The Risk-Return Relationship in Housing Markets: Financial Risk versus Consumption Insurance*

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Abstract

Standard risk-return tradeoff theory cannot explain why housing return varies with risk positively in some markets but negatively in other markets. This paper addresses this issue by incorporating two unique features of housing into a standard consumption-based asset pricing model: (1) intertemporal hedging incentives and (2) a kinked housing supply function. The model nests two competing effects of price risk on housing return: a financial risk effect associated with owning risky housing, and a consumption insurance effect associated with using the current house to hedge against future housing cost risk. The empirical findings confirm several equilibrium predictions implied by the model. In particular, the variation in the housing risk-return relationship across markets is driven both by households' hedging incentives and by housing supply constraints.

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1 Introduction

The importance of house price risk in today's economy is obvious. According to the Case-Shiller 10-city composite price index, real house prices rose by over 70 percent between 2001 and 2006 and fell by over 30 percent between 2006 and 2009. A significant difference between housing and other financial assets is that much of the variation in house prices is local, not national. Looking at data from local housing markets, an interesting pattern emerges. In some markets, there is a positive risk-return relationship for housing. This is consistent with the standard risk-return tradeoff theory.[1] However, in other markets, the relationship is negative. Figure 1 illustrates this by plotting the correlation between the quarterly real housing return and the associated risk during 1978-2006 in four metropolitan areas: Cincinnati, Oakland, St. Louis and San Jose. Standard risk-return tradeoff theory cannot explain why housing return varies negatively with risk in Cincinnati and St. Louis. It also cannot explain the variation in the sign and magnitude of the risk-return relationship across different markets.

This paper provides a formal explanation of the cross-sectional variation in the intertemporal housing risk-return relationship. To do so, the paper develops and tests a simple consumption-based asset pricing model that nests two competing explanations for the housing risk-return relationship. Higher risk implies more uncertainty associated with home investment, and therefore requires a higher rate of return (*financial risk effect*). Higher risk may also increase the value of holding the current house as a hedge against future housing cost risk, hence requiring less return to compensate for the risk (*consumption insurance effect*). The relative strength of each effect is determined not only by households' hedging incentives, but also by the housing supply elasticity. This is because, unlike other assets, houses stock can be added relatively quickly but disappears slowly. The extent to which a change in house price risk can be capitalized into house prices depends crucially on how quickly housing supply can adjust to the demand shock.

The relevance of hedging considerations in pricing financial assets has been shown by Merton (1973), and has been subsequently pointed out by Campbell (1987), Scruggs (1998) and Guo and

[1] See, for example, French, Schwert and Stambaugh (1987), Harvey (1989) and Pastor, Sinha and Swaminathan (2008).

Whitelaw (2006). Under certain conditions, Merton (1980) indicates that the hedging component becomes negligible. This establishes the positive risk-return tradeoff relation for standard financial assets such as stocks.[2] However, housing is unique. It is not only the single largest asset, but also serves as an important part of the current and future consumption vector. Current institutional arrangements have not yet provided a widely accepted financial instrument to reduce the house price risk.[3] As a result, there is a substantial motive to use current housing to self-hedge against future housing consumption risk. Cocco (2000), Ortalo-Magne and Rady (2002), Sinai and Souleles (2005) and Han (2008) have shown that hedging incentives have profound implications for tenure choice and housing demand. The corresponding implications for house price dynamics, however, have never been explored.

To fill this gap in the literature, this paper presents a dynamic, rational expectation model that explains the relationship between the first and second moments of house prices. Following recent literature in household finance,[4] the paper develops a housing consumption-based capital asset pricing model in which the current value of housing is determined by the expected capital gains and rental services it provides in the future. The paper then extends the model by incorporating two unique features of housing: (1) intertemporal hedging incentives, and (2) a kinked housing supply function. The model establishes an equilibrium housing risk-return relationship in both an endowment economy (with exogenous housing supply) and a production economy (with endogenous housing supply). More importantly, it delivers cross-sectional variation in the house price dynamics endogenously through both intertemporal hedging incentives and housing supply constraints.

[2] For example, Merton (1973) shows that the conditional expected excess return on a financial asset is a linear function of its conditional variance plus a hedging component that captures the investor's motive to hedge for future investment opportunities. However, the latter part becomes negligible if the optimal consumption function is less sensitive to the state variables that describe investment opportunities than it is to wealth, or if the variance of the change in wealth is larger than the variance of the change in the state variables (Guo and Whitelaw (2006)).

[3] Flavin and Yamashita (2002) find a low correlation between housing and other financial assets, which suggests that the home investment risk cannot be easily diversified. Using S&P/Case-Shiller Home Price Indices, the Chicago Mercantile Exchange (CME) launched housing futures and options in a small number of MSAs in 2006. However, De Jong, Driessen and Hemert (2008) find that the economic benefits for homeowners of having access to these housing futures is small.

[4] In his Presidential Address to the American Finance Association, John Campbell defines household finance as a field that "asks how households use financial instruments to attain their objectives." He further points out that this is an emerging field that has attracted much recent interest. Examples of household finance literature that feature housing include Lustig and Nieuwerburgh (2005), Yogo (2006) and Piazzesi, Schneider and Tuzel (2007).

First and foremost, the model predicts that the sign and magnitude of the housing risk-return relationship depend crucially on households' hedging incentives, which in turn depend on the correlation between the current and future desired housing markets. For households facing uncertain house prices, the more positive the correlation between their current and future housing markets, the larger the hedging value of owning the current home, and hence the less the return required to compensate the risk.

To further examine the cross-sectional variation of the housing risk-return relationship, this paper extends the analysis by modeling housing supply as a function of house price changes.[5] Standard asset pricing models tend to assume away the potential supply effects by taking supply either as fixed (Lucas (1978)) or as perfectly elastic (Cox, Ingersoll and Ross (1985)). While these assumptions are appropriate for financial markets, they are not suitable for modeling housing markets where new housing can be constructed in response to rising prices (Spiegel (2001)). Complicating the matter further, the extreme longevity of housing implies that houses can be built relatively quickly but disappears very slowly through depreciation (Glaeser and Gyourko (2005)). To address these issues, the paper incorporates a kinked housing supply function that is elastic with respect to upward shocks, but inelastic with respect to downward shocks.

Under this framework, a second prediction of the model concerns the variation of the risk-return relationship with supply elasticity. When the financial risk effect dominates, an increase in the house price risk reduces the demand for existing housing stock. Given the longevity of houses, the declining housing demand lowers house prices (thereby increasing housing return) rather than reducing housing stock. In contrast, when the consumption insurance effect dominates, an increase in the price risk increases the hedging demand for housing, which in turn affects house prices and returns only if the supply is slow to catch up. The opposite occurs if the house price risk decreases. Thus, the model predicts an asymmetric impact of supply elasticity on the net risk-return relations: when price risk increases, the relative strength of the consumption insurance effect decreases with supply elasticity;

[5]The classical asset-market approach treats housing supply much like the supply of any other asset and assumes that new construction depends on the level of house prices (e.g. Poterba (1984)). This assumption ignores perhaps the most unique aspect of housing supply – the importance of land as an input. Land is fundamentally different from other production factors in that land is inelastically supplied. The nature of spatial equilibrium suggests that housing supply depends on changes in house prices, rather than levels of house prices (Mayer and Somerville (2000)).

when price risk decreases, the relative strength of the financial risk effect decreases with supply elasticity.

To test these implications, the paper estimates a GARCH(1,1)-in-mean model using the MSA-level repeat sales house price indices from the Office of Federal Housing Enterprise Oversight (OFHEO) and migration data from the Census of Housing and Population (5% PUMS). The out-migration rate is used as a proxy for the correlation between an average household's current housing market and future housing cost. Intuitively, the lower the out-migration rate, the more likely it is that households would stay within the same market, and the higher the average hedging incentives of local households.

The paper finds that the estimated intertemporal risk-return relationship is positive in some markets, but negative in other markets. This provides evidence for the presence of both the financial risk effect and the consumption insurance effect. A cross-sectional investigation of the GARCH estimates provides a number of insights for explaining the dispersion in the sign and magnitude of the risk-return relationship for housing. First, across markets, the higher the out-migration rate, the less positive the correlation between the housing returns and risks. This confirms the model's prediction that the relative strength of the consumption insurance effect increases with local households' hedging incentives. Second, the less elastic the local housing supply, the stronger the relative strength of the financial risk effect versus the consumption insurance effect under a decrease in the price risk. This confirms the model's prediction about the asymmetric impact of the housing supply constraint on the housing risk-return effects. Finally, house price volatility is more persistent in areas where the consumption insurance effect dominates. This is consistent with the idea that house price volatility tends to reinforce itself through the mechanisms of self-hedging and leverage. The results remain robust after taking into account the positive hedging incentives for those who move between correlated MSAs and after controlling for potential bias due to possible endogeneity, sample selection and specification problems.

To further evaluate the validity of the empirical results, the paper tests the model using two alternative data sources. The first is the Standard & Poor's Case-Shiller index from 1987-2008. This allows one to confirm that the risk-return relationship identified in the main empirical analysis is not merely applied to home purchases with conforming loans. The second is the 1989-2007 condominium and single-family-house price data provided by the National Association of Realtors (NAR). The results show that condominium return varies more positively with the condominium price risk in markets where the condominium-house correlation is weaker. To the extent that a large number of condominium buyers plan to move into single family houses in the same market, the results again support the equilibrium predictions implied by the model.

The paper is organized as follows. Section 2 briefly discusses other related literature. Section 3 describes the essence of the model. Section 4 describes the data. Section 5 and 6 contain the empirical findings and robustness checks. Section 7 concludes. The Appendix contains proofs for the results in the model.

2 Related Literature

This paper is the first to examine and rationalize the cross-sectional variation in the intertemporal risk-return relationship for housing. The paper builds on several strands of the literature. First, financial economists have long been interested in studying the risk-return relationship for financial assets. For example, Chou (1988), Harvey (1989), Goyal and Santa-Clara (2003), and Bali (2008) find a positive risk-return relationship, whereas Campbell (1987), Baillie and DeGennaro (1990), Glosten, Jagannathan and Runkle (1993) find the opposite.[6] Missing from this line of the literature is an analysis of the risk-return relationship for housing. The omission is surprising, given that housing is the single largest asset in a typical American household's portfolio and that housing markets are highly incomplete. This paper contributes to the literature by extending the analysis of the risk-return relationship to housing. To explain the behavior of house prices, the model presented here focuses both on households' preferences (e.g. hedging incentives) and on housing supply constraints. The former is consistent with the existing finance literature that emphasizes the desire to hedge

[6] In addition, Nelson (1991), Whitelaw (1994) and Whitelaw (2000) find mixed evidence. See Bekaert and Wu (2000) for a review of the literature.

(Scruggs (1998) and Guo and Whitelaw (2006)) or the willingness to pay for insurance (Adrian and Rosenberg (2008)). The latter is consistent with the idea that asset supply has a significance effect on explaining asset prices (Spiegel (1998), Braun and Larrain (2009)).

Second, recent real estate studies have uniformly focused on the financial risk effect in studying the housing risk-return relationship. For example, treating housing as a financial asset, Meyer and Wieand (1996) show that increased price risk lowers the current house price and increases expected housing returns in an asset pricing context. To test this, Crone and Voith (1999) and Cannon, Miller and Pandher (2006) regress average housing returns across markets on return volatilities, where volatilities are measured by the standard deviation of returns within each market. Both results favor a positive risk-return relationship for housing. Unlike the current paper, their measures of housing return and risk do not capture households' time-varying expectation about future housing markets. More recently, Case, Cotter and Gabriel (2009) provide a novel test of a housing asset pricing model and find a positive beta risk-return relationship. Unlike the current paper, they take a beta pricing approach, where house price risk is measured by the amount of exposure of local housing markets to the national housing markets. In contrast, Dolde and Tirtiroglu (1997) observe time-varying housing returns and volatilities, and find a negative relation between conditional variance and returns in Connecticut but a positive relation in San Francisco. This is consistent with the basic findings presented here. However, their goal is to test the information diffusion in these two housing markets. The current paper makes two contributions to the housing risk-return relationship literature. On the theory side, this paper provides a simple theoretical framework that nests both the financial risk effect and the consumption insurance effect. While the former is consistent with the conventional wisdom, the latter contributes a new explanation to the understanding of house price dynamics. On the empirical side, this paper provides a first examination of the cross-market dispersion in intertemporal housing risk-return relationship. The empirical framework introduces two new factors – hedging incentives and supply constraints, both of which contain significant information for identifying the empirical housing risk-return relationship.

A third strand of the related literature emphasizes the hedging benefits associated with home ownership. Using a tenure choice model, Sinai and Souleles (2005) show that hedging demand could bid up house prices, thereby generating a negative rent risk premium. Sinai and Souleles (2008) further show that the hedging value is substantial even for households moving to different MSAs. Built on their insight, this paper presents an asset pricing model that formalizes the role of hedging incentives in explaining the housing risk-return relationship. Unlike their model which treats supply as fixed, this paper shows that supply elasticity plays an important role in predicting the relative strength of the hedging effect. This is an important step, as Glaeser, Gyourko and Saiz (2008) point out, "models of housing price volatility that ignore supply miss a fundamental part of the housing asset."[7]

Finally, there has been increasing amount of work that incorporates housing into asset pricing models. This line of literature aims to understand the importance of housing in explaining the time-series and the cross-sectional variation in asset returns. For example, focusing on the role of housing as a collateral asset, Lustig and Nieuwerburgh (2005) show that households demand a larger risk premium in times when the housing collateral ratio is low, and for assets whose returns are tightly correlated with aggregate consumption growth shocks when collateral is scarce. Alternatively, focusing on the non-separability in housing and non-housing consumption, more recent studies show that households command a larger risk premium if returns and rental price growth are more positively correlated (Piazzesi et al. (2007)) or during recessions when durable consumption falls (Yogo (2006)).[8] None of these studies examines how house price risk affects housing returns. This paper complements previous work in that it studies the behavior of house prices, but abstracts from stock assets and mortgage choices.

[7]Housing studies that incorporate a housing production sector include Davis and Heathcote (2005), which models a real business cycle, Ortalo-Magne and Rady (2006), which models prices and volume in the housing markets, and Chambers, Garriga and Schlagenhauf (2009), which explains the recent boom in homeownership. Unlike this present paper, none of these studies models the risk-return relationship.

[8]Other examples include Fernandez-Villaverde and Krueger (2002), Yao and Zhang (2005), Cocco (2005) and Flavin and Nakagawa (2008), which take a different focus on portfolio choice in the presence of housing.

3 Model

The theoretical framework begins with the standard housing consumption-based CAPM. The model consists of a large number of infinitely-lived households that enjoy both housing services h_t and a nondurable, numeraire consumption good c_t. Given the discount factor β, preferences are

$$E\left(\sum_{t=0}^{\infty}\beta^t u(c_t, h_{t-1})\right) \tag{1}$$

where $E(\cdot)$ indicates the expectation conditional on the information at time 0. The period utility, $u(c_t, h_{t-1})$, is increasing and concave in both c_t and h_{t-1}. In addition, I assume the separability in utility to isolate the financial risk effect and consumption insurance effect from the composition risk effect introduced in Piazzesi et al. (2007). There are two assets in the economy. One is a bond s_t, which represents a risk-free financial asset. The riskless return at time t is denoted as r_t^f. Following the convention, I assume that the bonds are in positive net supply. The other is housing capital h_t, which represents a real asset. One unit of housing stock a household purchased at the end of $t-1$ produces one unit of housing services at the beginning of t. Thus households consume h_{t-1} at time t.[9] Housing asset trades at the price P_t and depreciates at the rate δ. Let H_t denote the aggregate supply of housing at time t. In what follows, I consider both an endowment economy with exogenously fixed housing supply (in Section 3.1) and a production economy with endogenous housing supply (in Section 3.2). For convenience, the model assumes away transaction costs.

There are N_t identical households. All households enter period 0 with initial bond endowment b_0 and housing stock h_0. At time t, each household is endowed with labor income y_t, which is defined in units of perishable non-durable consumption. The budget constraint is given by

$$c_t + P_t h_t + s_t = y_t + s_{t-1}(1 + r_t^f) + P_t h_{t-1}(1-\delta) \tag{2}$$

The stochastic house price and labor income are the key sources of fundamental uncertainty in this

[9]The timing convention, although non-standard, is innocuous (see Piazzesi, Schneider and Tuzel (2003)). It is made to allow an analytical solution that characterizes the risk-return relationship.

model, with properties to be specified later as needed. Markets are incomplete in the sense that we close down other financial asset markets, including insurance markets. To finance future durable and non-durable consumption and insure against house price risk and income risk, households can transfer wealth over time through bond and housing holdings. The assumption that the model consists of only two assets is made to maintain a tight focus on the risk-return relationship for housing. While incorporating a richer set of financial assets is an important issue for future research, it is less likely to be a concern for modeling the house price risk premium. The low correlation between housing and existing financial assets suggest that housing markets are highly incomplete and that adding other assets does not help in lowering the risk associated with house price.[10]

An equilibrium is a collection of processes (c_t, s_t, h_t) and price process $\{r_t^f, P_t\}$ such that (i) (c_t, h_t, s_t) solve the household's problem of maximizing utility (1) subject to the budget constraint (2) and (ii) markets clear for bonds, housing and non-durable consumption.

3.1 Exogenous Housing Supply

Following the standard capital asset pricing models with fixed asset supply, I first consider an endowment economy where housing supply, H_t, is exogenous and fixed. The Lagrangian for the household's problem is

$$L = E\left(\sum_{t=0}^{\infty}\left(\beta^t u(c_t, h_{t-1}) - \mu_t\left[c_t + s_t + P_t \cdot h_t - (1+r_t^f)s_{t-1} - P_t h_{t-1}(1-\delta)\right]\right)\right) \tag{3}$$

where μ_t is the multiplier on the budget constraint. The first order conditions are

$$\begin{aligned}\frac{\partial L}{\partial s_t} &= -\mu_t + \beta E_t \mu_{t+1}(1+r_{t+1}^f) = 0 \\ \frac{\partial L}{\partial c_t} &= \beta^t\left(u_1(c_t, h_{t-1}) - \mu_t\right) = 0\end{aligned}$$

[10] Using different methods to measure housing returns, all three studies find a low correlation between the returns to housing and existing financial assets – Goetzmann (1993) uses indices for four U.S. cities estimated by Case and Shiller (1989); Gatzlaff (2000) uses indices for 20 MSAs in Florida estimated by similar techniques; Flavin and Yamashita (2002) use panel information on the owner's own assessments of house values. Even after accounting for recently developed housing futures and options, De Jong et al. (2008) find that the economic benefits of having access to these housing securities are small.

$$\frac{\partial L}{\partial h_t} = \beta^t \left\{-\mu_t P_t + \beta E_t \left[\mu_{t+1}\left((1-\delta)P_{t+1} + u_2(c_{t+1}, h_t)\right)\right]\right\} = 0$$

For an equilibrium to exist, the absence of arbitrage is necessary. This in turn implies the existence of a strictly positive pricing kernel, which represents the present value of a unit of numeraire one period ahead:

$$M_{t+1} = \beta \frac{u_1(c_{t+1}, h_t)}{u_1(c_t, h_{t-1})} \tag{4}$$

M_{t+1} is also called the "stochastic discount factor" in the asset pricing literature. After substituting the pricing kernel into first order conditions, we arrive at expressions for the risk-free interest rate and house price:

$$1 = E_t\left[(1 + r^f_{t+1})M_{t+1}\right] \tag{5}$$

$$P_t = E_t\left[M_{t+1}\left(P_{t+1}(1-\delta) + \frac{u_2(c_{t+1}, h_t)}{u_1(c_{t+1}, h_t)}\right)\right] \tag{6}$$

Equation (5) defines the interest rate in equilibrium. Equation (6) is the central housing pricing formula. The price of housing asset, P_t, is the expected present value of future price, plus the marginal utility derived from housing consumption in the following period, discounted back by M_{t+1}. While the first term in the bracket represents the financial gains associated with the current home ownership, the second term can be interpreted as the rent savings from holding one unit of housing at time $t+1$. To show this, assume there is a rental market in which the time-$(t+1)$ rent for one unit of housing service is Q_{t+1}. Hypothetically, at time $t+1$ the household rents housing service h_t for just one period. In this case, the household chooses the optimal level of h_t to minimize expenditure $c_{t+1} + Q_{t+1}h_t$ to achieve a desired utility level, $u(c_{t+1}, h_t) = \bar{u}$. This yields the following first order condition:

$$Q_{t+1} = \frac{u_2(c_{t+1}, h_t)}{u_1(c_{t+1}, h_t)} \tag{7}$$

Thus, the pricing relation can be written as

$$P_t = E_t\left[M_{t+1}\left(P_{t+1}(1-\delta) + Q_{t+1}\right)\right] \tag{8}$$

Hence, the price P_t of the housing asset is the expected present value of its future sale price, adjusted for depreciation, plus one-period rent at time $t+1$. The housing pricing relation (8) represents a close parallel to the basic asset pricing relation in the standard CCAPM of Lucas (1978), in which the role of future dividends associated with financial securities is replaced by future rents associated with home ownership in our context. Defining the total housing return as $R_{t+1} = \frac{P_{t+1}(1-\delta)+Q_{t+1}}{P_t}$, Equation (8) can now be rewritten as

$$E_t(R_{t+1}M_{t+1}) = 1 \tag{9}$$

Note that Equation (9) does *not* depend on any specific assumption about the utility functional form or price/rent distributions. It holds for any housing investment opportunity and any investor with a monotone and concave utility function.

The model presented so far is fairly unrestrictive. To derive the equilibrium housing risk-return relationship, one approach would be to specify the form of the utility function and the time-series processes for the per capita income and price level and obtain the induced time-series processes for the pricing kernel and housing return. The alternative approach taken here is to directly assume a time-series process for the pricing kernel, house price and rents, which yields plausible implications about housing markets. This is consistent with the spirit of standard asset pricing models (e.g., Constantinides (1992) and Constantinides and Duffie (1996)).[11] The advantage of this approach is that it permits an analytical solution for characterizing the equilibrium risk-return relationship.

I start by assuming a martingale process to describe the behavior of the pricing kernel.

$$M_{t+1} = exp\left(-r_t^f - \frac{1}{2}\sigma_{M,t}^2 + \epsilon_{M,t+1}\right) \tag{10}$$

where $\epsilon_{M,t+1}|I_t \sim N(0, \sigma_{M,t}^2)$. This functional form has been used by financial economists in pricing stocks and options.[12] In our context, this particular functional form of the pricing kernel may be

[11]For example, Cochrane (2005) states that if the statistical models for consumption and asset returns are right, they should coincide with the equilibrium consumption and return process generated by the true economy.

[12]For example, Amin and Ng (1993) use it to price an option, while Wu (2001) uses it to develop a volatility feedback model for stock markets.

obtained as the equilibrium implication in an economy with certain restrictions. Consider a simple example where preferences are $u(c_t, h_{t-1}) = \gamma \frac{c_t^{1-\tau_1}}{1-\tau_1} + (1-\gamma) \frac{h_{t-1}^{1-\tau_2}}{1-\tau_2}$ and where the income process follows $\ln \frac{y_{t+1}}{y_t} = \frac{1}{\tau_1}(\ln \beta + r_t^f) + \frac{1}{2\tau_1}\sigma_{y,t}^2 + \frac{1}{\tau_1}\epsilon_{y,t+1}$.[13] The preference structure and market clearing condition imply that $M_{t+1} = \beta \left(\frac{y_{t+1}}{y_t}\right)^{-\tau_1}$. Substituting this into the stochastic income process implies (10), where $\epsilon_{M,t+1} = -\epsilon_{y,t+1}$ and $\sigma_{M,t}^2 = \sigma_{y,t}^2$. Thus, given the separability in the utility function, $\sigma_{M,t}^2$ is driven purely by uncertainties in per capita labor income, which is exogenous in this economy.

Let $g_t^P = \ln \frac{P_{t+1}}{P_t}$ be the growth rate of the current house's price. Let $g_t^Q = \ln \frac{Q_{t+1}}{Q_t}$ be the growth rate of the future house's rent. Both follow a stochastic process:

$$g_{t+1}^j = \alpha_0^j + \alpha_1^j g_t^j + \epsilon_{j,t+1} \tag{11}$$

$$\epsilon_{j,t+1} \sim N(0, \sigma_{j,t}^2) \tag{12}$$

where $0 < \alpha_1^j < 1$ and $j = \{P, Q\}$. It is well known that housing returns exhibit positive autocorrelation in many markets; see Case and Shiller (1989, 1990) for U.S. cities and Englund and Ioannides (1997) for international comparative data. This provides empirical justification for the AR(1) process specified in Equation (11).

To pin down the risk-return relationship, I make further assumptions about the joint distribution of house price, rent and pricing kernel. First, the shocks to the price growth and pricing kernel are negatively correlated. Specifically, $cov(\epsilon_{P,t+1}, \epsilon_{M,t+1}) = \rho_{PM}\sigma_{P,t}^2$, where $\rho_{PM} < 0$. This assumption is consistent with the previous finding that house price varies positively with labor income (e.g., Davidoff (2006)). High labor income growth increases non-housing consumption growth, leading to a lower marginal utility from consuming non-housing consumption. Thus, the assumption $\rho_{PM} < 0$ implies that house price growth is slower in states where non-housing consumption is more valuable. The variance risk of the house price is systematic and must be priced. This implies a standard risk correction term for the risk premium – a component that is common to all assets. Second, the shocks

[13] My choice of the utility function follows Chambers, Garriga and Schlagenhauf (2009), which provides an economic interpretation for parameters in this type of preference structure. For illustration purpose, I make specific assumptions on the preference structure and the labor income process. But I have no objection to viewing the model as incorporating other assumptions. As it turns out in the empirical section, the implications from the model that incorporates these specific assumptions I made are supported by the data.

to the price growth and rental growth are positively correlated: $cov(\epsilon_{P,t+1}, \epsilon_{Q,t+1}) = \rho_{PQ}\sigma^2_{P,t}$, where $\rho_{PQ} > 0$. Underlying this assumption is another important empirical observation – households tend to move among correlated housing markets. According to the 1990 American Community Survey, 54% of movers move within the same county. Even among those who move from one MSA to another, the median effective price correlation between their current and future markets is as high as 0.67 (Sinai and Souleles (2008)). Thus, the positive correlation between the random components of the price process of the current house and of the future house is well supported. Unlike the first assumption, the price rent correlation implies a hedging correction term for the housing risk premium – a component that is unique to housing. Finally, for simplicity, I assume $corr(\epsilon_{Q,t+1}, \epsilon_{M,t+1}) = 0$.

To this end, I have presented a stylized model for pricing housing asset under uncertainty. The model makes specific assumptions about the time series process of the pricing kernel and housing prices. If these statistical processes are supported in an equilibrium of this economy, they should satisfy the pricing relation (8) derived from households' optimal investment behavior. The proposition below shows the existence of an equilibrium that supports the given statistical processes and yields an analytical solution for the housing risk-return relationship.

Proposition 3.1 *Define $p_t = \ln P_t$, $q_t = \ln Q_t$ and $r_{t+1} = \ln R_{t+1}$. Given the model specified by Equations (1), (2), (10)-(12), there exists a linear solution to the log price rent ratio*

$$p_t - q_t = c_0 + c_1 g^P_t + c_2 g^Q_t + c_3\sigma^2_{P,t} + c_4\sigma^2_{Q,t} \tag{13}$$

where the parameters are

$$\begin{aligned}
c_0 &= \frac{-r^f_t + k + \gamma\ln(1-\delta) + \gamma\alpha^P_0 + (1-\gamma)\alpha^Q_0}{1-\gamma}\\
c_1 &= \frac{\gamma\alpha^P_1}{1-\gamma}\\
c_2 &= \alpha^Q_1\\
c_3 &= \frac{\frac{1}{2}\gamma^2 + \gamma\rho_{PM} + \gamma(1-\gamma)\rho_{PQ}}{1-\gamma}\\
c_4 &= \frac{1}{2}(1-\gamma)
\end{aligned}$$

Furthermore, there exists a linear solution to the log return process

$$r_{t+1} = r_t^f + \gamma\epsilon_{P,t+1} + (1-\gamma)\epsilon_{Q,t+1} - \left(\frac{1}{2}\gamma^2 + \gamma\rho_{PM} + \gamma(1-\gamma)\rho_{PQ}\right)\sigma_{P,t}^2 - \frac{1}{2}(1-\gamma)^2\sigma_{Q,t}^2 \qquad (14)$$

The proof is contained in Appendix I. The solution method is consistent with Campbell and Shiller (1988) and Wu (2003). The first part of Proposition 3.1 shows that the log price-rent ratio $(p_t - q_t)$ depends on the risk-free rate, expected house price growth and rent growth, and their conditional volatilities. The results are fairly intuitive. For example, a higher risk-free rate represents a higher opportunity cost of housing investment, which makes current housing holding less attractive. In contrast, higher price growth and rent growth increase the value of housing, leading to a higher price-rent ratio. Finally, the parameter c_3 characterizes the net effect of price risk on the price-rent ratio. The economic implications of this term are the focus of the next proposition.

The second part of Proposition 3.1 shows that the log realized housing return r_{t+1} also follows a simple and intuitive pattern. It depends on the risk-free rate, shocks to price growth and rent growth, and their conditional volatilities. The positive dependence of r_{t+1} on $\epsilon_{P,t+1}$ and $\epsilon_{Q,t+1}$ confirms the conventional wisdom that housing is valuable not only as a financial asset but also as a consumption good (e.g., Henderson and Ioannides (1983), Brueckner (1997)).

Proposition 3.2 *Assuming* $-1 < \alpha_1^P < 1$ *and* $-1 < \alpha_1^Q < 1$.

(i) In an economy with an exogenous housing supply, the equilibrium housing risk-return relationship is characterized by

$$\frac{\partial \mathrm{E}_t(r_{t+1})}{\partial \sigma_{r,t}^2} = -\frac{\frac{1}{2}\gamma + \rho_{PM} + (1-\gamma)\rho_{PQ}}{\gamma + 2(1-\gamma)\rho_{PQ}} \qquad (15)$$

where $\sigma_{r,t}^2 = \mathrm{VAR}_t(r_{t+1}) = \left(\gamma^2 + 2\gamma(1-\gamma)\rho_{PQ}\right)\sigma_{P,t}^2 + (1-\gamma)^2\sigma_{Q,t}^2$.

(ii)

$$\begin{aligned} \frac{\partial \mathrm{E}_t(r_{t+1})}{\partial \sigma_{r,t}^2} &< 0 \;\; \textit{if} \;\; \rho_{PQ} > \frac{-\rho_{PM} - \frac{1}{2}\gamma}{1-\gamma} \\ \frac{\partial \mathrm{E}_t(r_{t+1})}{\partial \sigma_{r,t}^2} &> 0 \;\; \textit{if} \;\; \rho_{PQ} < \frac{-\rho_{PM} - \frac{1}{2}\gamma}{1-\gamma} \end{aligned} \qquad (16)$$

(iii)

$$\frac{\partial^2 \mathrm{E}_t(r_{t+1})}{\partial \sigma_{r,t}^2 \partial \rho_{PQ}} < 0 \tag{17}$$

The proof is contained in Appendix II. Proposition 3.2 (i) provides a clean characterization of the net risk-return relationship. Following this, Proposition 3.2 (ii) shows that the consumption insurance effect dominates in markets with sufficiently large ρ_{PQ}, whereas the financial risk effect dominates in other markets. In this sense, the model delivers cross-sectional variation in the *sign* of the risk-return relationship through the relative strength of ρ_{PQ} and ρ_{PM}. Proposition 3.2 (iii) further delivers cross-sectional variation in the *magnitude* of the risk-return relationship. That is, across markets, the marginal expected return required to compensate for the risk decreases with ρ_{PQ}. This provides the first testable implication for our empirical work.

To complete the solution of our model economy, we need to find housing and nondurable consumption in term of exogenous forces. The results will of course depend on what the rest of the economy looks like. Since households are identical with respect to their preferences and endowments, it follows from the market clearing conditions that in equilibrium $h_t^* = \frac{H_t}{N_t}$ and $c_t^* = y_t$. Since housing endowment is exogenous, the house price adjusts so that an exogenous shock in house price risk is fully translated into changes in house prices and returns. The equilibrium risk-return relationship is therefore fully characterized by Equation (15).

3.2 Endogenous Housing Supply

Thus far, housing supply has been taken as exogenous to the model. In this section, I endogenize housing supply. By doing so, I examine the extent to which supply mutes or exacerbates the relative strength of two competing risk-return effects.

The theoretical underpinnings of housing supply come from one of two sources: the asset-market approach (e.g., Poterba (1984)) and the urban spatial approach (e.g., Capozza and Helsley (1990)). The asset market approach assumes that the building industry is composed of competitive firms and that the aggregate supply of housing depends on the level of house prices. Despite their convenience, these assumptions ignore the unique aspect of housing supply – residential structure must be combined with land to produce new houses. Unlike other production factors, land is inelastically supplied. To ensure a spatial equilibrium for households within a city, new housing construction occurs only when the city makes the transition from one equilibrium to another equilibrium, a period identified by an increase in price level and an expansion of the city area. The nature of spatial equilibrium suggests that the aggregate housing supply depends on changes in house prices, rather than levels of house prices. In addition, since housing stock can increase rather quickly through new construction but decreases slowly through depreciation, housing supply is elastic only with respect to positive price changes.

To simplify the analysis, I do not model the details of the housing construction industry. Rather I assume a housing supply function that addresses both the inelasticity of land supply and the slow depreciation of housing structure.[14] In particular, housing supply at time t depends on the previous housing stock H_{t-1}, the depreciation rate, δ, the supply elasticity, ξ, and house price changes, $P_t - P_{t-1}$.

$$H_t = (1-\delta)H_{t-1} + \xi(P_t - P_{t-1}) \qquad \text{for} \quad P_t > P_{t-1} \tag{18}$$

$$H_t = (1-\delta)H_{t-1} \qquad \text{for} \quad P_t \leq P_{t-1} \tag{19}$$

Equations (18) and (19) capture two distinct features of the housing construction industry. First, land is inelastically supplied. In a monocentric urban growth model, new housing construction adjusts the housing stock from one spatial equilibrium to another following a positive demand shock. Such a demand shock would generate a permanent increase in house prices at all interior locations of the city. The size of a price change will therefore measure the magnitude of the demand shock. Second, houses can be built relatively quickly but disappears slowly. This implies a kinked housing supply function, where the kink occurs at the existing quantity of housing stock. If demand rises, housing can be added at the rate of local supply elasticity. If demand falls, housing stock does not respond, given its extreme longevity.[15] In both cases, housing declines at the rate of physical depreciation.

[14] Underlying the supply function I assumed here is a residential construction model based on the theory of urban development. See Mayer and Somerville (2000) for details.

[15] More generally, one could relax this assumption by assuming that housing supply adjusts to negative demand

The kinked housing supply function is important, for it fundamentally determines the extent to which house prices and returns vary with changes in the risk. This point can be illustrated through the following equation,

$$\frac{\partial r_{t+1}}{\partial \sigma_{r,t}^2} = \frac{\partial r_{t+1}}{\partial P_t}\frac{\partial P_t}{\partial \sigma_{r,t}^2} + \frac{\partial r_{t+1}}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial h_t}\frac{\partial h_t}{\partial \sigma_{r,t}^2} \tag{20}$$

The first term in Equation (20) captures how much price risk is capitalized into the return through the current house price changes. As shown in Appendix III, this term is equivalent to $-\frac{\frac{1}{2}\gamma+\rho_{PM}+(1-\gamma)\rho_{PQ}}{\gamma+2(1-\gamma)\rho_{PQ}}$. When the housing supply is exogenous, the supply market does not adjust; hence, the second term equals zero. The risk-return relationship in Equation (20) thus collapses to Equation (15), which is the basic risk-return relationship derived from an endowment economy.

However, with an endogenous housing supply, an initial shock in price risk affects returns not only through the current price changes, but also through the changes in demand for new construction. The latter are captured by the second term $\frac{\partial r_{t+1}}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial h_t}\frac{\partial h_t}{\partial \sigma_{r,t}^2}$. This term represents a general equilibrium effect in the housing market – an exogenous change in price risk affects current housing demand ($\frac{\partial h_t}{\partial \sigma_{r,t}^2}$), which in turn affects next period's house price through the supply side ($\frac{\partial P_{t+1}}{\partial h_t}$), leading to further changes in the realized returns ($\frac{\partial r_{t+1}}{\partial P_{t+1}}$). Two observations follow. First, since the housing supply is elastic with respect to upward shocks but inelastic with respect to downward shocks, only positive demand changes can be translated into returns through the next period's prices. This is because when demand drops, the next period's price does not adjust ($\frac{\partial P_{t+1}}{\partial h_t} = 0$ by Equation (19) and market clearing condition), and the second term therefore diminishes. Second, in markets where demand changes are positive, their impact on housing return is magnified by multiplier $\frac{\partial P_{t+1}}{\partial h_t}$, which is a decreasing function of ξ according to equation (18). In this case, the strength of the net risk-return relationship decreases with the supply elasticity.

Proposition 3.3 *In an economy with an endogenous housing supply, the supply elasticity, measured by ξ, has an asymmetric impact on the net risk-return relationship.*

(i) When $\sigma_{r,t}^2$ <u>increases</u>, lower supply elasticity amplifies the consumption insurance effect, but leaves

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shocks at a rate that is strictly positive but slower than ξ. What matters here is not the absolute value of elasticity, but the relative magnitude of the supply elasticity with respect to upward shocks versus downward shocks.

the financial risk effect unaffected. That is,

$$\frac{\partial r_{t+1}}{\partial \sigma^2_{r,t}} > 0 \quad \text{and} \quad \frac{\partial^2 r_{t+1}}{\partial \sigma^2_{r,t} \partial \xi} = 0 \quad \text{when} \quad c_3 < 0 \tag{21}$$

$$\frac{\partial r_{t+1}}{\partial \sigma^2_{r,t}} < 0 \quad \text{and} \quad \frac{\partial^2 r_{t+1}}{\partial \sigma^2_{r,t} \partial \xi} > 0 \quad \text{when} \quad c_3 > 0 \tag{22}$$

(i) When $\sigma^2_{r,t}$ decreases, lower supply elasticity amplifies the financial risk effect, but leaves the consumption insurance effect unaffected. That is,

$$\frac{\partial r_{t+1}}{\partial \sigma^2_{r,t}} > 0 \quad \text{and} \quad \frac{\partial^2 r_{t+1}}{\partial \sigma^2_{r,t} \partial \xi} < 0 \quad \text{when} \quad c_3 < 0 \tag{23}$$

$$\frac{\partial r_{t+1}}{\partial \sigma^2_{r,t}} < 0 \quad \text{and} \quad \frac{\partial^2 r_{t+1}}{\partial \sigma^2_{r,t} \partial \xi} = 0 \quad \text{when} \quad c_3 > 0 \tag{24}$$

The proof is provided in Appendix III. Why might the two risk-return effects vary differently with supply elasticity? The intuition is simple. With an endogenous housing supply, an increase in the house price risk increases housing demand if the consumption insurance effect dominates, and decreases housing demand if the financial risk effect dominates. In contrast, a decrease in the house price risk decreases housing demand if the consumption insurance effect dominates, and increases housing demand if the financial risk effect dominates. With a kinked housing supply function, the net risk-return relationship responds asymmetrically to these demand changes. In markets with positive demand changes, the larger ξ is, the more the housing supply would mute changes in house prices, and the weaker the net risk-return relationship is. In this case, the net risk-return relationship decreases with the value of ξ. This is described in Equations (22) and (23). In contrast, in markets with negative demand changes, housing supply does not adjust, and house prices and returns would have to adjust fully to clear the market. Thus, the net risk-return relationship is independent of the value of ξ. This is described in Equations (21) and (24). The latter finding is consistent with the notion in Glaeser et al. (2008) that "elasticity was uncorrelated with either price or quantity changes during the bust." Putting these together, we obtain a second testable prediction for cross-sectional variation in the risk-return relationship: when the price risk increases, the relative strength of the consumption insurance effect versus the financial risk effect decreases with housing supply elasticity; the opposite occurs if the price risk decreases.

In addition to delivering cross-sectional variation in the housing risk-return relationship, the model also helps explain the heterogeneity in the persistence of house price volatility across markets. Stein (1995) documents the presence of a leverage effect in the housing markets. According to the leverage effect hypothesis, a decline in housing return makes households more constrained and impairs their ability to buy another home. This leads to a decrease in housing demand, which in turn generates a further decrease in house prices. This makes housing assets more risky, leading to an increase in price volatility. Taken literally, the model presented here has nothing to say about how the leverage effect would interact with the risk-return relationship derived above. However, we can still draw some implications by laying out the basic intuition.

The idea is that, in areas where both the leverage effect and the consumption insurance effect are at work, an exogenous increase in price risk will continue to cause price fluctuations over subsequent years. When the consumption insurance effect dominates the financial risk effect, an increase in the price risk leads to an immediate drop in the housing returns, which by the leverage effect hypothesis brings about a further increase in the future price risk. Thus, house price volatility tends to reinforce itself in the simultaneous presence of the leverage effect and the consumption insurance effect. Intuitively, this yields the third testable prediction: all else being equal, house price volatility is more persistent in markets where the relative strength of the consumption insurance effect is stronger.

4 Data and Variable Construction

4.1 Data

To test the implications of the model, this paper examines the risk-return relationship using three sets of house price data: OFHEO price indices, Case-Shiller price indices and National Association of Realtors (NAR) median home values. The first two datasets focus on the single family house markets, while the last one focuses on the condominium markets. In all three datasets, markets are defined at the metropolitan statistical area (MSA) level. Table 1 displays summary statistics for the quarterly housing returns in these datasets.

The OFHEO house price indices are provided by the U.S. Office of Federal Housing Enterprise Oversight, which tracks average house price changes in repeat sales or refinancing on the same single-family properties. Using these indices, I construct a panel of quarterly housing returns in 90 MSAs between 1975 and 2004.[16,17] My choice of a quarterly holding period is made for two reasons. First, given the short sample period, constructing housing returns on the quarterly basis increases the number of observations in each MSA significantly. Second, while transaction costs are important in home sales, their impact on the holding period is offset by the impact of the idiosyncratic housing components. Thus, the relative short holding period is unlikely to be a concern for studying housing risks and returns (Goetzmann (1993)). The quarterly housing returns are then made real by deflating with net-of-shelter consumer price indices (CPIs) published by the Bureau of Labor Statistics. The large number of MSAs covered in the OFHEO dataset makes it the best dataset available for exploiting the cross-sectional variation in the risk-return relationship for housing.

One limitation of the OFHEO price indices is that the data exclude loans too big (those exceeding $417,000) or too shaky (the riskiest of the subprime) for Fannie and Freddie to guarantee. To address this issue, I consider an alternative dataset – the Standard & Poor's Case-Shiller indices. Using the same repeat sales methods, Case-Shiller indices provide house price changes based on a large set of transaction records, including properties with risky loans excluded from the OFHEO. The data cover 20 major metropolitan areas between 1987 to 2008.

A third house price dataset focuses on the condominium price dynamics. It contains the 1989-2007 condominium price and single family house price data provided by the NAR. Given its focus on condominium markets, the NAR dataset provides a useful complementary test for the main empirical analysis based on the single family house markets.

[16] Metropolitan areas are defined based on the 1990 boundaries. To conduct meaningful time series analysis, the sample is restricted to 90 MSAs in which the minimum number of quarterly observations is 100.

[17] Starting from 2005, the OFHEO used the 2006 definition of MSAs based on the 2000 Census. This definition is not compatible with the 1990 definition of MSAs. Since the data required to impute hedging incentives and supply constraints are all based on the 1990 definition of MSAs, I choose to select the series of OFHEO house price indices based on the 1990 definition of MSAs, which are available only until 2004.

4.2 Variable Construction

The key to testing the model lies in our ability to measure hedging incentives for an average household in a given market. In the model, hedging incentives are defined as a positive correlation between the growth of the current house's price and the growth of future rents, where rents can be interpreted more broadly as future housing costs. In the data, future housing markets are unobserved. To proxy the hedging incentives, I consider three distinct measures: the local out-migration rate, expected price correlation accounting for where people are likely to move, and the house-condominium price correlation, all constructed at the MSA level. The first two measures are constructed for households that plan to move from one house to another. The last measure is constructed for households that plan to move from a condominium to a house.

I start by describing how to use the local out-migration rate to proxy hedging incentives. Conceptually, the lower the out-migration rate, the more likely households would be to move within the same MSA, and the more likely current and future desired houses are positively correlated. Empirically, I use data from the Census of Housing and Population (5% PUMS) in 1990 and in 2000, and compute the fraction of recent emigrants who moved from a specific MSA to other areas in the last five years for each MSA, adjusted for the population weight. If a resident lived in a different MSA than the current one five years ago, that resident is an emigrant from the host MSA five years previously.[18] Table 2 presents a list of ten cities with highest out-migration rates and a list of ten cities with lowest out-migration rates. Table 3 displays summary statistics of weighted out-migration rates for both 1990 and 2000. In both years, there is a significant amount of cross-market dispersion in the out-migration rates. For example, in 2000, the out-migration rates in Cincinnati and St. Louis were 0.018 and 0.046, respectively; while the out-migration rates in Oakland and San Jose were 0.15 and 0.19, respectively. The large difference in the out-migration rates implies that the average hedging

[18] The same definition has been used by Gyourko, Mayer and Sinai (2006). While they restrict the sample to 1990 only, I compute the out-migration rate for both 1990 and 2000. There are two problem in computing this variable using the census data. First, the 1990 PUMS data do not report the "MSA-five-years-ago" variable. Rather, they report "PUMA-five-years-ago". Second, the MSA boundaries change over time between 1990 and 2000. In order to construct a consistent measure of the MSA-level out-migration rate, I use a newly created geographic variable in the PUMS, "CONSPUMA", to first redefine the MSA market in the 2000 data, and second, to create an "MSA-five-years-ago" variable for the 1990 data. The resulting measures of out-migration rate are therefore comparable over time.

incentives of households in the first two cities are much higher than the average hedging incentives in the last two cities. This provides a preliminary answer for why housing returns vary with risks positively in Oakland and San Jose, but negatively in Cincinnati and St. Louis, as described in the beginning of this paper.

Despite being conceptually appealing, using the local out-migration rate to proxy hedging incentives implicitly treats owning the current home as a poor hedge for households that plan to move across MSAs. As shown by Sinai and Souleles (2008), even for cross-MSA movers, the hedging benefit of homeownership could be substantially high, as households tend to move among correlated markets. For each of the 40 largest MSAs, Sinai and Souleles (2008) compute the expected house price correlation between the given MSA and each of the other MSAs, where the expectation is weighted by the probability that an average household in this MSA moves to each of the other MSAs.[19] After accounting for where households are likely to move, they find that the expected correlation faced by the median household is as high as 0.67. Using their estimates, I generate a rank variable that separates these MSAs into four groups base on the median of the expected price correlation with other MSAs: below 0.25, between 0.25 and 0.50, between 0.50 and 0.75 and between 0.75 and 1. This ranked price correlation variable provides a useful complement to the local out-migration rates. While using local out-migration rates implicitly treats the current homeownership as a good hedge against future housing cost risk only for those who move within the same MSA, using the ranked price correlation measure allows us to further explore the variations in hedging incentives among those who move across MSAs.

Unlike the first two measures, a third measure of hedging incentives is constructed for households that plan to move from a condominium to a single family house within the same MSA. Assuming that most condominium owners plan to trade up to a house in the future, then the hedging benefit of owning a condominium would increase with the condominium-house price correlation in the same market.[20] To impute this condominium-house price correlation, I draw the data from the NAR,

[19]In Sinai and Souleles (2008), the house price correlation among MSAs is computed using real annual growth in the OFHEO MSA-level house price index over the 1980-2005 period; the moving probability is measured by the fraction of taxpaying households in an MSA moving from that MSA to each of the others, based on the data from the U.S. Department of the Treasury. Note that their imputation excludes within-MSA moving.

[20]Of course, households may also consider purchasing a single family house first and then moving to a smaller

which provides the median prices for both single family houses and condominiums at the MSA level between 1989 and 2007.

Finally, to measure the degree to which housing supply in each market is constrained, I use the Wharton Residential Land Use Regulation Index (WRLURI) developed by Gyourko, Saiz and Summers (2007).[21] Table 4 presents the indices of supply constraints for the ten most constrained and least constrained metropolitan areas. Using the WRLURI, I divide the metropolitan areas in the sample equally into two groups: more constrained and less constrained markets.

5 Testing the Implications of the Model

5.1 Single Family House Markets: OFHEO Price Indices

The economic model naturally lends itself to the GARCH-in-mean model developed by Engle, Lilien and Robins (1987) and Bollerslev, Engle and Wooldridge (1988). For each market i, I estimate the quarterly log housing return $r_{i,t}$ and its conditional volatility $\sigma_{i,t}^2$ as:

$$r_{i,t} = a_{i,0} + a_{i,1} r_t^f + \theta_i \sigma_{i,t}^2 + u_{i,t} \tag{25}$$

$$\sigma_{i,t}^2 = b_{i,0} + b_{i,1} u_{i,t-1} + \lambda_i \sigma_{i,t-1}^2 \tag{26}$$

where $u_{i,t} \sim N(0, \sigma_{i,t}^2)$. Note that $\sigma_{i,t}^2$ is the variance of the error term $u_{i,t}$, conditioned on information available at time $t-1$.

The GARCH(1,1)-in-mean model is attractive in this setting for several reasons. First, the parameter, θ_i, permits a natural test for the strength of the consumption insurance effect relative to the financial risk effect. Second, the parameter, λ_i, allows one to examine the tendency for volatility clustering. In order to accommodate a relatively short time series in each market, I choose

condominium at a later stage of the life-cycle. Han (2008) shows that hedging considerations affect home purchase decisions only when households move up the housing ladder. A move from a single family house to a condominium is more likely to be a downsize move. Consequently, the condominium-house price correlation is less likely to affect the single family house price dynamics.

[21] The Wharton Residential Land Use Regulation Index is an aggregate measure that comprises of eleven subindices, each of which summarizes information on a different aspect of the regulatory environment. Nine pertain to local characteristics, while two reflect state court and state legislative/executive branch behavior, respectively.

a parsimonious specification in which all the other time-varying factors that could possibly affect housing returns are captured by the error term u_{it}.

The maximum likelihood estimates of the model (25) and (26) are obtained for 78 MSAs.[22] Columns (1) - (5) in the top panel of Table 5 report the cross-sectional summary of the model parameters in the full sample. The results can be summarized as follows. First, ARCH parameters ($b_{i,1}$) are statistically significant in 69 MSAs, while GARCH parameters (λ_i) are statistically significant in 66 MSAs. This suggests that ignoring the time-varying conditional volatility in modeling housing price dynamics could have led to a bias in estimating the risk-return relationship.

Second, the key parameter of interest is θ_i, which describes the net risk-return relationship in each housing market. Two findings emerge. First, the value of θ_i ranges from -47.15 to 37.77, with a median of -2.33. The standard deviation of θ_i across markets is 1.55, representing about a 28% deviation from the mean. The large cross-market dispersion in θ_i provides an important motivation for this study. Second, θ_i is statistically significant negative in 24 MSAs. Surprisingly, only five MSAs are found to have significantly positive estimates of θ_i. While the positive estimates of θ_i in some areas can be explained by the standard financial risk effect, the negative estimates of θ_i are suggestive of the presence of the consumption insurance effect. In the remaining MSAs, θ_i is insignificantly different from zero. However, this does not necessarily imply that the conditional volatility has no effect on the mean of housing return. Rather, it may indicate that both mechanisms are at work and offset each other.

The most robust prediction of the model is that the relative strength of the consumption insurance effect versus the financial risk effect increases with hedging incentives. To test this, nonparametric procedures are employed to examine the hypothesis of no association between the GARCH-in-mean parameter in the model and the price-rent correlation, proxied by a market's out-migration rate in 2000. These procedures are particularly useful when the form of distributions is a priori unknown or when there is a problem in the unit of measurement. For subsequent analysis the Spearman rank

[22]The estimates for each individual MSA are not reported to conserve space but are available from the author upon request. There are 12 MSAs for which the GARCH-in-mean model does not converge. However, this does not necessarily imply that the conditional variance is constant in these markets. An alternative explanation could be that the series of real housing returns in these MSAs are so noisy that that a systematic pattern of conditional heteroscedasticity does not hold given the relatively short time-horizon under consideration.

correlation test is used. Consider a set of paired data $(\theta_i, m_i); i = 1, 2, \cdots, n$, the Spearman rank correlation coefficient r_s is given by

$$r_s = \frac{Cov\,(rank(\theta_i), rank(m_i))}{\sigma_{\theta_i}\sigma_{m_i}} \tag{27}$$

where $rank(\theta_i)$ is the rank assigned to θ_i with standard deviation σ_{θ_i} and $rank(m_i)$ is the rank assigned to m_i –out-migration rate– with standard deviation σ_{m_i}. Under the null hypothesis of no correlation, r_s has zero mean and variance $\frac{1}{n-1}$. When n is large, the distribution of r_s is approximately normal.

Column (6) in Panel A of Table 5 reports the Spearman rank correlation coefficients. Note that a pair of ranked variables represent ranked out-migration rates in 2000 and the ranked parameters estimated in the model (25) and (26). The estimated correlation coefficient is 0.24 in the full sample, with a highly significant p-value of 0.04. This confirms the first implication of the model: across markets, the relative strength of the financial risk effect versus the consumption insurance effect decreases with average hedging incentives. More specifically, in metropolitan areas with higher out-migration rates, it is less likely that an average household's current market would be positively correlated with its future market. The resulting weaker hedging incentives make the relative strength of the financial risk effect stronger, leading to a larger value of θ_i. As out-migration rates decrease, the relative strength of the consumption insurance increases, requiring less return to compensate for the risk. Together, the results provide evidence for the presence of both the financial risk effect and the consumption insurance effect in housing markets.

The choice of a non-parametric method prevents me from employing the variations in mobility over time. The time-invariant nature of the mobility measure may add to measurement error, leading to a biased estimate of the correlation between hedging incentives and the risk-return relationship. To control for this, I also compute the metropolitan-area-level out-migration rates based on the 1990 5% PUMS. Column (7) of Table 5 reports the Spearman correlation coefficient of the model estimates and the 1990 out-migration rates. The estimated correlation coefficient is 0.34 in the full sample, which is statistically significant at the 2 percent level. The evidence is again strongly consistent with the first implication implied by the theory.

Lower out-migration rates establish higher hedging value of owning a home for those who plan to move within the same MSA, but not for those who plan to move across MSAs. As noted earlier, households tend to move between correlated markets. To address this, I use the estimates from Sinai and Souleles (2008) and impute a variable that ranks MSAs based on their expected price correlation with other MSAs, where the expectation takes into account where people are likely to move. Column (8) reports the Spearman correlation coefficient of this rank variable and the model estimates. The significantly negative correlation coefficient provides further support for the theory. In areas where households are more likely to move to highly positively correlated markets, strong hedging incentives make the relative strength of the financial risk effect weaker, leading to a less positive θ_i. Compared with local out-migration rate proxies, the price correlation rank variable generates a Spearman correlation coefficient that is much smaller in magnitude. This is consistent with the fact that the hedging demand for households that move across markets is considerably smaller than for those who move within the same market (where the price correlation would equal one).

The second implication of the theory is an asymmetric impact of the housing supply elasticity on the net risk-return relationship. To test this, Column (9) of Table 5 reports the Spearman correlation coefficient of the model estimates and the WRLURI supply constraint indices. Note that the indices are designed so that a low value indicates a less restrictive regulation environment. Higher values of θ_i, representing stronger financial risk effect, are significantly positively associated with a higher value of WRLURI indices, which represents a more supply constrained area with a lower value of supply elasticity, ξ. Thus, the statistically significant positive correlation (with a Spearman coefficient of 0.36) suggests that the less elastic the local housing supply, the stronger the financial risk effect. Combined with the fact that house price risk has been decreasing over the sample period,[23] the finding here is consistent with the hypothesis that, as the price risk decreases, the relative strength

[23]The evidence from the OFHEO (1975 - 2004) suggests that the average price risk across MSAs has declined over time. The OLS coefficient on the time variable (year and quarter) in the house price risk regression is significantly negative, with a t-value of -13.41. Similar patterns have also been found in most of the individual MSAs between 1975 and 2004.

of the financial risk effect decreases with supply elasticity.

Using the WRLURI, I further divide the sample equally into two groups. Panel B of Table 5 presents the estimates of model (25) and (26) in a subsample of relatively less constrained metropolitan areas. Panel C presents the estimates of the same model in a subsample of relatively more supply constrained metropolitan areas. The mean estimate of θ_i is -9.83 in Panel B and -1.26 in Panel C. The median estimate of θ_i is -6.45 in Panel B and -1.09 in Panel C. Thus, in general, the net risk-return relationship is more positive in more supply constrained areas, suggesting that the financial risk effect is amplified by a lower supply elasticity. This provides further support for the second implication of the theory. Moreover, consistent with the first implication of theory, the correlation coefficients between θ_i and the out-migration rates in both panels are positive. The insignificance of the correlation coefficients in Panels B and C could be due to the small sample size in both subsamples.[24]

Table 5 also reports the estimated GARCH parameter, λ_i, in Equation (25). The average cross-sectional mean of λ_i in the full sample is 0.61 while the median is 0.70. The positive signs indicate the presence of a volatility clustering effect – large volatility follows another large volatility. This allows us to examine the final implication of the model: all else equal, house price volatility tends be more persistent in areas where the consumption insurance effect dominates. To test this, I generate two MSA-level dummy variables: CI_i and PI_i. Let CI_i equal to one if the estimated θ_i in MSA i is significantly negative. Likewise, let PI_i equal to one if the estimated λ_i in MSA i is significantly positive. Thus, $CI_i = 1$ indicates the dominance of the consumption insurance effect in MSA i and $PI_i = 1$ indicates the presence of volatility persistence in MSA i. The estimated Spearman correlation between CI_i and PI_i in the full sample is 0.28, with a p-value of 0.02. The finding provides strong support for the third implication of the theory.

To further explore the cross-market dispersion in the values of λ_i, Panel A of Table 5 shows that the Spearman correlation between the 2000 out-migration rate and λ_i is -0.27, while the correlation between the 1990 out-migration rate and λ_i is -0.24. In addition, the correlation between the price

[24] The size of subsamples is even smaller after being merged with the expected price correlation data. For this reason, Table 5 does not report the Spearman correlation coefficients of model estimates and the ranked price correlation variable in subsamples.

correlation rank variable and λ_i is 0.14. All coefficients are statistically significant, providing further evidence for the hypothesis that house price volatility tends to reinforce itself in areas with stronger hedging incentives. To see this, note that the lower the local out-migration rates and the more positively the current and future markets are correlated, the more likely the consumption insurance effect is to dominate, and the more persistent is house price volatility. Furthermore, the Spearman correlation coefficient between λ_i and the WRLURI indices is significantly negative, with a value of -0.35. Recall that lower WRLURI indices represent more elastic housing supply, which in turn weakens the financial risk effect when the price risk decreases. Thus a negative correlation between λ_i and WRLURI indices indicates that the stronger the relative strength of the consumption insurance effect, the more persistent the price volatility. This again confirms the third implication of the theory. Finally, consistent with what has been discussed so far, the Spearman correlation coefficients between λ_i and the out-migration rates remain negative in both Panel B and Panel C. Given the small number of the MSAs covered in each subsample, it is not surprising that none of these correlation coefficients is statistically significant.

5.2 Single Family House Markets: Case-Shiller Indices

While the OFHEO price indices cover a large number of MSAs, they exclude houses financed by non-conforming loans. In this section, I repeat the main empirical analysis in Section 5.1 on an alternative house price dataset: Standard & Poor's Case-Shiller indices. The small number of markets in this dataset prevents me from obtaining a significant relationship between out-migration rates and model parameters. Nonetheless, the results still provide a number of useful insights.

Table 6 displays the cross-sectional summaries of the estimated parameters. Several observations follow. First, the housing return varies with its risk positively in some areas but negatively in other areas. For example, the estimated θ_i is -14.45 in Detroit, on the one hand, and 11.81 in San Diego, on the other hand. This is consistent with a lower out-migration rate in Detroit (0.09) and a higher out-migration rate in San Diego (0.16). The Spearman correlation between the out-migration rates and θ_i is 0.40. Thus, the estimated risk-return relationship varies with the out-migration rates in a way that is consistent with the first prediction of the theory.

Second, the estimated Spearman correlation between θ_i and the WRLURI indices is highly positive and statistically significant, with a value of 0.70. In addition, the estimates of θ_i in less constrained areas are much more negative than those in constrained areas. These findings confirm the second prediction of the theory: as the price risk decreases, the financial risk effect is amplified by the lower supply elasticity. Finally, the estimates of λ_i in the full sample range from 0.11 to 0.92, indicating that the house price volatility is more persistent in some areas but less so in other areas. The negative correlation between the estimated λ_i and out-migration rates, although insignificant, is consistent with the third implication of the theory. Taken together, these results are consistent with the main findings based on the OFHEO sample.

5.3 Condominium Markets

So far the empirical analysis has been based on single family house markets where the average hedging incentives are inversely measured by the out-migration rates. In this section, I focus on the condominium price dynamics by using the median home values provided by the National Association of Realtors (NAR). The dataset covers two housing types: condominiums and single family houses. To the extent that a larger fraction of condominium owners are considering moving to single family houses in the same market, the price rent correlation in the model can be proxied by the correlation between condominium price changes and single family house price changes. If the model is correct, the condominium returns should vary more positively with the price risk in areas where the condominium-house price correlation is less positive.

The NAR provides single family median prices for 155 metropolitan areas and condominium median prices for 55 metropolitan areas on a quarterly basis from 1989 to 2007. To obtain a useful comparison between condominium prices and single family prices, I first eliminate 100 MSAs for which quarterly condominium prices are not available. To conduct a sensible time series analysis, I then drop 42 MSAs for which the number of quarters observed is less than 70. This leaves us with only 13 MSAs. The resulting sample size is so small that the large sample properties of the Spearman correlation may not apply. Consequently, the results presented in the condominium markets should be viewed as illustrative rather than definitive evidence.

For each MSA, I first compute the quarterly returns in condominium prices and single family house prices. I then reestimate the model (25) and (26) using log condominium returns. In addition, I compute the correlation between single family house price changes and condominium price changes. Table 7 reports the estimated GARCH-in-mean parameters, the condominium-house price correlation in each market and the resulting Spearman correlation coefficient.

The main finding is a statistically significant negative relationship between the net risk-return relationship and condominium buyers' hedging incentives (Spearman's $\rho = -0.5385$, $p = 0.0576$). On the one hand, Philadelphia has a strongly negative risk-return relationship (-19.82) and a positive condominium-house price correlation (0.54). On the other hand, Bismark has a strongly positive risk-return relationship (7.00) and a negative condominium-house price correlation (-0.03). This is consistent with the hypothesis that average hedging incentives among condominium buyers are high in Philadelphia but low in Bismark. Thus, households not only understand the insurance value of their condominium purchase but also respond to the price risk in a rational way. In areas where the condominium-house price correlation is strongly positive, young households consider their condominium purchase as a hedge against the future house cost risk. As a result, less return is required to compensate for the risk. In contrast, in areas where the correlation between condominium and single family house prices is weaker or even negative, the initial condominium purchase provides little insurance value for future home purchase. Hence, the standard financial risk effect dominates.

6 Robustness Check

6.1 Leverage Effect Hypothesis

There is a long tradition in finance that attributes the negative risk-return relationship for stocks to the leverage effect. A leverage effect exists if a drop in the value of the stock (negative return) increases financial leverage, which could potentially make the stock riskier and increase its volatility.

To the extent that this effect operates in the housing market through downpayment constraints (Stein (1995)), the negative risk-return relationship for housing could be induced by the leverage effect rather than hedging incentives. This subsection discusses how I control for the leverage effect in estimating the housing risk-return relationship.

To address the leverage concern, I adopt an EGARCH(1,1)-in-mean specification to test for the leverage effect in housing returns. In particular, equation (26) can be modified as:

$$\ln(\sigma^2_{i,t}) = b_{i,0} + b_{i,1} z_{i,t-1} + b_{i,2}(|z_{i,t-1}| - E|z_{i,t-1}|) + \lambda_i \ln(\sigma^2_{i,t-1}) \tag{28}$$

where $z_{i,t} = u_{i,t}/\sigma_{i,t}$. This is a model that is proposed by Nelson (1991) and allows an asymmetric effect of unanticipated changes in housing returns. Equation (28) specifies the log conditional variance as an asymmetric function of last period's standardized innovation, $z_{i,t-1}$, and last period's log conditional variance, $\ln(\sigma^2_{i,t-1})$. The term $(|z_{i,t-1}| - E|z_{i,t-1}|)$ represents the size effect and the term $z_{i,t-1}$ represents the sign effect. The coefficient $b_{i,1}$ should be negative if leverage induces an inverse relation between the house price and volatility. The estimate of θ_i therefore serves as a direct estimate of the risk-return relationship after controlling for the possible leverage effect.

Equations (25) and (28) are jointly estimated for each individual MSA in the OFHEO sample. Out of 90 MSAs, the model converges in 51 MSAs. Panel A in Table 8 reports the summary of estimates for the full sample. The coefficient, $b_{i,1}$, measuring the asymmetric impact of past innovation on current volatility (leverage effect), is statistically significant at the 10% level for 27 MSAs. Among these MSAs, the sign of $b_{i,1}$ is significantly negative in 11 MSAs, indicating that in these areas negative innovations (price declines) increase volatility more than positive innovations (price advances). The sample presents rich cross-market variation in the strength of the leverage effect. The more negative the value of $b_{i,1}$, the stronger is the leverage effect.

Having controlled for the leverage effect, we find that the values of the risk-return relationship coefficient, θ_i, now range from -67.80 to 58.03, with a mean of -1.97 and a median of -2.91. Compared with Table 5, the estimates of θ_i remain dispersed across MSAs. In particular, the sign of θ_i is significantly negative in 15 MSAs, out of which only five MSAs are found to have leverage effects. Thus, the finding of a negative risk-return relationship cannot be attributed to the leverage hypothesis. Moreover, the Spearman correlation between the estimates of θ_i and the local out-migration rates is 0.23 (versus 0.24 in Table 5). This cannot be explained by the leverage effect either. Thus, the evidence of the consumption insurance effect is robust to the leverage considerations. Likewise, the Spearman correlation between the estimates of the volatility persistence parameter, λ_i, and the local out-migration rates is -0.21 (versus -0.27 in Table 5). This is consistent with the third implication of the theory: the stronger the average hedging incentives, the more persistent the house price volatility. Note that both correlation coefficients are less significant than in Table 5. This is possibly due to a much smaller sample size after merging the EGARCH estimates with the out-migration data.[25] Finally, the Spearman correlation between θ_i and the WRLURI indices is highly positive (with a value of 0.53) and statistically significant, consistent with the second implication about an asymmetric impact of the supply elasticity. Furthermore, the estimates of θ_i in more supply constrained areas are significantly more positive than those in less supply constrained areas, indicating that the relative strength of the financial risk effect is much stronger in supply constrained areas. Taking these findings together, I conclude that the results from the main empirical analysis remain strong even after controlling for the leverage effect.

A reading of the estimates in Panel A of Table 8 also reveals another interesting finding: the Spearman correlation between the estimates of θ_i and the estimates of $b_{i,1}$ is strongly positive (0.69) and statistically significant at almost any level. As mentioned earlier, a lower and negative value of $b_{i,1}$ indicates a stronger leverage effect. Using the American Housing Survey data, Lamont and Stein (1999) demonstrate that a strong leverage effect is associated with a high loan-to-value ratio; the latter is measured by the fraction of all owner-occupants with loan-to-value ratios exceeding 80% in a given MSA. Given the option to default, households with higher loan-to-value ratios could be less exposed to the financial risk associated with future housing capital gains, and hence, less returns may be required to compensate for the risk. This explains the observed strong positive correlation between θ_i and $b_{i,1}$.

[25] The EGARCH(1,1)-in-mean model converges in 51 MSAs. After merging with the out-migration data, the number of MSAs in the full sample is reduced to 37.

6.2 Emerging City Hypothesis

In the main empirical analysis, the evidence of the consumption insurance effect comes from the observation that the strength of the consumption insurance effect relative to the financial risk effect is positively associated with a market's out-migration rate. Presumably, a high out-migration rate is driven by changes in economic opportunity, as proxied by metropolitan-area-level unemployment differentials and housing cost differentials. This is not modeled in our theory, but it almost surely exists to some extent in every housing market. To the extent that the out-migration rate is driven by housing market conditions, the causation relationship hypothesized in the theory could be reversed. This subsection discusses how I control for the associated endogeneity problem.

To see concretely how an endogeneity bias might arise, consider the following hypothesis. The basic idea is that some areas are in the process of undergoing fundamental transitions. Moreover, such transitions are purported to have two distinct effects. First, strong economic growth encourages local households to stay within the same market rather than move to other markets. Second, for markets in the process of transition, current economic shocks, such as job growth, contain more information about future housing growth prospects and hence, change households' expectations about the future house price. Particularly, one might forecast that a strong economy will be accompanied by high return and low risk in the local housing market. If these two assertions are both correct, there would be a positive correlation between the out-migration rate and the estimated risk-return relationship, even if hedging incentives were absent.

I make two attempts to distinguish between the consumption insurance effect and this alternative. One approach is to assume that the extent to which a market can be characterized as "emerging" is more or less fixed over the twenty-nine-year duration of the sample period. If this identifying assumption is correct, the empirical method completely controls for the emerging-city phenomenon by allowing each metropolitan area to have its own coefficient on $\sigma^2_{i,t}$. Thus, if some metropolitan areas are more "emerging" than others over the entire sample, and hence have house returns that are more negatively correlated with risk, this would be picked up in the market-specific $\sigma^2_{i,t}$ coefficients, but not in the correlation term between θ_i and the intermetropolitan mobility.

One concern with this method is that the emerging market characteristic is not fixed for markets over the entire twenty-nine-year period. For example, an MSA that was not emerging in 1984 may begin to emerge in 1990. If this is the case, I will control directly for the observable variables that are proxies for the extent to which an MSA is emerging. One natural candidate variable is the local unemployment rate.

Table 9 repeats the estimation in Panel A (full sample) in Table 5, with one additional variable in the mean equation: the unemployment rate. I then use the non-parametric method to test the correlation between the estimated parameter and the out-migration rate. I ask whether the previous interaction results are truly due to the consumption insurance effect, or merely due to the fact that the out-migration rate is correlated with job growth. In most markets, the estimated coefficients on the unemployment rate are significantly negative, indicating that lower employment rates are associated with higher housing returns. However, the mean of the GARCH-in-mean parameter remains negative. Furthermore, the Spearman correlation coefficient remains significantly positive. Thus, the results are robust to the emerging city hypothesis.

7 Conclusion

The risk-return relationship is fundamental to finance. While a large and growing body of work focuses on the risk-return relationship for financial assets like stocks, little work has examined the risk-return relationship for housing. The omission seems surprising given the magnitude of the housing market and the significance of the house price risk in the current economy. More importantly, standard financial risk-return theory cannot explain why the housing return varies with risk positively in some areas but negatively in other areas.

This paper contributes to the literature by providing a consistent economic explanation for cross-sectional variation in the intertemporal risk-return relationship for housing. To do so, the paper establishes an equilibrium housing risk-return relationship in a consumption based asset pricing model. The model incorporates two unique features of housing: (1) self-hedging incentives, and (2) a kinked housing supply function. The paper starts by showing that the positive risk premium associated with owning a risky housing asset may be offset by the negative risk premium associated with using the current house to hedge against the future housing cost risk. The net risk-return relationship depends on the strength of the average hedging incentives in the local market. Moreover, the kinked housing supply function implies an asymmetric impact of supply elasticity on the relative strength of the two risk-return effects. Finally, given that housing is a highly leveraged asset, the model implies that the price risk tends to reinforce itself through the mechanisms of self-hedging and leverage.

Using the U.S. panel data, the paper finds support for these implications in both single family house markets and condominium markets. In single family house markets, the lower the local out-migration rate, the less the return required to compensate for the risk. To the extent that lower out-migration rates reflect stronger hedging incentives among local households, the finding confirms the presence of the consumption insurance effect implied by the theory. In addition, the relative strength of the financial risk effect versus the consumption insurance effect decreases with the local supply elasticity over the sample period. Finally, the price risk tends to be more persistent in areas where the consumption insurance effect dominates. These results are robust to using different datasets (OFHEO and Case-Shiller) and different specifications (GARCH and EGARCH). Turning to the condominium markets, the net risk-return relationship is negatively correlated with the correlation between condominium price changes and single family house price changes. This evidence reinforces the notion that many condominium buyers consider the holding of a condominium as a hedge against the price risk for the future purchase of single family houses, hence, requiring less return to reward the condominium price risk. Taken together, the empirical results confirm the prediction that local hedging incentives and supply constraints are the main forces that drive cross-market dispersion of the intertemporal housing risk-return relationship.

To conclude, the two-dimensional aspects of the housing risk-return relationship, financial risk and consumption insurance, make the analysis of housing unique. By modeling and testing the relative strength of these two effects in a unified framework, this paper provides a first step towards understanding the cross-sectional variation in the intertemporal risk-return relationship for housing.

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APPENDIX

7.1 Proof of Proposition 3.1

Proof: To solve the model, I apply Campbell and Shiller (1988)'s approximation. Let $p_t = \ln P_t$ and $q_t = \ln Q_t$.

$$\begin{aligned} r_t &= \ln R_{t+1} \\ &= k + \gamma \ln(1-\delta) + \gamma g^P_{t+1} + (1-\gamma) g^Q_{t+1} - (1-\gamma)(p_t - q_t) \end{aligned} \tag{A-1}$$

where parameter γ is the average ratio of the house price to the sum of the house price and the rent, a number slightly smaller than one, and k is a constant related to γ.

Substituting equation (4) and equation (A-1) into equation (9), we obtain

$$E_t exp\left[-r^f_t - \frac{1}{2}\sigma^2_{M,t} + \epsilon_{M,t+1} + k + \gamma \ln(1-\delta) + \gamma g^P_{t+1} + (1-\gamma) g^Q_{t+1} - (1-\gamma)(p_t - q_t)\right] = 1 \tag{A-2}$$

I first postulate a solution to the log price rent ratio in terms of the state variables. I then verify this solution and solve for the parameters of the solution. The linear solution takes the following form.

$$p_t - q_t = c_0 + c_1 g^P_t + c_2 g^Q_t + c_3 \sigma^2_{P,t} + c_4 \sigma^2_{Q,t} \tag{A-3}$$

Substituting this solution to equation (A-2), we get

$$1 = E_t exp[A(\cdot)] \tag{A-4}$$

where

$$\begin{aligned} A(\cdot) &= -r^f_t - \frac{1}{2}\sigma^2_{M,t} + \epsilon_{M,t+1} + k + \gamma \ln(1-\delta) + \gamma g^P_{t+1} + (1-\gamma) g^Q_{t+1} \\ &\quad -(1-\gamma)(c_0 + c_1 g^P_t + c_2 g^Q_t + c_3 \sigma^2_{P,t} + c_4 \sigma^2_{Q,t}) \\ &= \left(-r^f_t + k + \gamma \ln(1-\delta) - (1-\gamma)c_o + \gamma \alpha^P_0 + (1-\gamma)\alpha^Q_0\right) - \frac{1}{2}\sigma^2_{M,t} + \epsilon_{M,t+1} \\ &\quad + \left(\gamma \alpha^P_1 + (1-\gamma)c_1\right) g^P_t + \left((1-\gamma)(\alpha^Q_1 - c_2)\right) g^Q_t - (1-\gamma)c_3 \sigma^2_{P,t} - (1-\gamma)c_4 \sigma^2_{Q,t} \\ &\quad + \gamma \epsilon_{P,t+1} + (1-\gamma)\epsilon_{Q,t+1} \end{aligned} \tag{A-5}$$

This leads to

$$\begin{aligned} E_t A(\cdot) &= const. - \frac{1}{2}\sigma^2_{M,t} + \left(\gamma\alpha_1^P + (1-\gamma)c_1\right) g_t^P + \Big((1-\gamma)(\alpha_1^Q - c_2)\Big) g_t^Q - (1-\gamma)c_3\sigma^2_{P,t} - (1-\gamma)c_4\sigma^2_{Q,t} \\ Var_t A(\cdot) &= \sigma^2_{m,t} + \gamma^2\sigma^2_{P,t} + (1-\gamma)^2\sigma^2_{Q,t} + 2\gamma\rho_{PM}\sigma^2_{P,t} + 2\gamma(1-\gamma)\rho_{PQ}\sigma^2_{P,t} \end{aligned} \tag{A-6}$$

where $const. = -r_t^f + k + \gamma\ln(1-\delta) - (1-\gamma)c_o + \gamma\alpha_0^P + (1-\gamma)\alpha_0^Q$

Since $A(\cdot)$ is a normal random variable, we must have

$$E_t A(\cdot) + \frac{1}{2} Var_t A(\cdot) = 0 \tag{A-7}$$

Substituting equations (A-6) into equation (A-7), we obtain

$$\begin{aligned} 0 &= (const.) + \left(\gamma\alpha_1^P - (1-\gamma)c_1\right) g_t^P + \Big((1-\gamma)(\alpha_1^Q - c_2\Big) g_t^Q \\ &\quad + \left(-(1-\gamma)c_3 + \frac{1}{2}\gamma^2 + \gamma\rho_{PM} + (1-\gamma)\gamma\rho_{PQ}\right)\sigma^2_{P,t} + \left(\frac{1}{2}(1-\gamma)^2 - (1-\gamma)c_4\right)\sigma^2_{Q,t} \end{aligned} \tag{A-8}$$

Note that for this equation to hold, the terms in the five brackets must be identically zero, since g_t^j and $\sigma^2_{j,t}$ are random variables. Solving the resulting equations produces the solutions below:

$$p_t - q_t = c_0 + c_1 g_t^P + c_2 g_t^Q + c_3\sigma^2_{P,t} + c_4\sigma^2_{Q,t} \tag{A-9}$$

where

$$\begin{aligned} c_0 &= \frac{-r_t^f + k + \gamma\ln(1-\delta) + \gamma\alpha_0^P + (1-\gamma)\alpha_0^Q}{1-\gamma} \\ c_1 &= \frac{\gamma\alpha_1^P}{1-\gamma} \\ c_2 &= \alpha_1^Q \\ c_3 &= \frac{\frac{1}{2}\gamma^2 + \gamma\rho_{PM} + \gamma(1-\gamma)\rho_{PQ}}{1-\gamma} \\ c_4 &= \frac{1}{2}(1-\gamma)\sigma^2_{Q,t} \end{aligned}$$

Substituting equation (A-9) into equation (A-1), we then obtain

$$r_{t+1} = r_t^f + \gamma\epsilon_{P,t+1} + (1-\gamma)\epsilon_{Q,t+1} - \left(\frac{1}{2}\gamma^2 + \gamma\rho_{PM} + \gamma(1-\gamma)\rho_{PQ}\right)\sigma^2_{P,t} - \frac{1}{2}(1-\gamma)^2\sigma^2_{Q,t} \quad \blacksquare$$

7.2 Proof of Proposition 3.2

(i) Proof: It follows from Equation (14) that

$$E_t(r_{t+1}) = r_t^f - \left(\frac{1}{2}\gamma^2 + \gamma\rho_{PM} + \gamma(1-\gamma)\rho_{PQ}\right)\sigma_{P,t}^2 - \frac{1}{2}(1-\gamma)^2\sigma_{Q,t}^2 \tag{A-10}$$

$$\sigma_{r,t}^2 = \left(\gamma^2 + 2\gamma(1-\gamma)\rho_{PQ}\right)\sigma_{P,t}^2 + (1-\gamma)^2\sigma_{Q,t}^2 \tag{A-11}$$

$$\begin{aligned}\frac{\partial E_t(r_{t+1})}{\partial \sigma_{r,t}^2} &= \frac{\partial E_t(r_{t+1})}{\partial \sigma_{P,t}^2}\frac{\partial \sigma_{P,t}^2}{\partial \sigma_{r,t}^2} \\ &= -\left(\frac{1}{2}\gamma^2 + \gamma\rho_{PM} + \gamma(1-\gamma)\rho_{PQ}\right)\frac{1}{\gamma^2 + 2\gamma(1-\gamma)\rho_{PQ}} \\ &= -\frac{\frac{1}{2}\gamma + \rho_{PM} + (1-\gamma)\rho_{PQ}}{\gamma + 2(1-\gamma)\rho_{PQ}} \end{aligned} \tag{A-12}$$

where the last step comes from Equations (A-10) and (A-11). It follows from Equation (A-12) that

$$\begin{aligned}\frac{\partial E_t(r_{t+1})}{\partial \sigma_{r,t}^2} &< 0 \text{ if } \rho_{PQ} > \frac{-\rho_{PM} - \frac{1}{2}\gamma}{1-\gamma} \\ \frac{\partial E_t(r_{t+1})}{\partial \sigma_{r,t}^2} &> 0 \text{ if } \rho_{PQ} < \frac{-\rho_{PM} - \frac{1}{2}\gamma}{1-\gamma} \qquad \blacksquare\end{aligned}$$

(ii) Proof: Following Equation (A-12), we have

$$\begin{aligned}\frac{\partial^2 E_t(r_{t+1})}{\partial \sigma_{r,t}^2 \partial \rho_{PQ}} &= -\frac{1}{(\gamma + 2(1-\gamma)\rho_{PQ})}(1-\gamma) + \frac{\frac{1}{2}\gamma + \rho_{PM} + (1-\gamma)\rho_{PQ}}{(\gamma + 2(1-\gamma)\rho_{PQ})^2}2(1-\gamma) \\ &= \frac{2(1-\gamma)\rho_{PM}}{(\gamma + 2(1-\gamma)\rho_{PQ})^2} < 0 \qquad \blacksquare\end{aligned}$$

where the inequality follows from the assumption that $\rho_{PM} < 0$.

7.3 Proof of Proposition 3.3

$$\begin{aligned} r_{t+1} &\equiv \ln R_{t+1} \\ &= \ln \frac{(1-\delta)P_{t+1} + Q_{t+1}}{P_t} \end{aligned} \tag{A-13}$$

Taking the derivatives of r_{t+1} with respect to $\sigma^2_{P,t}$ yields

$$\begin{aligned}\frac{\partial r_{t+1}}{\partial \sigma^2_{r,t}} &= \frac{1}{R_{t+1}}\frac{\partial R_{t+1}}{\partial \sigma^2_{r,t}} \\ &= \frac{1}{R_{t+1}}\{\frac{\partial R_{t+1}}{\partial P_t}\frac{\partial P_t}{\partial \sigma^2_{r,t}} + \frac{\partial R_{t+1}}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial \sigma^2_{r,t}}\}\end{aligned} \tag{A-14}$$

That is, the house price risk affects the housing return through both the current price and future price. The first term in Equation (A-14) describes how much the price risk is capitalized into the current price. In particular, the optimal investor behavior characterized by Proposition 2.1 requires that,

$$\frac{\partial P_t}{\partial \sigma^2_{r,t}} = P_t \frac{\partial p_t}{\partial \sigma^2_{P,t}}\frac{\partial \sigma^2_{P,t}}{\partial \sigma^2_{r,t}} = P_t \frac{\frac{1}{2}\gamma + \rho_{PM} + (1-\gamma)\rho_{PQ}}{\gamma + 2(1-\gamma)\rho_{PQ}} \tag{A-15}$$

Thus, the first term in Equation (A-14) is reduced to

$$\frac{1}{R_{t+1}}\frac{\partial R_{t+1}}{\partial P_t}\frac{\partial P_t}{\partial \sigma^2_{r,t}} = -\frac{1}{R_{t+1}}\frac{R_{t+1}}{P_t}\frac{\partial P_t}{\partial \sigma^2_{r,t}} = -\frac{\frac{1}{2}\gamma + \rho_{PM} + (1-\gamma)\rho_{PQ}}{\gamma + 2(1-\gamma)\rho_{PQ}} \tag{A-16}$$

The second term in Equation (A-14) describes how much the price risk is capitalized into the next period's price. In particular,

$$\frac{1}{R_{t+1}}\frac{\partial R_{t+1}}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial \sigma^2_{r,t}} = \frac{1}{R_{t+1}}\frac{\partial R_{t+1}}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial h_t}\frac{\partial h_t}{\partial \sigma^2_{r,t}} \tag{A-17}$$

This term captures the general equilibrium effect in the housing markets. That is, changes in the price risk affect housing demand, which, in turn, affects the housing supply market, leading to changes in the next period's house prices.

Below I examine these two terms in four different cases: (1) an increase in $\sigma^2_{r,t}$ when $c_3 < 0$; (2) an increase in $\sigma^2_{r,t}$ when $c_3 > 0$; (3) a decrease in $\sigma^2_{r,t}$ when $c_3 < 0$ and (4) a decrease in $\sigma^2_{r,t}$ when $c_3 > 0$. Note that in all four cases, the first term is identical, but the second term is different depending on the value of $\frac{\partial h_t}{\partial \sigma^2_{r,t}}$. When $c_3 < 0$, i.e., when the financial risk effect dominates, $\frac{\partial h_t}{\partial \sigma^2_{r,t}} < 0$, whereas when $c_3 > 0$, i.e., when the consumption insurance effect dominates, $\frac{\partial h_t}{\partial \sigma^2_{r,t}} > 0$.[26] Since the housing supply is elastic with respect to upward shocks, but inelastic with respect to downward shocks, the impact of supply elasticity on the net risk-return relationship is asymmetric.

Case 1: Effects of an increase in $\sigma^2_{r,t}$ on r_{t+1} when $c_3 < 0$.

In this case, an exogenous increase in $\sigma^2_{r,t}$ leads to a negative demand shock. However, equation (18) shows that the housing supply is inelastic with respect to downward shocks. The market clearing condition requires that $h_t = \frac{H_t}{N_t}$. Thus, in equilibrium, $\frac{\partial h_t}{\partial \sigma^2_{r,t}} = 0$. The second term in the bracket in equation (A-14) diminishes. Equation (A-14) now

[26]See Han (2008) for detailed discussion and proof of the net effect of house price risk, $\sigma^2_{r,t}$, on housing demand h_t.

reduces to

$$\begin{aligned}\frac{\partial r_{t+1}}{\partial \sigma^2_{r,t}} &= \frac{1}{R_{t+1}}\frac{\partial R_{t+1}}{\partial P_t}\frac{\partial P_t}{\partial \sigma^2_{r,t}} \\ &= -\frac{\frac{1}{2}\gamma + \rho_{PM} + (1-\gamma)\rho_{PQ}}{\gamma + 2(1-\gamma)\rho_{PQ}}\end{aligned} \tag{A-18}$$

This is equivalent to the result in equation (14), which is derived from an endowment economy characterized by fixed housing supply. Clearly, the net risk-return relationship described in (A-18) is independent of the supply elasticity, measured by ξ.

Case 2: Effects of an increase in $\sigma^2_{r,t}$ on r_{t+1} when $c_3 > 0$.

In this case, an exogenous increase in $\sigma^2_{P,t}$ leads to a positive demand shock. Denote $\tau_1 \equiv \frac{\partial h_t}{\partial \sigma^2_{r,t}} > 0$. The housing supply is elastic with respect to the upward shocks, as described in equation (18). Combining this with the housing market clearing condition, we obtain

$$P_{t+1} = P_t + \frac{1}{\xi}(N_{t+1}h_{t+1} - (1-\delta)N_t h_t) \tag{A-19}$$

This yields

$$\begin{aligned}\frac{\partial P_{t+1}}{\partial \sigma^2_{r,t}} &= \frac{\partial P_{t+1}}{\partial h_t}\frac{\partial h_t}{\partial \sigma^2_{r,t}} \\ &= -(1-\delta)N_t\frac{\tau_1}{\xi}\end{aligned} \tag{A-20}$$

Substituting Equation (A-18) and (A-20) into (A-14), we obtain

$$\frac{\partial r_{t+1}}{\partial \sigma^2_{r,t}} = \left(-\frac{\frac{1}{2}\gamma + \rho_{PM} + (1-\gamma)\rho_{PQ}}{\gamma + 2(1-\gamma)\rho_{PQ}}\right) - \left(\frac{1}{R_{t+1}P_t}(1-\delta)^2 N_t\frac{\tau_1}{\xi}\right) < 0 \tag{A-21}$$

where the inequality follows from $c_3 > 0$ and $\tau_1 > 0$. The marginal dependence of the net risk-return relationship on the supply elasticity is given by

$$\frac{\partial^2 r_{t+1}}{\partial \sigma^2_{r,t}\partial \xi} = \frac{1}{R_{t+1}P_t}(1-\delta)^2 N_t\frac{\tau_1}{\xi^2} > 0 \tag{A-22}$$

where the inequality follows from $\tau_1 > 0$. Thus, in this case, the net housing risk-return relationship decreases with supply elasticity, measured by ξ.

Case 3: Effects of a decrease in $\sigma^2_{r,t}$ on r_{t+1} when $c_3 < 0$.

In this case, an exogenous decrease in $\sigma^2_{r,t}$ leads to a positive demand shock. This is the same as in case 2 except that now we have $\tau_2 \equiv \frac{\partial h_t}{\partial \sigma_{r,t}^2} < 0$. Following the logic in case 2, we obtain

$$\frac{\partial r_{t+1}}{\partial \sigma_{r,t}^2} = \left(-\frac{\frac{1}{2}\gamma + \rho_{PM} + (1-\gamma)\rho_{PQ}}{\gamma + 2(1-\gamma)\rho_{PQ}}\right) - \left(\frac{1}{R_{t+1}P_t}(1-\delta)^2 N_t \frac{\tau_2}{\xi}\right) > 0 \tag{A-23}$$

where the inequality follows from $c_3 < 0$ and $\tau_2 < 0$. The marginal dependence of the net risk-return relationship on the supply elasticity is given by

$$\frac{\partial^2 r_{t+1}}{\partial \sigma_{r,t}^2 \partial \xi} = \frac{1}{R_{t+1}P_t}(1-\delta)^2 N_t \frac{\tau_2}{\xi^2} < 0 \tag{A-24}$$

where the inequality follows from $\tau_2 < 0$. Thus, in this case, the net housing risk-return relationship decreases with supply elasticity, measured by ξ.

Case 4: Effects of a decrease in $\sigma_{r,t}^2$ on r_{t+1} when $c_3 > 0$.

In this case, an exogenous decrease in $\sigma_{r,t}^2$ leads to negative demand shock. This is the same as in case 1. In equilibrium, $\frac{\partial h_t}{\partial \sigma_{r,t}^2} = 0$. The second term in the bracket in equation (A-14) diminishes. Equation (A-14) now reduces to

$$\frac{\partial r_{t+1}}{\partial \sigma_{r,t}^2} = -\frac{\frac{1}{2}\gamma + \rho_{PM} + (1-\gamma)\rho_{PQ}}{\gamma + 2(1-\gamma)\rho_{PQ}} \tag{A-25}$$

Clearly, the net risk-return relationship described in (A-25) is independent of the supply elasticity, measured by ξ. ■

Figure 1: Correlation Between Housing Return and Risk

Source: Housing return and risk are measured by the average of real quarterly housing return and its standard deviation over a moving window of 3 years. House price data are obtained from the OFHEO and are made real by deflating with net-of-shelter consumer price indices (CPIs) published by the Bureau of Labor Statistics. The t-statistics in these linear regressions is -16.90 for Cincinnati, -9.97 for St. Louis, 2.76 for Oakland, and 4.52 for San Jose.

Table 1: Summary Statistics of Nominal Quarterly Housing Returns

Dataset	Property Type	Time Span	Mean	Std. Dev.	Min	Max	Obs.
OFHEO	Single Family House	1975-2004	0.014	0.028	-0.403	1.075	9337
Case-Shiller	Single Family House	1987-2008	0.012	0.025	-0.122	0.161	1574
NAR	Condominium	1989-2007	0.021	0.093	-0.483	0.657	800
NAR	Single Family House	1989-2007	0.017	0.046	-0.220	0.230	800

a. This table shows summary statistics of nominal quarterly housing returns computed from three different datasets: OFHEO, Case-Shiller and NAR. b. Obs. indicates the number of observations defined in terms of the combination of (MSA, year, quarter).

Table 2: Metropolitan Areas with Highest and Lowest Out-Migration Rates in 2000

Highest Out-Migration Rates		Lowest Out-Migration Rates	
Index Value	Area Name	Index Value	Area Name
0.407	Jacksonville, NC	0.094	Philadelphia, PA-NJ
0.318	Fayetteville, NC	0.094	Detroit, MI
0.314	Killeen-Temple, TX	0.093	New Bedford, MA
0.309	Bryan-College Station, TX	0.089	Scranton–Wilkes-Barre–Hazleton, PA
0.296	Iowa City, IA	0.085	Hickory-Morganton-Lenoir, NC
0.288	Anchorage, AK	0.081	Pittsburgh, PA
0.272	Bloomington, IN	0.057	Ponce, PR
0.271	Flagstaff, AZ-UT	0.036	Aguadilla, PR
0.270	Gainesville, FL	0.026	Chattanooga, TN-GA
0.264	Colorado Springs, CO	0.025	San Juan-Bayamon, PR

a. The out-migration rate is computed as the fraction of recent emigrants who moved from a specific MSA to other areas in the last five years for each MSA, adjusted for population weight. If a resident lived in a different MSA than the current one five years ago, that resident is an emigrant from the host MSA five years previously. b. The data source is the 2000 Census of Housing and Population.

Table 3: Comparison of MSA-Level Out-Migration Rates in 1990 and 2000

Out-Migration Rate	Numbers of Observations	Mean	S.D.	Min	Max
1990	152	.147	.079	.009	.395
2000	289	.155	.048	.025	.407

a. The out-migration rate is computed as the fraction of recent emigrants who move from a specific MSA to other areas in the last five years for each MSA, adjusted for the population weight. If a resident lived in a different MSA than the current one five years ago, that resident is an emigrant from the host MSA five years previously. b. The data source is 1990 and 2000 Census of Housing and Population.

Table 4: Ten Most and Least Constrained Metropolitan Areas

Most Regulated		Least Regulated	
Index Value	Area Name	Index Value	Area Name
1.79	Providence-Fall River-Warwick, RI-MA	-0.80	Kansas City, MO-KS
1.54	Boston, MA-NH	-0.76	Indianapolis, IN
1.21	Monmouth-Ocean, NJ	-0.72	St. Louis, MO-IL
1.03	Philadelphia, PA	-0.56	Cincinnati, OH-KY-IN
1.01	Seattle-Bellevue-Everett, WA	-0.50	Dayton-Springfield, OH
0.90	San Francisco, CA	-0.41	Oklahoma City, OK
0.85	Denver, CO	-0.35	Dallas, TX
0.80	Nassau-Suffolk, NY	-0.27	Fort Worth-Arlington, TX
0.71	Bergen-Passaic, NJ	-0.24	San Antonio, TX
0.70	Fort Lauderdale, FL	-0.19	Houston, TX

a. This table is derived from Gyourko et al. (2007), who present a more complete table of the metropolitan-area level WRLURI index. b. Factor analysis is used to create the aggregate index, which is then standardized so that the sample mean is zero and the standard deviation equals one. The index is designed so that a low value indicates a less restrictive regulation environment.

Table 5: GARCH-in-Mean Estimates for Single Family House Markets (OFHEO Sample)

	(1) mean	(2) se(mean)	(3) median	(4) max	(5) min	(6) $r_s(*, m_{i,2000})$	(7) $r_s(*, m_{i,1990})$	(8) $r_s(*, corr_i)$	(9) $r_s(*, sup_i)$
Panel A: Full Sample									
θ_i	-5.56	1.55	-2.33	37.77	-47.15	0.24*(0.04)	0.34*(0.02)	−0.11*(0.06)	0.36*(0.002)
λ_i	0.61	0.03	0.70	0.95	-0.09	−0.27*(0.02)	−0.24(0.09)	0.14(0.10)	−0.35*(0.00)
$a_{i,0}$	0.01	0.00	0.01	0.03	-0.01	−0.05(0.66)	−0.26(0.07)	−0.02(0.96)	−0.03(0.75)
$b_{i,0}$	0.00	0.00	0.00	0.00	0.00	0.27*(0.03)	0.22(0.14)	−0.13(0.70)	0.49*(0.00)
$a_{i,1}$	0.73	1.68	0.61	1.33	-0.29	0.35(0.66)	0.18(0.81)	0.33(0.56)	0.28(0.12)
$b_{i,1}$	0.37	0.03	0.29	1.50	0.03	0.32*(0.00)	0.26(0.08))	−0.10(0.75)	0.32*(0.01)
Panel B: Subsample – Less Supply Constrained									
θ_i	-9.83	2.55	-6.45	11.81	-47.16	0.21(0.22)	0.29(0.16)		−0.25(0.13)
λ_i	0.70	0.03	0.73	0.94	0.03	−0.07(0.68)	−0.11(0.60)		−0.22(0.18)
$a_{i,0}$	0.01	0.001	0.01	0.02	-0.00	−0.08(0.65)	−0.12(0.55)		0.13(0.43)
$b_{i,0}$	0.00	0.00	0.00	0.00	-0.00	0.17(0.33)	0.18(0.40)		0.41*(0.01)
$a_{i,1}$	0.78	0.45	0.70	0.95	-0.18	0.22(0.72)	0.21(0.67)		0.33(0.35)
$b_{i,1}$	0.27	0.03	0.24	0.69	0.04	0.15(0.39)	0.23(0.28)		0.18(0.28)
Panel C: Subsample – More Supply constrained									
θ_i	-1.26	1.83	-1.09	37.78	-18.05	0.09(0.61)	0.09(0.61)		0.31(0.06)
λ_i	0.52	0.05	0.62	0.96	-0.09	−0.10(0.57)	−0.10(0.57)		0.10(0.52)
$a_{i,0}$	0.01	0.00	0.01	0.03	-0.01	−0.05(0.80)	−0.04(0.80)		−0.26(0.11)
$b_{i,0}$	0.00	0.00	0.00	0.00	-0.00	0.05(0.79)	0.05(0.79)		0.03(0.84)
$a_{i,1}$	0.51	0.62	0.59	1.33	-0.29	0.42(0.27)	−0.12(0.37)		0.21(0.44)
$b_{i,1}$	0.46	0.05	0.41	1.50	0.03	0.21(0.23)	0.21(0.24)		−0.09(0.59)

a. The estimates are the cross-metropolitan-area averages of the GARCH-in-mean model.

$$
\begin{aligned}
r_{i,t} &= a_{i,0} + a_{i,1} r_t^f + \theta_i \sigma_{i,t}^2 + u_{i,t} \\
\sigma_{i,t}^2 &= b_{i,0} + b_{i,1} u_{i,t-1} + \lambda_i \sigma_{i,t-1}^2
\end{aligned}
$$

where $u_{i,t} \sim N(0, \sigma_{i,t}^2)$. b. se(mean) indicates standard errors of the means. c. r_ss denote the Spearman rank correlation coefficients of the model estimates and market conditions, with their significance level in parentheses. d. The correlation coefficients significant at the 5% level or lower are marked with a star. e. $m_{i,2000}$ indicates the out-migration rate imputed from the 2000 PUMS; $m_{i,1990}$ indicates the out-migration rate imputed from the 1990 PUMS; $corr_i$ indicates the categorical variable that ranks the MSAs based on their expected price correlation with other MSAs; $supp_i$ indicates the WRLURI housing supply constraints indices. f. The GARCH(1,1)-in-mean model converges in 78 MSAs. The number of MSAs in the sample is reduced to 69 after being merged with the 2000 migration data; 49 after being merged with the 1990 migration data; 74 after being merged with the WRLURI data; and 35 after being merged with the expected price correlation data from Sinai and Souleles (2008).

Table 6: GARCH-in-Mean Estimates for Single Family House Markets (Case-Shiller Sample)

Panel A: Full Sample							
	mean	se(mean)	median	max	min	$r_s(*, m_{i,2000})$	$r_s(*, sup_i)$
θ_i	-11.95	4.22	-10.68	11.81	-47.16	0.40(0.12)	0.70*(0.001)
λ_i	0.69	0.06	0.77	0.92	0.11	$-0.26(0.33)$	$-0.33(9.17)$
$a_{i,0}$	0.008	0.001	0.007	0.022	0.001	$-0.24(0.37)$	$-0.45(0.06)$
$b_{i,0}$	0.000	0.000	0.000	0.0001	0.000	0.40(0.12)	0.45(0.06)
$a_{i,1}$	0.78	0.57	0.68	1.21	-0.13	0.42(0.38)	0.15(0.77)
$b_{i,1}$	0.27	0.07	0.19	1.14	0.05	0.23(0.40)	0.36(0.14))
Panel B: Subsample – Less Supply Constrained							
θ_i	-19.81	4.16	-16.85	-6.88	-46.86	$-0.38(0.35)$	
λ_i	0.83	0.03	0.86	0.93	0.68	0.40(0.32)	
$a_{i,0}$	0.01	0.002	0.008	0.021	0.004	0.67(0.07)	
$b_{i,0}$	0.0000	0.0000	0.0000	0.00001	-0.0000	$-0.10(0.82)$	
$a_{i,1}$	0.81	0.32	0.65	1.21	-0.07	0.34(0.59)	
$b_{i,1}$	0.14	0.03	0.11	0.26	0.05	0.38(0.35)	
Panel C: Subsample – More Supply Constrained							
θ_i	-4.09	6.44	0.008	11.81	-47.15	0.02(0.95)	
λ_i	0.56	0.10	0.64	0.90	0.11	0.00(0.99)	
$a_{i,0}$	0.005	0.001	0.003	0.011	0.001	$-0.19(0.65)$	
$b_{i,0}$	0.00002	0.00001	0.0000	0.0001	-0.0000	0.31(0.46)	
$a_{i,1}$	0.64	0.18	0.67	0.89	-0.13	0.30(0.18)	
$b_{i,1}$	0.46	0.10	0.31	1.14	0.06	$-0.14(0.74)$	

a. The estimates are the cross-metropolitan-area averages of the GARCH-in-mean model.

$$r_{i,t} = a_{i,0} + a_{i,1}r_t^f + \theta_i\sigma_{i,t}^2 + u_{i,t}$$
$$\sigma_{i,t}^2 = b_{i,0} + b_{i,1}u_{i,t-1} + \lambda_i\sigma_{i,t-1}^2$$

where $u_{i,t} \sim N(0, \sigma_{i,t}^2)$. b. se(mean) indicates standard errors of the means. c. r_s denotes the Spearman rank correlation coefficient of the metropolitan area mobility level and the model parameter, with its significance level in parentheses. d. The correlation coefficients significant at the 5% level or lower are marked with a star. e. The data are from the Case-Shiller indices, the 2000 5% PUMS and WRLURI housing supply constraints indices. The GARCH(1,1)-in-mean model converges in 18 MSAs. The number of MSAs in the sample is 16 after being merged with the 2000 migration data; 18 after being merged with the WRLURI data; 10 after being merged with the 1990 migration data. Given the small sample size, the Spearman coefficients between the model estimates and the 1990 out-migration rates are not reported. Similarly, the Spearman coefficients between the model estimates and WRLURI indices in the subsamples are not reported.

Table 7: Estimates for Condominium Markets (NAR Sample)

MSA	θ_i	Condo-house Correlation
Bismark, ND	7.0027	-0.0261
Cincinnati-Middletown, OH-KY-IN	3.3142	0.1189
Las Vegas-Paradise, NV	10.4342	0.4809
Los Angeles-Long Beach-Santa Ana, CA	-0.1855	0.2468
Miami-Fort Laudelale-Miami Beach, FL	-0.5499	0.2281
New Haven-Milford, CT	0.3262	0.3588
New Orleans-Metairie-Kenner, LA	-7.2977	0.3957
Philiadelphia-Camden-Wilmington, PN-NJ-DE-MD	-19.824	0.5356
Providence-New Bedford-Fall River, RI-MA	72.0742	0.0831
Richmond, VA	4.7390	0.2806
Riverside-San Bernardino-Ontario, CA	-0.1657	0.2749
Sacramento-Arden-Arcade-Rosewille, CA	-24.2508	0.5178
San Francisco-Oakland-Fremont, CA	0.8161	0.3691
Spearman's Correlation	-0.5385	

a. θ_i indicates the GARCH-IN-MEAN parameter in each condominium market. b. Condo-house correlation indicates the correlation between condominium median prices and single family house median prices for each MSA. c. The quarterly price data on the condominium market and single family house market come from the National Association of Realtors. d. Spearman correlation indicates the rank correlation between the estimated volatility feedback effect and the cross-market correlation between condominium and single family house market. The correlation coefficient is significant at the 10% level, with p-value of 0.0576.

Table 8: EGARCH-in-Mean Estimates for Single Family House Markets (OFHEO Sample)

Panel A: Full Sample								
	mean	se(mean)	median	max	min	$r_s(*, m_{i,2000})$	$r_s(*, WRLURI_i)$	$r_s(*, b_{i,1})$
θ_i	-1.97	3.80	-2.91	58.03	-67.80	0.23(0.15)	0.53*(0.00)	0.69*(0.00)
λ_i	0.93	0.01	0.95	0.99	0.49	$-0.21(0.20)$	$-0.16(0.33)$	0.15(0.39)
$b_{i,1}$	-0.01	0.02	-0.02	0.18	-0.27	0.04(0.83)	0.29(0.08)	1.00
$b_{i,2}$	0.40	0.03	0.35	0.98	0.14	0.36*(0.03)	$-0.03(0.87)$	0.02(0.92)
$a_{i,0}$	0.002	0.002	0.004	0.013	-0.016	$-0.11(0.51)$	-0.46*(0.00)	-0.71*(0.00)
$b_{i,0}$	-0.5	0.104	-0.36	0.11	-3.92	$-0.20(0.25)$	$-0.14(0.39)$	0.20(0.24)

Table 8: EGARCH-in-Mean Estimates for Single Family House Markets (OFHEO Sample) (cont'd)

Panel B: Subsample – Less Supply Constrained					
θ_i	-13.92	4.38	-12.39	13.97	-67.80
λ_i	0.96	0.006	0.96	0.99	0.89
$b_{i,1}$	-0.05	0.03	-0.06	0.16	-0.27
$b_{i,2}$	0.38	0.03	0.34	0.72	0.22
$a_{i,0}$	0.006	0.001	0.006	0.013	-0.002
$b_{i,0}$	-0.36	0.05	-0.35	0.108	-0.817
Panel C: Subsample – More Supply constrained					
θ_i	11.31	4.78	0.93	58.03	-12.31
λ_i	0.92	0.027	0.96	0.99	0.49
$b_{i,1}$	0.04	0.03	0.06	0.18	-0.22
$b_{i,2}$	-0.43	0.06	0.47	0.99	0.14
$a_{i,0}$	-0.002	0.002	0.002	0.011	-0.016
$b_{i,0}$	-0.66	0.21	-0.36	0.12	-3.92

a. The estimates are the cross-metropolitan-area averages of the EGARCH(1,1)-in-mean model.

$$
\begin{aligned}
r_{i,t} &= a_{i,0} + \theta_i \sigma^2_{i,t} + u_{i,t} \\
\ln(\sigma^2_{i,t}) &= b_{i,0} + b_{i,1} z_{i,t-1} + b_{i,2}(|z_{i,t-1} - E|z_{i,t-1}|) + \lambda_i \ln(\sigma^2_{i,t-1})
\end{aligned}
$$

where $z_{i,t} = u_{i,t}/\sigma_{i,t}$. b. se(mean) indicates standard errors of the means. c. $r_s(*, m_i)$ denotes the Spearman rank correlation coefficient of the metropolitan area mobility level and the model parameter, with its significance level in parentheses. Similarly, $r_s(*, b_{i,1})$ denotes the Spearman correlation coefficient between the estimated parameters and the EARCH(1) parameter, $b_{i,1}$. The sample size is significantly reduced after the sample splitting. Consequently, the Spearman correlation coefficients are only reported for the full sample. d. The correlation coefficients significant at the 5% level or lower are marked with a star. e. The data are from the OFHEO, the 2000 5% PUMS and WRLURI housing supply constraints indices. The EGARCH(1,1)-in-mean model converges in 51 MSAs. The number of MSAs is reduced to 38 after being merged with the WRLURI indices; and 37 after being merged with the 2000 data.

Table 9: GARCH-in-Mean Estimates with Unemployment Rates (OFHEO Sample)

	mean	se(mean)	median	max	min	r_s
θ_i	-3.451	4.823	.152	13.496	-38.202	0.43 (0.10)
$a_{i,2}$	-0.003	0.001	-0.003	-0.001	-0.012	$-0.51(0.09)$

a. Table 9 repeats the estimation in Panel A (full sample) in Table 5, with one additional variable: the unemployment rate. $a_{i,2}$ is the coefficient on the unemployment rate in the mean equation. b. The estimates are the cross-metropolitan-area averages of the GARCH-in-mean model. c. se(mean) indicates standard errors of the means. d. r_s denotes the Spearman rank correlation coefficient of the metropolitan area mobility level and the model parameter, with its significance level in parentheses. e. The data are from the OFHEO, the 2000 5% PUMS and the BLS unemployment rate.