From Defined Benefit to Defined Contribution and Back: A Derivative Valuation of the Florida Pension Election

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Abstract

During the year 2002, The State of Florida's 600,000 public employees were given the unprecedented ability to convert their traditional Defined Benefit (DB) pension plan, into a self-managed Defined Contribution (DC) plan, where the individual controls asset allocation and investment decisions. To mitigate some of the personal risk associated with investment self-determination, the State of Florida granted each and every employee electing the DC plan, the right but not the obligation, to switch back into the DB plan at any point prior to retirement. The strike price of this option-to-exchange is their accumulated benefit obligation (ABO) in the DB plan. Motivated by these reforms, our paper presents some analytic results relating to the optimal time, and financial value, of this unique pension guarantee. We start with a simple deterministic model to provide intuition, and conclude with a full-fledged stochastic model that derives a formal economic value. Our paper goes beyond some of the recent literature by providing simple analytic guidance on the optimal time to 'elect'. Our main conceptual contribution is that we conclude that this guarantee behaves less like a traditional financial option and more like a forward contract. Furthermore, we conclude that the value of the so-called option is at most worth 30% of the DC contribution rate; and only when exercised at the 'optimal time'. Furthermore, for most Florida State employees above the age of 45, the guarantee has little economic value, since the DB plan dominates the DC plan from day one. Of course, it remains to be seen what percent of Florida's 600,000 state employees will 'elect' to behave rationally with their newfound pension autonomy.

1 The Floridian Pension Option:

During the year 2002, The State of Florida's 600,000 public employees were given the unprecedented ability to convert their traditional Defined Benefit (DB) pension plan, into a self-managed Defined Contribution (DC) plan, where the individual controls asset allocation and investment decisions. This new Public Employee Optional Retirement Program (PEORP) has been the focus of intense scrutiny by local and national media because it is the largest such pension conversion in the history of the U.S. and is being viewed by some observers as a potential laboratory for Social Security reform.

To mitigate some of the personal risk associated with investment self-determination, the State of Florida granted each and every employee electing the DC plan, the right but not the obligation, to switch back into the DB plan at any point prior to retirement. The strike price of this 'option to return' is the employee's accumulated benefit obligation (ABO) in the DB plan. The ABO is essentially the present value of that portion of the life annuity (pension) to be received at retirement, based on the number of service and salary at the time of computation. For future employees – i.e. those not in the plan at the time the PEORP was initiated – the buy back price will be the accumulated actuarial liability (AAL).

Motivated by these reforms, our paper presents some analytic results relating to the optimal time, and financial value, of this unique pension option; which is labeled the *2nd election*. We start with a simple deterministic model to provide intuition, and conclude with a full-fledged stochastic model that derives a formal option value. We are careful to distinguish between the financial 'economic value' of the option to the employee – which is the focus of this paper – versus the more vague and controversial pension 'funding cost' of providing the option to the employee. While the former is related to portfolio replication and dynamic hedging of guarantees, the latter depends on various actuarial standards of practice, assumptions and cost methods that are beyond the scope (and interest) of this analysis. We refer the interested reader to the work by Haberman and Sung (1994) as well as O'Brien (1986) for stochastic models of pension plans that are focused on actuarial funding methods. We will return to the important distinction between *economic value* and *funding costs* as it related to the option value, later in the paper.

Our paper goes beyond some of the recent academic literature on pension choices and guarantees – for example Sherris (1995), Pennacchi (1999) and Lachance, Mitchell and Smetters (2002), as well as Childs, Fore, Ott, and Lilly (2002) – by providing simple analytic guidance on the optimal time to 'elect' to switch from a DB to a DC plan, and vice versa. We make (what we believe are) some innocuous and simplifying assumptions – that can be changed from liberal to conservative depending on the need – in order to shed intuitive light on the essence of the decision. A by-product of our analysis is a sound pedagogical tool that explains the risks and rewards from the various choices.

Furthermore, in contrast to the estimates by Lachance, Mitchell and Smetters (2002), we conclude that the *economic value* of the Florida pension option is at most 30% of the DC plan contribution rate; and only when it is exercised at the 'optimal time'. In other words, a rational Floridian would be willing to forfeit the option in exchange for a 30% increase in the DC contribution rate. This 30% is an upper bound which only applies to (very) young participants. Indeed, for most Florida employees above the age of 45, the *2nd election* option has little economic value, since the DB plan dominates the DC plan from day one. To be clear, we are not saying that Floridian's above the age of 45 should not be opting into the State's DC plan. Rather, for those that intend to retire (with 100% certainty) from the State plan, the DB choice is optimal, since they will accumulate greater retirement wealth.

From a broader academic literature perspective, this paper attempts to bring the tools of financial economics and No Arbitrage valuation to the field of pensions, retirement planning and longevity insurance. This is in the same spirit as recent papers by Boyle and Hardy (2002) as well as Sundaresan and Zapatero (1997).

1.1 Agenda for the paper:

The next section develops a complete intuition for the deterministic case, where interest rates, investment returns, retirement dates and employment horizons are known with perfect certainty. We then move to the stochastic case where, interestingly, we demonstrate that the linearity of the payoff structure collapses the problem to the results applicable under the deterministic case. The subsequent section provides some numerical examples and actual data from the State of Florida. We refer the interested reader to the work by Bodie (1990) as well as Bodie, Marcus and Merton (1988) for a refresher on the basic differences between defined contribution and defined benefit pension plans

2 Deterministic Case:

We start by assuming that all economic parameters are known with perfect certainty. We let C_t denote the value of the Defined Contribution (DC) account at time t, and let B_t denote the value of the accumulated benefit obligation (ABO) – or the accumulated actuarial liability[1] (AAL) – in the Defined Benefit (DB) plan, at time t. While the ABO and AAL are distinct quantities, most of our comments and model apply to either metric (buy back price).

Our underlying framework revolves around a participant currently aged$-y$, with τ years

[1]The AAL captures an average contribution rate on the part of the plan sponsor, and is therefore only loosely tied to any one individual's pension benefit.

of service, that has just been given the choice at time $t = 0$, (upon launching the PEORP) to move to the DC plan. The participant is also granted a 2nd election to buy back into the DB plan – if they actually left the DB plan to begin with – at any future switching time s, where $s \leq T$, which is the retirement horizon. Our main focus of analysis is two-fold. *Should the participant make the first election to switch into the DB plan? And if so, when should they elect – if ever – to switch back?*

Exhibit #1 provides a graphical illustration of the various decision points. The participant starts in the DB plan when the PEORP is inaugurated and must decide if to switch into the DC plan (today). Assuming the participant does decide to elect the DC plan, he or she has the option to make the *2nd election* and return to the DB plan anytime prior to retirement. If, on the other hand, the participant decided not to elect the DC plan at inception of the program, they have a second (and last) chance to switch to the DC plan prior to retirement.

There is an alternative choice – that we do not explicitly address here – in which the employee is given the option to leave the vested benefits in the DB plan, and direct new contributions only, into the DC plan. As we shall see later, this strategy is implicitly dominated by one of the two alternatives. In other words, either it is optimal to *switch* everything into the DC plan, or it is optimal to *keep* everything in the DB plan.

Formally, given the availability of the 2nd election (a.k.a. buy-back) option, at retirement time T, the participant will accumulate a total wealth of:

$$W(s) = B_T + (C_s - B_s)e^{\mu(T-s)}, \tag{1}$$

where μ denotes the instantaneous (deterministic) investment return (a.k.a. force of interest) in the DC account. The wealth at retirement, $W(s)$, is then converted into actual retirement income by computing $W(s)/a_x$, where x, is the retirement age and a_x is the relevant actuarial annuity factor which provides the indexed DB pension. For example, if the unisex price of a (3% or CPI indexed) \$1 of income per year for life, starting at age $x = 60$, in an $\rho = 8\%$ (valuation) interest rate environment is $a_x = \$14.75$, then a participant with $W_T(s) = \$100,000$ in his or her retirement account, would be entitled to a pension of $100,000/14.75 = \$6,779$ per annum in real terms.

Without any loss of generality, we assume that an optimal $s = 0$ implies that *either* the participant immediately switches back to the DB account, or, never bothers electing the DC account (PEORP Plan) to begin with. Note, also, that if the (2nd election) option is *not* available, the participant's wealth at retirement will simply be $W = \max[C_T, B_T]$, since in a world of perfect determinism, the participant will select the plan with the greatest value at retirement.

Thus, our metric for the value of the Florida option to switch – in a deterministic setting, of course – is the proportional increase in retirement wealth that is obtained by exercising the option. In this spirit, we define the quantity:

$$v(s) := \frac{W(s) - \max[C_T, B_T]}{\max[C_T, B_T]} = \frac{B_T + (C_s - B_s)e^{\mu(T-s)}}{\max[C_T, B_T]} - 1, \tag{2}$$

assuming the election is made at time s. Quite naturally, if $s = T$, the value of $v(T) = 0$, since there is no gain from switching. We will return to the behavior of $v(s)$ – as a function of the various underlying parameters – later in the analysis.

An alternative representation of the value of the option, is to locate the contribution rate in the DC plan that would generate the same level of retirement wealth, assuming the second election (option) were not available.

One point worth noting is that at retirement, which is at time T and age x, we do not distinguish between accumulated wealth in the DC account, and the value of the DB account. In other words, throughout this analysis we are implicitly assuming that pure economic value is all that matters. This is regardless of whether the funds are sitting in a DB or DC account. Of course, participants with a very strong (psychological) preference for a lump sum benefit, will subjectively value a dollar in the DC account – which does not force annuitization – much higher than a dollar in the DB account. The value of life annuities versus systematic withdrawal plans has been the subject of intense research in the academic literature, and we refer the interest reader to the recent paper by Brown and Poterba (2000) where they discuss the welfare gains from annuitization, and how intra-family risk sharing might influence the choice of particular pension (annuity) benefits. We also recommend the related papers by Benartzi and Thaler (2001, 2002) for a discussion of some of the behavioral aspects of pension choice.

Likewise, we abstract from group, versus individual annuity, pricing – *vis a vis* the potential anti-selection issues – by treating the actuarial factor a_x, independently of whether it originates in the DC or DB plan.

In the same spirit, we ignore mortality – during the employment years – as well as possible termination probabilities, disability, etc. These assumptions are justified because we are focusing on the *economic value* of the option to the rational employee who expects to work in the Florida Retirement System (FRS) for his or her entire life. Indeed, for those with a shorter employment horizon the optimal policy will differ, but the economic value of the 2nd election option will be lower as well. In contrast, the actuarial funding method would take account of retirement probabilities, mortality, disability and other employment decrements. We view the economic value – which does not account for any termination probabilities – as an upper bound on the funding cost. We repeat, once again, that our optimality criteria is predicated on a participant that wants to maximize the 'option' value, as opposed to being concerned with early termination and liquidity concerns. *These real* *world, and risk averse concerns, would only serve to reduce the cost of providing the option to the employee.*

2.1 The Dynamics and Account Value of C_s and B_s

Both C_s (the funds in the individual DC account) and B_s (the present value of the DB benefit) evolve at different rates over time. The accumulated actuarial liability (AAL) and the accumulated benefits obligation (ABO) will also have completely different dynamics, and thus the optimal time to switch will depend on the relevant price to re-enter. It is a fact that the B_s value (price) for the ABO will be much lower than the B_s value (price) for the AAL. For now we will focus on the ABO case, which as we mentioned earlier, would be applicable to all participants that joined the Florida plan prior to the implementation date.

2.1.1 The Dynamics of C_s

First, we examine the Defined Contribution individual account. Let I_0 denote the salary (income) at time $t = 0$ – i.e. the current point in time at which the analysis is being conducted – and assume it grows at a (constant) rate of g per annum. The employer contributes to the DC account at a rate of $\widehat{c}$ percent of salary per annum, which then translates into a nominal dollar contribution of:

$$c_s := \widehat{c} I_0 e^{gs}, \tag{3}$$

per annum. For example, if the (current) initial salary is $I_0 = \$30,000$, the growth rate is $g = 4.75\%$, and the contribution rate is $\widehat{c} = 9\%$, then in the tenth year, the employer's contribution would be approximately $c_{10} = \$4,342$, in nominal dollars. If the contributions are invested in an account earning μ per annum, the accumulated value of these contributions at time s, would be:

$$\begin{aligned} C_s &= C_0 e^{\mu s} + \int_0^s c_t e^{\mu(s-t)} dt = C_0 e^{\mu s} + \int_0^s \widehat{c} I_0 e^{gt} e^{\mu(s-t)} \\ &= C_0 e^{\mu s} + \widehat{c} I_0 e^{\mu s} \int_0^s e^{(g-\mu)t} dt = C_0 e^{\mu s} + \frac{\widehat{c} I_0 (e^{gs} - e^{\mu s})}{g - \mu}, \end{aligned} \tag{4}$$

where the (constant) symbol C_0 denotes the funds that are already in the DC account – as a result of past contributions and gains – at the current time $t = 0$. In the special case where $\mu = g$, equation (4) converges to a limiting value of $(C_0 + \widehat{c} I_0 s) e^{\mu s}$.

For example, if $C_0 = 5,000$, (current market value of the DC account), $\widehat{c} = 0.09$ (annual contributions equal to 9% of salary), $I_0 = 30,000$ (current salary of $30,000 per annum), $g = 0.0475$ (nominal salary increase of 4.75% per annum) and $\mu = 0.10$ (a 10% growth rate of investments in the DC account), then in ten years, the (projected) value in the DC account is $C_{10} = \$70,691$, and in twenty years the (projected) value is: $C_{20} = \$283,974$.

Finally, it is relatively easy to show – and will be of later use – that:

$$C'_s = c_s + \mu C_s, \tag{5}$$

where the prime symbol denotes a derivative with respect to the variable s. The rate of change in the value of the DC account, is the sum of the contributions to the DC account, c_s and the instantaneous investment growth of the DC account, μC_s.

2.1.2 The Dynamics of B_s

The evolution and economic value of the funds in the Defined Benefit account at time B_s, depend on the pension accrual rate $\widehat{b}$, the number of years of service $(s + \tau)$, the salary $I_0 e^{gs}$, and the present discounted value of the annuity factor $e^{-\rho T} a_x$. The valuation discount rate ρ is clearly not the risk-free rate of interest in the economy, and is the topic of some controversy in the pension literature. In the Florida State pension plan, ρ is currently 8% which is much higher than the 5% risk-free Treasury rate prevailing at the time of writing. It appears the pension plan discounts (values) liabilities at the expected rate of return on assets, which obviously depends on the equity/bonds asset mix held by the State. This approach is driven by the Governmental Accounting Standards Board (GASB 25) and is somewhat alien to financial economic valuation techniques. Nevertheless, it appears that a large fraction of public and private pension plans in the U.S. are currently discounting their actuarial liabilities at a flat 8% – and have been doing so for quite a while. We refrain from debating this particular controversy in the paper, and refer the interested reader to the work by Gold (2000) for a discussion of the issues surrounding the valuation of liabilities in publicly sponsored pension plans.

In either event, this valuation rate ρ is a critical input in the decision process, and the actual formula relating DB values is specified by:

$$B_s = \widehat{b}(s + \tau) \left(I_0 e^{gs}\right) \left(a_x e^{-\rho(T-s)}\right) = k(s + \tau) I_0 e^{s(g+\rho)}, \tag{6}$$

where I_0 is the current salary, and where the new variable $k := \widehat{b} a_x e^{-\rho T}$, which roughly captures the accrual rate of each pension dollar, for each year of service. It has similar characteristics to the DC contribution rate parameter, $\widehat{c}$, which we will discuss later. Technically, there is a 6 year vesting period during which the employee does not own the funds in the DB account, but this does not affect the optimal behavior in any meaningful way, since we can define years of service τ, in terms of pensionable years of service. Intuitively, the 6 year vesting period in the DB plan will induce employees to select the DC plan if their employment horizon is less than 6 years. However, once again, our representative agent is one that intends to remain within the Florida retirement system throughout his or her entire career

– which is the costliest for the State – and thus provides an upper bound for the aggregate option value.

Also, we deliberately abstract from reality – in addition to the various deterministic assumptions – by assuming that the DB pension is a function of the final year of pay, as opposed to the *average* of the final five years of pay. This assumption was made in order to gain intuition for the deterministic optimality criteria. However, in practice, it could be trivially dealt with by letting I_0 represent the *average* salary as opposed to the *current* salary in equation (6).

For example, imagine a $y = 40$ year-old employee with $\tau = 7$ years of service, currently earning $I_0 = \$30,000$, which is expected to grow by $g = 4.75\%$ per annum. The employee earns $\widehat{b} = 1.6\%$ of salary for each year of service, and plans to retire at age $x = 60$, which is in $T = 20$ years. In a $\rho = 8\%$ valuation environment, where the annuity factor $a_{60} = 14.75$, one obtains a current ABO of $B_0 = \$10,005$. The ABO upon retirement – assuming, once again, that all relevant – constants remain the same, will be $B_{20} = \$494,284$. This retirement value of the ABO can also be obtained by simply multiplying the 27 years of service, time 1.6% times the final salary of \$77,571 times the annuity factor of 14.75.

Upon introduction of the PEORP, the above-mentioned employee would be given the choice to transfer the $B_0 = \$10,005$ into the DC plan, thus earning a rate of return of μ on the initial transfer and all future contributions. The question becomes whether to move into the DC plan at all, as well as if-and-when to switch back.

Now, similar to the model for the evolution of the Defined Contribution account, we are interested in expressing the value of the DB account in terms of an instantaneous pseudo employer contribution, and interest earnings. The value of the DB account at any time s, should be the sum of the two components. We stress the word *pseudo* contribution, since the actual funding of the DB pension plan would depend on a variety of actuarial assumptions and methods – for example, the entry age cost method – and thus would not necessarily reflect the contribution of the employer *per se*. Nevertheless, we define a new variable b_s, akin to the variable c_s, to be:

$$
\begin{aligned}
b_s = B'_s - \rho B_s &= k I_0 e^{s(g+\rho)} + gk(s+\tau) I_0 e^{s(g+\rho)} \\
&= \left(\frac{1}{s+\tau} + g \right) B_s. \qquad (7)
\end{aligned}
$$

For any given time s, the instantaneous pseudo contribution b_s, is an increasing function of the service record τ, an increasing function of the salary growth rate g, and an increasing function as one gets closer to retirement, (i.e. as T decreases, since k will increase.) These observation are all consistent with basic intuition in the matter.

Finally, similar to the relationship between the DC contribution amount c_s, and the value of the DC account C_s in equation (4), one can easily show that:

$$B_s = B_0 e^{\rho s} + \int_0^s b_t e^{\rho(s-t)} dt, \tag{8}$$

which means that the value of the DB account (or the ABO) is the sum of all the pseudo contributions made on behalf of the participant – to date – accumulated at the valuation rate of interest ρ.

2.2 Maximizing Wealth

From a theoretical point of view, the objective of a (rational) participant should be to locate the *best* time s to switch, in order to maximize the value of wealth at retirement. In practice, of course, one might gain substantial utility from the non-monetary benefits of having a DC account over a promised DB pension, but we ignore those for now.

From a purely mathematical point of view, the optimal time to switch is located by differentiating equation (1) with respect to s, and then using the first order condition of $W'(s) = 0$, to isolate s^*. The asterisk atop of the s, distinguishes the general time s, from the optimal time s^*. In theory, we could do the same with equation (2), and locate the switching time that maximizes the value of the option. The algebra is identical since the optimal time will be invariant to scaling.

In any event, elementary calculus leads to:

$$W'(s) = (C'_s - B'_s)\, e^{\mu(T-s)} - \mu(C_s - B_s) e^{\mu(T-s)}. \tag{9}$$

The optimal switching point – when it is interior to the region $s \in [0, T]$ – is obtained at the point in time of s^*, for which:

$$W'(s^*) = (C'_{s^*} - B'_{s^*})\, e^{\mu(T-s^*)} - \mu(C_{s^*} - B_{s^*}) e^{\mu(T-s^*)} = 0. \tag{10}$$

This is the first order condition for optimizing the value of wealth at retirement.

Now, dividing both sides by common factors, substituting from equations (7) and (4), we get that s^* satisfies:

$$b_{s^*} - c_{s^*} = (\mu - \rho) B_{s^*} \tag{11}$$

The second order condition, namely that $W''(s^*) < 0$, must still be confirmed, but is a by-product of our analysis. Under normal parameter conditions, $W(s)$ starts off as a concave function, attains its maximum value at $W(s^*)$, and then starts to decline until it reaches a second (and final) point where $W'(s) = 0$, which is a local minima.

Note, from equation (11), that the optimal time to switch does not depend on the actual value of the DC account C_s. Some have erroneously suggested *heuristic* second election rules, based on the size of the account. For example, some have argued that a good candidate for the switching time, is when the ABO is about to overcome the money in the DC account. (See the FRS website, for example.) But this is a complete optical illusion since it is growth and investment returns – not size – that matter. Instead, it depends on the 'gap' between the pseudo contribution rate b_s on the DB account, and the actual contribution c_s on the DC account. Intuitively, the value of b_s starts out quite low, and then overtakes c_s. Once the 'gap' is large enough, it is optimal to switch. Furthermore, when $\mu = \rho$, which means that the valuation rate is equal to the rate of return in the DC investment account, the optimal time to make the second election, is as soon as b_s overtakes c_s.

Interestingly, in the event that $\rho > \mu$, for example when the valuation interest rate is unnaturally high, and the ABO price is artificially low, the optimal time to make the second election, is even earlier, which is when $b_{s^*} > c_{s^*}$.

One can re-write the critical first order condition, using equations (7) and (3), as:

$$\left(\frac{1}{s^* + \tau} + g\right) B_{s^*} - \widehat{c} I_0 e^{gs^*} = (\mu - \rho) B_{s^*}, \tag{12}$$

which, after dividing by B_{s^*}, and some algebra, leads us to:

$$\frac{1}{s^* + \tau} - \frac{\widehat{c}}{k(s^* + \tau)e^{s^*\rho}} = \mu - (\rho + g) \tag{13}$$

Finally, collecting terms – and waving our hands a bit – the optimality condition can be heuristically stated as the first value of s, for which:

$$\phi(s) := \frac{1 - \frac{\widehat{c}}{k} e^{-\rho s}}{(s + \tau)} \geq \mu - (\rho + g), \tag{14}$$

where, we remind the reader once again, that $k = \widehat{b} a_x e^{-\rho T} = \widehat{b} a_x e^{-\rho(y-x)}$.

Equation (14) contains the main contribution of this section, and possibly the paper, so we note the following observations.

- For younger employees the ratio $\widehat{c}/k > 1$. Thus, $1 - \widehat{c}/k < 0$, and $\phi(0) < 0$. In this case, the global (qualitative) behavior of the function $\phi(s)$ is as follows: It starts in negative territory – since $\phi(0) < 0$. It then increases and crosses zero once, before reaching a maximum value in the positive quadrant. Then, it starts to decline and eventually converges to zero in the limit. Figure #1 (in the appendix) illustrates the behavior of this function.

- If k is large relative to $\widehat{c}$ – which would be applicable to older employees with more service credit – there is a possibility that $\phi(s) \geq \mu - (\rho + g)$, for all values of s, which would imply that $s^* = 0$. The optimal policy is an immediate switch to (or staying in) the DB plan. This, once again, should make sense given the fact that longer periods of service are associated with a higher ABO and thus a greater incentive to remain in the DB plan.

- See Figure #1 and Figure #2 for a global and local picture of the behavior of $\phi(s)$, for three different examples. They are all based on an employee that has $\tau = 3$ years of service, who plans to retire at age $x = 60$. In the first example the employee is $y = 45$ years-old, and is $T = 15$ years from retirement. In the second example the employee is $y = 43$ years-old, and has $T = 18$ years to retirement, and in the third example the employee is $y = 40$ years-old, and has $T = 20$ years to retirement. The valuation rate is $\rho = 8\%$, the investment (expected) return within the DC plan is $\mu = 12\%$, the salary growth rate is $g = 4.75\%$, the DB rate is $\widehat{b} = 1.6\%$, and the DC contribution rate is $\widehat{c} = 9\%$ of salary.

- If, at some point over it's employment domain $s \in [0, T]$, the function $\phi(s)$ 'hits' the value of $\mu - (\rho + g)$, we have an interior solution to our first order condition, and that is the optimal time s^* to make the second election. This is consistent with our earlier observation in equation (11), that in most cases, the b_s (pseudo contributions to the DB plan) start at a lower value than c_s (contributions to the DC plan). The optimal switch (second election) point, occurs if and when, $b_s - c_s \geq (\mu - \rho)B_s$.

- We have completely moved away from expressing the optimal switching rule as a function of either the balance in the DC account C_s, or the actual value of the ABO, denoted by B_s. In fact, under our assumptions, the size or magnitude of the account is irrelevant to the decision. In fact, as we shall see clearly in the numerical examples, the optimal switching time s^*, occurs *prior* to the time when $C_s = B_s$. And, in some cases, by quite a number of years. Intuitively, the participant makes the second election – back into the DB plan – and keeps the difference $C_{s^*} - B_{s^*}$ invested and growing in the DC plan.

- All else being equal, the parameter value of $k = \widehat{b} a_x e^{-\rho T}$ will increase, the closer the participant is to retirement, since the implied value of T will be lower. Thus, a shorter time to retirement, will have the same impact at a longer service record τ. They both act to increase that value of the function $\phi(s)$, and thus accelerate the point at which the second election should be made.

- It is relatively easy to prove that $\phi(s) + \rho$ is an increasing function of the valuation interest rate ρ. Thus, the function $\phi(s)$ will shift closer to $\mu - (\rho + g)$, as ρ increases, and thus the switching time will be earlier. This, once again, should be consistent with intuition. As the valuation interest rate increases, there is a greater incentive to be in the DB plan.

- Finally, note from equation (14), that as long as $\phi(s) < \mu - (\rho + g)$, the participant is *better off* staying in the DC plan. Thus, one can invert equation (14), and express the entire decision as a function of μ, as opposed to a function of s. Stated differently, we can then locate the *required rate of return* needed to justify staying in the DC plan. This can be formally represented as:

$$\mu > \phi(s) + (\rho + g) \tag{15}$$

  As long as the value of μ is large enough, one should stay in the DC plan. But, as soon as μ *slips* under $\phi(s) + (\rho + g)$, it is optimal to switch. Recall that $\phi(s)$ starts at a negative value, and then moves into the positive region. At some point, $\phi(s) + (\rho + g)$ might exceed μ. If this occurs within the range $s \in [0, T]$, the second election is made. If, this occurs, mathematically, outside the range $s \in [0, T]$, or if this never occurs, the participant should stay in the DC plan until retirement.

As a prelude to the stochastic model – in which the investment return in the DC account is uncertain – one can envision giving advice in the following manner. At each point in time, the participant in the DC plan is given a threshold investment return μ that is required to justify staying in the DC plan. For example, he or she might be told that "you must earn at least 6% in your DC account to justify staying in the DC plan for the next month". Table #1 (in the appendix) provides some numerical estimates of this 'threshold return' under a variety of entry ages and initial service credit years.

In the same manner, the participant could be given a statistically (simulated) distribution for the (instantaneous) return in their DC account, from which they can observe the probability of earning the threshold return. Thus, on a more sophisticated level, one might be informed that "There is a 72% chance your portfolio will earn enough to justify staying in the DC plan".

2.3 The AAL and New Employees

While most of the above analysis centres around employees currently in the Florida retirement system – for whom the price of returning is based on the ABO - a similar analysis can be conducted for new employees and the accumulated actuarial liability (AAL). Without getting into the technical details, we can model the evolution of the *price of re-entry* into the DB plan, by modeling the (new) B_s as an integral of the (new) b_s. Obviously, some of the subjectivity involved in the actuarial method of funding would reduce the precision by which we could project B_s. In any event, we would then locate the optimal time s, that maximizes equation (1), using the new definition of B_s and b_s. In fact, this would lead to a first order condition that is identical to equation (11). Our numerical results indicate the given the high cost of the AAL, for somebody in the DC plan, the optimal time to make the second election – if at all – is right before retirement. We deliberately refrain from pursuing this line of research since our objective is to analyze the option value for the current 600,000 employees, and because the AAL is highly dependent on (subjective) actuarial assumptions.

2.4 Numerical Examples.

We now present some actual numbers to get a practical sense of the optimal timing and value of Florida's *2nd election* option. Our parameters are mostly based on actual Florida State actuarial assumptions.

2.4.1 Case #1.

We start with a $y = 40$ year-old employee with $\tau = 7$ years of service, currently earning $I_0 = \$30,000$, which is expected to grow by $g = 4.75\%$ per annum. The employee earns $\widehat{b} = 1.6\%$ of salary for each year of service, and plans to retire at age $x = 60$, which is in $T = 20$ years. In a $\rho = 8\%$ valuation environment, where the annuity factor is $a_{60} = 14.75$, one obtains a current ABO of $B_0 = \$10,005$. All of these numbers are grounded in the current economic variables used by the Florida Retirement System.

This employee rationally makes the initial election to switch into the DC plan, and rightfully transfers the ABO of $\$10,005$ into the DC plan, so that $C_0 = \$10,005$, as per the discussion in the earlier section. Accordingly, if we assume a $\widehat{c} = 9\%$ of pay contribution rate, and an investment return of $\mu = 8\%$ (same as the valuation rate) in the DC account, the retirement value of the DC account will be $C_{20} = \$246,225$, assuming a second election is not made. If, however, employee were to remain in the DB plan, the retirement value of the ABO would be $\$494,284$, as we calculated earlier.

Thus, without the availability of the second election, it would be suboptimal – under these parameter values – to ever elect the DC plan, since $\$246,225$ is clearly less than $\$494,284$. However, with ability to make a second election, it initially becomes optimal to move into the DC plan, and then return at time $s^* = 3.06$ years. This number comes directly from equation (14). The participant will then rightfully spend the last 17 years of his or her working life in the DB plan. Note that at time $s^* = 3.06$, the value of the ABO will be $B_{3.06} = \$21,242$, while the money in the DC account will have grown to $C_{3.06} = \$22,827$. The remaining $\$$ $1,585$ in the DC account upon the second election, will grow during the remaining 17 years, to $\$6,146$. The retirement wealth will consist of the DB account $\$494,284$, plus the left over money in the DC plan; all of which accumulates to $W(3.06) = \$500,430$.

Note, once again, that the optimal time to switch is not when the value of the ABO is about to exceed the value of the DC account. In fact, that would not occur until $s = 6$ years. Indeed, if the participant were to wait until $s = 6$ to make the second election, he or she would be no better off than staying in the DB plan to begin with, since $B_6 = C_6 = \$40,000$. Indeed, the value of the option comes from being able to buy into the ABO, *and* keep some leftovers. The greater the leftovers, and the longer the period over which it accumulates, the higher the value of the option.

To be more precise, and consistent with our definition in equation (2), the value of the option is:

$$v(3.06) = \frac{\$500,430}{\$494,284} - 1 = 1.24\%. \tag{16}$$

This captures the relative gain from being able to move into, and then out of, the DC plan. Notice that the option value is relatively small at 1.24% of DB retirement wealth.

We emphasize, once again, that were it not for the ability to make the second election, it would be suboptimal for this employee to participate in the DC plan to begin with. The reason is that with $\tau = 7$ years of service, and a relatively low $\mu = 8\%$ (subjective estimate for the) rate of return in the DC plan, there is no reason to leave the DB plan.

2.4.2 Case #2.

We continue with the same employee set-up, except that in this case we assume the DC account balance will grow at $\mu = 12\%$, instead of 8%. Thus, while the retirement value of the ABO remains at $B_{20} = \$494,284$, the retirement value of the DC account, assuming the initial ABO of $\$10,006$ – and all future contribution – are invested in the plan, is $C_{20} = \$424,520$. Once again, in the absence of the second election, it would be suboptimal to ever move into the DC plan. However, once the second election is granted, the optimal time to switch back is (later) in $s^* = 6.725$ years. In this case, retirement wealth will be $w(6.725) = \$535,542$, and the so-called option value is a more substantial:

$$v(6.725) = \frac{535542}{494284} - 1 = 8.3\% \tag{17}$$

Upon the second election, the value of the DC account is $C_{6.7} = 54,630$, while the value of the ABO in the DB account is $B_{6.7} = 46,244$.

2.4.3 Case #3.

We assume an employee age $y = 36$, with $\tau = 1$ year of service and an expected retirement age of $x = 60$, or in $T = 24$ years. The employee is currently earning $I_0 = \$30,000$, which is expected to grow by $g = 4.75\%$ per annum. The employee earns $\widehat{b} = 1.6\%$ of salary for each year of service. In a $\rho = 8\%$ valuation environment, when the annuity factor $a_{60} = 14.75$, one obtains a current ABO of $B_0 = \$1,038$. If this employee stays in the DB plan until retirement, his or her ABO will be $B_{24} = \$553,438$. If this employee decides to participate in the PEORP and move his ABO to the DC plan, his retirement wealth will be $C_{24} = \$565,474$, in a $\mu = 12\%$ environment.

This employee should therefore switch into the DC plan, and then return at $s^* = 10.88$. His total retirement wealth will be $w(10.88) = \$695,555$, which leads to an option value of:

$$v(10.88) = \frac{\$695,555}{\$565,474} - 1 = 23\% \tag{18}$$

One way to put this option value in context, is that to create the same level of wealth at retirement – without the buyback option – the employer would have to increase the contribution rate by 23%, i.e. from 9% of current salary to 11.1% of current salary.

2.4.4 The Hurdle Rate approach for the Optimal Switching Time

Consistent with the earlier idea inverting our optimality condition, and expressing the optimal (second election) switching time as a function of μ, we offer the following Table #1. The assumptions are identical to the earlier cases. The employee earns $\widehat{b} = 1.6\%$ of salary for each year of service. In a $\rho = 8\%$ valuation environment, when the annuity factor $a_{60} = 14.75$. The DC contribution rate is $\widehat{c} = 9\%$, and the retirement age is $x = 60$.

For example, a 35 year-old employee that joined the Defined Contribution plan at age 30 – who had 5 years of service at that time – would have to earn at least 6.8% to justify staying in the DC plan. However, once this same employee reaches age 50, the required threshold increases to 13.3%, which is more difficult to achieve, and will have a higher probability of failure. Notice that at young enough ages, the threshold investment return is *negative!* This is because the 9% contributions of the employer to the DC plan are so high, relative to the pseudo contributions to the DB plan, that a participant would be able to lose money in the DC account, and still come out ahead.

We must reiterate, once gain, that in the simplified model of this section, we are not taking into account the extra risk that may be incurred in the DC portfolio. As a practical matter the employee would have to consider any additional risk taken to earn the higher threshold investment return, in addition to their capacity for borrowing outside of the DC plan. Thus, for example, *even* if the employee were confident that they could earn the threshold return described above, it is by no means clear that they should continue in the DC plan. They may be better off switching into the DB plan, and transferring other investments, or borrowed funds, into the riskier assets with the higher (subjective) expected returns. And, for an employee who is liquidity constrained and can not borrow, they too might be better off switching to avoid the excessive risk which is present in the DC plan.

Indeed, all of these factors are taken into account in Section 3, where we discuss the more realistic stochastic approach, which leads to a risk-neutral valuation.

2.4.5 The Impact of μ on the Value of the Option.

Interestingly, when we raise the investment return in Case #3, to $\mu = 14\%$, we obtain that although $C_{24} = \$778,948$, $s^* = 14.67$ years, and $W(14.67) = \$817,514$, the actual option value, or increase in final wealth from having access to the second election, is only $v(14.67) = 4.9\%$, which is much lower than for the case $\mu = 12\%$. At first, this might seem puzzling. If the return from the DC account is greater, then how the can the value of the option value to switch back, be lower? However, this apparent paradox is resolved by realizing that although the final value of W is much higher, compared to when $\mu = 12\%$, the incentive to *ever leave* the DC plan is reduced as well. In fact, when $\mu = 16\%$, which is unrealistically high, of course, it is never optimal to make the second election. Rather, the employee stays in the DC plan until retirement. This is a direct result of equation (14). When, $\mu - (\rho + g)$ is large enough, the function $\phi(s)$ will never reach $\mu - (\rho + g)$ in the time interval $s \in [0, T]$.

Indeed, on a global level, there is a relatively narrow range of returns for which the second election option has value. If μ is large enough, the participant switches to the DC plan, and remains there until retirement. In Case #3, the threshold μ for never switching back, would be 15.3%.

In the extreme case, an employee with $\tau = 1$ years of service, who is currently aged $y = 31$, and who expects to retire at age $x = 60$, would make the second election at time $s^* = 13.43$ (which is age 43) and would derive an option value of $v(13.43) = 30.8\%$. In other words, his final retirement wealth would increase 30.8% by electing to switch at the optimal time. Once again, this translates into an equivalent DC contribution rate of $\widehat{c} = 10.75\%$.

In sum, we are hard pressed to find realistic cases in which the option value is worth more than 30%, or a case for which the equivalent defined contribution rate is much higher than $\widehat{c} = 11\%$.

3 Stochastic Case: Dealing with Uncertainty.

There are two distinct methods of dealing with the case of parameter uncertainty, for selecting the best time to switch.

3.1 Practical Method.

The first is not really a method, but more of a pedagogical solution. It involves simulating the evolution of the DC portfolio during the next instant, and then illustrating the probability it will exceed the return threshold μ, needed to justify remaining in the DC plan. This is consistent with our earlier discussion following equation (14). The computational algorithm must *solve for* (and show) the threshold return investment required to stay in the DC plan during the next instant in time. Then it must compute the probability of actually earning that return. The individual would then decide whether to stay – or elect to return – based on the simulated probabilities and their own risk aversion.

3.2 Financial Economic Equilibrium

A more rational financial economic valuation approach, is to locate the optimal time to switch that maximizes the value of the 2nd election option, akin to an optimal stopping problem. Interestingly, as we shall see in a moment, the payoff structure is linear which greatly simplifies the analysis, and actually collapses the problem to the deterministic case.

Before we embark on this path, a few points are in order.

- We abstract from asset allocation issues, by assuming the employee has a pre-existing portfolio outside the pension system. This portfolio will be used to supplement the purchase of the ABO benefit (if needed) and will also be used to fine-tune the asset mix between risky and risk-free investments. In other words, we assume the participant is not liquidity constrained and can therefore invest in (and sell from) the same assets that are available in the DC plan.

- We emphasize that the valuation rate ρ is quite distinct from (and likely higher than) the risk-free rate. Thus, part of the benefit of participating in the DB plan is that benefits grow at a (artificially) higher rate. As we illustrated earlier, the greater the valuation rate ρ, the greater the incentive to move (back) into the DB plan.

- We now make the critical argument that in economic equilibrium, the actual drift, or expected investment return μ within the DC plan, will *not* impact the optimal time to make the second election. This is because the opportunity cost of funds should be treated as true risk free-rate r. In essence, we are adopting the classical risk-neutrality argument from option pricing, and will locate the optimal time to switch that will maximize the risk-neutral expected payoff at retirement.

- Once we are in a risk-neutral framework, all that matters is the expected wealth at retirement, assuming all DC assets are invested in the risk-free rate. While this might seem perverse at first reading – and one might feel more comfortable maximizing end of period utility – we believe that μ should not impact the optimal time to switch. This is quite different from the deterministic case where μ was central to the optimal policy, and the greater it's value, the greater the incentive to stay in the DC plan. In the full equilibrium case, one can always access an investment with an expected rate of return μ outside the pension plan. Thus, if μ is relatively high, one can rebalance assets outside the plan, to take advantage of the risk-premium. The decision to switch should be based on the time value of money, which is dictated by r.

Assuming the reader accepts these argument, we can proceed in a manner that is similar to the deterministic case. The expected wealth at retirement – which is the payoff from the option plus underlying security – assuming the second election is made at time s, is:

$$\begin{aligned} E_Q\left[\widetilde{W}(s)\right] &= E_Q\left[\widetilde{B}_T + (\widetilde{C}_s - \widetilde{B}_s)e^{(\widetilde{X}_T - \widetilde{X}_s)}\right] \\ &= E_Q\left[\widetilde{B}_T\right] + E_Q\left[\widetilde{C}_s e^{(\widetilde{X}_T - \widetilde{X}_s)}\right] - E_Q\left[\widetilde{B}_s e^{(\widetilde{X}_T - \widetilde{X}_s)}\right] \\ &= E_Q\left[\widetilde{B}_T\right] + E_Q\left[\widetilde{C}_s e^{(\widetilde{X}_T - \widetilde{X}_s)}\right] - E_Q\left[\widetilde{B}_s\right] E_Q\left[e^{(\widetilde{X}_T - \widetilde{X}_s)}\right]. \end{aligned} \tag{19}$$

The tildes denote the random nature of the pension quantities, $E_Q[.]$ denotes expectations under the risk-neutral measure, the new variable $\widetilde{X}_t$ is a stochastic process capturing the investment returns in the DC account, and the final equality comes from the independence assumption between the DB account value, and the behavior of the investments in the DC account.

Consistent with the classical option pricing literature, as originally employed by Black and Scholes (1973), Margrabe (1978) and Fischer (1978), we will assume that $\widetilde{X}_t$ is a non-standard Brownian motion, with drift $r - 0.5\sigma^2$ and volatility, or standard deviation, σ. Algebraically:

$$\widetilde{X}_t = (r - 0.5\sigma^2)t + \sigma B_t, \tag{20}$$

so that the process $e^{\widetilde{X}_t}$ is a Geometric (exponential) Brownian Motion. The drift term in equation (20) is consistent with techniques and notation from stochastic calculus, so that we end-up with a risk-neutral expectation of:

$$E_Q[e^{(\widetilde{X}_T - \widetilde{X}_s)}] = e^{r(T-s)}. \tag{21}$$

See Hull (2002, Chapter 11) for more details about the underlying stochastic process used in option pricing theory. Nevertheless, as before, the value of the DC account can be modeled by:

$$\widetilde{C}_s = \widetilde{C}_0 e^{\widetilde{X}_s} + \int_0^s \widehat{c}_s e^{(\widetilde{X}_s - \widetilde{X}_t)} dt, \tag{22}$$

and the value of the DB account will be:

$$\widetilde{B}_s = \widetilde{B}_0 e^{\rho s} + \int_0^s \widehat{b}_s e^{\rho(s-t)} dt. \tag{23}$$

If we further make the assumption that valuation rates (ρ), salary growth rates (g) and accrual rates ($\widehat{b}$) are constant, the expected value of the Defined Benefit ABO is the same as in the deterministic case, which was presented in equation (6).

Now, by combining equation (19) and equation (22), we obtain:

$$\begin{aligned} E_Q\left[\widetilde{C}_s e^{(\widetilde{X}_T - \widetilde{X}_s)}\right] &= E_Q\left[\int_0^s \widehat{c} I_0 e^{gt} e^{(\widetilde{X}_T - \widetilde{X}_t)} dt\right] \\ &= \int_0^s \widehat{c} I_0 e^{gt} E\left[e^{(\widetilde{X}_T - \widetilde{X}_t)}\right] dt = \int_0^s \widehat{c} I_0 e^{gt} e^{r(T-t)} dt \\ &= e^{rT} \int_0^s \widehat{c} I_0 e^{(g-r)t} dt = \frac{\widehat{c} I_0 e^{rT}}{g-r}\left(e^{(g-r)s} - 1\right) \end{aligned} \tag{24}$$

The second equality, which interchanges the expectation and integral, is allowed by Fubini's Theorem. In essence, we are computing the risk-neutral expectation of an Asian option with a zero strike price, which is essential an Asian forward contract. See Milevsky and Posner (1998) for more information on Asian options and the techniques for computing the relevant expectations. This leaves as with a value for $E_Q[\widetilde{W}(s)]$ that is identical to the deterministic situation in equation (1) and (9).

In other words, the optimal financial economic policy in the presence of stochastic investment return is identical to the deterministic case, but with μ replaced with the risk-free (opportunity cost) rate r.

Formally, the optimal 2nd election switching time (a.k.a. stopping time) is the first time that s satisfies:

$$\phi(s) := \frac{1 - \frac{\widehat{c}}{k} e^{-\rho s}}{(s+\tau)} \geq r - (\rho + g). \tag{25}$$

Those familiar with the financial option pricing literature might wonder where the volatility σ is taken into account in the option value and optimal policy. The fact is, we are not really dealing with a traditional non-linear payoff structure. Rather, our payoff profile is linear in the underlying stochastic variable – akin to a futures contract – and the option value is invariant to return volatility. The intuition for this follows directly from the strike price of this so-called option. The cost of making the 2nd election – which is the ABO – is the same regardless of the performance of the underlying DC account value. Thus, the participant (option holder) is not shielded from downside risk and participates dollar for dollar in any market declines. This is quite different from a true option where any fluctuations in the underlying security, beyond the strike price, have no impact on the holders welfare.

In summary, assuming a constant interest rate, valuation rate and salary growth rate, the optimal time to make the second election in the presence of DC return uncertainty is when the left-hand side of equation (25) equals or exceeds the quantity $r-\rho-g$. The option value is defined equal to the proportional increase in retirement wealth that would be applicable in the absence on the 2nd election.

3.2.1 A Precise Value for the Florida Option .

Table #2 and Table #3 (in the appendix) provide a snapshot of the Florida retirement system in late 2001. The matrix displays the number of employees in various 'age bucket' as a function of their years of service. For example, 2.75% of the labor force is less than 22 years old and have less than 2 years of service. In contrast, 1.78% are between the ages of 22 and 27, and have between 2 and 7 years of service, etc. In the upper corner we observe that only 0.02% of the Florida public labor force are between the ages of 57 and 62 and have between 37 and 42 years of service. Table #3 displays the average (current) salaries in each of these 5-year buckets. Quite intuitively, the older employees (with more service credit years) have higher salaries.

Finally, Table #4 provides option values – as per equation (2) – for the employees at the upper corner of the various buckets. Once again, these option values represent the increase in equivalent retirement wealth that is attributable to the 2nd pension election. Thus, for example, an employee that is 22 years of age, and that has 2 years or credited service, would gain 23.4% from having the 2nd election. In contrast, employees that are 47 years of age, or older, would assign zero value to the option since they would be better off staying in the DB plan, or never electing the DC plan to begin with. Once again we stress that a zero option value does not imply that it is irrational to elect the DC plan at advanced ages. If the participant decides to stay in the DC plan, or defer the 2nd election, the State will save money.

3.2.2 Comparison with Competing Estimates

In a directly related paper that appeared in the *Journal of Risk and Insurance*, Lachance, Mitchell and Smetters (2002) analyzed the same Florida 2nd pension election using Monte Carlo Simulation (MCS) techniques. They derived a variety of employer cost estimates using a range of ages and employment termination probabilities. They concluded that "the market value of this option could represent up to 100 percent of the DC contributions over the worklife".

While the current paper differs in focus and methodology – for example, we did not resort to simulations, nor did we look at employer costs – we are hard pressed to find cases in which the value of this option exceeds 30% of the DC contribution rate. Furthermore, the value of this (pseudo) option is actually highest for employees that are most likely to terminate employment prior to retirement. Thus, from an actuarial funding perspective, the cost of providing this option to a large group of young employees – for which the law of large numbers would apply – would be much smaller than 30%.

And, while the Lachance, Mitchell and Smetters (2002) paper randomized the risk free rate, the salary growth rate, the investment returns in the DC and DB plan as well as the contribution rate to the DB plan, our analytic model can be used on any given (extreme) range of parameter value. *Furthermore, as we argued above, the linearity of the payoff structure implies that uncertainty in the stochastic economic variables will not necessarily add value to the 2nd election option, since all that matters are future expectations.*

It remains to be seen whether a fully stochastic interest rate (yield curve) model would add any substantial value to our estimates of the 2nd election option. Furthermore, we strongly believe that the basic intuition provided by our simpler, but fully analytic, model is worth the cost in assumptions relative to very opaque simulation approach.

3.2.3 What Buy-Back Price Will Create a Zero Option Value?

Instead of charging the ABO to an employee wishing to switch from the DC plan to the DB plan, there is an alternative scheme which would effectively reduce the option cost to zero. This alternative is to demand a payment of (or set the strike price to be) all the contributions that had been made to the DC plan, accumulated with interest, *at the valuation rate* ρ. In other words, the employer would restore the employee to the DB plan, assuming they return all the funds received, but accumulated at the employer's valuation rate.

Of course, if the employee managed to earn a rate of return that is higher than the valuation rate in the DB plan, the employee will retain the excess earnings. The possibility of extra earnings was, after all, the incentive to participate in the DC plan in the first place. On the other hand, if the employee has earned less than the valuation rate, they will have to pay back more than what they have in their fund, which should make intuitive sense. Under this scheme the employer's position is that the employee who choses to seek extra earnings by participating in the DC plan must also bear the risk of loss, and cannot expect the employer to "bail them out" (or retain a put option) when they decide to return to the DB plan.

Looking at the mathematics of the situation, the cost to the employee wishing to switch from DC to DB at time s would be:

$$B_s = B_0 e^{\rho s} + \int_0^s c_t e^{\rho(s-t)} dt$$

Note that this is the same as B_s defined in equation (8) except with c_t replacing b_t. Following the derivation as above leads to a revised equation (11) where the left hand side is zero. We therefore see, as was intuitively clear, that when $\mu = r$, the equation is satisfied by all s^*. In other words, the final wealth at retirement is constant regardless of the time of switching.

4 Conclusion.

This paper has analyzed an intriguing financial option that has been offered to all 600,000 employees in the State of Florida's newly privatized pension system. The so-called buy-back option allows the employee to initially switch from a Defined Benefit pension plan to a Defined Contribution plan, and then back again, at any point prior to retirement, by paying the accumulated benefit obligation (ABO). The ABO is essentially the present value of their pension benefit based on years of credited service.

The Florida pension reforms are being viewed by many observers as a precursor to Social Security reform in U.S., which elevates this problem to more than just a quirk in a particular pension plan.

Our simple, yet robust, model provides us with the following economic insights.

- The optimal (option maximizing) time to make the 2nd election is a function of the *rate of change* in the accumulated benefit obligation – compared to the growth rate of the DC account – and is not a function of the size of the account *per se*.

- Undoubtedly, this option is valuable – especially to young employees – and is likely to be exercised by most participants who initially elect the DC plan. In fact, in our basic model which (deliberately) ignores mortality and early termination probabilities, *nobody* retires from the DC plan; rather, they all eventually return to the DB plan.

- While we – and the educational material developed by the State – repeatedly labeled the 2nd pension election an option, in truth, it behaves more like a (linear) forward contract as opposed to a (non-linear) call or put option. Thus, for example, increasing the volatility of the underlying securities within the DC plan will not increase the value of the option – because the participants stand to lose as much as they stand to gain – which is in contrast to traditional option pricing models.

- We are hard pressed to find realistic combinations of parameter values that induce a 2nd election option value that is greater than 30%. When translated into Defined Contribution terms, one would have to raise the contribution rate from 9% to at the absolute most 12% – for those who elect the DC plan – in order to create a final wealth profile that is equivalent to the absence of the buy-back option.

- When one factors in the probabilities associated with reaching the optimal switching time – and the rationality involved in making this decision – we believe that the actuarial funding cost of the buy-back option is much lower than 30% of the DC plan contribution rate. Indeed, according to State of Florida estimates, a 34 year old male with 2 years of service has less than a 25% probability of making it to normal retirement age.

Ongoing research is examining the impact of randomizing the quantities that we held fixed thru-out the paper, such as interest rates r, salary growth rates g, discount rates ρ, and retirement dates T. However, despite the substantial (and enjoyable) intellectual capital that is spent in this endeavour, we find that greater uncertainty does little to add value to the 'second election', which we believe is mainly due to the linearity of the payoff profile.

Finally, given the abundance of empirical evidence – see for example Benartzi and Thaler (2001, 2002) – that many employees are not making fully rational and utility maximizing choices within their pension plan, it will be fascinating to observe what fraction of Floridians will behave rationally with their 2nd election. Indeed, according to preliminary evidence reported in the *Tampa Tribune* in early September 2002, a very small fraction of state employees are electing to switch into the DC plan.

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Exhibit #1

The Florida Decision Tree

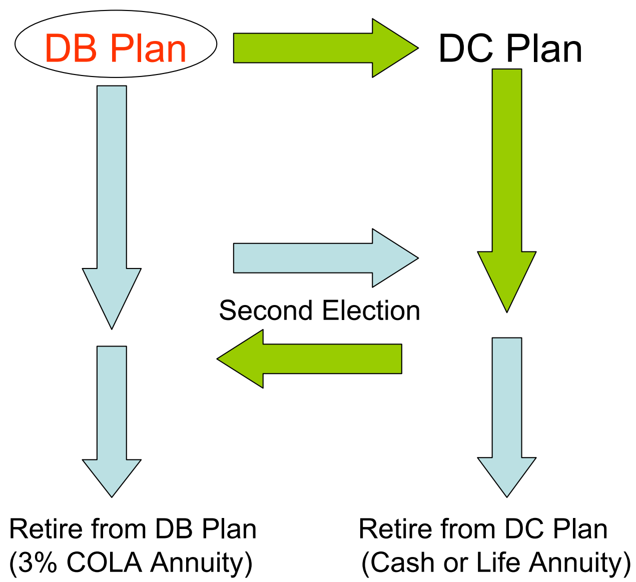


Figure #1

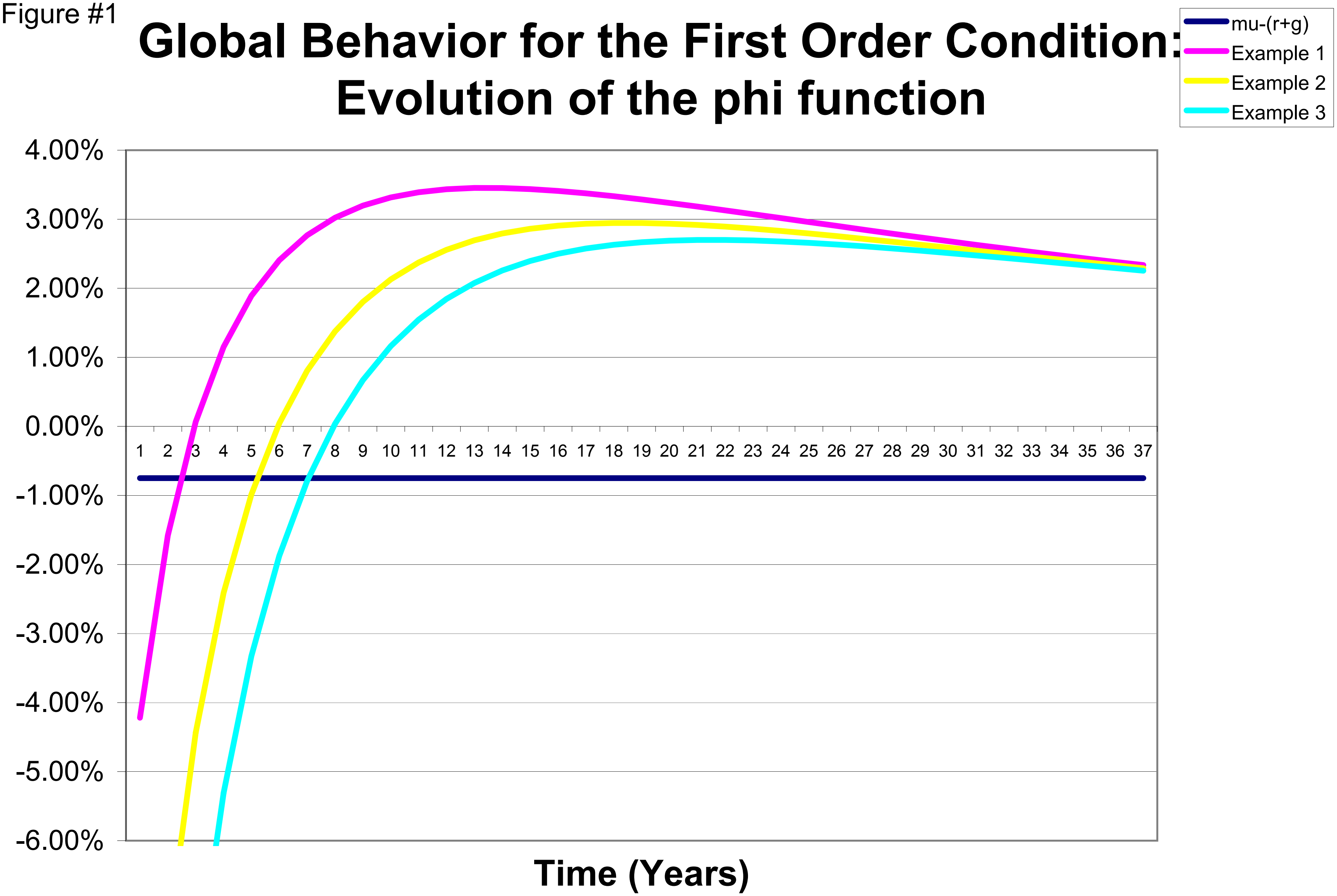
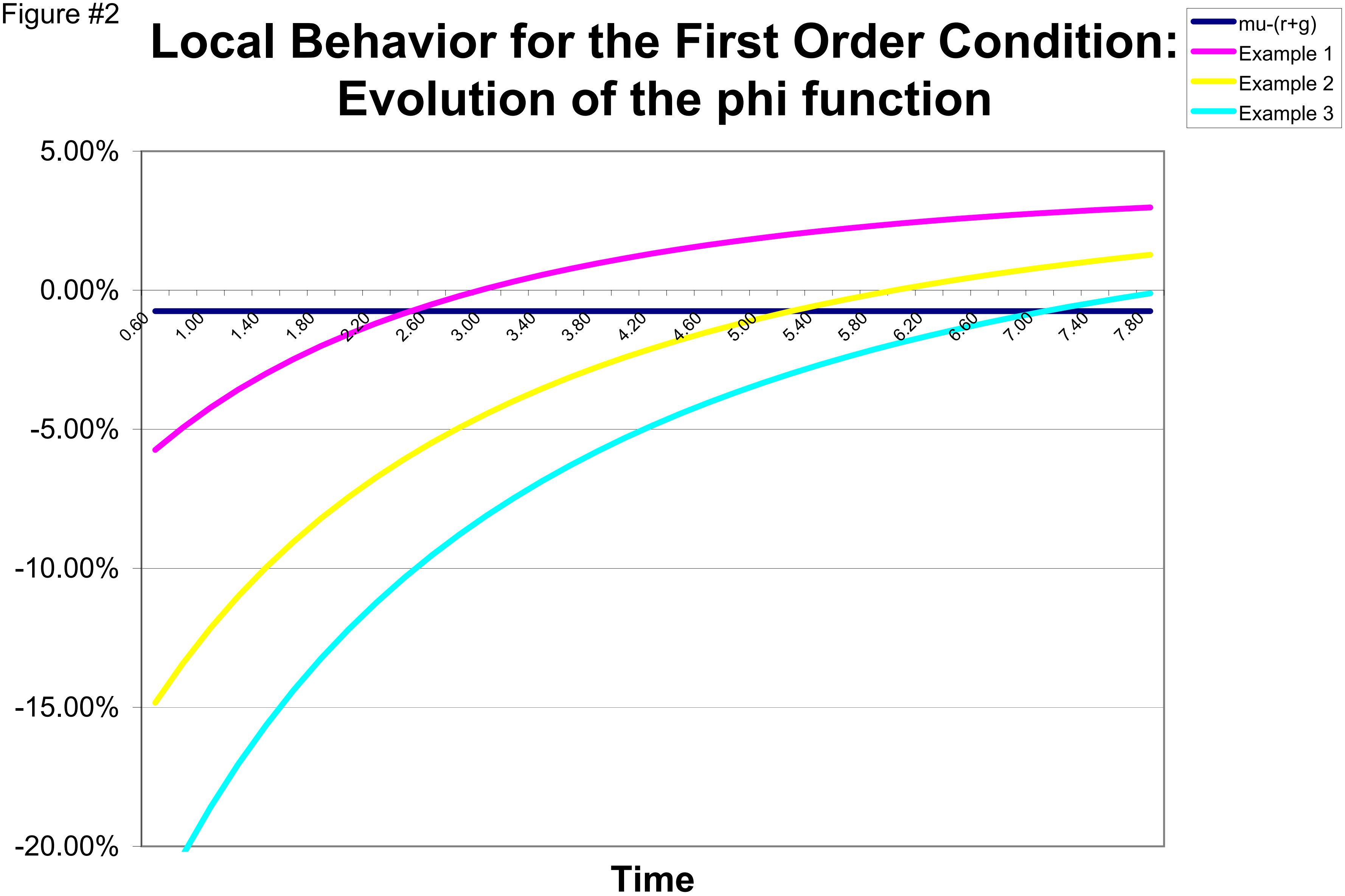

Global Behavior for the First Order Condition:
Evolution of the phi function
mu-(r+g)
Example 1
Example 2
Example 3
4.00%
3.00%
2.00%
1.00%
0.00%
-1.00%
-2.00%
-3.00%
-4.00%
-5.00%
-6.00%
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37
Time (Years)

Figure #2



Local Behavior for the First Order Condition: Evolution of the phi function
mu-(r+g)
Example 1
Example 2
Example 3
5.00%
0.00%
-5.00%
-10.00%
-15.00%
-20.00%
0.60
1.00
1.40
1.80
2.20
2.60
3.00
3.40
3.80
4.20
4.60
5.00
5.40
5.80
6.20
6.60
7.00
7.40
7.80
Time

Table 1

The Threshold Investment Return (mu), Required to Justify Staying in the DC Plan.

Entry Age	*Initial Service*	*Age at which the second election is being contemplated* 30	35	40	45	50	55
50	**5**					15.7%	17.1%
	10					14.3%	15.6%
	15					13.8%	14.9%
	20					13.5%	14.5%
	25					13.4%	14.2%
40	**5**			-5.0%	10.0%	13.8%	14.9%
	10			3.9%	10.1%	13.5%	14.5%
	15			6.8%	11.4%	13.3%	14.2%
30	**5**	-51.3%	-5.4%	6.8%	11.4%	13.3%	14.2%
	10	-19.3%	0.6%	8.3%	11.6%	13.2%	13.9%

Notes: Assuming retirement at age 60, DB rate of 1.6% of salary, and DC rate of 9% of salary

Table #2

Number of Florida State Employees in Various Age Buckets:

SVC	Current age…. 62	57	52	47	42	37	32	27	22
42	*127*								
37	*352*	*230*							
32	*955*	*2,458*	*2,602*						
27	*3,611*	*7,431*	*13,852*	*8,407*					
22	*4,056*	*7,664*	*11,444*	*14,581*	*6,860*				
17	*4,640*	*8,235*	*12,320*	*14,099*	*15,040*	*6,897*			
12	*7,743*	*10,613*	*16,505*	*19,395*	*19,425*	*19,796*	*8,321*		
7	*9,645*	*9,473*	*14,429*	*18,076*	*18,751*	*19,139*	*20,685*	*9,457*	
2	*8,216*	*8,737*	*13,450*	*17,304*	*21,548*	*23,278*	*24,921*	*33,312*	*14,654*

Percent of Florida State Employees in Various Age Buckets:

SVC	Current age…. 62	57	52	47	42	37	32	27	22
42	*0.02%*								
37	*0.07%*	*0.04%*							
32	*0.18%*	*0.46%*	*0.49%*						
27	*0.68%*	*1.39%*	*2.60%*	*1.58%*					
22	*0.76%*	*1.44%*	*2.15%*	*2.74%*	*1.29%*				
17	*0.87%*	*1.55%*	*2.31%*	*2.65%*	*2.82%*	*1.29%*			
12	*1.45%*	*1.99%*	*3.10%*	*3.64%*	*3.65%*	*3.72%*	*1.56%*		
7	*1.81%*	*1.78%*	*2.71%*	*3.39%*	*3.52%*	*3.59%*	*3.88%*	*1.78%*	
2	*1.54%*	*1.64%*	*2.52%*	*3.25%*	*4.04%*	*4.37%*	*4.68%*	*6.25%*	*2.75%*

Source: Florida State Board of Administration; 2001

Table #3

Average Salary of Florida State Employees in Various Age Buckets:

Current age….

SVC	62	57	52	47	42	37	32	27	22
42	*$ 52,756*								
37	*$ 50,000*	*$ 45,652*							
32	*$ 44,712*	*$ 48,779*	*$ 46,925*						
27	*$ 40,072*	*$ 43,736*	*$ 46,650*	*$ 38,670*					
22	*$ 35,528*	*$ 38,570*	*$ 41,227*	*$ 42,171*	*$ 36,633*				
17	*$ 31,853*	*$ 34,608*	*$ 36,583*	*$ 37,230*	*$ 36,769*	*$ 32,840*			
12	*$ 27,496*	*$ 30,934*	*$ 32,002*	*$ 32,075*	*$ 31,619*	*$ 32,158*	*$ 28,686*		
7	*$ 22,385*	*$ 26,243*	*$ 27,362*	*$ 27,086*	*$ 26,340*	*$ 27,076*	*$ 28,040*	*$ 24,923*	
2	*$ 14,922*	*$ 19,629*	*$ 20,914*	*$ 20,828*	*$ 19,626*	*$ 19,473*	*$ 21,287*	*$ 22,595*	*$ 15,504*

Source: Florida State Board of Administration; 2001

Table #4

Theoretical Option Value for Florida State Employees:

Current age….

SVC	62	57	52	47	42	37	32	27	22
42	*0.0%*								
37	*0.0%*	*0.0%*							
32	*0.0%*	*0.0%*	*0.0%*						
27	*0.0%*	*0.0%*	*0.0%*	*0.0%*					
22	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*0.0%*				
17	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*0.5%*			
12	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*1.3%*	*4.6%*		
7	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*0.3%*	*2.8%*	*7.0%*	*12.3%*	
2	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*1.6%*	*5.5%*	*10.6%*	*16.6%*	*23.4%*

Assumptions: 4.75% salary growth rate, and 8% actuarial valuation rate.