





Recent Developments and Challenges of Single Agent Dynamic Models

Andrew Ching
Rotman School of Management
University of Toronto

First version: June 3, 2009
This version: September 1, 2009
(typos corrected))



Roadmap

Roadmap Introduction Learning Stockpiling Technology adoption/Switching Costs Measuring the Discount Factor

- Dynamic Structural Models in Marketing: Learning, Stockpiling, Technology Adoption.
- Measuring Time Preference and Time Discounting.
- Conclude.



Introduction



Roadmap Introduction Learning Stockpiling Technology adoption/Switching Costs Measuring the Discount Factor

- The basic framework of brand choice models started by Guadagni and Little (1983) assumes (i) consumers have complete information, (ii) consumers are myopic.
- But many consumers don't know the quality of all brands in a store. Ignoring this could lead to biased inference.
- Keane (1997) points out the importance of modeling consumer heterogeneity and forward-looking behavior.

Bayesian Learning

- To address problem (i), academic research in marketing and economics has extended traditional static choice models to explicitly allow for consumer learning.
- Eckstein et al. (1988), Erdem and Keane (1996), Akerberg (2003), Mehta et al. (2004), Mehta et al. (2008), Crawford and Shum (2005), Anand and Shachar (2005), Erdem, Keane and Sun (2008) explicitly model consumers as bayesian learners, and estimate their models using micro-level data.
- Ching (2000), Narayanan, Manchanda, Chintagunta (2005), Ching and Ishihara (2009a, 2009b), Ching (forthcoming) etc., also model consumers as bayesian learners and allow for learning from others.

Consumption experience signal: $q_{ijt}^e = q_j + \delta_{ijt}$, where $\delta_{ijt} \sim N(0, \sigma_\delta^2)$.

Expected quality: $E[q_j|I(t+1)] = E[q_j|I(t)] + \nu_j^e(t)(\bar{q}_{jt}^e - E[q_j|I(t)])$.

Challenging Issues

- If we assume consumers are forward-looking, we face the curse of dimensionality in the state space.
- the complexity of the likelihood/GMM objective function makes it difficult to search for the global maximum/minimum.
- Some used approximation methods. Some opted to assume consumers are myopic (e.g., Mehta et al. 2004, Mehta et al. 2008, Coscelli and Shum 2004)
- When estimating learning models using product level data, standard BLP estimation method may not apply because there are two unobserved product characteristics: $(E[q_j|I(t)], \xi_{jt})$
$$E[U_{ijt}|I(t)] = E[q_j|I(t)] + \alpha p_j + \xi_{jt} + e_{ijt}.$$
- Ching (2000), Ching (2008), Ching and Ishihara (2009a, 2009b) used a pseudo-policy approach to address this problem.

Stockpiling

- Psendorfer (2002), Hendel and Nevo (2006a) find evidence that consumers stockpile and make purchases strategically.
- Challenge: The size of the state space is huge because there are many possible combinations of brands and quantities.
- Erdem, Imai and Keane (2003) allows for discrete type unobserved heterogeneity, and assume consumers consume their inventories proportionally across brands in each period.
- Hendel and Nevo (2006b) reduce the size of the state space by assuming that brand effects only takes place when you buy the product, and the inventory effect is the same across brands.

Technology Adoption and Switching Costs

Roadmap Introduction Learning Stockpiling Technology adoption/Switching Costs Measuring the Discount Factor

- Adoption of a new technology or product often times generate a flow of benefits over time.
- There could also be uncertainty about how the technology might change over time.
- One challenge is that per period benefits are hardly observed. As a result, one cannot measure the adoption costs/switching costs in monetary terms.
- Recent works: Song and Chintagunta (2003), Ryan and Tucker (2007), Yang and Ching (2009).

An example: Yang and Ching (2009)

Roadmap Introduction Learning Stockpiling Technology adoption/Switching Costs Measuring the Discount Factor

- Consider adoption decisions of ATM cards for consumers in Italy (mainly a cash economy).
- Once you learn how to use ATM machines, you can enjoy its benefits every period.
- There is uncertainty about how many ATM machines will become available in the future.
- The benefits depend on consumption, income, opportunity costs of time, and age.
- We don't directly observe the per period benefits of adopting ATM cards. But we observe consumers' usage patterns.

Yang and Ching (2009) (cont'd)

Roadmap Introduction Learning Stockpiling Technology adoption/Switching Costs Measuring the Discount Factor

- In Italy, checking account pays interests. By observing the cash withdrawal patterns before and after adopting an ATM card and how much cash is needed to finance one's consumption, we can model consumers' cash demand, and use the data to infer the monetary benefits of adopting an ATM card.
- Combining it with the dynamic model of consumer adoption decisions, it is possible to measure the adoption costs in monetary terms.
- The model also provides an alternative explanation about why the elderly are reluctant to adopt new technology.

Approximation Methods

Roadmap Introduction Learning Stockpiling Technology adoption/Switching Costs Measuring the Discount Factor

- Keane and Wolpin (1994) – interpolation method, needs a terminal period, and solve the model using backward induction. Examples: Erdem and Keane (1996), Crawford and Shum (2005), Sun (2005).
- Rust (1997) – random grid approximation, needs the state variables to evolve stochastically over time, applies to infinite horizon problem. Example: Sriram et al. (forthcoming).
- Akerberg (2001) – makes use of importance sampling, need to allow all parameters to be heterogeneous. Examples: Akerberg (2003), Hartmann (2006), Hartmann and Viard (2008), Rossi (2007).
- Imai, Jain and Ching (forthcoming) – approximate the value function using the past outcomes of the algorithm, can be implemented in Bayesian approach, *no need to maximize the likelihood*, applies to infinite horizon problems. Examples: Osborne (2008), Osborne (2009), Ching, Imai, Ishihara and Jain (2009a, 2009b).

Estimating the Discount Factor

- Can the discount factor and preferences be nonparametrically identified? In general, one needs to make functional form assumptions, or rely on exclusion restrictions (Rust 1994, Magnac and Thesmar 2002).
- Rust (2006) argues that “nonparametric assumptions” such as conditional independence and rational expectation are no weaker than functional form assumptions, such as CARA, error is log normal.
- Many people argue that the discount factor is not well-identified and thus set β according to the interest rate (it is not clear which interest rate one should pick).
- With enough structure in the model, the discount factor can be recovered, along with preference parameters.
- Does it necessarily mean that we make very strong assumptions to draw inference? Let’s consider the following example.

Model (An Example)

A simple dynamic model of store choice with reward programs (Ching, Imai, Ishihara and Jain 2009a, Hartmann and Viard 2008).

- There are two supermarkets in a city ($j = 1, 2$).
- Each store offers a stamp card, which can be exchanged for a gift upon completion.
- Consumers get one stamp for each visit with a purchase.
- Reward programs at the two supermarkets differ in terms of: (i) the number of stamps required for a reward (\bar{S}_j), and (ii) the mean value of the reward (G_j).
- $G_{ij} \sim N(G_j, \sigma_{G_j}^2)$, i indexes consumers.
- Once consumers receive a gift, they will start with a blank stamp card again in the next period.

Model (cont'd)

- $p_{ijt} \sim i.i.d. N(\underline{p}, \sigma_p^2)$.
- Let $s_i = (s_{i1}, s_{i2})$ be the number of stamps collected.
- Consumer i 's single period utility of visiting supermarket j is:

$$U_{ijt}(s_{it}, p_{it}; \theta) = \begin{cases} \alpha_j - \gamma p_{ijt} + G_{ij} + \epsilon_{ijt} & \text{if } s_{ijt} = \bar{S}_j - 1, \\ \alpha_j - \gamma p_{ijt} + \epsilon_{ijt} & \text{otherwise.} \end{cases}$$

- The utility of “not shopping” is normalized to zero, i.e., $U_{i0t}(s) = 0$.
- We assume that ϵ follows i.i.d. extreme value distribution.

Model (cont'd)

- The consumer i 's objective is to choose a sequence of store choices to maximize the sum of the present discounted future utility.

$$\max_{\{d_{ijt}\}_{t=1}^{\infty}} E \left[\sum_{t=1}^{\infty} \beta^{t-1} d_{ijt} U_{ijt}(s_{it}, p_{it}; \theta) \right]$$

where $d_{ijt} = 1$ if consumer i chooses store j in period t and $d_{ijt} = 0$ otherwise, and β is the discount factor.

- The evolution of the state, s_{it} , is as follows.

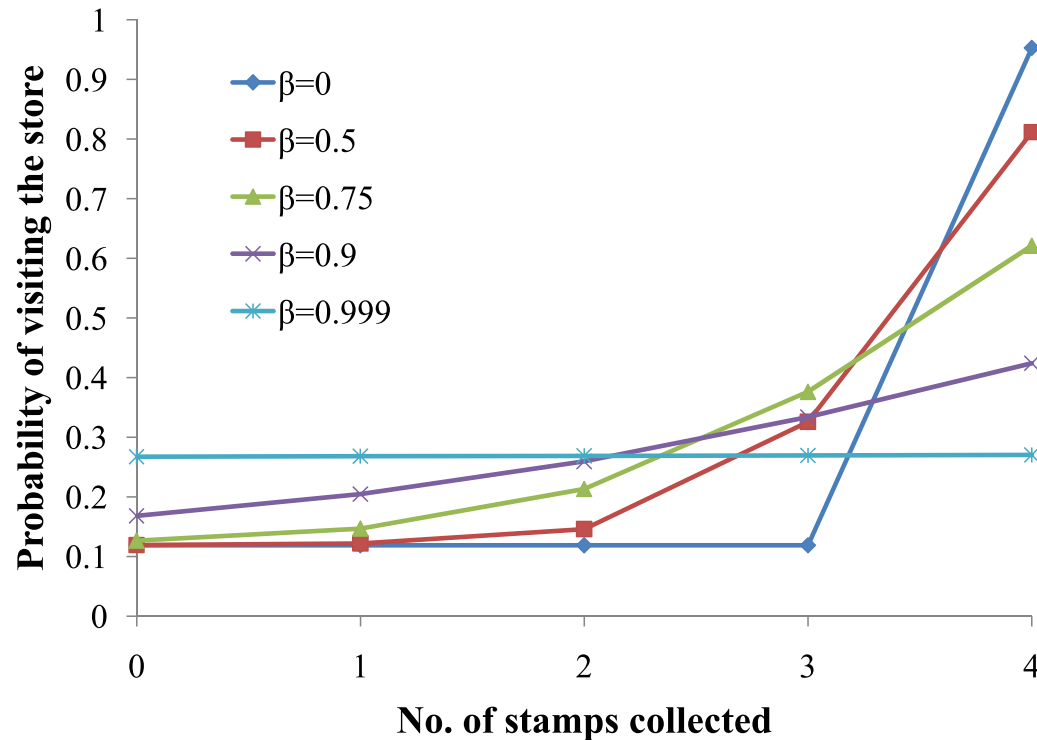
$$s_{ijt+1} = \begin{cases} s_{ijt} + 1 & \text{if } d_{ijt} = 1 \text{ and } s_{ijt} < \bar{S}_j - 1 \\ 0 & \text{if } d_{ijt} = 1 \text{ and } s_{ijt} = \bar{S}_j - 1 \\ s_{ijt} & \text{if } d_{ijt} = 0. \end{cases}$$

Implications of the Model

Roadmap Introduction Learning Stockpiling Technology adoption/Switching Costs Measuring the Discount Factor

- The main dynamics of the model is the intertemporal trade-off created by the reward program.
- Suppose that a consumer is close to completion of the stamp card for store 1, and store 2 offers a lower price today.
- If the consumer chooses store 2 today, he/she will get a better deal today but will delay the completion of the stamp card for store 1.
- If β is less than one, the delay will lower the present discounted value of the gift.
- The incentive not to delay the completion will increase as he/she gets closer to the completion of the stamp card.

Choice Probabilities



- Only one store and the outside option.
- Other parameters are set as $\bar{S}_1 = 5$, $\alpha_1 = -2.0$, $G_1 = 3.0$, $\sigma_{G_1} = 0$, and $\gamma = -1.0$.
- β can be identified because the model has exclusion restrictions.

Estimation

- We simulate samples based on the solution of the dynamic model with and without unobserved consumer heterogeneity.
- True parameters of the simulation
 - ◆ Store intercepts: $\alpha_1 = \alpha_2 = 0$.
 - ◆ Population mean of rewards: $G_1 = 1.0$, $G_2 = 5.0$.
 - ◆ Population variance of rewards: $\sigma_{G_1} = 0$, and $\sigma_{G_2} = 0$ (no heterogeneity) or 1.0 (heterogeneity).
 - ◆ Number of stamps for rewards: $\bar{S}_1 = 2$, $\bar{S}_2 = 4$.
 - ◆ Price coefficient: $\gamma = -1.0$.
 - ◆ Discount factor: $\beta = 0.6$ or 0.8.
- Estimate the model using the IJC method (see Ching, Imai, Ishihara and Jain (2009a) for details).

Results: Homogeneous Model

		$\beta = 0.6$		$\beta = 0.8$	
parameter	TRUE	mean	sd	mean	sd
α_1 (intercept for store 1)	0.0	-0.001	0.019	-0.030	0.022
α_2 (intercept for store 2)	0.0	-0.002	0.019	-0.018	0.028
G_1 (reward for store 1)	1.0	0.998	0.017	1.052	0.021
G_2 (reward for store 2)	5.0	5.032	0.048	5.088	0.085
γ (price coefficient)	-1.0	-0.999	0.016	-0.996	0.019
β (discount factor)	0.6/0.8	0.601	0.008	0.800	0.010

Sample size: 100,000 (1,000 consumers for 100 periods)

Fixed parameters: $\bar{S}_1 = 2$, $\bar{S}_2 = 4$, $\bar{p} = 1.0$, $\sigma_p = 0.3$, $\sigma_{G_j} = 0$ for $j = 1, 2$

Tuning parameters: $N = 1,000$ (no. of past pseudo-value functions used for Emax function approximation), $h = 0.01$ (bandwidth).

Results: Heterogeneous Model

		$\beta = 0.6$		$\beta = 0.8$	
parameter	TRUE	mean	sd	mean	sd
α_1 (intercept for store 1)	0.0	-0.005	0.019	-0.022	0.022
α_2 (intercept for store 2)	0.0	0.010	0.021	0.005	0.037
G_1 (reward for store 1)	1.0	1.017	0.017	1.010	0.019
G_2 (reward for store 2)	5.0	5.066	0.065	4.945	0.130
σ_{G_2} (sd of G_2)	1.0	1.034	0.046	1.029	0.040
γ (price coefficient)	-1.0	-1.004	0.016	-0.985	0.019
β (discount factor)	0.6/0.8	0.595	0.005	0.798	0.006

Sample size: 100,000 (1,000 consumers for 100 periods)

Fixed parameters: $\bar{S}_1 = 2$, $\bar{S}_2 = 4$, $\bar{p} = 1.0$, $\sigma_p = 0.3$, $\sigma_{G_1} = 0$

Tuning parameters: $N = 1,000$ (no. of past pseudo-value functions used for Emax function approximation), $h = 0.01$ (bandwidth).

Conclude (Other Challenges)

- **Hyperbolic Discounting:** The discount factor for more imminent future is larger than those for more distant future (Loewenstein and Prelec 1992, Laibson 1997), e.g., $f(t) = \beta\delta^t$. Laibson et al. (2007), Fang and Wang (2008) are attacking the identification of hyperbolic discounting. Quantitative Marketing needs to do more research in this area.
- Consumers may not know prices (Dickson and Sawyer 1990, Vanhule and Dreze 2001, Monroe and Lee 1999). Anderson and Simester (1998) (2002) ran a series of field experiments to investigate this. Essentially no dynamic structural models address this issue (see the reduced form models by Ching, Erdem and Keane 2009a, 2009b).
- Should take advantage of new estimation methods – more complicated dynamic structural models can be estimated now!
- Want to know more estimation methods? See Aguirregabiria and Mira (forthcoming).

References

Ackerberg, Daniel (2001), "A New Use of Importance Sampling to Reduce Computational Burden in Simulation Estimation," working paper, UCLA.

Ackerberg, Daniel (2003), "Advertising, Learning, and Consumer Choice in Experience Good Markets: An Empirical Examination," *International Economic Review* **44**(3): 1007-1040.

Aguirregabiria, Victor and Pedro Mira (forthcoming), "Dynamic Discrete Choice Structural Models: A Survey," *Journal of Econometrics*.

Anand, Bharat and Ronald Shachar (2005), "Advertising, the Matchmaker," working paper, Faculty of Management, Tel Aviv University.

Anderson, Eric and Duncan Simester (1998), "The Role of Sale Signs," *Marketing Science* **17**(2): 139-155.

Anderson, Eric and Duncan Simester (2002), "Are Sales Signs Less Effective When More Products Have Them?" *Marketing Science* **20**(2): 121-142.

Ching, Andrew (2000), *Dynamic Equilibrium in the U.S. Prescription Drug Market After Patent Expiration*, Ph.D. dissertation, University of Minnesota.

Ching, Andrew (forthcoming), "A Dynamic Oligopoly Structural Model for the Prescription Drug Market After Patent Expiration," *International Economic Review*.

Ching, Andrew (2008), "Consumer Learning and Heterogeneity: Dynamics of Demand for Prescription Drugs After Patent Expiration," working paper, Rotman School of Management, University of Toronto.

Ching, Andrew, Tülin Erdem, and Michael Keane (2009a), "The Price Consideration Model of Brand Choice," *Journal of Applied Econometrics* **24**(3): 393-420.

Ching, Andrew, Tülin Erdem, and Michael Keane (2009b), "How Much Do Consumers Know About the Quality of Products? Evidence from the Diaper Market," work-in-progress.

Ching, Andrew and Masakazu Ishihara (2009a), "The Effects of Detailing on Prescribing Decisions under Quality Uncertainty," working paper, University of Toronto.

Ching, Andrew and Masakazu Ishihara (2009b), "Measuring the Informative and Persuasive Roles of Detailing on Prescribing Decisions," working paper, University of Toronto.

Ching, Andrew, Susumu Imai, Masakazu Ishihara, and Neelam Jain (2009a), "A Practitioner's Guide to Bayesian Estimation of Dynamic Programming Models," working paper, Rotman School of Management, University of Toronto.

- Ching, Andrew, Susumu Imai, Masakazu Ishihara, and Neelam Jain (2009b), "A Dynamic Model of Consumer Learning and Forgetting," work-in-progress.
- Coscelli, Andrea and Matthew Shum (2004), "An Empirical Model of Learning and Spillover in New Drug Entry," *Journal of Econometrics* 122: 213-246.
- Crawford, Gregory S. and Matthew Shum (2005), "Uncertainty and Experimentation in Pharmaceutical Demand," *Econometrica* 73: 1137-1174.
- Dickson, Peter and Alan Sawyer (1990), "The Price Knowledge and Search of Supermarket Shoppers," *Journal of Marketing* 54(3): 42-53.
- Eckstein, Zvi, Dan Horsky, and Y. Raban (1998), "An Empirical Dynamic Model of Optimal Brand Choice," working paper, Tel-Aviv University.
- Erdem, Tülin and Michael Keane (1996), "Decision-making Under Uncertainty: Capturing Dynamic Brand Choice Processes in Turbulent Consumer Goods Markets," *Marketing Science* 15(1): 1-20.
- Erdem, Tülin, Susumu Imai, and Michael Keane (2003), "Brand and Quantity Choice Dynamics Under Price Uncertainty," *Quantitative Marketing and Economics* 1(1): 5-64.
- Erdem, Tülin, Michael Keane, and Baohong Sun (2008), "A Dynamic Model of Brand Choice When Price and Advertising Signal Product Quality," *Marketing Science* 27(6): 1111-1125.
- Fang, Hanming and Yang Wang (2008), "Estimating Dynamic Discrete Choice Models with Hyperbolic Discounting, with an Application to Mammography Decisions," working paper, Duke University.
- Loewenstein, George and Darzen Prelec (1992), "Anomalies in Intertemporal Choice: Evidence and an Interpretation," *Quarterly Journal of Economics* 107(2): 573-597.
- Guadagni, Peter and John D.C. Little (1983), "A Logit Model of Brand Choice Calibrated on Scanner Data," *Marketing Science* 2:203-238.
- Hartmann, Wesley (2006), "Intertemporal Effects of Consumption and Their Implications for Demand Elasticity Estimates," *Quantitative Marketing and Economics* 4(4): 325-349.
- Hartmann, Wesley and Brian Viard (2008), "Do Frequency Reward Programs Create Switching Costs?" *Quantitative Marketing and Economics* 6(2): 109-137.
- Hendel, Igal and Aviv Nevo (2006a), "Sales and Consumer Inventory," *The RAND Journal of Economics* 37(3): 543-561.
- Hendel, Igal and Aviv Nevo (2006b), "Measuring the Implications of Sales and Consumer Stockpiling," *Econometrica* 74(6): 1637-73.

Imai, Susumu, Neelam Jain, and Andrew Ching (forthcoming) “Bayesian Estimation of Dynamic Discrete Choice Models,” *Econometrica*.

Keane, Michael (1997), “Current Issues in Discrete Choice Modeling,” *Marketing Letters* **8**: 307-322.

Keane, Michael and Kenneth Wolpin (1994), “The Solution and Estimation of Discrete Choice Dynamic Programming Models by Simulation and Interpolation: Monte Carlo Evidence,” *Review of Economics and Statistics* **74**(4): 648-672.

Laibson, David (1997), “Golden Eggs and Hyperbolic Discounting,” *Quarterly Journal of Economics*, 112(2): 443-477.

Laibson, David, Andrea Repetto, and Jeremy Tobacman (2007), “Estimating Discounting Functions with Consumption Choice Over the Lifecycle,” working paper, Harvard University.

Magnac, Thierry and David Thesmar (2002): “Identifying Dynamic Discrete Decision Processes,” *Econometrica*, **70**: 801-816.

Mehta, Nitin, S. Rajiv, and K. Srinivasan (2004), “The Role of Forgetting in Memory-based Choice Decisions,” *Quantitative Marketing and Economics* **2**(2): 107-140.

Mehta, Nitin, Jack Chen, and Om Narisimhan (2008), “Informing, Transforming and Persuading: Disentangling the Multiple Effects of Advertising on Brand Choice Decisions,” *Marketing Science* **27**(3): 334-355.

Monroe, Kent and Angela Lee (1999), “Remembering Versus Knowing: Issues in Buyers’ Processing of Price Information,” *Journal of Academy of Marketing Science* **27**(2): 207-225.

Narayanan, Sridhar, Puneet Manchanda, and Pradeep Chintagunta (2005), “Temporal Differences in the Role of Marketing Communication in New Product Categories,” *Journal of Marketing Research* **42**(3): 278-290.

Osborne, Matthew (2008), “Consumer Learning, Switching Costs, and Heterogeneity: A Structural Examination,” working paper, U.S. Department of Justice.

Osborne, Matthew (2009), “Consumer Stockpiling Behavior, Price Sensitivity and Its Implications For Price Elasticities and Consumer Welfare,” work-in-progress.

Pesendorfer, Martin (2002), “Retail Sales: A Study of Pricing Behavior in Supermarket,” *Journal of Business* **75**(1): 33-66.

Song, Inseong, and Pradeep Chintagunta (2003) “A Micromodel of New Product Adoption with Heterogeneous and Forward Looking Consumers: Application to the Digital Camera Category.” *Quantitative Marketing and Economics*, 1, 371–407.

Rossi, Federico (2007), “\$1 Discount or \$1 Reward? The Effect of Consumers’ Preferences on Reward Programs,” working paper, UNC.

Rust, John (1994) "Structural Estimation of Markov Decision Processes," in *Handbook of Econometrics*, Vol. 4, ed. by R. Engle and D. McFadden. Amsterdam: North Holland, pp. 3081-3143.

Rust, John (1997), "Use Randomization to Break the Curse of Dimensionality," *Econometrica* **65**(3): 487-516.

Rust, John (2006), "Comments on 'Structural vs. Atheoretic Approaches to Econometrics' by Michael Keane," mimeo, University of Maryland.

Ryan, Stephen and Catherine Tucker (2007) "Heterogeneity and the Dynamics of Technology Adoption", working paper, MIT

Sriram, S., Manoj Agarwal, and Pradeep Chintagunta (forthcoming), "Investigating Consumer Adoption of Related Technology Products," *Marketing Science*.

Sun, Baohong (2005), "Promotion Effect on Endogeneous Consumption," *Marketing Science* **24**(3): 430-443.

Vanhuele, Marc and Xavier Dreze (2001), "Measuring the Price Knowledge Shoppers Bring to the Store," *Journal of Marketing* **66**(4): 72-85.

Yang, Botao and Andrew Ching (2009), "Dynamics of Consumer Adoption of Financial Innovations: The Case of ATM Cards," working paper, University of Toronto.