, and their role cannot be captured simply by allowing the value-based boundary to depend on liquidity. Moreover, the importance of liquidity varies depending on the availability of external financing. To improve the accuracy of structural models, it is necessary to account not only for the firm's value of assets and its optimal cash management policy, but also for factors that affect the costs of accessing external cash. Because these costs depend on both firm-level and economy-wide characteristics, capturing all relevant factors while maintaining analytical tractability is challenging. Even for relatively simple settings, models incorporating endogenous cash holdings in the dynamic framework are complex and typically necessitate numerical solutions (Acharya et al. (2006);

Anderson and Carverhill (2007)). Furthermore, even taken together, value and liquidity fail to explain a substantial part of the variation in the observed timing of default.

In light of the findings presented in this paper, it may be beneficial to admit that default cannot be predicted perfectly based on the information available to investors. The CreditGrades model, which is popular among practitioners, assumes that the default boundary is a random variable drawn from a given distribution (Finger et al. (2002)). This makes the distance to default uncertain and increases spreads on short-maturity bonds, which most structural models struggle to explain. A more refined approach is developed in Duffie and Lando (2001), Jarrow and Protter (2004), and Çetin et al. (2004). These papers construct structural models in which investors lack some important information that would allow them to replicate firms' decision to default. In many such models, corporate securities can be priced as if, conditional on the information available to investors, default were an unpredictable Poisson event with a hazard rate determined by the firm's economic fundamentals (Giesecke (2006)). This result makes it possible to apply convenient asset pricing techniques developed in the reduced-form credit risk literature (e.g., Duffie and Singleton (1999)) while preserving the structural view of default as driven by deteriorating economic fundamentals. Currently, structural models with asymmetric information assume that default is triggered by the asset value crossing a deterministic (albeit possibly unknown) boundary. Extending such models and exploring the nature of information imperfections that make the timing of default unpredictable for investors may be a promising way to advance our understanding of credit risk.

Appendix A: Computing the market value of the firm

I estimate the market value of the firm as the sum of market values of its bonds, bank debt, and equity. The firm's bond structure is inferred from the history of outstanding bond amounts in the FISD database for each bond issued by the firm and its wholly owned subsidiaries. The market value of bonds included in the Merrill Lynch High Yield II Index (MLI) is calculated by multiplying the currently outstanding amount by the bond price. Bonds with remaining maturity of less than one year or face value under \$100 million are not included in the MLI. The market value of these bonds is calculated assuming that their yield equals the weighted-average yield of all quoted bonds of the same issuer on each date. If price quotes are unavailable in MLI for any of the firm's bonds, the firm-month observation is excluded from the sample.

Estimates of bank loan prices are based on quotes provided by the LSTA/LPC Mark-to-Market Pricing service, available from May 1998. On average, for each loan-month, the data base provides a mean price quote from 3 dealers. When there are several loans outstanding for a firm, I use their mean price, resulting in 7.5 dealer quotes per bank debt price on average (median is 4). A total of 305 sample firms have loan prices included in the LSTA/LPC database, including 106 of the 213 defaulted firms. For firm-months not included in the database, the market price of bank debt is estimated as a quadratic function of the weighted-average bond price, as follows:

$$
\begin{array}{rcl}
P_{bank} & = & \underset{(14.2)}{40.18} + \underset{(12.9)}{1.045} \times P_{bond} - \underset{(-8.45)}{0.00461} \times P_{bond}^2,
\end{array}
$$

where P_{bank} and P_{bond} are average loan and bond prices in cents on the dollar, respectively, and t-statistics adjusted for firm clustering are reported in parentheses. The quadratic term controls for nonlinearities due to different seniority of loans and bonds. The regression produces an R^2 of 75.5% and is not substantially improved by the inclusion of additional firm-specific or macroeconomic controls.

Preferred equity is rarely important in the sample; its par value is below 1% of the face value of debt for 83.6% of firms. Preferred stock is worth little in default, and thus its par value is likely to vastly overstate its market value in distress. Varma (2003) finds mean recovery rates for preferred stock of 15.3%, compared with 36.1% for senior unsecured bonds (the most common bond type by far). Hence, to approximate the market value of preferred stock, I assume that its price relative to par is equal to the constant fraction 15.3/36.1=0.424 of the firm's current bond price. Sensitivity analysis shows that this approximation has a negligible effect on my estimates.

For the median firm in the sample, bonds and bank loans together constitute 97.8% of total debt. Firms may make use of other debt types, such as capital lease obligations, commercial paper, mortgages, and project finance debt. Because commercial paper (rare in the sample) has short maturity and is backed by credit lines, and most other debt types are secured, I assume that all such debt obligations are similar to bank debt and have the same price-to-par ratio. These types of debt are not frequently used by high-yield firms, so this approximation affects only a small fraction of sample firms.

Potentially more important is the accurate measurement of the split of the firm's total debt between bonds and bank debt, which is especially relevant when their market prices are significantly different. I use 10-Q and 10-K filings to manually collect data on the composition of debt for 240 sample firms whose difference between the average bond and loan price exceeds 10 cents on the dollar. This subsample includes most defaulted and very distressed firms, which are of primary interest for this study. For other firms, I rely on FISD data to infer the total amount of bonds outstanding for the firm and its wholly owned subsidiaries, and estimate the face value of bank debt as the difference between total debt from Compustat and the FISD-implied par bond amount.[31] Once par values and market prices of all debt instruments are found the market value of debt can be computed. The value of the firm is the sum of the value of debt, the approximated value of preferred stock, and the market value of equity, found from CRSP. On a few occasions, firms are delisted from the stock exchange a few months prior to default (typically because the share price falls below the exchange's threshold). Where possible, I obtain market equity prices for defaulted firms from quarterly Compustat data and interpolate them using a simple linear approximation to yield monthly equity price estimates. The potential errors from this approximation are immaterial, because total equity values for these firms are a small fraction of firm values.

[31] The median difference between the ratio of bonds to total debt implied by FISD and that found from 10-K filings is only 7%. However, the two can be very different for large firms with complex debt structures and many bond-issuing subsidiaries. By using the FISD-based split only for firms with similar prices of bonds and loans, I limit the potential error due to possible mismeasurement of the bond to loan ratio.

Appendix B: Computing the value of assets at default

When default is costly, the value of the firm prior to default is affected by expected default costs, and hence differs from the value of the firm's productive assets. This section outlines the procedure used to estimate the unobserved value of assets from the observed value of the firm. It is adapted from Davydenko, Strebulaev, and Zhao (2010), who provide a detailed discussion of the procedure, its assumptions, and estimated costs of default.

A. Assumptions and pricing equations

As is common in reduced-form credit risk models, default is modeled as a doubly-stochastic (Cox) process. I make the following specific assumptions:

1. The market value of the firm's productive assets V_t (i.e., the unlevered value of the firm) follows a geometric Brownian motion. Under the risk-neutral measure,
$$dV_t = rV_t dt + \sigma V_t dW_t^Q,$$
2. Conditional on the history of V_t, default is the first jump of a heterogenous Poisson process with the following intensity under the real probability measure:
$$\lambda_t^P = e^{\beta_0 + \beta_1 \log \frac{V_t}{B}}, \tag{B1}$$
where β_0 and β_1 are parameters common to all firms, which are to be estimated as part of the procedure.
3. The risk-neutral default intensity is a multiple of the real-measure intensity:
$$\lambda_t^Q = \mu \lambda_t^P, \tag{B2}$$
where $\mu \geq 1$ is the risk premium, assumed constant for all firms and time periods.
4. The "recovery" value of the firm L_t (i.e., the value of its assets upon a hypothetical default at time t) is a constant (although possibly firm-specific) fraction of its continuation value:
$$L_t = (1 - \alpha)V_t. \tag{B3}$$
5. The face value of debt B and the risk-free rate of interest r are constant.

Davydenko, Strebulaev, and Zhao (2010) show that under the above assumptions, the value of the firm at any time prior to default (or maturity, whichever comes first) satisfies

$$M_t = L_t + (1 - L_t/V_t)e^{-r(T-t)} E_t^Q \left[V_T e^{-\int_t^T \lambda_u^Q \, \mathrm{d}u} \right]. \tag{B4}$$

When M_t and L_t are both known, this equation can be solved for the unobserved V_t. If the proportional cost of default α is known instead of the recovery value L_t, this equation can be re-written as:

$$M_t = (1 - \alpha)V_t + \alpha e^{-r(T-t)} E_t^Q \left[V_T e^{-\int_t^T \lambda_u^Q \, \mathrm{d}u} \right]. \tag{B5}$$

B. The estimation procedure

To find the value of assets from Eq. (B4), parameter values β_0 and β_1 of the function λ_t^Q specified in Eq. (B1) are required. But to estimate these parameters, one needs to know the asset values of the sample firms. To avoid the joint estimation problem, Davydenko, Strebulaev, and Zhao (2010) suggest the following sequential procedure:

Step 1. As an initial approximation for V_t, choose $V_t^{(1)} = M_t$.

Step 2. Applying standard tools of parametric survival analysis to all firm-month observations for defaulting and nondefaulting firms, estimate the parameters of the hazard function given by Eq. (B1), with V_t replaced by $V_t^{(1)}$. This yields initial coefficient estimates $\beta_0^{(1)}$ and $\beta_1^{(1)}$.

Step 3. To obtain the risk-neutral default hazard λ_t^Q, multiply the real-probability hazard by a constant risk-premium coefficient μ: $\lambda_t^{Q\ (1)} = \mu\lambda_t^{P\ (1)}$. Driessen (2005) and Berndt et al. (2008) estimate average jump-to-default risk premia around 2. Berndt et al. find that the premium is lower for lower-rated firms, but do not provide estimates for very risky firms, such as those that constitute our defaulted sample. The results of Huang and Huang (2003) also suggest that default risk premia are considerably lower for riskier firms, at around 1.11 for B firms and 1.17 for BB firms (see Table VI in Berndt et al. (2008)). Hull, Predescu, and White (2005) estimate risk premia embedded in CDS spreads at 1.2 for B firms and 1.3 for CCC and lower-rated firms. Based on these studies, I choose a risk premium of $\mu = 1.3$. Estimates of the default boundary are fairly insensitive to this parameter.[32]

Step 4. For defaulted firms, approximate the value of the firm at default M_τ (where $t = \tau$ is the default time) by its value at the end of the last calendar month prior to default. Also, approximate the recovery value L_τ by the value of the firm at the end of the calendar month of default, adjusted for the market return in the month of default. Then, solve Eq. (B4) for V_τ using simulations. This yields a new approximation for the value of assets at default, $V_\tau^{(2)}$, and the proportional cost of default $\alpha^{(2)} = 1 - L_\tau/V_\tau^{(2)}$. For firms not observed to default, assume that the cost of default equals the sample average of $\alpha^{(2)}$.

Step 5. Given the current approximation for α, solve Eq. (B5) for V_t using all nondefaulting firm-months. The solution for nondefaulting firm-months, combined with the solution for firms at default obtained in the previous step, forms a new approximation for asset values in all firm-months. This approximation is denoted $V_t^{(2)}$.

Step 6. Go back to step 2 and re-estimate hazard function coefficients using the current approximation $V_t^{(2)}$. Repeat steps 2 through 5 until changes in the hazard function parameters $\beta_0^{(1)}$ and $\beta_1^{(1)}$ and in estimated asset values at default $V_\tau^{(j)}$ become negligible.

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[32]To illustrate, in the base case of $\mu = 1.3$ the estimated median default boundary equals 61.6%. Varying μ from 1.0 to 1.8 changes the median boundary from 60.4% to 63.4%.

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Table 1
Sample composition

Panel A reports the number of defaults in the sample by year and by the type of the first default event (bankruptcy filing, payment omission or delay, or distressed bond exchange). Panel B reports the composition of the defaulted sample by broad industry category.

Panel A: Number of defaults by year

	Bankruptcy filings	Payment defaults	Distressed exchanges	Total
1997	3	1	0	4
1998	3	7	2	12
1999	7	14	5	26
2000	9	18	1	28
2001	24	32	6	62
2002	9	21	1	31
2003	11	11	2	24
2004	5	7	1	13
2005	8	4	1	13
All	79	115	19	213

Panel B: Industry composition

	Defaulted firms		Control sample	
Consumer goods	28	13.1%	82	10.2%
Business equipment	12	5.6%	49	6.1%
Steel	13	6.1%	26	3.2%
Other manufacturing	30	14.1%	127	15.8%
Telecommunications	43	20.2%	102	12.7%
Wholesale and retail trade	35	16.4%	123	15.3%
Transportation	12	5.6%	27	3.3%
Energy & Utilities	13	6.1%	98	12.2%
Other industries	27	12.7%	172	21.3%
All	213	100%	806	100%

Table 2
Descriptive statistics

This table reports descriptive statistics for firms at default, and for firms that do not default for at least one year after the observation date. *Volatility of assets* is the annualized median standard deviation of monthly firm returns in the industry, using Fama-French's 50 industries. *Market to book ratio* is the ratio of the market value of equity plus the book value of total liabilities, divided by the book value of total assets. *Corporate tax rate* is the marginal tax rate estimated using the simulation method of Graham (1996). *Original High Yield issuers* refers to firms whose outstanding bonds were never rated investment grade. *Firm value* is the total market value of all of the firm's financial claims. *Debt maturity* is the weighted average of maturities of all debt instruments, assuming that all bank debt has a maturity of one year. *Short-term/Total debt* is the ratio of the debt in current liabilities to total debt, lagged one quarter for firms at default. *Debt interest rate* is the firm's interest expense divided by the average outstanding debt in the same quarter. Statistics for nondefaulting firms are calculated using firm means for each variable.

	Firms at default				Nondefaulting firms			
	Mean	Median	Std.dev.	N	Mean	Median	Std.dev.	N
Panel A: General descriptive statistics								
Book assets ($ Mil.)	2,617	810	6,239	203	2,725	1,196	4,474	801
Sales/Book assets	0.256	0.198	0.243	201	0.264	0.221	0.221	801
Volatility of assets	0.295	0.282	0.076	213	0.278	0.266	0.067	801
Market to book ratio	1.240	1.127	0.439	187	1.600	1.390	0.814	791
Corp.tax rate (%)	17.0%	13.1%	14.7%	62	29.0%	32.4%	8.7%	556
Nominal share price ($)	1.30	0.83	1.59	191	22.11	19.30	13.69	795
Debt price (¢ per $1)	49.61	48.99	19.78	173	96.88	99.85	10.39	754
% negative book equity	58.6%			203	12.5%			800
% original HY issuers	85.3%			211	69.2%			800
Panel B: Leverage								
Market debt/Firm value	0.876	0.917	0.119	158	0.483	0.475	0.207	729
Book debt/(Book debt + Market equity)	0.930	0.959	0.081	158	0.489	0.482	0.211	729
Book debt/Book assets	0.814	0.764	0.398	203	0.483	0.452	0.221	796
Panel C: Debt characteristics								
Bonds/Total debt	0.628	0.626	0.234	173	0.646	0.656	0.216	754
Debt maturity	4.43	4.02	2.25	173	5.32	4.91	2.47	754
Short-term/Total debt	0.205	0.065	0.283	203	0.097	0.053	0.124	793
Debt interest rate	10.5%	10.3%	2.9%	195	9.3%	8.7%	3.6%	789
No. of bond issues	3.78	2	5.91	176	3.58	1.89	6.09	778
No. of loan covenants	3.40	3	1.63	141	3.16	3	1.17	626

Table 3
The market value of assets

Firm value is the total market value of all of the firm's financial claims. *Market assets* is the firm value adjusted for expected default costs, as described in Appendix B. *Face debt* is the book value of all outstanding debt. *Expected default cost* is the difference between the value of assets and the value of the firm. Statistics for nondefaulting firms are calculated using firm means for each variable.

	Firms at default						Nondefaulting firms					
	Mean	Median	Std.dev.	5%	95%	N	Mean	Median	Std.dev.	5%	95%	N
Panel A: The market value of assets												
Market assets/Face debt	0.660	0.616	0.252	0.303	1.221	148	3.112	2.250	2.906	1.086	8.168	707
Panel B: The value of the firm												
Firm value/Face debt	0.587	0.555	0.263	0.239	1.161	158	4.866	2.215	44.079	1.045	7.991	729
Firm value/Market assets	0.887	0.920	0.168	0.571	1.122	144	0.990	0.996	0.021	0.959	1.000	707
Panel C: Equity values and distress costs												
Equity value/Market assets	0.085	0.043	0.126	0.005	0.324	158	3.895	1.248	44.075	0.157	6.960	729
Expected default cost/Market assets	0.113	0.080	0.168	-0.122	0.429	144	0.010	0.004	0.021	0.000	0.041	707

Table 4a

Regressions of the default boundary

The dependent variable is the market value of assets at the end of the month preceding default, divided by the face value of debt. *Asset volatility* is the annualized median standard deviation of monthly firm returns in the industry, using Fama-French's 50 industries. *Cost of default* is the proportion of the firm's asset value lost in default. *Avg. maturity* is the weighted average of maturities of all debt instruments, assuming that all bank debt has a maturity of one year. *Shortest maturity* is the remaining maturity of the bond that is scheduled to be repaid first. *Payout ratio* is total debt and equity payouts in each quarter divided by the average book value of assets in the that quarter. *Coupon rate* is the firm's interest expense divided by the average outstanding debt in the same quarter. *Risk-free rate* is the five year constant maturity Treasury rate. *Tax rate* is the firm's marginal tax rate estimated using the simulation method of Graham (1996). The regressions are estimated using the Heckman two-stage procedure, with the quick ratio added at the first stage to each specification's regressors. *1/Mills* is the inverse of the Mills' ratio from first-stage Heckman regressions. The values of z-statistics are reported in parentheses. Coefficients marked ***, **, and * are significant at the 1%, 5%, and 10% significance levels, respectively.

		Heckman regressions										Heckman + IV			
	pred.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Asset volatility	–	-0.87***								-0.91***	-0.83***				
		(-3.20)								(-2.97)	(-2.80)				
Cost of default	+		0.16**							0.14**	0.13*				
			(2.36)							(2.03)	(1.90)				
Avg. maturity	–			0.0087						0.010		-0.58			
				(0.81)						(0.90)		(-0.18)			
Shortest matur.	–				-0.020**						-0.015*		-0.060		
					(-2.38)						(-1.70)		(-0.84)		
Payout ratio	–					-0.69*				-0.38	-0.48			-1.40	
						(-1.82)				(-0.77)	(-1.00)			(-0.53)	
Coupon rate	+						-1.18*			0.18	0.38				-6.52
							(-1.66)			(0.20)	(0.41)				(-0.89)
Risk-free rate	–							-1.63		-2.34	-2.15				
								(-0.85)		(-1.20)	(-1.11)				
Tax rate	–								0.68*						
									(1.95)						
1/Mills		-0.050	-0.074	-0.11	-0.071	-0.090	-0.090	-0.10	-0.011	-0.045	-0.019	1.97	-0.022	-0.078	-0.044
		(-0.79)	(-1.09)	(-1.45)	(-1.00)	(-1.23)	(-1.20)	(-1.41)	(-0.11)	(-0.68)	(-0.31)	(0.17)	(-0.20)	(-0.89)	(-0.40)
const.		1.06***	0.84***	0.93***	0.95***	0.98***	1.05***	1.04***	0.63**	1.09***	1.09***	-2.32	1.00***	1.01***	1.47**
		(4.94)	(4.35)	(4.69)	(4.71)	(4.48)	(4.51)	(4.66)	(2.31)	(4.88)	(4.95)	(-0.13)	(3.95)	(4.19)	(2.34)
N		148	144	147	147	139	145	148	47	134	134	147	138	139	145
R^2		0.067	0.047	0.015	0.050	0.037	0.031	0.020	0.122	0.122	0.137				

Table 4b

Regressions of the default boundary, ctd.

The dependent variable is the market value of assets at the end of the month preceding default, divided by the face value of debt. *Networth covenants* and *Coverage covenants* are count variables, equal to the number of balance sheet-based (cash flow-based, respectively) loan covenant types listed for the firm in DealScan. *Covenant violated* and *Covenant waived* are dummy variables, equal to one for firms that violated a covenant (respectively, for which a covenant was waived) some time before default, according to news reports in Factiva. *GDP growth* is the seasonally-adjusted quarterly GDP growth rate. *Default rate* is the percentage of all U.S. bond issuers that default in each calendar year. *Quick ratio* is the sum of cash and accounts receivable, divided by current liabilities. *Current ratio* is the ratio of current assets to current liabilities. Regressions (8) and (9) are estimated using OLS. Other regressions are estimated using the Heckman two-stage procedure, with the quick ratio added at the first stage to each specification's regressors. *1/Mills* is the inverse of the Mills' ratio from first-stage Heckman regressions. The values of z-statistics are reported in parentheses. Coefficients marked ***, **, and * are significant at the 1%, 5%, and 10% significance levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Networth covenants	0.053*								
	(1.76)								
Coverage covenants		0.0041							
		(0.23)							
Covenant violated			-0.067						
			(-0.93)						
Covenant waived				-0.11					
				(-1.55)					
Loans/Total debt					0.064				
					(0.56)				
GDP growth						0.021**			
						(2.36)			
Default rate							-0.045**		
							(-1.99)		
Quick ratio								-0.025	
								(-0.69)	
Current ratio									0.013
									(0.54)
Asset volatility	-0.90***	-0.90***	-0.96***	-0.98***	-0.86***	-0.86***	-0.83***	-0.84***	-0.88***
	(-2.96)	(-2.94)	(-3.04)	(-3.16)	(-2.68)	(-2.84)	(-2.72)	(-2.73)	(-2.91)
Cost of default	0.13*	0.14**	0.13*	0.12*	0.15**	0.13*	0.14**	0.14*	0.15**
	(1.89)	(2.04)	(1.79)	(1.70)	(2.09)	(1.97)	(2.06)	(1.96)	(2.09)
Avg. maturity	-0.27	-0.39	-0.38	-0.44	-0.35	-0.40	-0.40	-0.38	-0.36
	(-0.56)	(-0.80)	(-0.78)	(-0.90)	(-0.72)	(-0.84)	(-0.82)	(-0.78)	(-0.75)
Payout ratio	0.0099	0.0100	0.011	0.011	0.013	0.010	0.0075	0.0085	0.0059
	(0.86)	(0.85)	(0.92)	(0.91)	(1.01)	(0.89)	(0.66)	(0.81)	(0.56)
Coupon rate	0.15	0.21	0.21	0.39	0.17	0.12	0.075	0.19	0.15
	(0.17)	(0.23)	(0.23)	(0.42)	(0.18)	(0.13)	(0.082)	(0.21)	(0.16)
Risk-free rate	-2.15	-2.35	-2.34	-2.98	-2.50	-3.30*	-3.22	-2.49	-2.43
	(-1.11)	(-1.20)	(-1.19)	(-1.48)	(-1.27)	(-1.69)	(-1.64)	(-1.28)	(-1.25)
1/Mills	-0.041	-0.044	-0.060	-0.067	-0.039	-0.036	-0.027		
	(-0.62)	(-0.66)	(-0.77)	(-0.88)	(-0.60)	(-0.55)	(-0.41)		
const.	1.04***	1.08***	1.17***	1.23***	1.03***	1.06***	1.20***	0.98***	0.97***
	(4.66)	(4.74)	(4.42)	(4.73)	(4.38)	(4.80)	(4.81)	(6.52)	(6.44)
N	134	134	134	134	134	134	134	134	134
R^2	0.143	0.122	0.125	0.137	0.123	0.156	0.146	0.122	0.121

Table 5
Cash flows and liquid assets

Profit margin is the ratio of the pretax income to sales. *% Making losses* is the proportion of firms with negative pretax income. *EBIT* is the sum of pretax income and interest expenses. *% Negative cash flow* is the proportion of firms for which the operating cash flow, defined as income before extraordinary items plus depreciation, is negative. *Interest coverage ratio* is EBIT plus amortization and depreciation, divided by the interest expense. *Quick ratio* is the sum of cash and accounts receivable divided by current liabilities. *% Quick ratio < industry median* is the proportion of firms for which the quick ratio is below its median value for all Compustat firms in the same industry, using Fama-French's 50 industries. *Current ratio* is the ratio of current assets to current liabilities. *Cash ratio* is the ratio of cash and equivalents to current liabilities. *Defensive interval* is the sum of cash and accounts receivable divided by the sum of the cost of goods sold and selling and administrative expenses. Statistics for nondefaulting firms are calculated using firm means for each variable.

	Firms at default				Nondefaulting firms			
	Mean	Median	Std.dev.	N	Mean	Median	Std.dev.	N
Panel A: Profitability								
Profit margin	-2.389	-0.214	12.433	203	-0.203	0.017	1.214	799
% making losses	93.1%			204	33.8%			801
EBIT/Total assets	-0.103	-0.022	0.270	193	0.013	0.016	0.032	790
% negative cash flow	81.1%			196	17.1%			797
Interest coverage ratio	-2.893	-0.168	7.468	193	6.874	3.271	31.317	790
Panel B: Balance sheet liquidity								
Quick ratio (QR)	0.553	0.354	0.605	195	1.266	0.922	1.338	772
% QR < 1	85.1%			195	58.8%			772
% QR < industry median	80.0%			195	53.6%			772
Current ratio	0.953	0.739	0.814	196	1.916	1.597	1.350	770
Cash ratio	0.243	0.073	0.513	196	0.621	0.224	1.326	773
Defensive interval	1.090	0.769	1.030	179	1.615	0.983	2.188	719

Table 6

Value, liquidity, and traditional default predictors

This table reports proportional Cox hazard regressions of default. *MVA/D* is market value of assets divided by the face value of debt. *WC* is working capital, *TA* is the book value of total assets, *CA* is current assets, *CL* is current liabilities, *RE* is retained earnings, *ME* is the market value of equity, *TL* is the book value of total liabilities, *S* is sales, *NI* is net income, *DD* is the distance to default, and *EDF* is the Expected Default Frequency. Values of z-statistics are reported in parentheses. Coefficients marked ***, **, and * are significant at the 1%, 5%, and 10% significance levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
MVA/D	-7.75***		-7.02***		-7.20***				-6.66***		-6.44***		-6.57***
	(-14.1)		(-9.30)		(-12.5)				(-9.62)		(-7.97)		(-8.25)
WC/TA		-2.99***	-2.97***							-3.65***	-3.44***		
		(-4.49)	(-4.51)							(-5.19)	(-4.82)		
CA/CL				-0.66***	-0.50***							-0.48***	-0.54***
				(-3.83)	(-3.35)							(-3.11)	(-3.49)
RE/TA		-0.34	0.020							-0.42	-0.19	-0.46	-0.23
		(-1.38)	(0.075)							(-1.19)	(-0.55)	(-1.33)	(-0.66)
EBIT/TA		-8.07***	-2.96							-4.48	2.70	-4.72	2.15
		(-4.19)	(-1.46)							(-1.14)	(0.65)	(-1.24)	(0.53)
ME/TL		-16.3***	0.18							-8.89***	0.90	-8.78***	1.51
		(-9.20)	(0.087)							(-3.59)	(0.37)	(-3.56)	(0.61)
S/TA		-1.02	0.050							-0.40	0.47	-0.56	0.39
		(-1.58)	(0.075)							(-0.57)	(0.67)	(-0.83)	(0.58)
NI/TA				-15.7***	-5.12***					-3.11	-5.97	-3.82	-6.26*
				(-12.2)	(-2.94)					(-0.87)	(-1.56)	(-1.11)	(-1.67)
TL/TA				2.30***	-0.51					-0.98	-1.02	-0.67	-0.84
				(6.85)	(-1.07)					(-1.49)	(-1.58)	(-1.04)	(-1.32)
DD						-0.34***		-0.10***	-0.011	0.025	0.028	0.0058	0.0059
						(-19.2)		(-4.57)	(-0.42)	(0.77)	(0.84)	(0.18)	(0.18)
EDF							0.36***	0.31***	0.090**	0.20***	0.097**	0.19***	0.098**
							(11.3)	(9.19)	(2.20)	(5.09)	(2.22)	(4.97)	(2.26)
χ^2	721	641	752	316	746	403	570	591	728	681	763	664	753
Pseudo-R^2	0.56	0.50	0.58	0.25	0.58	0.31	0.44	0.46	0.57	0.53	0.59	0.52	0.59
N	25,018	25,018	25,018	25,018	25,018	25,018	25,018	25,018	25,018	25,018	25,018	25,018	25,018

Table 7
The role of financing constraints

This table reports proportional Cox hazard regressions of default. *Cash shortage* is a dummy variable that equals one if the quick ratio is below the industry median, and zero otherwise. Regressions (4) to (8) use the following proxies for financing constraints: In regression (4), it is the number of bond covenant classes that restrict the firm's ability to raise cash; in (5), the ratio of outstanding secured debt to fixed assets; in (6), one minus undrawn credit lines divided by current liabilities; in (7), one minus the normalized de-trended quarterly par amount of all new high-yield loans in DealScan; in (8), the difference between average Baa and Aaa bond yields. In each of these regressions, *Constrained* and *Unconstrained*) are dummy variables that equal one if the proxy for constraint used in that regression is above (below) the sample median. Values of z-statistics are reported in parentheses. Coefficients marked ***, **, and * are significant at the 1%, 5%, and 10% significance level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Market assets/Face debt	-7.88***		-7.90***	-7.75***	-7.84***	-7.86***	-7.75***	-7.76***
	(-15.8)		(-15.7)	(-15.2)	(-15.6)	(-12.3)	(-15.3)	(-15.4)
Cash shortage		0.90***	0.79***					
		(3.98)	(3.29)					
Cash short. × Constrained				0.65***	0.72***	0.81***	0.83***	0.79***
				(2.83)	(3.12)	(3.16)	(3.68)	(3.41)
Cash short. × Unconstrained				0.47	0.30	0.37	-0.26	0.19
				(1.41)	(1.03)	(1.00)	(-0.74)	(0.67)
Proxy for constraints				0.025	-0.030	0.032	-1.57**	-0.85
				(0.18)	(-0.42)	(0.23)	(-2.30)	(-1.64)
χ^2	903	19.0	915	912	909	539	922	916
Pseudo-R^2	0.55	0.012	0.56	0.56	0.56	0.50	0.56	0.56
N	29,808	29,808	29,808	29,808	29,692	9,821	29,808	29,808

Table 8

Firm-specific boundaries

Panel A reports statistics on observed market value of assets and predicted levels of the default boundary, for defaulting firms at the end of the calendar month preceding default, all expressed as a proportion of the face value of debt. V_b^{GJL} is calculated from Eq. (32) in Goldstein, Ju, and Leland (2001). V_b^{LT} is given by Eq. (11) in Leland and Toft (1996). V_b^{KMV} is the KMV boundary, given by Eq. (**??**) of the main text. Panel B reports regressions of the empirically observed default boundary on its levels predicted by each model. The dependent variable is the ratio of the market value of assets at the end of the month preceding default to the face value of debt. The regressions are estimated using the Heckman two-stage procedure, with the quick ratio added at the first stage to each specification's regressor. The values of z-statistics are reported in parentheses. Panel C reports the area under the ROC (receiver operating characteristic) curve (sensitivity vs. 1-specificity) for tests that classify all firm-month observations as defaulting and not defaulting at least until after the next scheduled bond payment.

Panel A: The location of the boundary

	Mean	Median	Std.Dev.	5%	95%	N
Observed asset value/Face debt	0.660	0.616	0.252	0.303	1.221	148
V_b^{GJL}/*Face debt*	0.735	0.713	0.311	0.360	1.186	182
V_b^{LT}/*Face debt*	0.743	0.744	0.154	0.489	1.002	156
V_b^{KMV}/*Face debt*	0.602	0.533	0.142	0.500	0.986	203

Panel B: Regressions of *Observed asset value/Face debt*

	Coefficient	1/Mills	const.	R^2	N
V_b^{GJL}/*Face debt*	0.19**	-0.091	0.79***	0.053	133
	(2.31)	(-1.22)	(3.37)		
V_b^{LT}/*Face debt*	0.29**	-0.065	0.63**	0.044	139
	(2.03)	(-0.87)	(2.42)		
V_b^{KMV}/*Face debt*	-0.31*	-0.15*	1.28***	0.028	147
	(-1.78)	(-1.76)	(4.07)		

Panel C: Area under the ROC curve

	Area under ROC	Std.Err.	[95% Conf.	Interval]	N
Asset value/Face debt	0.976	0.004	0.968	0.984	29,767
Asset value/V_b^{GJL}	0.938	0.009	0.921	0.954	23,961
Asset value/V_b^{LT}	0.972	0.005	0.962	0.982	27,053
Asset value/V_b^{KMV}	0.975	0.004	0.968	0.983	29,758
z-score	0.893	0.015	0.863	0.922	26,582
Distance to default	0.930	0.009	0.912	0.948	28,938