

Estimation of Discrete Choice Models with Strategic Interactions

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Roadmap

- Motivations.
- Discrete Games with Incomplete Information.
 - Classical approach: Two step method.
 - Bayesian approach.
- Static Discrete Games with Complete Information.
- Future research directions.

Motivations

- Analysis of many economics, marketing and social phenomena involves dynamic strategic interactions between a relatively small number of agents.

- Examples:

Ellickson and Misra (2007)

- Supermarket pricing strategies.

Aguirregabiria, Mira and Roman (2007)

- Entry and growth of retail markets.

Liran (2003)

- Movie Release Date Timing Game

Motivations (cont'd)

- In order to address to take into account the strategic interactions, one tractable way is to apply game theory to model the equilibrium.
- This allows us to address two main fundamental research questions (Berry and Tamer, 2006):
 - Prediction of equilibrium market structure under policy experiments.
 - Nature of competition.

Difficulties of Estimating Discrete Games

- Until recently, econometric models of discrete games had been limited to relatively simple static games.
- Problems:
 - Computational burden in the solution of dynamic discrete games;
 - The indeterminacy problem associated with multiple equilibria.

Reasons behind these problems

- Best response functions are non-linear in other players' actions – leads to multiple equilibria.
- The dimension of the state space, and the cost of computing an equilibrium, increases exponentially with the number of heterogeneous players.
- Also need to determine how to select an equilibrium.

Complete information games vs. Incomplete information games

- Suppose a player's utility is characterized by two vector of state variables: x_t and ε_t .
- Suppose that there are N players, and let $x_t = (x_{1t}, x_{2t}, \dots, x_{Nt})$, $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \dots, \varepsilon_{Nt})$.
- Games with complete information assumes that both x_t and ε_t are common knowledge.
- Games with incomplete information assumes that x_t is common knowledge. But player i only knows the realization of ε_{it} , she does not observe ε_{-it} (but knows their distributions).

Games with Incomplete Information

- Private information state variables help introduce unobservables in the econometric model.
- Under certain regularity conditions, dynamic games of incomplete information have at least one equilibrium – not necessarily the case in dynamic games of complete information (Doraszelski and Satterthwaite, 2003).

Games with Incomplete Information using Classical econometrics

- Two-step methods (Aguirregabiria and Mira (2007), Bajari, Benkard and Levin (2007), Jofre-Bonet and Pesendorfer (2003), Pakes, Ostrovsky and Berry (2007), and Pesendorfer and Schmidt-Dengler (2007)).
- Extend Hotz-Miller estimator to the estimation of dynamic games.
- Under the assumption that the data is generated from ONE equilibrium, this estimator avoids solving multiple equilibria.

Basic ideas of Two-step methods

- Interpret players' choice probabilities as their belief about the behavior of their opponents.
- Given these beliefs, interpret each players' problem as a single player game against nature.
- Although equilibrium probabilities are correspondences (instead of functions) of structural parameters, each player's best response is a function of structural parameters and players' beliefs under some regularity conditions.
- The expected future values can also be expressed as a function of individual's choice probabilities.
- Estimate players' beliefs nonparameterically from the data.
- Data requirement is demanding – needs to assume data from all markets are generated from ONE equilibrium.

Pros and Cons of 2-step method

- Pros
 - Simple and easy to implement! It does not require solving the dynamic solution of the game explicitly.
 - Allow us to obtain structural parameters and learn the nature of the competition: strategic complements or substitutes?
- Cons
 - Data requirement is demanding – needs to assume data from all markets are generated from ONE equilibrium.
 - Cannot allow for unobserved market specific heterogeneity.
 - If this assumption is satisfied, this is an advantage from estimation point of view.
 - However, if the model has multiple equilibria, it seems likely that different equilibria could be played in different markets.
 - Silent about the equilibrium selection process.
 - When doing policy experiments or what-if analysis, we do not know if the same equilibrium will be played.

Games with Incomplete Information using Bayesian econometrics

- Ferrall, Houde, Imai, Pak (2008).
- Allow for presence of multiple equilibria in the data.
- Can accommodate unobserved heterogeneity.
- Applicable in both static and dynamic games.
- Monte Carlo results suggest that this method is promising!

Key ideas behind FHIP estimator

- If we allow the data to be generated from multiple equilibria, the usual way to form the likelihood function requires estimating the equilibrium selection probability, and solving ALL equilibria.
- However, under certain regularity conditions, it can be shown that the inverse mapping of a market specific structural parameter (c) and equilibrium choice probabilities (p) is a function.
- To form the likelihood, one needs to integrate out a market specific heterogeneity parameter, c .
- One can use a change-of-variable technique to convert the integration problem wrt c , to an integration problem wrt to p .

Pros and Cons of FHIP

- Pros:
 - Because the mapping from p to c is a function, there is no need to solve for equilibria in forming the likelihood after doing the change-of-variable.
 - Likelihood based -- could use either maximum likelihood or Bayesian methods. But with high dimensional integration, it is usually easier to do it in Bayesian.
- Cons:
 - Still not able to recover the equilibrium selection process.

Static Games with Complete Information

- All the players observe the error terms in the model. Also have multiple equilibria problems.
- Bresnahan and Reiss (1990) and Berry (1992).
 - Impose conditions in the heterogeneity parameters such that the model predicts a unique number of entrants.
- Pakes, Porter, Ho and Ishii (2005) and Ciliberto and Tamer (2007)
 - Consider general form of heterogeneity.
 - Consider partial identification (set of parameters).

Dynamic Games of Complete Information

- So far, no body has attacked this problem yet!

Future Research Directions

- Do players always make their decisions every period?
 - Ching, Erdem and Keane (2007) consider a brand choice model where consumers only actively make purchase decision probabilistically.
- How do we identify a set of players?
- Without knowing the equilibrium selection process, how do we do policy analysis?
- Dynamic games with continuous choice, e.g., Pakes and McGuire (2001), Benkard (2004), Ching (2004).
- Want to know more? Aguiregabiria and Mira (2007).