Over-Confidence May Reduce Negotiation Delay*

Alberto Galasso

University of Toronto†

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Abstract

When a seller negotiates with multiple buyers, how does over-confidence affect the timing of trade? In this paper we distinguish between over-confidence about trade opportunities and over-confidence about the terms of trade. In bargaining environments without externalities both types of over-confidence can cause delays in agreement. If externalities are present the two forms of subjective bias have very different impacts on delay. In particular, over-confidence about trade opportunities may reduce bargaining delay.

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†Rotman School of Management, University of Toronto, 105 St. George Street Toronto ON, Canada. Email: alberto.galasso@rotman.utoronto.ca.

1 Introduction

It is common wisdom among economists, management scientists and legal scholars that negotiator over-confidence reduces concessionary behavior and generates bargaining delays. Intuitively, when negotiators are excessively optimistic about the shares they will get tomorrow, it may be impossible to find a settlement today that satisfies all parties' expectations.

This cause of bargaining delays is well known in law and economics[1] and in applied psychology (Bazerman and Neal 1982, 1985). Only recently has game theory started exploring the connection between optimism and delay. Yildiz (2003, 2004) shows that excessive optimism may generate delays in finite horizon two-player negotiation games if the horizon is not too long. Ali (2006) shows that in multilateral bargaining games, extreme optimism may generate delays even in arbitrarily long finite games.

This paper studies how over-confidence affects negotiations between one seller and multiple buyers in the presence of multilateral externalities. We show that in these settings it is necessary to introduce a distinction between two different types of over-confidence. First, buyers may overestimate the likelihood of receiving an offer from the seller and be excessively optimistic about the opportunity to trade. Second, buyers may overestimate the content of the offer and therefore be excessively optimistic about the terms of trade. The main contribution of this paper is to show that, while over-confidence about the terms of trade tends to generate delay, over-confidence about the opportunities to trade may be beneficial and reduce negotiation delays.

To develop the intuition for this result, consider the following scenario. A noted scholar bargains for a tenured position with two universities located in the same geographical area. Each university would like to hire the scholar and prefers having the professor hired by the competitor to having him hired abroad. The scholar's decision has to be taken before a deadline; if no agreement is reached before the deadline, the professor will accept a job at a foreign university. Consider now the very last period before the deadline. At this point the scholar can extract from one of the two universities an amount of surplus that renders the school indifferent between hiring him and having him work abroad. Notice that in all previous negotiating periods he cannot extract as much surplus because he needs to compensate the negotiating counterpart for the positive externality arising if he signs at the competing institution. In other words, only in the last period does the threat of working abroad become real. Therefore, the presence of positive externalities gives an incentive to the scholar to delay the agreement. Assume now that the two universities overestimate the probability of hiring the scholar. In the extreme case, each school assigns probability zero of having the scholar work for the competing institution. In this case the scholar will not be required to compensate the school for the positive externality and will have no incentive to delay the agreement. This simple example illustrates how in environ-

[1] Among others see Landes (1971) and Posner (1972).

ments in which there are externalities for non-traders over-confidence may reduce the externalities' impact and facilitate agreement.

Our theoretical model involves the trade of an indivisible object between one seller and N buyers in the presence of multilateral externalities and negotiation delay. There are several managerial applications that fit into this general framework.[2] One of these applications involves mergers and acquisitions. Usually, a merger of competing firms imposes externalities on the other firms in the industry. For example, if the industry becomes less competitive and more concentrated, the merger creates positive externalities for those not included in it. Empirical evidence described in Walsh (1989) shows extensive negotiations between the target company and its potential buyers.

Similarly, when a firm obtains a cost-reducing innovation protected by a patent, it imposes a negative externality on competing firms (Katz and Shapiro, 1986). The results in Jehiel and Moldovanu (1995a) suggest that bargaining delay may arise because of these externalities. Gans, Hsu and Stern (2008) provide empirical evidence of the existence of delays in the licensing process.

Finally, in various industries (e.g. automobiles) retailers negotiate exclusive dealings and agree to sell only one manufacturer's brand. Typically, an exclusive dealing allows the manufacturer to obtain a larger market share than its rivals and therefore it imposes a negative externality on competitors (Besanko and Perry, 1993).

Our results suggest that in all these settings over-confidence about trade opportunities and over-confidence about terms of trade may have different effects on bargaining delay.

The plan of the paper is as follows. Section 2 provides a simple example that illustrates how over-confidence may reduce settlement delays. Section 3 studies general finite horizon negotiation games with positive externalities. Extensions and robustness are examined in section 4. Section 5 concludes. All proofs are in the appendix.

2 A Motivating Example

Consider a large corporation, S, and two cities $i \in \{A, B\}$. The corporation is choosing where to locate its operations and it bargains with the cities over regulations and municipal ordinances in exchange for the resources it brings to their local economies. An example of such negotiations is the recent choice faced by General Motors of where to locate the first lithium-ion battery pack manufacturing facility in the United States. Early GM press releases indicated that the lab would be located in Michigan and that the exact location was subject to negotiations with state and local government authorities.[3]

For simplicity, we model these ordinances as a transfer from the city to the cor-

[2] For additional examples see Segal (1999) and Genicot and Ray (2006).

[3] See GM press release on 12 January 2008. In June 2009 the company officially opened a 3,000-square-meter battery lab in Warren, MI.

poration. We study a bargaining procedure composed of two stages. In stage one the corporation makes a take it or leave it offer, p, to city A. If city A accepts the offer it transfers p to the corporation and the game ends. The utility of the corporation is given by p and the utility of the city is given by $1 - p$.

If city A refuses the proposal, the game moves to stage two. In this stage both cities have the same probability (1/2) of meeting with the corporation. If S meets i, then with probability 1/2 the corporation will propose a transfer and with probability 1/2 the city will make the offer. If city i accepts then it transfers p to the corporation and the game ends. All players discount the future with a common discount factor δ.

We solve this game (with no externalities and no subjective bias) using backward induction. Each city knows that in period two it will get a positive payoff only if it will be matched with the corporation and will be able to make an offer. This event happens with probability 1/4. So in period one the corporation will offer city A a transfer $p = 1 - \delta/4$ in order to make the city indifferent between accepting and rejecting the offer. There will always be immediate agreement.

Subjective Bias

Following Yildiz (2003, 2004) and Ali (2006), we consider an extension of the above bargaining game where it is common knowledge that city A overestimates its bargaining power. This subjective bias may be modeled in two ways. First, city A may overestimate its probability of being matched with the corporation in the second period. Alternatively, it can overestimate its ability of making an offer to the corporation once matched with it. Let us assume that city A believes its probability of being matched with the corporation is $b > 0.5$ and its probability of making an offer once matched is $\rho > 0.5$.

Notice that the two dimensions of bargaining power may be affected by different factors. For example the probability of being approached by the corporation may depend on specific conditions of the local labour market or on possible synergies between local firms and the corporation. Differently, the content of the ordinances may depend on the city budget or on political conflicts inside the local government. Asymmetry in beliefs may arise because the city has more precise information about one dimension then the other. For example, a city may be very optimistic about the possible synergies between local firms and the corporation but perceive correctly how the city budget will affect the content of the ordinances.[4]

In this case the corporation, to trade in period one, will need to offer city A a transfer equal to $1 - \delta b\rho$ and this offer will be profitable for the corporation only if

[4]Similarly, in the example described in the introduction the likelihood that a department will be approached by a famous scholar may depend on the quality of its faculty members. Differently the terms of the offer may depend on school resources. Even if the two dimensions may well be correlated (departments with lots of resources tend to have better faculty) the fact that they depend on different factors may generate different levels of optimism. For example, the head of the department may overestimate the quality of the faculty but perceive correctly the resources of the department.

$1-\delta b\rho > \delta/2$. It follows that it is profitable for the corporation to delay agreement when:

$$\frac{2-\delta}{2\delta} < b\rho. \tag{1}$$

Inspection of formula (1) leads to the following result.

Remark 1 *If* $1 \geq \delta \geq 2/3$ *bargaining delay arises if over-confidence is large enough. Both an increase in* b *and an increase in* ρ *may generate bargaining delays.*

From this simple example we observe the well known result that over-confidence may be source of bargaining delay. In addition, we notice that the way of modeling subjective bias (b or ρ) is irrelevant: it is the product of the two biases that determines if there is delay or not.

Externalities

Following Jehiel and Moldovanu (1995a, 1995b), we consider now the case in which if the corporation locates in one city the other city is subject to an external effect. Let us consider the case of a positive externality. More specifically, we assume that a non-contracted city gets a positive payoff, θ, when the corporation locates in the other city. Without subjective biases, the maximum payoff that the corporation can obtain in period one is: $1-\delta/4-\delta\theta/2$.

It follows that it is optimal for the corporation to delay agreement if:

$$\theta > \frac{4-3\delta}{2\delta}. \tag{2}$$

Formula (2) implies:

Remark 2 *Bargaining delay arises if positive externalities are large enough.*

The intuition for this delay is simple: because city A needs to be compensated for the positive externality, it may be too costly to induce the corporation to agree immediately.

Subjective Bias and Externalities

We introduce now subjective bias in the framework with externalities. In this case the maximum payoff that the corporation will be able to obtain is $1-\delta b\rho-\delta(1-b)\theta$. It follows that to delay agreement is profitable for the corporation whenever:

$$\theta > \underline{\theta}(\delta, b, \rho) \equiv \frac{1}{1-b}\left(\frac{1}{\delta} - \frac{1}{2} - b\rho\right) \tag{3}$$

that leads to the following result.

Remark 3: *Bargaining delay arises if* $\theta > \underline{\theta}(\delta, b, \rho)$. *The cutoff* $\underline{\theta}(\delta, b, \rho)$ *decreases in* ρ *and may increase or decrease in* b.

The previous result implies that in the presence of multilateral externalities it is crucial to distinguish between the two types of subjective biases. In particular, it shows that if cities are over-confident about their opportunities to approach the corporation, negotiation delays may be reduced. The intuition for this result is the following: over-confidence about the probability of being matched reduces city perception of the externalities and this allows the corporation to propose a (profitable) acceptable offer.

3 Model

We now generalize the previous example adopting a setting similar to the one in Jehiel and Moldovanu (1995a, 1995b).[5]

There is one seller, $N > 2$ buyers and the bargaining procedure is composed of T stages. The first stage is called stage T, the second stage $T-1$ and so on until the end of the game at stage 1. All players discount the future with a common discount factor δ. At the beginning of any stage the seller randomly meets one of the N buyers. All buyers have the same probability (i.e. $1/N$) of meeting the seller. If the seller, S, meets one of the buyers, $i \in \{1, 2, .., N\}$, then with probability $1/2$ he proposes a transaction price p and with probability $1/2$, i proposes a price. If the proposal is accepted then the buyer obtains the good, pays price p to the seller and the game ends. The utility of the seller is given by p, the utility of buyer i by $1-p$ and the utility of buyer $j \neq i$ is given by θ. We allow externalities to be positive ($\theta > 0$) or negative ($\theta < 0$). If there is no agreement, then there are two possibilities. If the game has already reached the last stage then the game ends, otherwise the game continues to the next stage. This stage has the same structure as the one described above. If the game ends with no trade, then the utility of all players is equal to zero.

In the next proposition, we study how the externalities affect delay in equilibrium.

Proposition 1 *For each T, N and δ there are $\underline{\theta}(\delta, N)$ and $\overline{\theta}(\delta, N, T) \geq \underline{\theta}(\delta, N)$ such that:*

(i) if $\theta \leq \underline{\theta}(\delta, N)$ there is immediate trade;

(ii) if $\underline{\theta}(\delta, N) < \theta < \overline{\theta}(\delta, N, T)$ there exists a $\widetilde{t} > 1$ such that trade occurs at stage $\widetilde{t}$ or earlier;

(iii) if $\theta \geq \overline{\theta}(\delta, N, T)$ trade occurs at the last period.

Proposition 1 states that it is possible to identify two externality cutoffs and therefore determine parameter values for which there is either absence of delay or

[5] The main difference with Jehiel and Moldovanu (1995a, 1995b) is that in our model we focus on symmetric buyers (and hence, in terms of Jehiel and Moldovanu (1995a, 1995b) to *non-generic* economic situations). In section 4.3 we show that our main results do not depend on the assumption of symmetric buyers.

agreement only at the last period. Both thresholds decrease in δ and N meaning that it is more beneficial to delay the agreement when players are not impatient and when buyers have a low probability of trading.

If externalities are low enough, i.e. $\theta \leq \underline{\theta}(\delta, N)$, there is immediate agreement. In this region the seller prefers compensating the contracting buyer for the externality than waiting to extract a larger surplus. Notice that the cut-off does not depend on the length of the game T. In fact, in the proof we show that if the condition is verified for two-period games, it is verified for any other finite horizon game. Because $\underline{\theta}(\delta, N) > 0$, proposition 1 has the following immediate corollary.

Corollary 1 *If* $\theta \leq 0$ *agreement is immediate.*

Corollary 1 implies that in our model there is never delay when externalities are negative or when there are no externalities. It is important to notice that this result is different from the one in Jehiel and Moldovanu (1995a) where delay arises with negative externalities because of asymmetries among buyers. In section 4.3, we show that our main result does not depend on the symmetry among buyers.

If the externality exceeds $\underline{\theta}(\delta, N)$ delay may arise. In the intermediate region, the exact length of the delay is difficult to pin down and the equilibrium play has the feature that periods of waiting alternate with periods of activity (Jehiel and Moldovanu (1995b) refer to this aspect as "cyclical delay"). The intuition for this dynamics is the following. In this region, if the game reaches stage 2 the presence of externalities induce the seller to wait and delay trade to stage 1 (the last period). Because of the absence of trade at stage 2, the stage 3 expected payoff of the buyer will be the present value of his payoff at stage 1. Therefore, at stage 3 the seller will choose whether to offer the buyer this payoff or to wait until stage 1. If trade is delayed the same choice is taken in stage 4. For t large enough, the present value of buyer payoff will be so small that it will not be profitable for the seller to wait and trade occurs in stage t. However, in stage $t+1$, because trade is expected in stage t, buyers will need to be compensated for the externality arising at t. The presence of this externality may induce the seller not to trade at stage $t+1$. Therefore, agreement can be expected at t but not at $t+1$ and $t-1$. Despite this cyclicality, we can still pin down an upper bound in delay. In fact, we can identify a stage, $\widetilde{t} > 1$, such that trade occurs at stage $\widetilde{t}$ at the latest.

Finally, if externalities are large enough, i.e. $\theta \geq \overline{\theta}(\delta, N, T)$, trade occurs at the last period. This threshold increases with the length of the game T meaning that it is less profitable to wait until the last period when the game is very long. Moreover, as the discount factor tends to one, the two thresholds get closer and either trade occurs immediately or it occurs at the last stage.

The region $[-\infty, \underline{\theta}]$ represents the minimum range of externalities in which agreement is immediate whereas the interval $[\overline{\theta}, \infty]$ is the minimum range of externalities in which agreement is delayed. When both thresholds shift to the right immediate agreement is observed for a larger set of parameters. Conversely, when they shift to the left the region where delay occurs expands. This simple comparative statics will be used to explore the impact of subjective bias on delay.

3.1 Subjective Bias and Externalities

We now extend the game introducing subjective biases. We assume that at each $t \leq T$ buyer i believes that he will be matched with the seller with probability $b > 1/N$ and if matched he will make an offer with probability $\rho > 1/2$. These beliefs (and seller's beliefs) are common knowledge.[6] From each player subjective point of view, his belief is true and he perceives the others' beliefs as over-confident. As in Ali (2006), we assume that agents beliefs are stationary to focus on the effect of over-confidence and not on the impact of learning.

We first study the game without externalities. Notice that the multilateral bargaining environment differs from the one studied in Ali (2006) where players divide a dollar according to unanimity of super-majority voting rules. Nevertheless, if there is only one buyer, both our model and the model of Ali (2006) are equivalent to the two-player negotiation game studied in Yildiz (2003). The next proposition shows that if $\theta \leq 0$ and the game is long enough we obtain immediate agreement as in Yildiz (2003).

Proposition 2 *If $\theta = 0$, there exists an $L(\delta)$ such that there is immediate agreement if $T \geq L(\delta)$.*

Proposition 2 says that if the bargaining game is long enough there is immediate agreement. As in Yildiz (2003) the intuition is that if the game is long enough, it is not worthwhile for players to delay the agreement until the last period (where over-confidence does not have a bite). Moreover, if agreement is expected at stage t, the rent extracted in that stage will be so low that players are also expected to agree in stage $t+1$. Because agreement at stage t implies agreement at stage $t+1$, the game will display either immediate agreement or delay until the last stage. The following corollary provides a condition for delay.

Corollary 2 *If $\theta = 0$ either there is immediate agreement or agreement is delayed until the last period. Delay occurs only if*

$$b\rho > \frac{1}{\delta^{T-1}} - \frac{1}{2}. \tag{4}$$

Condition (4) has an important implication. It shows that in games without externalities, if δ^{T-1} is not too small, both types of over-confidence can generate delay. Moreover, because it is only the product $b\rho$ that matters, modeling over-confidence through b or ρ has the *same impact* on the timing of agreement. We now turn to the analysis of the game in the presence of externalities. The next proposition shows that also in this case we can identify the no-delay and the T-period delay regions.

[6] See Yildiz (2004) for a discussion about the relation between rationality and common prior.

Proposition 3 *For each T, b, ρ and δ there are $\underline{\theta}(\delta, b, \rho)$ and $\overline{\theta}(\delta, b, \rho, T) \geq \underline{\theta}(\delta, b, \rho)$ such that:*

(i) if $\theta \leq \underline{\theta}(\delta, b, \rho)$ there is immediate trade;

(ii) if $\underline{\theta}(\delta, b, \rho) \leq \theta < \overline{\theta}(\delta, b, \rho, T)$ and $\theta > 0$ there exists a $\widetilde{t} < T$ such that trade occurs at stage $\widetilde{t}$ or earlier, if $\underline{\theta}(\delta, b, \rho) < \theta < \overline{\theta}(\delta, b, \rho, T)$ and $\theta \leq 0$ there is immediate agreement;

(iii) if $\theta \geq \overline{\theta}(\delta, b, \rho, T)$ trade occurs at the last period.

Proposition 3 shows that also in the presence of over-confidence it is possible to derive externality thresholds that identify regions of delay and immediate agreements. Differently from Proposition 1, both thresholds may now be negative. In particular, it is possible to show that if condition (4) is satisfied then $\overline{\theta}(\delta, b, \rho, T) < 0$ and agreement occurs at the last stage even in the absence of externalities.[7]

When externalities are positive, in the intermediate region $\underline{\theta}(\delta, b, \rho) < \theta < \overline{\theta}(\delta, b, \rho, T)$ the exact length of the delay is difficult to pin down and the equilibrium play has the feature that periods of waiting alternate with periods of activity (there may be "cyclical delay").[8]

Next corollary shows that, because of the externalities, the two types of over-confidence have now different impact on the threshold values.

Corollary 3 *$\underline{\theta}(\delta, 1/N, \rho)$ and $\overline{\theta}(\delta, 1/N, \rho, T)$ decrease in ρ whereas $\underline{\theta}(\delta, b, 1/2)$ and $\overline{\theta}(\delta, b, 1/2, T)$ increase in b.*

If buyers overestimate the terms of trade but perceive their opportunity to trade without bias the two thresholds *decrease* with over-confidence. This result is consistent with the common intuition that over-confidence tends to generate bargaining delays. However, if buyers perceive correctly the probability of making an offer but overestimate the likelihood of being matched with the seller, both thresholds *increase* with over-confidence. Counter-intuitively, delays are less likely the greater is buyer over-confidence. The reason is that over-confidence about trade opportunities reduces buyer perception of the externalities and allows the negotiating parties to trade immediately.

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[7] To proof Proposition 3 we apply backward induction to our finite horizon bargaining model. Following Jehiel and Moldovanu (1995a) and Ali (2006) it is possible to show that the game allows for multiple SPE but all SPE are payoff equivalent. Because there is a unique continuation value at the first period, the criterion used to determine whether the game has immediate agreement is to check if the sum of continuation values exceeds $1/\delta$.

Moreover, adopting the proof of proposition 1 in Yildiz (2003) to our setting, it is possible to show that the unique continuation values may be derived by iterative application of conditional dominance, thus relaxing the role of the equilibrium solution (see Dekel, Fudenberg and Levine, 2004 and Ali, 2009 for a discussion of the role of equilibrium solution in behavioural models).

[8] Moreover, it is possible to show that as $\delta \to 1$ the lower bound and the upper bound converge (i.e. $\underline{\theta}(\delta, b, \rho) \to \overline{\theta}(\delta, b, \rho, T)$) and hence at the limit there is either immediate agreement or delay until the last period.

If buyers overestimate both terms of trade and opportunities to trade the impact of an increase in b becomes ambiguous. The intuition is that over-confidence about trade opportunities has two opposite effects on seller's incentives to trade. On one hand, it reduces the impact of externalities and therefore renders trade more appealing. On the other hand, it increases buyer expected payoffs and therefore reduces the rent that the seller can extract with immediate trade. Intuitively, the greater the over-confidence about the terms of trade the less the seller can benefit from a reduction in the perceived externality. In the next corollary we provide a sufficient condition under which an increase in b always reduces the likelihood of delay.

Corollary 4 *If* $2/3 > \delta$ *both* $\underline{\theta}(\delta, b, \rho)$ *and* $\overline{\theta}(\delta, b, \rho, T)$ *increase in* b *for every* ρ. *As* $\delta \to 1$ $\underline{\theta}(\delta, b, \rho)$ *and* $\overline{\theta}(\delta, b, \rho, T)$ *decrease in* b *for every* ρ. $\underline{\theta}(\delta, b, \rho)$ *and* $\overline{\theta}(\delta, b, \rho, T)$ *always decrease in* ρ.

Intuitively, if δ is small, the present value of the rent that buyers expect to extract by making the last offer is small. In this circumstance, the impact of ρ on delays is much smaller than the impact of the externalities. Therefore, over-confidence over trade opportunities renders trade more likely. Conversely, when δ is very large, the rent from making the last offer becomes very relevant and an increase in b renders delay more likely. Finally, overconfidence about about the terms of trade always renders delay more likely (independently of the specific value of b).[9]

4 Extensions and Robustness

In this section, we discuss some natural extensions of our model.

4.1 Infinite Horizon

Following Jehiel and Moldovanu (1995b) and Björnerstedt and Westermark (2009), we now study the infinite horizon extension of the model presented in section 3. As Björnerstedt and Westermark (2008, 2009), we focus on the class of stationary subgame perfect equilibria. In our framework, a pure strategy stationary equilibrium exists in which all buyers trade in the first period if the following conditions are satisfied:

[9] The discount factor in corollary 4 is far below those traditionally used in structural estimation of bargaining games ($\delta \approx 0.96$). It is important to notice that corollary 4 provides only a sufficient condition in order to have that the cutoffs increase in b for every value of ρ, and that the cutoff values may increase even if the discount factor is not so small. For example, corollary 3 shows that when $\rho = 1/2$ the cutoffs increase for any $\delta < 1$, thus the effect may present for more plausible values of the discount factor.

$$\begin{aligned}\pi^S &= \frac{1}{2}\left(1-\delta\pi^B\right)+\frac{1}{2}\delta\pi^S \\ \pi^B &= b\rho\left(1-\delta\pi^S\right)+b(1-\rho)\delta\pi^B+(1-b)\theta \\ \pi^S+\pi^B &\leq 1/\delta \qquad (5)\end{aligned}$$

where π^S and π^B are the history independent expected payoffs of the seller and the buyers. The following proposition derives a condition for which agreement is immediate.

Proposition 4 *In the infinite horizon bargaining game immediate agreement occurs as a stationary subgame perfect equilibrium if $\theta \leq \widehat{\theta}(b,\delta)$. The threshold $\widehat{\theta}(b,\delta)$ is positive and non-decreasing in b.*

This proposition shows that immediate agreement occurs over a larger set of parameters when buyers over-estimate the likelihood of being matched with the seller. This finding implies that our main result does not depend on the assumption of finite horizon.

Björnerstedt and Westermark (2009) show that absence of immediate trade may also occur in a mixed strategy equilibrium that they call "Hold-up" equilibrium. In this equilibrium when buyer j meets the seller, an acceptable offers is made only with some probability $p_j < 1$. The next proposition shows the existence of a symmetric Hold-up equilibrium in which the likelihood of trade after being matched is the same across buyers p^*. We show that this equilibrium exists only if externalities are large enough. In addition, p^* increases in b so the expected bargaining delay decreases with overconfidence about trade opportunities.[10]

Proposition 5 *If $\theta \geq \theta^H(b,\delta) > 0$ a Symmetric Hold-Up equilibrium exists in which each buyer, when matched with the seller, trades with probability $p^*(b,\delta,\theta)$. The probability $p^*(b,\delta,\theta)$ increases in b.*

The thresholds in proposition 4 and 5 are positive and indicate that in our setting delay may occur only if there are large positive externalities. In this sense our findings are similar to those in Björnerstedt and Westermark (2009) where delay only occurs with positive externalities. In Jehiel and Moldovanu (1995b) there is delay in the infinite horizon when externalities are negative but the equilibria supporting this delay are non-stationary.

[10] Björnerstedt and Westermark (2009) also show that in the infinite horizon model there exists a variety of asymmetric stationary equilibria.

4.2 Seller Over-confidence

In the baseline model we assumed that the seller estimates correctly the likelihood of making the offer (equal to $1/2$). We now relax this assumption and consider the case in which the seller overestimates his bargaining power. Specifically, we assume that he believes he will make the offer with probability equal to $s > 1/2$.

It is important to notice that, differently from the buyers, for the seller the subjective bias affects only the probability of making the offer. This is because in our model the seller is always matched with one of the N (identical) buyers. In the next proposition we characterize externality thresholds similar to those in Proposition 3.

Proposition 6 *For each* T, b, ρ, s *and* δ *there are* $\underline{\theta}(\delta, b, \rho, s)$ *and* $\overline{\theta}(\delta, b, \rho, s, T) \geq \underline{\theta}(\delta, b, \rho, s)$ *such that:*

(i) *if* $\theta \leq \underline{\theta}(\delta, b, \rho, s)$ *there is immediate trade;*

(ii) *if* $\underline{\theta}(\delta, b, \rho, s) < \theta < \overline{\theta}(\delta, b, \rho, s, T)$ *there exists a* $\widetilde{t} < T$ *such that trade occurs at stage* $\widetilde{t}$ *or earlier;*

(iii) *if* $\theta \geq \overline{\theta}(\delta, b, \rho, s, T)$ *trade occurs at the last period.*

The previous proposition shows that also in the presence of seller over-confidence it is possible to derive externality thresholds that identify regions of delay and immediate agreements. Exploiting these thresholds, in the next corollary we show that seller over-confidence always reduces the likelihood of trade.

Corollary 5 $\underline{\theta}(\delta, b, \rho, s)$ *and* $\overline{\theta}(\delta, b, \rho, s, T)$ *decrease in* s.

The intuition for the previous result is that seller over-confidence only affects the perceived terms of trade and not buyers perceived likelihood of trading. Because an increase in s increases seller expected payoff without affecting the perception of the externalities, it renders immediate agreement less likely.

4.3 Asymmetric Buyers

In our baseline model we assumed that buyers were identical. In this section we relax this assumption and show that over-confidence about trade opportunities may reduce bargaining delay even in the presence of asymmetric buyers.

We follow Jehiel and Moldovanu (1995a, 1995b) and assume that buyers may differ in two respect. First, they may have different valuations for the object: V_i with $i \in \{1, 2, .., N\}$. Second, they may be subject to different externalities. Specifically, we assume that if the seller trades with buyer i at price p the utility of i is $V_i - p$ and the utility of buyer $j \neq i$ is given by θ_{ij}. As in the previous section we assume that at each $t \leq T$ buyer i believes that he will be matched with the seller with probability $b > 1/N$ and that he will make the offer with probability $\rho > 1/2$.[11]

[11] For notational simplicity we assume that buyers do not differ in their beliefs. Example 8 in Section 4.4 shows that the main result holds with asymmetric beliefs as well.

There is an important difference between the symmetric and the asymmetric case. When buyers are symmetric (as in the previous sections), absence of immediate trade occurs only if no-buyer is willing to trade at the first stage. When buyers are asymmetric, absence of immediate trade may occur for two reasons. First, it may happen because no buyer is willing to trade with the seller at the first stage. Second, it may occur because the subset of buyers that are willing to trade immediately are not matched with the seller at the first stage. Let us call I^k the set of buyers willing to trade at stage k and denote by C^k its cardinality. To remain consistent with our previous analysis, we will say that there is absence of immediate trade if the set of buyers willing to trade at the first stage is empty. (i.e. $I^T = \phi$).

The following example shows that overconfidence about trading opportunities may reduce bargaining delay even when buyers are asymmetric.

Example 1 *Let* $N = 2$, $T = 2$, $V_1 = 1.5$ and $V_2 = 1$. *Let* $\theta_{21} = 2.5$, $\theta_{12} = 1$, $\delta = .75$ and $\rho = 1/2$. *If the game reaches the last period the expected payoffs for the seller, buyer 1 and buyer 2 are:*

$$\pi^S = \frac{1}{2}\left(\frac{V_1 + V_2}{2}\right) \tag{6}$$

$$\pi^{B_1} = \frac{1}{2} bV_1 + (1-b)\theta_{21} \tag{7}$$

$$\pi^{B_2} = \frac{1}{2} bV_2 + (1-b)\theta_{12}. \tag{8}$$

In the first period agreement with buyer 1 and 2 occurs as long as $\pi^S + \pi^{B_1} \leq V_1/\delta$ *and* $\pi^S + \pi^{B_2} \leq V_2/\delta$. *With the parameters of the example none of the buyers is willing to trade immediately if* $b = 0.5$ *(*$I^T = \phi$*) and there is immediate agreement with both buyers if* $b = 0.75$ $\left(I^T = \{1, 2\}\right)$.

With buyer asymmetries, as in our baseline model, over-confidence about trade opportunities reduces the perception of the externalities. In the previous example, because externalities are the source of bargaining delay, a larger b reduces their impact and allows immediate agreement.

We now consider general finite horizon bargaining games with asymmetric buyers. Because of the asymmetries, it is not easy to identify externality bounds as those derived in section 3.[12] To deal with this technical difficulty, we study the impact of over-confidence through a different lens. Let us indicate as $\widetilde{i}$ the buyer with the maximum valuation, V_i. In the following proposition we show that when the discount factor is not too large, if over-confidence about trade opportunities is large enough, there is never delay because the buyer with the largest valuation trades every time he is matched with the seller.

Proposition 7 *There exists a* $\widetilde{b} \in [1/N, 1)$ *such that if* $b \geq \widetilde{b}$ *and* $\delta < 2/3$ *then* $\widetilde{i} \in I^t$ *for* $t = 1, ..., T$.

[12] Jehiel andMoldovanu (1995a) characterize these thresholds only for an example.

Intuitively, as in the setting with symmetric buyers, over-confidence about trade opportunities reduces bargaining delay by reducing the impact of the externalities. When b is large and δ small, the perceived externalities are so small that only buyers' valuations become relevant for the trade decision. The seller will therefore always trade if he is matched with the buyer with the largest valuation.

4.4 Endogenous Choice of Bargaining Partner

In section 3, following previous literature on bargaining with externalities (Jehiel and Moldovanu 1995a, 1995b; Björnerstedt and Westermark 2008, 2009) we assumed that at the beginning of any stage the seller randomly meets one of the buyers. Specifically, in our baseline model we assumed that all buyers have the same probability of meeting the seller (equal to $1/N$) and we investigated the impact of optimistic beliefs ($b > 1/N$) on negotiation delay. In this section we extend the baseline model removing the assumption of exogenous matching between the seller and the buyer and allowing the seller to choose which buyer to negotiate with. We will refer to the extensions of the model in which the choice of bargaining partner is endogenous as "extended" games.

We begin by showing that the equilibrium outcome of our baseline model is equivalent to the one of a particular extended game. Assume that at each period there is uncertainty about the gains from trade. Specifically, at each t there is only one buyer that obtains utility equal to 1 by consuming the good, all the other buyers obtain utility zero from consumption (and utility θ if some other buyer trades). The probability of being the "valuable" buyer at time t is equal to $1/N$ and it is i.i.d. across periods and buyers. Buyers are overconfident and assign probability $b > 1/N$ to the event of being the valuable buyer. In each period the seller chooses which buyer to approach and the identity of the valuable buyer is commonly observed before the seller decides with whom to negotiate. If the seller negotiates with buyer j, with probability 1/2 he proposes a transaction price p and with probability 1/2, j proposes a price. Each buyer j believes that if approached by the seller he will make an offer with probability $\rho > 1/2$. In the next proposition we show that the outcome of this game is equivalent to the one of our baseline model.

Proposition 8 *In the extended game with endogenous trade in which:*

(i) each buyer is the "valuable" buyer at time t with probability $1/N$;

(ii) each buyer assigns probability $b > 1/N$ to the event of being the valuable buyer;

has externality thresholds $\underline{\theta}(\delta, b, \rho)$ and $\overline{\theta}(\delta, b, \rho, T)$ identical to those of the baseline model.

Because there is no difference in the externality bounds, the previous proposition shows that our main result (i.e. overconfidence may reduce negotiation delay) may hold also in settings where the seller chooses what buyer to negotiate with.

Nevertheless, the setting described above is very special because the choice of bargaining partner is only driven by the (random) identity of the "valuable" buyer. We now study a more general set-up in which only at the last period there is a single valuable buyer and in all previous periods all buyers obtain utility 1 from trading.[13]

We assume that buyers have asymmetric beliefs about the probability of being the valuable buyer in the last period (b_i is now the probability assigned by buyer i of being the valuable buyer in the last period) and investigate the impact on last period optimism on the likelihood to trade in earlier periods.

First, in the following example we show that even in this setting overconfidence may reduce bargaining delay.

Example 2 *There are two periods, two buyers and no overconfidence about terms of trade $\rho = 1/2$. Each buyer is the last-period valuable buyer with probability 1/2 and buyers beliefs are $b_1 > b_2 > 1/2$. Externalities are positive and satisfy:*

$$2\left(\frac{1}{\delta} - \frac{3}{4}\right) < \theta < \frac{1}{1-b_1}\left(\frac{1}{\delta} - \frac{1}{2} - \frac{b_1}{2}\right).$$

In equilibrium there is no delay and the seller trades in the first period with buyer 1 (intuitively, because $\theta > 1/2$ the seller prefers buyer 1 to buyer 2 and because θ is not too large there is immediate trade). In the absence of overconfidence ($b_1 = b_2 = 1/2$) trade occurs in the last period with the valuable buyer.

In the previous example the seller chooses to negotiate with the buyer that is more optimistic about the likelihood to trade in the last period, so optimism about trade in the last period is associated with high probability of trading in the first period. In the next example, we show that for some parameter configurations we may observe the opposite outcome: optimism at the last period is associated with low probability of trade in previous periods.

Example 3 *There are two periods, two buyers and overconfidence about terms of trade $\rho > 1/2$. Each buyer is the last-period valuable buyer with probability 1/2 but their beliefs are $b_1 > b_2 > 1/2$. Externalities are positive and satisfy:*

$$2\left(\frac{1}{\delta} - \frac{3}{4}\right) < \theta < \min\left\{\frac{1}{1-b_2}\left(\frac{1}{\delta} - \frac{1}{2} - b_2\rho\right), \rho\right\}.$$

In equilibrium the seller is going to trade in the first period with buyer 2 (because $\rho > \theta$). In the absence of overconfidence ($b_1 = b_2 = 1/2$) trade occurs in the last period with the valuable buyer.

In example 3, buyer 1 optimism about the likelihood of trading in the last period induces the seller to trade with buyer 2. In other words, overconfidence about trading in the last period is associated with low probability of trading in previous periods.

[13] We thank an anonymous referee for suggesting this framework.

Example 3 indicates a limitation of our model and, more generally, of the literature of contracting with externalities. More specifically, this simple example shows that not all the exogenous matching probabilities (and the corresponding beliefs) can be micro-founded by a model in which the choice of bargaining partner is endogenous and beliefs are derived in equilibrium.

This in turn indicates that our behavioral assumption (exogenous overconfidence about likelihood to trade) may not be appropriate in certain environments. A number of behavioral models in which biased beliefs are treated as exogenous share this limitation. Recent literature in economics and psychology (Fudenberg, 2006 and Ali, 2009) has started to address this problem by providing learning theoretic foundations for behavioral assumptions.

Example 3 has similar implications for the literature on bargaining with externalities (without optimism). In these papers, the practice has been to treat both matching probabilities and externalities as exogenous (Jehiel and Moldovanu 1995a, 1995b; Björnerstedt and Westermark 2008, 2009). Also in these models not all the specifications of matching probabilities and externalities can be micro-founded by a game in which the choice of bargaining partner is endogenous. In fact, buyers expecting a large externality in the last period may be unattractive for the seller in previous periods.[14]

The main objective of this paper is to study the impact of overconfidence in negotiations involving externalities. It is standard practice in the literature of bargaining with externalities to assume exogenous matching, therefore a complete analysis of endogenous matching is outside the scope of this paper. Nonetheless, in the next proposition we derive a sufficient condition that guarantees that the more optimistic buyer at time T will be the one to trade in earlier periods.

Proposition 9 *Consider an extended game in which N buyers have the same value of θ and ρ but different beliefs about being valuable in the last period b_i with $1 \leq i \leq N$. If $\theta > \rho$ there exists a $\widetilde{\delta}$ such that when $\delta < \widetilde{\delta}$ the seller bargains with the valuable buyer in the last period and with the buyer with the largest b_i in earlier periods.*

The above proposition focuses on settings where buyers differ only for their beliefs about being the valuable buyer in the last period. The condition indicates that overconfidence about being the valuable buyer in the last period is associated with high probability of trading in previous periods if externalities are large and positive and the discount factor is small. Intuitively, in these settings last period overconfidence, by reducing the bite of the externalities, renders the most optimistic buyer most attractive to the seller. In this case, buyer overconfidence is compatible with what is observed (or expected to happen) over time.

[14] It is possible to see this conceptual difficulty in example 3.1 in Jehiel and Moldovanu (1995a). In that example there are 3 buyers all have the same probability (1/3) of meeting the seller even if one of them (buyer 3) is very unattractive for the seller.

In the previous analysis, we focused on "extended games" in which buyers differ in their beliefs about being the last period valuable buyer. An alternative approach to study endogenous choices of bargaining partner would be to extend the baseline model assuming that buyers differ in their overconfidence about terms of trade. Specifically, we now consider a setting in which (i) all buyers are valuable in all periods, (ii) each buyer has a different value of ρ_i , and (iii) the seller chooses which buyer to bargain with. Despite the asymmetry in buyer beliefs, if the game reaches the last period, the seller will be indifferent among the buyers because they all make the offer with probability 1/2. This indifference generates a multiplicity of subgame perfect Nash equilibria.

The next proposition shows that when externalities are large and the discount factor is small, the most preferred equilibrium for the seller involves immediate trade with the least overconfident buyer. We also show that, for the same values of θ and δ, there is an equilibrium in which the seller trades immediately with the most overconfident buyer.

Proposition 10 *If all buyers are valuable in all periods and buyers have different beliefs about terms of trade, ρ_i with $i = 1, ..., N$, and the seller choice of bargaining partner is endogenous there are multiple equilibria. If $\theta > \max_i \{\rho_i\}$ and δ is low enough:*

(i) the most preferred equilibrium for the seller involves immediate trade with the least overconfidence buyer;

(ii) there is an equilibrium in which the seller trades immediately with the most overconfident buyer.

Despite the difference in the two models, proposition 10 has implications that are similar to those derived from examples 2 and 3 and proposition 9. Proposition 10 indicates that not all the exogenous matching probabilities (and their corresponding beliefs) can be micro-founded by a game in which the seller chooses the bargaining partner endogenously. Interestingly, the fact that in his most preferred equilibrium the seller trades with the least overconfident buyer suggests that, in some applications, it may be questionable to assume that a buyer is both overconfident about the terms of trade and overconfident about trade opportunities.

As noted above, a complete analysis of games in which the seller chooses his bargaining partner is outside the scope of this paper. The two frameworks sketched in this section can be useful to explore a variety of additional issues that we leave for future research.

5 Conclusion

A large number of studies in applied psychology have documented the presence of over-confidence. People report having above the median driving skills (Svenson, 1981), ability to solve trivia quizzes (Moore and Cain, 2007) and chances to get the job they want (Weinstein, 1980). This self-serving bias can dramatically affect important business decisions. For example, Astebro (2003) finds that fifty percent of inventors that are advised not to commercialize their inventions by the Canadian Inventor's Assistance Program exert further development efforts.

Negotiation is a key managerial activity influenced by over-confidence. In fact, results from experimental and field research indicate a precise connection between optimism and bargaining delay (for a survey of this literature see Babcock and Loewenstein, 1997). These studies show not only that over-confidence reduces negotiator success in reaching agreement but also that training designed to decrease over-confidence improves the effectiveness of dispute resolution (Barzerman and Neale, 1982). Building on these findings, popular managerial literature encourages executives to use various tactics to remove optimism from the bargaining counterpart (e.g. HBS Press, 2005).

In this paper we show that there are environments in which over-confidence does not have such a negative impact on negotiations. Specifically, we show that when a single principal bargains with several agents in the presence of multilateral externalities over-confidence may reduce bargaining delay. Our setting encompasses a large number of managerial negotiations. For example, patent licensing, exclusive dealing contracts and takeovers are all multi-agent games with multilateral externalities. In all these settings over-confidence about trade opportunities may be beneficial and reduce negotiation delays.

There are several useful directions for further research. The first is to extend the model to a multi-principal multi-agent game to study the interplay between upstream and downstream optimism. Second, experimental and empirical evidence could be useful in assessing the quantitative impact of over-confidence on multilateral negotiations. Finally, following previous literature we assumed that player's beliefs are common knowledge. Dropping this assumption can help understanding the interplay of optimism with signalling and screening and their impact on bargaining delay.

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Appendix: Proofs

Proof of Proposition 1

Let us define π_t^S and π_t^B as the expected payoffs for the seller and each buyer if the game reaches stage t. At stage 1 trade is going to occur and these expected payoffs are:

$$\begin{aligned}
\pi_1^S &= 1/2 \\
\pi_1^B &= \frac{1}{N}\frac{1}{2} + \theta\frac{(N-1)}{N}.
\end{aligned}$$

This implies that agreement occurs at time 2 if $\delta\left(\pi_1^S + \pi_1^B\right) \leq 1$ or $\theta \leq \underline{\theta}(\delta, N)$ where:

$$\underline{\theta}(\delta, N) \equiv \frac{1}{N-1}\left(\frac{N}{\delta} - \frac{1}{2}(N+1)\right). \tag{9}$$

We will now show that if $\delta\left(\pi_t^S + \pi_t^B\right) \leq 1$ for a generic $t < T$ and that $\theta \leq \underline{\theta}(\delta, N)$ then $\delta\left(\pi_{t+1}^S + \pi_{t+1}^B\right) \leq 1$. First notice that:

$$\begin{aligned}
\pi_{t+1}^S &= \frac{1}{2}\left(1 - \delta\pi_t^B\right) + \frac{1}{2}\delta\pi_t^S \\
\pi_{t+1}^B &= \frac{1}{2N}\left(1 - \delta\pi_t^S\right) + \frac{1}{2N}\delta\pi_t^B + \frac{\theta(N-1)}{N}
\end{aligned}$$

that imply

$$\pi_{t+1}^S + \pi_{t+1}^B = \frac{1}{2}\left(1 + \frac{1}{N}\right) + \frac{\theta(N-1)}{N} + \frac{\delta}{2}\left(1 - \frac{1}{N}\right)\left(\pi_t^S - \pi_t^B\right)$$

and

$$\pi_{t+1}^S - \pi_{t+1}^B = \frac{1}{2}\frac{(N-1)}{N} - \theta\frac{(N-1)}{N} + \frac{\delta}{2}\frac{(N+1)}{N}\left(\pi_t^S - \pi_t^B\right).$$

Applying these formulas recursively we rewrite this difference as

$$\pi_{t+1}^S - \pi_{t+1}^B = \frac{(N-1)}{N}\left(\frac{1}{2} - \theta\right)\sum_{j=0}^{t}\left(\frac{(N+1)}{N}\frac{\delta}{2}\right)^j$$

that implies

$$\pi_{t+1}^S + \pi_{t+1}^B = \frac{(N+1)}{2N} + \frac{\theta(N-1)}{N} + \frac{\delta}{2}\left(\frac{N-1}{N}\right)^2\left(\frac{1}{2} - \theta\right)\sum_{j=0}^{t-1}\left(\frac{(N+1)}{N}\frac{\delta}{2}\right)^j$$

We will now show that $g(\theta, \delta, N, t) \equiv \delta\left(\pi_{t+1}^S + \pi_{t+1}^B\right) \leq 1$. First, notice that $g(\theta, \delta, N, t)$ is increasing in θ :

$$\frac{\partial g}{\partial \theta} = \frac{(N-1)}{N}\left[\delta - \delta^2\frac{(N-1)}{2N}\left(\frac{1 - \left(\frac{(N+1)}{2N}\delta\right)^{t-1}}{1 - \frac{(N+1)}{2N}\delta}\right)\right] \geq 0.$$

Second, notice that

$$g(\underline{\theta}, \delta, N, t) = 1 + \frac{\delta^2}{2}\frac{N-1}{N}\left(1 - \frac{1}{\delta}\right)\sum_{j=0}^{t-1}\left(\frac{\delta}{2}\frac{(N+1)}{N}\right)^j \leq 1.$$

Therefore, there is immediate agreement for any $\theta \leq \underline{\theta}(\delta, N)$, that proves part *(i)* of the proposition. If $\theta > \underline{\theta}(\delta, N)$ there is disagreement at time 2 and at time 3 continuation values will be equal to the discounted continuation values of time 2. Therefore there will be agreement at time 3 only if $\delta^2\left(\pi_1^S + \pi_1^B\right) \leq 1$. Let us define $\widetilde{t}$ as the minimum time for which $\delta^{\widetilde{t}-1}\left(\pi_1^S + \pi_1^B\right) \leq 1$. This condition rewrites as:

$$\delta^{\widetilde{t}-1}\left(\frac{(N+1)}{2N} + \frac{\theta(N-1)}{N}\right) \leq 1.$$

Therefore, if $\widetilde{t} < T$ then agreement occurs at $\widetilde{t}$ at the latest, that proves part *(ii)* of the proposition. Finally, if $\widetilde{t} \geq T$ agreement occurs at the last period. It is easy to see that $\widetilde{t} \geq T$ whenever:

$$\theta \geq \overline{\theta}(\delta, N, T) \equiv \frac{N}{N-1}\left(\frac{1}{\delta^{T-1}} - \frac{N+1}{2N}\right)$$

that proves part *(iii)* of the proposition.

Proof of Corollary 1

Because $N/\delta > (N+1)/2$ the lower bound in (9) is always positive. This implies that there cannot be delay if $\theta \leq 0$.

Proof of Proposition 2

The expected payoffs of the seller and each buyer from trade at time 1 are:

$$\begin{aligned} \pi_1^S &= 1/2 \\ \pi_1^B &= b\rho. \end{aligned}$$

Define $S_1 \equiv \pi_1^S + \pi_1^B$. Agreement occurs at time 2 only if $\delta S_1 \leq 1$. Similarly, indicate as $S_t = \pi_t^S + \pi_t^B$ the perceived continuation value of the seller and one buyer at time t. Agreement at time $t+1$ occurs if $\delta S_t \leq 1$. We will now show that $\delta S_t \leq 1$ implies $\delta S_{t+1} \leq 1$. Notice that agreement at $t+1$ implies:

$$\begin{aligned} \pi_{t+1}^S &= \frac{1}{2}\left(1 - \delta\pi_t^B\right) + \frac{1}{2}\delta\pi_t^S \\ \pi_{t+1}^B &= b\rho\left(1 - \delta\pi_t^S\right) + b(1-\rho)\delta\pi_t^B \end{aligned}$$

and

$$\begin{aligned} S_{t+1} &= S_1 + \delta S_t\left(\frac{1}{2} - b\rho\right) - \delta\pi_t^B(1-b) \\ &\leq S_1 + \frac{1}{2} - b\rho - \delta\pi_t^B(1-b) \\ &\leq 1 - \delta\pi_t^B(1-b) \leq 1/\delta \end{aligned}$$

where the first inequality arises because $\delta S_t \leq 1$. Notice now that the maximum value for $\pi_1^S + \pi_1^B$ is $3/2$. Defining $L(\delta)$ by $\delta^{L(\delta)-1}3/2 \leq 1$, we have that the maximum interval of disagreement is $L(\delta)$ and that there is immediate agreement for any game with length $T \geq L(\delta)$.

Proof of Corollary 2

To have agreement in period $T < t < 1$ we need that $\delta S_t > 1$ and $\delta S_{t-1} \leq 1$. This is impossible because $\delta S_t \leq 1$ implies $\delta S_{t+1} \leq 1$. There is delay until period 1 if $\delta^{T-1}S_1 > 1$ or if $b\rho > \frac{1}{\delta^{T-1}} - \frac{1}{2}$.

Proof of Proposition 3

If the game reaches stage 1 trade is going to occur and the expected payoffs for the seller and the buyer are:

$$\begin{aligned} \pi_1^S &= 1/2 \\ \pi_1^B &= b\rho + (1-b)\theta. \end{aligned}$$

So, agreement occurs at time 2 if $\delta S_1 \leq 1$ or $\theta \leq \underline{\theta}(\delta, b, \rho)$ where

$$\underline{\theta}(\delta, b, \rho) \equiv \frac{1}{(1-b)}\left(\frac{1}{\delta} - \frac{1}{2} - b\rho\right). \tag{10}$$

To show *(i)*, we assume that $\delta S_t \leq 1$ for a generic $t < T$ and that trade occurs in any period before t. We will now show that if $\theta \leq \underline{\theta}(\delta, b, \rho)$ then $\delta S_{t+1} \leq 1$. Notice that

$$\pi_{t+1}^S = \frac{1}{2}\left(1 - \delta\pi_t^B\right) + \frac{1}{2}\delta\pi_t^S \tag{11}$$
$$\pi_{t+1}^B = b\rho\left(1 - \delta\pi_t^S\right) + b(1-\rho)\delta\pi_t^B + (1-b)\theta \tag{12}$$

that imply

$$S_{t+1} = S_1 + \delta S_t\left(\frac{1}{2} - b\rho\right) - \delta\pi_t^B(1-b). \tag{13}$$

We consider 3 sub-cases.

Subcase 1: $\theta \leq \min\{0, \underline{\theta}\}$

First notice that $\pi_1^B > \theta$. Now assume that $\pi_{t-1}^B > \theta$ for a generic $t-1$. Agreement at $t-1$ implies $\delta S_{t-1} \leq 1$ and $1 - \delta\pi_{t-1}^S \geq \delta\pi_{t-1}^B$ that combined with (12) imply $\pi_t^B \geq b\delta\pi_{t-1}^B + (1-b)\theta \geq b\delta\theta + (1-b)\theta \geq \theta$. So $\pi_t^B \geq \theta$ when $\theta \leq 0$. If $\delta S_t \leq 1$ formula (13) and $\pi_t^B \geq \theta$ imply that

$$\begin{aligned} S_{t+1} &\leq (1-b)\theta + 1 - \delta\pi_t^B(1-b) \\ &\leq (1-b)\theta(1-\delta) + 1 \\ &\leq \left(\frac{1}{\delta} - \frac{1}{2} - b\rho\right)(1-\delta) \leq \frac{1}{\delta}. \end{aligned} \tag{14}$$

Subcase 2: $0<\theta \leq \min\{\rho, \underline{\theta}\}$

We show now that if $\rho \geq \theta > 0$ then $\pi_t^B \geq (1-b)\theta + b\delta^{t-1}\theta$ for a generic t. Using $\pi_t^B \geq b\delta\pi_{t-1}^B + (1-b)\theta$ is easy to see that at $t = 1$ the relationship is satisfied. Assume now it is satisfied at a generic $t-1$. Then $\pi_{t-1}^B \geq (1-b)\theta + b\delta^{t-2}\theta \geq \delta^{t-2}\theta$ that implies $\pi_t^B \geq (1-b)\theta + b\delta^{t-1}\theta$. Replacing this lower bound on the right hand side of (14) we have that a sufficient condition for agreement is

$$(1-b)\theta + 1 - \delta[(1-b)^2\theta + b\delta^{t-1}\theta(1-b)] \leq 1/\delta \tag{15}$$

that can be rewritten as

$$(1-b)\theta + 1 \leq \frac{1}{\delta} + \delta(1-b)^2\theta + b\delta^t\theta(1-b)$$

where the right hand side is minimized at $\delta = 1$ when[15]

$$(1-b)\theta + 1 \leq 1 + (1-b)^2\theta + b\theta(1-b)$$

[15] To see this notice that the right hand side is minimized at $\delta = 1$ if for every $\delta \in [0, 1]$ we have

that is satisfied for every θ and implies $\delta S_{t+1} \leq 1$.

Subcase 3: $\rho<\theta \leq \underline{\theta}$

To study the case in which $\theta > \rho$ notice that agreements at $t+1$ occurs if

$$S_{t+1} = (1-b)\theta + b\rho + \frac{1}{2} + \delta S_t \left(\frac{1}{2} - b\rho\right) - \delta\pi_t^B (1-b) \leq \frac{1}{\delta}$$

that at $\theta = \underline{\theta}$ rewrites as:

$$S_t \left(\frac{1}{2} - b\rho\right) \leq \pi_t^B (1-b)$$

that is satisfied if

$$\frac{\pi_t^B}{\pi_t^B + \pi_t^S} \geq \frac{\frac{1}{2} - b\rho}{(1-b)}. \tag{16}$$

The left hand side (16) is less than 1/2 for every $\rho \geq 1/2$. This means that the condition is satisfied if $\pi_t^B \geq \pi_t^S$. It is easy to see that $\pi_1^B \geq \pi_1^S$ when $\theta > \rho \geq 1/2$. Assume now that $\pi_{t-1}^B \geq \pi_{t-1}^S$. Then (11) and (12) imply

$$\pi_t^B - \pi_t^S = \pi_1^B - \pi_1^S + \delta\pi_{t-1}^B \left[\frac{1}{2} + b(1-\rho)\right] - \delta\pi_{t-1}^S \left(\frac{1}{2} + b\rho\right) \tag{17}$$

and can be rewritten as

$$\pi_t^B - \pi_t^S = \pi_1^B - \pi_1^S + \delta\left(\pi_{t-1}^B - \pi_{t-1}^S\right)\left(\frac{1}{2} + b\rho\right) - \delta\pi_{t-1}^B \left[b(2\rho - 1)\right]. \tag{18}$$

Because (17) increases in π_{t-1}^B and decreases in π_{t-1}^S and we assumed $\pi_{t-1}^B \geq \pi_{t-1}^S$ formula (18) is minimized when $\pi_{t-1}^B = \pi_{t-1}^S$ that implies $\delta\pi_{t-1}^B \leq 1/2$. Because

$$\pi_1^B - \pi_1^S = (1-b)\theta + b\rho - 1/2 > \left[b(2\rho - 1)\right]/2$$

whenever $\theta > \rho$ and $\delta\pi_{t-1}^B \leq 1/2$ formula (18) imply $\pi_t^B - \pi_t^S > 0$ and $\delta S_{t+1} \leq 1$. This concludes the proof of *(i)*.

To show *(ii)* we follow the proof of Proposition 1, if $\theta > \underline{\theta}(\delta, b, \rho)$ we denote as $\widetilde{t}$ the minimum time for which $\delta^{\widetilde{t}-1}\left(\pi_1^S + \pi_1^B\right) \leq 1$. If $\widetilde{t} < T$ then agreement occurs at $\widetilde{t}$ at the latest. If $\theta = 0$ agreement at stage $\widetilde{t}$ implies immediate agreement because

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that

$$1/\delta + \delta(1-b)^2\theta + b\delta^t\theta(1-b) \geq 1 + (1-b)^2\theta + b\theta(1-b).$$

To see that this is satisfied notice that

$$\begin{aligned} 1/\delta - 1 &\geq (1-\delta)(1-b)^2\theta + b(1-\delta^t)\theta(1-b) \\ 1/\delta &\geq \theta(1-b)[(1-b) + b(1-\delta^t)/(1-\delta)] \end{aligned}$$

that is satisfied because the right hand side is less than one when $\rho \geq \theta$.

of the result in Corollary 2. When $\theta < 0$ agreement at stage $\widetilde{t}$ also implies immediate agreement as shown in formula (14). This proves part *(ii)* of the proposition.

If $\widetilde{t} \geq T$ then agreement will occur at the last period. Notice that $\widetilde{t} \geq T$ whenever:

$$\theta \geq \overline{\theta}(\delta, b, \rho, T) \equiv \frac{1}{1-b}\left(\frac{1}{\delta^{T-1}} - \frac{1}{2} - b\rho\right) \tag{19}$$

that proves part *(iii)* of the proposition.

Proof of Corollary 3

The threshold values are:

$$\underline{\theta}(\delta, b, \rho) \equiv \frac{1}{(1-b)}\left(\frac{1}{\delta} - \frac{1}{2} - b\rho\right)$$

$$\overline{\theta}(\delta, T, b, \rho) = \frac{1}{1-b}\left(\frac{1}{\delta^{T-1}} - \frac{1}{2} - b\rho\right) \tag{20}$$

it is easy to see that $\partial\underline{\theta}(\delta, 1/N, \rho)/\partial\rho < 0$, $\partial\overline{\theta}(\delta, 1/N, \rho, T)/\partial\rho < 0$ *and that* $\partial\underline{\theta}(\delta, b, 1/2)/\partial b > 0$ and $\partial\overline{\theta}(\delta, T, b, 1/2)/\partial b > 0$.

Proof of Corollary 4

From formula (20) it is easy to see that $\partial\underline{\theta}(\delta, b, \rho)/\partial\rho < 0$ and $\partial\overline{\theta}(\delta, T, b, \rho)/\partial\rho < 0$. Moreover

$$\frac{\partial\underline{\theta}(\delta, T, b, \rho)}{\partial b} > 0$$

only if

$$\frac{1}{\delta} - \frac{1}{2} > \rho \tag{21}$$

and because ρ is bounded by 1, the condition is always satisfied when $2/3 > \delta$. It is easy to see that when condition (21) the upper threshold $\overline{\theta}(\delta, T, b, \rho)$ also increases in b. Finally, because $\rho > 1/2$, for δ large enough condition (21) is violated and the thresholds decrease in b.

Proof of Proposition 4

Conditions (5) imply that agreement occurs if

$$\frac{(1-\delta)(1-b)\theta + 1 - b\delta}{1 - b\delta} \leq \frac{1}{\delta} \tag{22}$$

For $\delta < 1$ the condition becomes:

$$\theta \leq \widehat{\theta}(b, \delta) = \frac{1 - b\delta}{\delta(1-b)}.$$

It is easy to see that $\partial\widehat{\theta}(b,\delta)/\partial b = (1-\delta)/\delta(1-b)^2 \geq 0$.[16]

Proof of Proposition 5

We follow Björnerstedt and Westermark (2009) and indicate with $v_{S,i}$ and $w_{S,i}$ the value for the seller when making an offer and receiving an offer from buyer i. Similarly, $v_{i,S}$ and $w_{i,S}$ indicate the value for buyer i when making and receiving an offer. Let $p_{S,i}$ the probability that the seller gives an acceptable offer to i and $p_{i,S}$ the probability that the buyer makes an acceptable offer. Define $p^* = 0.5p_{S,i} + 0.5p_{i,S}$. As Björnerstedt and Westermark (2009) we have the following value equations:

$$\begin{aligned}
v_{S,i} &= (1-p_{S,i})w_{S,i} + p_{S,i}(1-w_{i,S}) \\
w_{S,i} &= \delta\left(\frac{1}{2}\frac{1}{n}\sum_{j\in N} v_{S,,j} + \frac{1}{2}\frac{1}{n}\sum_{j\in N} w_{S,,j}\right) \\
v_{i,S} &= p_{i,S}(1-w_{S,i}) + (1-p_{i,S})w_{i,S} \\
w_{i,S} &= \delta b\left(\rho v_{i,S} + (1-\rho)w_{i,S}\right) + \delta\frac{(1-b)}{N-1}(N-1)p^*\theta + \delta\frac{(1-b)}{N-1}(N-1)(1-p^*)w_{i,S}
\end{aligned}$$

In a Hold-up equilibrium the proposer must be indifferent between offering or not so $1 - w_{i,S} = w_{S,i}$. Moreover, Björnerstedt and Westermark (2009) show that in a Hold-up equilibrium $w_{S,i} = 0$ holds. These results imply that $v_{S,i} = 0$, $w_{i,S} = 1$ and that $v_{i,S} = 1$. Using these values in the last value equation we obtain

$$1 = \delta b\left(\rho + (1-\rho)\right) + \delta\frac{(1-b)}{N-1}(N-1)p^*\theta + \delta\frac{(1-b)}{N-1}(N-1)(1-p^*)$$

that implies

$$p^* = \frac{(1-\delta)}{\delta(1-b)(\theta-1)}.$$

that increases in b. Moreover $p^* \leq 1$ as long as

$$\theta \geq \theta^H(b,\delta) \equiv 1 + \frac{(1-\delta)}{\delta(1-b)}. \tag{23}$$

Formula (23) also indicates that, as in Björnerstedt and Westermark (2009), the equilibrium exists only if externalities are larger than the surplus.

Proof of Proposition 6

If the game reaches stage 1 trade is going to occur and the expected payoffs for the seller and the buyer are:

$$\begin{aligned}
\pi_1^S &= s \\
\pi_1^B &= b\rho + (1-b)\theta.
\end{aligned}$$

[16] For $\delta = 1$ condition (22) is always satisfied as in example 1 in Björnerstedt and Westermark (2009) but seller's payoff is non-negative only if $\theta \leq \widehat{\theta}(b,1)$.

So, agreement occurs at time 2 if $\delta S_1 \leq 1$ or $\theta \leq \underline{\theta}(\delta, b, \rho, s)$ where

$$\underline{\theta}(\delta, b, \rho, s) \equiv \frac{1}{(1-b)} \left(\frac{1}{\delta} - s - b\rho \right). \tag{24}$$

To show *(i)*, we follow the procedure used in the proof of Proposition 3 and consider the following recursive equations:

$$\begin{aligned} \pi_{t+1}^S &= s\left(1 - \delta\pi_t^B\right) + (1-s)\delta\pi_t^S \\ \pi_{t+1}^B &= b\rho\left(1 - \delta\pi_t^S\right) + b(1-\rho)\delta\pi_t^B + (1-b)\theta \end{aligned}$$

that imply

$$S_{t+1} = S_1 + \delta S_t \left(1 - s - b\rho\right) - \delta\pi_t^B \left(1 - b\right).$$

We consider 3 sub-cases.

Subcase 1: $\theta \leq \min\{0, \underline{\theta}\}$

As in Proposition 3 the fact that $\delta S_t \leq 1$and $\pi_t^B \geq \theta$ imply that

$$\begin{aligned} S_{t+1} &\leq (1-b)\theta + 1 - \delta\pi_t^B\left(1-b\right) \\ &\leq (1-b)\theta(1-\delta) + 1 \\ &\leq \left(\frac{1}{\delta} - (1-s) - b\rho\right)(1-\delta) \leq \frac{1}{\delta}. \end{aligned}$$

Subcase 2: $0<\theta \leq \min\{\rho, \underline{\theta}\}$

$\delta S_{t+1} \leq 1$ follows directly from Proposition 3 because the sufficient condition (15) does not depend on s.

Subcase 3: $\rho<\theta \leq \underline{\theta}$

To study the case in which $\theta > \rho$ notice that agreements at $t+1$ occurs if

$$\frac{\pi_t^B}{\pi_t^B + \pi_t^S} \geq \frac{1 - s - b\rho}{1 - b}. \tag{25}$$

and that the inequality is satisfied if $\pi_t^B \geq (1-s)\pi_t^S/s$ (because $(1-s) > (1-s-b\rho)/(1-b)$). It is easy to see that $\pi_1^B > (1-s)\pi_1^S/s$ when $\theta > \rho > 1-s$. Assume now that $\pi_{t-1}^B \geq (1-s)\pi_{t-1}^S/s$. Then

$$\pi_t^B-(1-s)\pi_t^S/s = \pi_1^B-(1-s)\pi_1^S/s+(1-s)\delta\left(\pi_{t-1}^B - (1-s)\pi_{t-1}^S/s\right)-\delta b\rho\pi_{t-1}^S+\delta b\pi_{t-1}^B(1-\rho) \tag{26}$$

that is is minimized when $\pi_{t-1}^B = (1-s)\pi_{t-1}^S/s$ that implies $\delta\pi_{t-1}^B \leq 1-s$. But for these values

$$\begin{aligned} \pi_1^B - (1-s)\pi_1^S/s - \delta b\rho\pi_{t-1}^S + \delta b\pi_{t-1}^B(1-\rho) &= \\ b\rho + (1-b)\theta - (1-s)(1-b(1-\rho)) &> 0 \end{aligned}$$

whenever $\theta > \rho > 1 - s$ and implies $\pi_t^B - (1-s)\pi_t^S/s > 0$ and $\delta S_{t+1} \leq 1$. This concludes the proof of *(i)*.

The proofs of *(ii)* and *(iii)* follow directly from Proposition 3 and

$$\theta \geq \overline{\theta}(\delta, b, \rho, s, T) \equiv \frac{1}{1-b}\left(\frac{1}{\delta^{T-1}} - s - b\rho\right). \tag{27}$$

Proof of Corollary 5

It follows immediately from differentiation of formulas (24) and (27).

Proof of Proposition 7

The recursive formulas in this setting are

$$\begin{aligned}
\pi_{t+1}^S &= \frac{1}{N}\sum_{i\in I^{t+1}}\left(\frac{1}{2}\left(V_i - \delta\pi_t^{Bi}\right) + \frac{1}{2}\delta\pi_t^S\right) + \frac{N - C^{t+1}}{N}\delta\pi_t^S \\
\pi_{t+1}^{Bi} &= b\left(\rho\left(V_i - \delta\pi_t^S\right) + (1-\rho)\delta\pi_t^{Bi}\right) + (1-b)\left(\frac{1}{N-1}\sum_{\substack{j\neq\widetilde{i}\\ j\in I^{t+1}}}\theta_j + \frac{N-C^{t+1}}{N-1}\delta\pi_t^{B\widetilde{i}}\right) \text{ if } i \in I^{t+1} \\
\pi_{t+1}^{Bi} &= \delta\pi_t^{Bi} \qquad \text{if } i \notin I^{t+1}.
\end{aligned}$$

Notice that for both formulas it is possible to characterize the following upper bounds

$$\begin{aligned}
\pi_{t+1}^S &\leq \frac{V_{\widetilde{i}}}{2} + \frac{\delta\pi_t^S}{2} \\
\pi_{t+1}^{B\widetilde{i}} &\leq b\left(\rho\left(V_{\widetilde{i}} - \delta\pi_t^S\right) + (1-\rho)\delta V_{\widetilde{i}}\right) + (1-b)\left(\frac{1}{N-1}\sum_{\substack{j\neq\widetilde{i}\\ j\in I^{t+1}}}\theta_j + \frac{N-C^{t+1}}{N-1}\delta\pi_t^{B\widetilde{i}}\right) \text{ if } i \in I^{t+1}
\end{aligned}$$

and that focusing on the largest $\widetilde{i}$ guaratees that $V_{\widetilde{i}} \geq \delta\pi_t^S$. Trade with buyer $\widetilde{i}$ occurs as long as $S_{t+1} \leq V_{\widetilde{i}}/\delta$. Using these upper bounds it is possible to see that

$$S_{t+1} \leq \frac{V_{\widetilde{i}}}{2} + \frac{\delta\pi_t^S}{2} + b\left(\rho\left(V_{\widetilde{i}} - \delta\pi_t^S\right) + (1-\rho)\delta V_{\widetilde{i}}\right) + (1-b)\left(\frac{1}{N-1}\sum_{\substack{j\neq\widetilde{i}\\ j\in I^{t+1}}}\theta_j + \frac{1}{N-1}\delta V_{\widetilde{i}}\right)$$

whose right hand side, as b gets large, converges to

$$\begin{aligned} S_{t+1} &\leq \frac{V_{\tilde{i}}}{2} + \frac{\delta\pi_t^S}{2} + \rho\left(V_{\tilde{i}} - \delta\pi_t^S\right) + (1-\rho)\delta V_{\tilde{i}} \\ &\leq \frac{V_{\tilde{i}}}{2} + \rho V_{\tilde{i}} + (1-\rho)\delta V_{\tilde{i}} \\ &< \frac{V_{\tilde{i}}}{\delta}. \end{aligned}$$

whenever $\delta < 2/3$. Because the two upper bounds are polynomials in b the inequality is not satisfied only at the limit but also for b large enough. So for each t it is possible to compute the minimum value $\widetilde{b}_t$ for which the inequality is satisfied. Setting $\widetilde{b}$ equal to the maximum $\widetilde{b}_t$ we obtain the result in the proposition.

Proof of Proposition 8

If the game reaches stage 1 trade is going to occur with the valuable buyer and the expected payoffs for the seller and the buyers are:

$$\begin{aligned} \pi_1^S &= 1/2 \\ \pi_1^B &= b\rho + (1-b)\theta. \end{aligned}$$

So, at time 2 the seller will approach the valuable buyer and agreement will occur if $\delta S_1 \leq 1$ or $\theta \leq \underline{\theta}(\delta, b, \rho)$ where

$$\underline{\theta}(\delta, b, \rho) \equiv \frac{1}{(1-b)}\left(\frac{1}{\delta} - \frac{1}{2} - b\rho\right). \tag{28}$$

Assume now that $\delta S_t \leq 1$ for a generic $t < T$ and that trade occurs in any period before t. At period $t+1$, before the identity of the valuable buyer is revealed, the expected payoff of the seller and the buyers are:

$$\pi_{t+1}^S = \frac{1}{2}\left(1 - \delta\pi_t^B\right) + \frac{1}{2}\delta\pi_t^S \tag{29}$$

$$\pi_{t+1}^B = b\rho\left(1 - \delta\pi_t^S\right) + b(1-\rho)\delta\pi_t^B + (1-b)\theta \tag{30}$$

that imply

$$S_{t+1} = S_1 + \delta S_t\left(\frac{1}{2} - b\rho\right) - \delta\pi_t^B\left(1-b\right). \tag{31}$$

The recursive condition is the same as the one of the baseline model. Moreover,as in the baseline model, if $\theta > \underline{\theta}(\delta, b, \rho)$ we denote as $\widetilde{t}$ the minimum time for which $\delta^{\widetilde{t}-1}\left(\pi_1^S + \pi_1^B\right) \leq 1$. If $\widetilde{t} < T$ then agreement occurs at $\widetilde{t}$ at the latest and $\widetilde{t} \geq T$ whenever:

$$\theta \geq \overline{\theta}(\delta, b, \rho, T) \equiv \frac{1}{1-b}\left(\frac{1}{\delta^{T-1}} - \frac{1}{2} - b\rho\right). \tag{32}$$

Proof of Proposition 9

At the last periods trade occurs with the valuable buyer because he is the only one with positive surplus. Without loss of generality let us refer to the buyer with the largest b_i as buyer 1. At period 2 (the one before the last) the seller will trade with buyer 1 if $\theta > \rho$ because in this case his expected payoff is going to be the smallest. Trade is going to occur as long as:

$$\theta < \frac{1}{(1-b_1)}\left(\frac{1}{\delta} - \frac{1}{2} - b_1\rho\right)$$

that is satisfied for δ small enough. We define as δ' the maximum value of δ for which the inequality is satisfied. Assume now that there is trade with buyer 1 at period $t-1$. The expected payoffs from trading in period t with the same buyer is:

$$\begin{aligned} \pi_t^{B1} &= \rho\left(1-\delta\pi_{t-1}^{S}\right) + (1-\rho)\delta\pi_{t-1}^{B1} \\ \pi_t^{S} &= \frac{1}{2}\left(1-\delta\pi_{t-1}^{B1}\right) + \frac{1}{2}\delta\pi_{t-1}^{S} \\ \pi_t^{Bj} &= \theta \quad \text{for } \ j > 1. \end{aligned}$$

The seller will prefer trading with buyer 1 as long as $\pi_{t-1}^{Bj} > \pi_{t-1}^{B1}$ that is satisfied if

$$\theta > \rho\left(1-\delta\pi_{t-2}^{S}\right) + (1-\rho)\delta\pi_{t-2}^{B1}. \tag{33}$$

Notice that if $\theta > \rho$ there exists a δ'' such that the condition is satisfied for any $t > 2$ if $\delta < \delta''$. The proposition follows by setting $\widetilde{\delta}$ equal to the minimum between δ' and δ''.

Proof of Proposition 10

At the last period the seller is indifferent among the N buyers, therefore any randomization among them gives him the same payoff. This randomization generates a multiplicity of equilibria. If in the last period the seller does not randomizes and approaches buyer i then $\pi_1^{B_i} = \rho_i$ and $\pi_1^{B_j} = \rho_j$ with $j \neq i$. In the previous period the payoff of the seller will be $1-\delta\rho_i$ if he approaches buyer i and $1-\delta\theta$ if he approaches any other buyer. Because $\theta > \rho_i$ for each i, the seller prefers to approach in period 2 the buyer that will be approached in period 1. Trade will occur immediately as long as the discount factor is low enough so that $1-\delta\rho_i > \delta/2$. This shows that for every buyer i there exists an equilibrium in which the seller trades immediately with buyer i and players expect him to trade with buyer i in all future periods. Among these N equilibria there are the equilibria described in (i) and (ii). It is easy to see that equilibrium of type (i) is the most preferred by the seller among the N equilibria that do not involve randomization. Because any randomization at the last period would imply an expected payoff for buyers larger than $\delta\rho_i$ the equilibrium is also the most preferred by the seller among all the equilibria of the game involving randomization.