

# Bargaining in Competing Supply Chains with Uncertainty

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## Abstract

Substantial literature has been devoted to supply chain coordination. The majority of this literature ignores competition between supply chains. Moreover, a significant part of this literature focuses on coordination that induce the supply chain members to follow strategies that produce the equilibria chosen by a vertically integrated supply chain. This paper investigates the equilibrium behavior of two competing supply chains in the presence of demand uncertainty. We consider joint pricing and quantity decisions and competition under three possible supply chain strategies: Vertical Integration (VI), Manufacturer's Stackelberg (MS), and Bargaining on the Wholesale price ( $BW(\alpha)$ ,  $\alpha$  is the bargaining parameter) over a single or infinitely many periods. We show that, in contrast to earlier literature, using VIVI (VI in both chains) is the *unique* Nash Equilibrium over one period decision, while using MSMS or  $BW(\alpha)BW(\alpha)$  may be Nash Equilibrium over infinitely many periods.

**Key Words:** competing supply chain, uncertain demand, bargaining, channel coordination.

# 1 Introduction

Supply chain coordination has gained considerable notice lately from both practitioners and researchers. In monopolistic markets with a single chain or markets with perfect competing retailers a vertically integrated supply chain maximizes the profit of the chain, e.g., Jeuland and Shugan (1983), Cachon (2003), and Bernstein and Federgruen (2005). (We informally define vertically integrated chain as well as other concepts from game theory in Appendix 1.) Therefore, many supply chain contracts try to induce retailers and suppliers to act according to the vertical integration strategy.

In this paper we focus on two supply chains operating in the same market. For example, in 2005 Coca-Cola (60.9%) and PepsiCo (36%) controlled 96.9% of the soft-drink sales in India (Times News network, 2005). Other examples include the cellular phone industry, internet and telephony services, the Canadian coffee shop market, fast food, car manufacturing and retailing, and crude oil and gasoline industries.

A market with two competing supply chains was investigated in the seminal work of McGuire and Staelin (1983). They consider a price (i.e., Bertrand) competition between two suppliers selling through independent retailers. They conclude that for highly substitutable products a decentralized supply chain Nash equilibrium is preferred by both manufacturers. Coughlan (1985) applied this research to the electrical industry and Moorthy (1988) further explains why the decentralized chains can lead to higher profits for the manufacturer and the entire chains. Bonanno and Vickers (1988), investigate a similar model, and use geometric insights to show that with franchise fees there are some settings in which the manufacturers' optimal strategy is to sell its products using an independent retailer. Thus, in these cases the manufacturer prefers a decentralized supply chain irrespective of the decision in the other supply chain. Neither of these work consider demand uncertainty. In the operation literature, Wu and Chen (2003) consider a quantity (i.e., Cournot) competition of a duopoly facing newsvendor demand, but they ignore pricing decisions. There are a few common features to all these papers, namely they ignore the important interaction between price and quantity decisions and they also consider competition over only a single period.

A few supply chain coordination mechanisms that induce the chain to act as if they are vertically integrated were investigated, e.g. buy back, Pasternack (1985), quantity flexibility, Tsay (1999), and revenue sharing Cachon and Larivierre (2005). See Cachon (2003) for a survey

of this literature including many recent references. Two more recent reviews are Kouvelis et al. 2006 that focuses on supply chain coordination literature papers published in *Production and Operations Management* journal during 1992-2006 and Tang (2005) who covers many recent literature on supply chain coordination. Lin and Kong (2002) consider a duopoly that has no demand uncertainty and investigate a symmetric Nash bargaining model. Similar to McGuire and Staelin (1983), they show that Nash bargaining can lead to higher supply chains profits than a vertically integrated chain.

In a recent work, Baron et al. (2007) investigate the Nash equilibrium of an industry with two supply chains by extending the seminal work of McGuire and Staelin (1983). Baron et al. (2007) show that both the traditional MS and the VI strategies are special cases of Nash Bargaining on the wholesale price when the demand is deterministic. They suggest that the supply chain coordination mechanisms that focus on inducing supply chains to act as if they are vertically integrated should be treated with caution.

Here we extend the work of Baron et al. (2007) to include uncertain demand. Each chain is composed of a single manufacturer and a single retailer. we consider a single and an infinite period models. In each period, the market share, and therefore profits, of each chain are based on the quantities brought to the market and the prices charged by these supply chains. For the infinite period game, we ignore the carrying of inventory from one period to the other that might be realistic when items are perishable or seasonal. We focus on a linear demand model with a simple uncertain demand distribution that includes only two realizations. We assert a similar result to the one used in Padmanabhan and Png (1997, 2004) for Monopolistic markets. Namely, that sales are determined by quantity ordered at the high demand state and by demand at the low demand state. This stylistic setting allows us to gain insight about the differences between various supply chain strategies for competing supply chains when the demand is uncertain. We consider three possible supply chain strategies: Vertical Integration (VI), Manufacturer's Stackelberg (MS), and Bargaining on the Wholesale price ( $BW(\alpha)$ ). In the latter strategy the participants negotiate the wholesale price and then the retailer maximizes its profit by choosing an order quantity and a retail price. We model this strategy using a—possibly non symmetric—Nash bargaining model.

This papers main contribution is to link and extend the above works by considering joint pricing and quantity decisions and competition over infinitely many period. We show that

repeated games are important because they capture issues that are ignored when attention is restricted to a single period game. We show that, in contrast to McGuire and Staelin (1983), in a competition over a single period the unique Nash equilibrium is for both chains to vertically integrate. This difference is due to the fact that we consider a Cournot competition while they consider a Bertrand competition. Importantly, this result, obtained for the uncertain demand case, holds also for the certain demand case. This suggests that insights generated when investigating the certain demand case could be applicable even when demand is uncertain. Moreover, similar to the empirical evidence given in Dez-Vial (2007) the uniqueness of the VI equilibrium shows that with competition the power of the different players does not affect the relative importance of VI. When the competition is over infinitely many periods additional Nash equilibria can appear. These equilibria can lead to higher profits for the manufacturers and retailers in each of the supply chains. This result is similar in essence to the equilibrium in the infinite period prisoner dilemma and it extends the result of McGuire and Staelin (1983) to settings of price and quantity competition over infinite horizon.

The outline of the paper is as follows. In the next section we present the Nash Bargaining for a single supply chain. Section 3 presents the modeling and solution approach for two competing supply chains. Section 4 analyzes operational and strategic decisions in competing supply chains with uncertain demand. Section 5 concludes the paper.

## **2 Bargaining on the Wholesale Price in a Single Supply Chain**

In this section we consider a supply chain composed of a manufacturer servicing a single retailer. We formulate the Bargaining on the Wholesale (BW) price between the manufacturer and the retailer as a Nash Bargaining game. We compare the results of the BW supply chain strategy with those of supply chains that operate with a Manufacturer Stackleberg (MS) or a Vertically Integrated (VI) strategies. Both are common in the literature (e.g., McGuire and Staelin (1983), and Bonanno and Vickers (1988)). We show that the MS strategy is a special case of a BW strategy and provide sufficient conditions for the VI strategy to be equivalent to a special case of BW as well. These conditions are different from those in Baron et al. (2007) for the deterministic case.

For a chain that uses a BW strategy the sequence of decisions is as follows: the manufacturer and the retailer bargain on the wholesale price,  $w$ , afterwards they agree on the desired retailer orders quantity,  $q$  (which is fully produced and delivered by the manufacturer). Then, the demand uncertainty is resolved and the retailer chooses price  $p$  and sales take place.

In a supply chain with a MS strategy the manufacturer first chooses the wholesale price to maximize its profits, ignoring the retailer's profits. Then, the retailer orders its desired quantity,  $q$ , demand uncertainty is resolved and the retailer chooses the retail price  $p$ , and sales take place. We model a VI supply chain as a retailer operated chain with the manufacturing cost  $c$  as the wholesale price it faces. Then, the retailer chooses  $q$ , demand uncertainty is resolved and the retail price is chosen, and sales take place. With these decisions, in a supply chain with a VI strategy all profits are obtained by the retailer.

We denote the per unit production cost for the manufacturer by  $c \geq 0$ . We let  $X$  be the state of the system, that is each realization of  $X = x$  implies a specific demand function for each supply chain. We allow the demand functions to depend on retail prices and substituting parameters. We assume that demand is decreasing with the retail price  $p$  for every realization of  $X$ . (We later specify and use an additive demand model that satisfies these conditions.) Let  $p_x$  denote the retail price when demand state is  $x$ ,  $s_x$  denote the sales when demand state is  $x$ , and  $\pi_x^{SC}$  and  $\pi^{SC}$  denote the supply chain profit at demand state  $x$  and the expected profit, respectively. Let  $\pi_x^R$  and  $\pi^R$  denote the retailer's profit at demand state  $x$  and the expected profit, respectively and  $\pi^M$  denotes the manufacturer's profit. Denoting the market demand faced by the chain at state  $x$  by  $d_x$ , we have

$$\begin{aligned} s_x &= \min(q, d_x), \\ \pi_x^{SC} &= s_x p_x - qc, \end{aligned} \tag{1}$$

$$\begin{aligned} \pi^{SC} &= E(\pi_x^{SC}), \\ \pi_x^R &= s_x p_x - qw, \end{aligned} \tag{2}$$

$$\begin{aligned} \pi^R &= E(\pi_x^R), \\ \pi^M &= q(w - c). \end{aligned} \tag{3}$$

Observe that the profits of the manufacturer do not depend on the demand realization as is captured in (3). Some of our notation is summarized in Table 1.

The Nash bargaining model, presented by Nash (1950) and discussed by Kalai and Smorodinsky

## Notation

Subscript $i$	denotes that a quantity is related to the $i^{th}$ supply chain
$p_{ix}$	retail price at demand state $x$
$q_i$	production and order quantity
$c_i$	per unit production cost
$w_i$	wholesale price (when this chain is not VI)
$s_{ix}$	sales by the retailer at demand state $x$
$\pi_{ix}^{SC}$ and $\pi_i^{SC}$	profit at demand state $x$ and expected profit, respectively
$\pi_i^M$	manufacturer's profit (when this chain is not VI)
$\pi_{ix}^R$ and $\pi_i^R$	retailer's profit at demand state $x$ and expected profit, respectively (when this chain is not VI)
$d_{ix}$	market demand faced by the chain at demand state $x$
$u$	probability of high demand state, $1 - u$ is the probability of low demand state

Table 1: Notation

(1975), can be formulated as a Nash Bargaining Product, e.g., Binmore et al. (1986). We let  $\alpha \in [0, 1]$  be the bargaining power,  $\tilde{\Phi}(w)$  denote the Nash bargaining product and  $\tilde{f}^M, \tilde{f}^R$  denote the manufacturer's and retailer's threat points, respectively. Then, the Nash Bargaining Product model for a manufacturer and a retailer choosing a wholesale price  $w$  is:

$$Max_w \left\{ \tilde{\Phi}(w) \right\} = Max_w \left\{ (\pi^M - \tilde{f}^M)^\alpha (\pi^R - \tilde{f}^R)^{1-\alpha} \right\}.$$

While we allow the bargaining parameter  $\alpha \in [0, 1]$ , we assume that it is exogenously given. It is clear that the cases  $\alpha = 0$  and  $\alpha = 1$  allow the retailer and manufacturer, respectively, to dictate the choice of  $w$ . Thus,  $\alpha = 0.5$  reflects a balance of power between the manufacturer and the retailer. For this reason our numerical results often consider the *BW(0.5)* case.

Binmore et al. (1986) explain (in the discussion following their Proposition 6) that the values of the threat points should reflect the status quo (e.g., not operating in the market) rather than external options (e.g., to coordinate the chain as if it is a VI one). Thus, to reflect the manufacturer's choice not to produce, assuming this option has a zero influence on the manufacturer's profit, we choose  $\tilde{f}^M = 0$ . Similarly, we choose  $\tilde{f}^R = 0$  reflecting the option not to sell in this market, which we assume has zero profit. Finally, to ensure individual rationality, e.g., Assumption 4 of Kalai and Smorodinsky (1975), we require  $w \geq c$ . We note that in our

model this constraint can only be active when  $\alpha = 0$ .

We let  $\Phi(w)$  be the adjusted Nash bargaining product, thus with the choices as above the adjusted BW( $\alpha$ ) model is:

$$Max_w \{\Phi(w)\} = Max_w \left\{ (\pi^M)^\alpha (\pi^R)^{1-\alpha} \right\} = Max_w \left\{ ((w-c)q)^\alpha (E(p_x s_x) - wq)^{1-\alpha} \right\}, \quad (4)$$

subject to  $w \geq c$ .

Baron et al. (2007) show for the certain demand case the BW(1) model is equivalent to the MS model and the BW(0) model is equivalent to the VI model when there is no demand uncertainty. The following proposition gives a condition under which these equivalents can be extended to uncertain demand cases.

**Proposition 1** (i) *The MS strategy is equivalent to BW(1).* (ii) *If*

$$\frac{dp_x}{dw} \in [0, 1] \quad \forall x, \quad (5)$$

*the retailer facing a BW(0) model maximizes the profits of a Vertically Integrated supply chain.*

**Proof.** (i) When  $\alpha = 1$  the manufacturer chooses the wholesale price without consideration of the retailer's profit as in MS. (ii) On the range  $w \geq c$ , we can exchange the expectation and derivative to get

$$\begin{aligned} \frac{d\Phi(w)}{dw} &= E \left( p_x \frac{ds_x}{dw} + s_x \frac{dp_x}{dw} \right) - q = E \left( p_x \frac{\partial s_x}{\partial p_x} \frac{dp_x}{dw} + s_x \frac{dp_x}{dw} \right) - q \\ &\leq E \left( s_x \frac{dp_x}{dw} \right) - q \leq E(s_x) - q \leq 0, \end{aligned}$$

where the second equality holds by the chain rule, the first inequality holds because we assume  $\frac{\partial s_x}{\partial p_x} \leq 0$  and (5), the second inequality holds by (5), and the last inequality because  $s_x \leq q \forall x$ . Thus, on the range  $w \geq c$ ,  $\Phi(w)$  is non increasing with  $w$  and the choice  $w = c$  maximizes  $\Phi(w)$ . Because choosing  $w < c$  is infeasible the retailer chooses  $q$  and  $p_x$  to maximize  $E(p_x s_x) - cq = \pi^{SC}$  as does a VI chain, completing the proof. ■

The requirement (5) in Proposition 1 is not too restrictive. Requiring that retail price is increasing with the chains' wholesale price,  $dp_x/dw \geq 0$ , is a very common assumption. For example, when there is no competition it is common to assume  $dp/dw \geq 0$  (Jeuland and Shugan, 1983). The requirement that the chain's retail price will increase less than the wholesale price  $dp/dw \leq 1$ , is also reasonable in most settings. For example, consider a decrease in the wholesale

price. Then, if only a part of this discount is transferable to the customer (i.e., the retailer decreases its price by only a fraction of the discount it receives), it follows that  $dp/dw \leq 1$ . This is a natural condition when demand is certain. However, verifying (5) is less trivial in the uncertain demand case. In fact, even with the simple uncertain demand structure we consider in this paper, we could not verify (5).

In the next sections we consider a stylistic market with two competing supply chains to demonstrate the use of  $BW(\alpha)$  as a supply chain coordination contract. It is important to note that we could not establish the condition in part (ii) of Proposition 1. Yet, numerically we found that when  $\alpha = 0$  the resulting solution is indeed identical to that of a VI chain.

### 3 Two Competing Supply Chains, Model and Solution Approach

We consider two competing supply chains, each with a manufacturer serving a single retailer. Each supply chain has a given  $\alpha$ , representing the relative market power of its manufacturer. Given these  $\alpha$ 's each supply chain makes a strategic and operational decision. We use the same notation as before adding a subscript  $i$  to denote quantities belonging to the  $i^{th}$  supply chain.

At the strategic level, the participants in each supply agree on the chain's structure. They have three possible structures:  $BW(\alpha)$ , VI (or  $BW(0)$ ), and MS (or  $BW(1)$ ). We consider the VI structure because, as mentioned earlier, this structure is optimal in many settings. We consider the MS structure because, in some settings, it is optimal for two competing supply chains, e.g. McGuire and Staelin (1983), Coughlan (1985), and Moorthy (1988).

At the operational level, after each chain chooses its structure, the participants in each chain choose quantity and price to maximize their profits. In the infinite period settings both the strategic and operational decisions are made in each period.

Note that we assume that none of the supply chains is a leader in either its strategic (supply chain strategy) choice or its prices or quantity decisions. For example, the retail prices are chosen simultaneously. We further assume that all of the information is common knowledge.

It is clear that if either VI, or MS, are dominant strategies, there would be no point in using  $BW(\alpha)$  for  $\alpha \in (0, 1)$  as a supply chain coordination mechanism. Thus, for our model, detailed below, we discuss conditions under which  $BW(\alpha)$  is beneficial for both the retailer and

the manufacturer of a chain.

### 3.1 Uncertainty and demand structures

To keep things simple (and tractable) we assume that the demand is uncertain with just two possible realizations: a high value,  $h$ , with probability  $u$ , and a low value,  $l$ , with probability  $1 - u$ . Note that when  $u = 1$  or  $u = 0$  we obtain the certain demand case. We consider additive demand as in Mills (1959) where the  $i^{th}$  retailer's demand depends on three elements: the primary uncertain demand  $a_{ix}$  at state  $x \in (h, l)$ , (we assume  $a_{il} \leq a_{ih}$ ) its own price,  $p_{ix}$ , and the competitor's price  $p_{jx}$  ( $j \neq i$ ) through a substituting coefficient  $b_i \in (0, 1)$ :

$$d_{ix} = a_{ix} - p_{ix} + b_i p_{jx}, \quad i = 1, 2; \quad j = 3 - i, \quad x \in \{h, l\}. \quad (6)$$

As reflected in (6),  $b_i = 0$  implies that the chains are independent of each other while  $b_i = 1$  implies that the products are not differentiable.

With this model we have

$$\begin{aligned} \pi_i^{SC} &= u s_{ih} p_{ih} + (1 - u) s_{il} p_{il} - q_i c_i, \quad i = 1, 2 \\ \pi_i^R &= u s_{ih} p_{ih} + (1 - u) s_{il} p_{il} - q_i w_i, \quad i = 1, 2. \end{aligned}$$

As before we assume that demand uncertainty is resolved before a retail price  $p_i$  is chosen.

### 3.2 Strategic choices and resulting channel structures

We denote the strategic choice of the  $i^{th}$  chain by  $ST_i \in \{VI, MS, BW(\alpha_i), 0 < \alpha_i < 1\}$ . To consider the strategic game, we use the profits based on the subgame-perfect Nash equilibrium strategy for the quantity and prices decisions of each chain in a single period. For each  $ST_1 ST_2$ , we let  $\pi_i^{SC}(ST_1 ST_2)$  denote the expected profit of the  $i^{th}$  supply chain in a single period. We present the strategic decisions in a table game where the two SCs' denote two players and the payoff to each player is the SC profit in Table 2. Note that there are a total of six industry structures to consider. We analyze the strategic decisions using this game representation in Section 4. Observe that even if a stable Nash equilibrium exists to the game in Table 2 some of the firms (e.g., a manufacturer or a retailer) might find it beneficiary to choose another structure. As in the existing supply chain literature we assume that the members in the supply

	<i>VI</i>	<i>MS</i>	<i>BW</i> ( $\alpha_2$ )
<i>VI</i>	$\pi_1^{SC}(VIVI), \pi_2^{SC}(VIVI)$	$\pi_1^{SC}(VIMS), \pi_2^{SC}(VIMS)$	$\pi_1^{SC}(VIBW(\alpha_2)), \pi_2^{SC}(VIBW(\alpha_2))$
<i>MS</i>	$\pi_2^{SC}(VIMS), \pi_1^{SC}(VIMS)$	$\pi_1^{SC}(MSMS), \pi_2^{SC}(MSMS)$	$\pi_1^{SC}(MSBW(\alpha_2)), \pi_2^{SC}(MSBW(\alpha_2))$
<i>BW</i> ( $\alpha_1$ )	$\pi_2^{SC}(VIBW(\alpha_1)), \pi_1^{SC}(VIBW(\alpha_1))$	$\pi_2^{SC}(MSBW(\alpha_1)), \pi_1^{SC}(MSBW(\alpha_1))$	$\pi_1^{SC}(BW(\alpha_1)BW(\alpha_2)), \pi_2^{SC}(BW(\alpha_1)BW(\alpha_2))$

Table 2: Strategic game for two supply chains

chain can be induced to act according to this Nash equilibrium. This can be done by bargaining on the profit within the chain, as suggested in Baron et al. (2007).

To model the multi period competition, let  $\beta \in (0, 1)$  be a common discount factor per period and  $GA(\beta, \infty)$  represent the repeated game, in which the game from Table 2 is played infinitely many periods. We assume items are perishable with no salvage value at the end of each period, thus inventory is not carried from a period to the other. Let  $ST_{ki} \in \{VI, BW(\alpha)\}$  denote a specific strategic choice of the  $i^{th}$  ( $i = 1, 2$ ) chain in period  $k$  ( $k = 0, 1, \dots$ ). (We do not explicitly consider the MS case because by Proposition 1, it is just the BW(1) case). Then, the net present value from a specific supply chains strategy is

$$\Pi_i^{SC} = \sum_{k=0}^{\infty} \beta^k \pi_i^{SC}(ST_{k1}ST_{k2}), \quad i = 1, 2.$$

For the infinite period competition the strategic decision of each chain maximizes  $\Pi_i^{SC}$ .

We note that even though we ignore the cost of changing strategies between periods, their inclusion can be easily treated in a similar manner. This requires adding an element that multiplies the cost of changing strategies by an indicator for such a change to the total profit. The rest of our analysis can then be carried through with this corrected profit.

### 3.3 Solution approach

To find equilibrium decisions for all of the participants in the supply chains, we search the subgame-perfect Nash channel equilibrium. For each demand state and chain structure, we first find Nash equilibrium retail prices strategies as a function of the order quantities and wholesale prices (if they exist), we then find the Nash equilibrium order quantity strategy (as a function of the wholesale prices if they exist). Thus, for the VIVI supply chain structure one backward induction step allows us to express the supply chains profits, whereas for all the other structures, we use two backward induction steps. That is, we find the wholesale prices for an MS chain using the First Order Condition (FOC) in the manufacturer's profit function

(3), and for a  $BW(\alpha)$  chain using the FOC for the Nash bargaining model in (4). In all cases we also verify the second order conditions, but omit this from the exposition.

We demonstrate this approach using the VIVI and MSMS structures in Section 4. The other strategies can be analyzed in a similar way. The reader is referred to Chapter 2 of Gibbons (1992) and chapter 3 of Fudenberg and Tirole (1991) for discussion on the notion of subgame-perfect Nash equilibrium and backward induction.

In some uses, the profits in the game presented in Table 2 result in a similar structure to the prisoner dilemma. In these cases we use “folk theorems”, e.g., Fudenberg and Tirole (1991), for repeated games. These theorems show that for an infinitely repeated game there is some discount factor  $\beta \in (0, 1)$  that supports equilibriums for any feasible, individually rational payoffs. Taking this approach we explicitly express  $\hat{\beta}$  such that for each  $\beta \in [\hat{\beta}, 1)$  a specific equilibrium is feasible.

To simplify the exposition and the insights generated by them, our analysis is focused on symmetric supply chain chains defined as:

**Definition 1** *Symmetric Supply Chains: supply chains are symmetric, if  $a_{ix} = a_x, b_i = b$  and  $c_i = c$  for all the supply chains for each  $x = l, h$ .*

We note that in Definition 1 we allow the participants in the supply chains to have different market power, i.e., different  $\alpha$ 's.

Before demonstrating our solution approach we state a result (proof is in Appendix 2) that substantially simplifies our analysis for the uncertain demand case.

**Proposition 2** *In symmetric competing supply chains, when demand uncertainty is resolved before the retail price decision is made, the retail price will be chosen such that: at the high demand state,  $s_{ih} = q_i$  and at the low demand state when  $\left(\frac{a_l(b+2)}{(b+2)(b-2)} + a_h\right) \frac{u}{1-b} < w$  the wholesale price  $w$  holds*

$$(b + 1)(b - 2)(ua_h - w + wb) > -ua_l(b + 2) \tag{7}$$

and  $s_{il} = d_i$ .

The main conclusion from Proposition 2 is that if (7) holds, i.e., if  $w$  is high enough, then in the low demand state sales are not constrained by the stock  $q_i$ . Note that the individual

rationality requires that  $c \leq w$  thus a sufficient condition for (7) to hold is that

$$\left( \frac{a_l(b+2)}{(b+1)(b-2)} + a_h \right) \frac{u}{1-b} < c. \quad (8)$$

Clearly, (8) is the requirement for a VI supply chain.

Padmanabhan and Png (1997, 2004) show that, in a monopoly market, sales equal the order quantity at the high demand state and equal the demand at the low demand state. Thus, Proposition 2 extends their results for a duopoly with two symmetric competing supply chains (provided that (7) holds). Unfortunately, we have no intuitive explanation for (7) and it is not simple to extend the results of Proposition 2 to asymmetric chains. Yet, we conjecture that a similar property holds for asymmetric chains as well. Thus, from now on we assume that (7) holds, which simplifies our analysis.

## 4 Competing SC with Uncertain Demand

### 4.1 Analysis of operational decision

In this section, we solve the operational decision problem in the uncertain demand case, which leads to a three-stage subgame-perfect equilibria in three game configurations. Note that the general BW( $\alpha_1$ )BW( $\alpha_2$ ) model cannot be solved analytically and therefore we solve it numerically, while we solve VIVI and MSMS analytically.

We first present in Tables 3 and 4 the equilibrium solutions of all the variables in symmetric chains respectively for VIVI, BW( $\alpha_1$ )BW( $\alpha_1$ ) (as the results for the  $\alpha_1 \neq \alpha_2$  cases are too cumbersome we only give the ones for  $\alpha_1 = \alpha_2$ ), VIBW( $\alpha_2$ ) and for VIMS, MSMS respectively (there are no closed form solutions to the MSBW ( $\alpha$ ) model for general  $\alpha$ ). (As for MS and BW( $\alpha$ ) chains  $\pi_i^{SC} = \pi_i^R + \pi_i^M$ , we only give  $\pi_i^{SC}$  explicitly for VI chains.) We further present the numerical solutions for these strategies when  $u = 0.5$  in Tables 5 and 6 for the case  $a_l = .5, a_h = 2, b = 0.5, \alpha_1 = \alpha_2 = 0.5$ ; we let the production cost be  $c = 0.3$  and  $c = 0$  in these tables respectively. We next present the procedure to solve the VIVI, MSMS and BW( $\alpha_1$ )BW( $\alpha_2$ ) models. (The entry NA in these and other tables stands for not applicable.)

### 4.1.1 The VIVI model

In VIVI, the manufacturer and the retailer are two branches of the same company and thus both face the same demand. The company sequentially decide on production quantity  $q$  and retailer price  $p_x$ . When the demand is low, the company determines the optimal price by:

$$\text{Max}_{p_{il}} \{ \pi_{il}^{SC} \} = \text{Max}_{p_{il}} p_{il} (a_l - p_{il} + bp_{jl}) - cq_i \quad i = 1, 2; \quad j = 3 - i, \quad (9)$$

where that demand is  $a_l - p_{il} + bp_{jl}$  follows from our assertion for the low demand case.

FOC with respect to  $p_{il}$  leads to the following reaction functions:

$$a_l + bp_{jl} - 2p_{il} = 0. \quad (10)$$

Solving the reaction functions leads to the optimal price for the low demand state (for both chains):

$$p_l = \frac{a_l}{2 - b}. \quad (11)$$

Thus, the  $i^{th}$  chain's optimal profit at the low demand state is:

$$\pi_{il}^{SC} = \frac{a_l^2}{(b - 2)^2} - q_i c. \quad (12)$$

When demand is high, we follow our mention that retail price is chosen to clear inventory. Thus, both chains determine optimal prices by solving the reaction functions that follow from:

$$q_i = a_h - p_{ih} + bp_{jh} \quad i = 1, 2; \quad j = 3 - i. \quad (13)$$

This leads to:

$$p_{ih} = \frac{a_h}{1 - b} - \frac{q_i + bq_j}{1 - b^2} \quad i = 1, 2; \quad j = 3 - i, \quad (14)$$

and

$$\pi_{ih}^{SC} = q_i \left( \frac{a_h}{1 - b} - \frac{q_i + bq_j}{1 - b^2} - c \right) \quad i = 1, 2; \quad j = 3 - i. \quad (15)$$

Therefore, the  $i^{th}$  SC's expected profit is:

$$\pi_i^{SC} = q_i u \left( \frac{a_h}{1 - b} - \frac{q_i + bq_j}{1 - b^2} \right) + (1 - u) \left( \frac{a_l}{2 - b} \right)^2 - q_i c \quad i = 1, 2; \quad j = 3 - i. \quad (16)$$

FOC with respect to  $q_i$  leads to the reaction functions:

$$c - ua_h - b^2c + 2q_i u - bua_h + bq_j u = 0, \quad i = 1, 2; \quad j = 3 - i, \quad (17)$$

whose solution gives the order quantity at the VI equilibrium:

$$q_i = q_j = (b + 1) \frac{ua_h + c(b - 1)}{u(b + 2)}. \quad (18)$$

Using the expressions above, the optimal price (for both chains) at high demand state is:

$$p_h = \frac{b^2c - c - ua_h}{u(b - 1)(b + 2)}.$$

#### 4.1.2 The MSMS model

While, by Proposition 1 this model is equivalent to the  $BW(1)BW(1)$ , we bring its solution here. This enables us to later contrast the results of the MSMS with the ones of the VIVI for our Bertrand competition structure. This comparison is similar to earlier work, e.g. McGuire and Staelin (1983) that uses the concept of Cournot competition.

In this case the retailer actions agree with the ones of a VIVI chain that faces a production cost  $w_i$  rather than  $c$ . Thus, the low price is identical to the one in (11). Moreover, substituting  $w_i$  instead of  $c$  in (16), the  $i^{th}$  retailer's expected profit is:

$$\pi^R = q_i u \left( \frac{a_h}{1 - b} - \frac{q_i + bq_j}{1 - b^2} \right) + (1 - u) \left( \frac{a_l}{2 - b} \right)^2 - q_i w_i \quad i = 1, 2; \quad j = 3 - i. \quad (19)$$

Taking the FOC gives the following reaction function

$$ubq_j + 2uq_i + w_i - ua_h - w_i b^2 - uba_h = 0, \quad i = 1, 2; \quad j = 3 - i.$$

whose solution is

$$q_i = \frac{ua_h b^2 + w_j b^3 - bw_j + 2w_i - 2ua_h - 2w_i b^2 - uba_h}{u(b^2 - 4)} \quad i = 1, 2; \quad j = 3 - i. \quad (20)$$

Substituting (20) into (3) and taking FOC with respect to  $w_i$  yields the manufacturers' reaction function:

$$ua_h b^2 + w_j b^3 - bw_j + 4w_i - 2ua_h - 2c + 2cb^2 - 4w_i b^2 - uba_h = 0 \quad i = 1, 2; \quad j = 3 - i. \quad (21)$$

Solving (21) leads to:

$$w_i = \frac{2c + 2ua_h - 2bc - bua_h}{(b - 1)(b - 4)} \quad i = 1, 2. \quad (22)$$

Hence, the order quantity at the MS equilibrium is

$$q_i = \frac{2(b+1)(bc-c+ua_h)}{u(b+2)(4-b)} \quad i = 1, 2. \quad (23)$$

leading to an optimal retail price at the high demand states is:

$$p_h = \frac{2c + 6ua_h - 2b^2c - b^2ua_h}{u(b-1)(b+2)(b-4)}.$$

### 4.1.3 The $BW(\alpha_1)BW(\alpha_2)$ model

Here, the manufacturer and the retailer are bargaining on the wholesale price. We remind that while we assume that from operational point of view both supply chains are symmetric, the bargaining power within each chain is allowed to differ. The supply chains first choose the wholesale price,  $w_i$ , in accordance with (4). Because given  $w$ , the retailers' profits are identical to the ones in the MSMS model, we substitute (19) and (20) into (4) and take the FOC with respect to  $w_i$ . This yields the retailers' reaction function.

Then  $\hat{w}_i$  at the equilibrium can be obtained by solving the retailers' reaction function. Thus, the optimal order quantity and retailer price at each demand status at the equilibrium can be computed. Then, similar to their calculation in the MSMS case, the expected profit given to the retailer, manufacturer and the SC can be obtained.

## 4.2 Analysis of strategic decisions

Here we investigate two strategic competition models (VIMS and  $VIBW(\alpha)$ ) first over a single period and then over infinitely many periods. An analysis considering all three strategy decisions can be done in a similar manner but it is cumbersome and adds no insight. Thus it is omitted. We first analyze the bi-matrix table game constructed with VI and MS. We present this model to contrast the equilibrium industry structure when using a Cournot rather than a Bertrand competition, e.g., McGuire and Staelin (1983). We then analyze the more general strategic game composed of VIVI and  $BW(\alpha)BW(\alpha)$ .

Decision		Supply Chain Structure	
Variable	VIVI	BW( $\alpha_1$ )BW( $\alpha_1$ )	VIBW( $\alpha_2$ )
$w_2$	NA	$\hat{w}_i$	$\hat{w}_2$
$q_1$	$\frac{(bc-c+ua_h)(b+1)}{u(b+2)}$	$(b+1) \frac{ua_h+(b-1)\hat{w}_i}{u(b+2)}$	$(b+1) \frac{2c-2ua_h-2bc+buah+(b^2-b)\hat{w}_2}{u(b-2)(b+2)}$
$q_2$			$(b+1) \frac{b^2c-2ua_h-bc+buah-2(b-1)\hat{w}_2}{u(b-2)(b+2)}$
$p_{il}$	$\frac{a_l}{2-b}$	$\frac{a_l}{2-b}$	$\frac{a_l}{2-b}$
$p_{1h}$	$\frac{b^2c-c-ua_h}{u(b-1)(b+2)}$	$\frac{(b^2-1)\hat{w}_i-ua_h}{(b-1)u(b+2)}$	$\frac{2c+2ua_h-b^2c+b^3c-2bc-buah-(b^2-b)\hat{w}_2}{u(b-1)(b-2)(b+2)}$
$p_{2h}$			$\frac{\hat{w}_2(b-1)(b^2-2)+2ua_h-b^2c+bc-buah}{u(b-1)(b-2)(b+2)}$
$\pi_2^R$	NA	$\frac{b^3ua_l^2-3b^2a_l^2-b^3a_l^2+4a_l^2+4ua_h^2-4ua_l^2-3b^2ua_h^2+b^3ua_h^2+3b^2ua_l^2}{u(1-b)(b-2)^2(b+2)^2}$ $+ \hat{w}_i(b+1) \frac{(1-b)\hat{w}_i-2ua_h}{u(b+2)^2}$	$\frac{(b+1)(b^2c-2ua_h-bc+buah-2(b-1)\hat{w}_2)(\hat{w}_2(b-1)(b^2-2)+2ua_h-b^2c+bc-buah)}{u^2(b-1)(b-2)^2(b+2)^2}$ $- \hat{w}_i(b+1) \frac{b^2c-2ua_h-bc+buah-2(b-1)\hat{w}_2}{u(b-2)(b+2)}$
$\pi_2^M$	NA	$(b+1) \frac{ua_h+(b-1)\hat{w}_i}{u(b+2)} (\hat{w}_i - c)$	$(b+1) (\hat{w}_2 - c) \frac{b^2c-2ua_h-bc+buah-2(b-1)\hat{w}_2}{u(b-2)(b+2)}$
$\pi_1^{SC}(VIBW(\alpha_2))$		$\frac{(b+1)(2c-2ua_h-2bc+buah+(b^2-b)\hat{w}_2)(2c+b^3c-2bc-buah-(b^2-b)\hat{w}_2)^2}{u^2(b-1)(b-2)^2(b+2)^2}$	$\frac{(b+1) \frac{2c-2ua_h-2bc+buah+(b^2-b)\hat{w}_2}{u(b-2)(b+2)} - c(b+1)}{u(b-2)(b+2)}$
$\pi_i^{SC}(VIVI)$		$\frac{4a_l^2-3b^2a_l^2-b^3a_l^2+4ua_h^2-4ua_l^2-3b^2ua_h^2+b^3ua_h^2+3b^2ua_l^2}{(1-b)(b-2)^2(b+2)^2} - c(b+1)(2ua_h-c+bc)$	

Table 3: Optimal solutions in three supply chain structures for symmetric chains

Decision Variable	<i>MSMS</i>	Supply chain structure
$w_2$	$\frac{2c+2ua_h-2bc-bua_h}{(b-1)(b-4)}$	<i>VIMS</i>
$q_1$	$\frac{2(b+1)(bc-c+ua_h)}{u(b+2)(4-b)}$	
$q_2$		
$p_{il}$	$\frac{a_l}{2-b}$	
$p_{1h}$	$\frac{2c+6ua_h-2b^2c-b^2ua_h}{u(b-1)(b+2)(b-4)}$	
$p_{2h}$		
$\pi_2^M$	$\frac{2(b-2)(ua_h+bc-c)^2(b+1)}{u(b-1)(b-4)^2(b+2)}$	
$\pi_i^R(MSMS) =$	$\frac{44b^2a_l^2-8b^3a_l^2-5b^4a_l^2+b^5a_l^2-64a_l^2+32ba_l^2-48ua_h^2+64ua_l^2+44b^2ua_h^2-12b^3ua_h^2-6b^4ua_h^2+2b^5ua_h^2-44b^2ua_l^2+8b^3ua_l^2+5b^4ua_l^2-b^5ua_l^2-32bua_l^2}{(b-1)(b-2)^2(b+2)^2(b-4)^2}$	
$\pi_2^R(VIMS) =$	$\frac{2c(b+1)(b^2-6)(bc-c+2ua_h)}{u(b+2)^2(b-4)^2}$	
$\pi_1^{SC}(VIMS) =$	$\frac{4b^3ua_l^2-12b^2a_l^2-4b^3a_l^2+16a_l^2+4ua_l^2-16ua_h^2-3b^2ua_h^2+b^3ua_h^2+12b^2ua_l^2-c(b+1)(bc-c+2ua_h)}{4(1-b)(b-2)^2(b+2)^2}$	
	$\frac{64a_l^2-48b^2a_l^2-16b^3a_l^2+64ua_l^2-64ua_h^2-44b^2ua_h^2-8b^3ua_h^2+5b^4ua_h^2+16b^3ua_l^2+32bua_l^2}{16(1-b)(b-2)^2(b+2)^2} - c(b+1)(b+4)^2 \frac{(bc-c+2ua_h)}{16u(b+2)^2}$	

Table 4: Optimal analytical solutions in two supply chain structures for symmetric chains

$a_l = 0.5, a_h = 2, b = 0.5, u = 0.5, \text{ and } c = 0.3$			
<b>Decision</b>	<b>Supply Chain Structure</b>		
<b>Variable</b>	<i>VIVI</i>	<i>BW(0.5)BW(0.5)</i>	<i>VIBW(0.5)</i>
$w_2$	NA	.662	.64
$q_1$	1.02	.80	1.088
$q_2$			.747
$p_{il}$	.33	.33	.33
$p_{1h}$	1.96	2.394	2.051
$p_{2h}$			2.278
$\pi_2^R$	NA	.485	.427
$\pi_2^M$	NA	.291	.254
$\pi_1^{SC}$	.749	.775	.844
$\pi_2^{SC}$			.681

Table 5: optimal decisions and profits in three strategies with uncertain demand

$a_l = 0.5, a_h = 2, b = 0.5, u = 0.5, \text{ and } c = 0$			
<b>Decision</b>	<b>Supply Chain Structure</b>		
<b>Variable</b>	<i>VIVI</i>	<i>BW(0.5)BW(0.5)</i>	<i>VIBW(0.5)</i>
$w_2$	NA	.419	.394
$q_1$	1.2	.948	1.278
$q_2$			.884
$p_{il}$	.33	.33	.33
$p_{1h}$	1.6	2.103	1.705
$p_{2h}$			1.967
$\pi_2^R$	NA	.655	.577
$\pi_2^M$	NA	.397	.348
$\pi_1^{SC}$	1.015	1.053	1.145
$\pi_2^{SC}$			.925

Table 6: Optimal decisions and profits in three strategies with uncertain demand

## 4.2.1 Competition over a single period

### 4.2.1.1 The VI, MS competition

We present sub-game perfect equilibrium solutions at the operational level in a bi-matrix similar to Table 2, whose entries are the profits of both SCs. For a bi-matrix table game composed of VI and MS equilibrium solutions in uncertain demand case, we have:

**Lemma 1**  $\pi_2^{SC}(VIVI) - \pi_2^{SC}(VIMS) > 0$ , and  $\pi_2^{SC}(MSVI) - \pi_2^{SC}(MSMS) > 0$ .

**Proof.** Given the channel strategy in one SC as VI, based on Tables 3 and 4 we have:

$$\pi_2^{SC}(VIVI) - \pi_2^{SC}(VIMS) = \frac{(b+1)(b^2+2)(bc-c+ua_h)^2}{8u(1-b)(b+2)^2} > 0.$$

Given the channel strategy in one SC as MS, using Table 4 we have:

$$\pi_2^{SC}(MSVI) - \pi_2^{SC}(MSMS) = \frac{(b+1)(4b+b^2+8)(b^2-4b+8)(bc-c+ua_h)^2}{16u(1-b)(b+2)^2(b-4)^2} > 0.$$

■

Thus, given any the channel strategy in one SC, the VI strategy is dominant for the other SC.

**Theorem 1** *In competing SC with the uncertain demand structure considered, Vertical Integration by both SCs is always a Perfect equilibrium to the bi-matrix table game composed of VI and MS equilibrium solutions. Moreover, this perfect equilibrium is unique for  $0 < b < 1$  and*

$$\begin{aligned} \pi^{SC*}(VIVI) &\geq \pi^{SC*}(MSMS) \quad \text{if } 0 < b \leq \frac{2}{3}, \\ \pi^{SC*}(MSMS) &> \pi^{SC*}(VIVI) \quad \text{if } \frac{2}{3} < b \leq 1. \end{aligned} \quad (24)$$

**Proof.** In Lemma 1, we showed that the best response strategy for the second SC is VI regardless of the strategy of the first SC. Similarly, we can show that the best response strategy for the first SC is VI no matter what the strategy of the second SC is. Therefore, (VI,VI) is the unique Nash Equilibrium solution of the strategic game in uncertain demand case.

Taking the difference between  $\pi^{SC*}(VIVI)$  and  $\pi^{SC*}(MSMS)$  leads to:

$$\pi^{SC*}(VIVI) - \pi^{SC*}(MSMS) = \frac{(2-b)(3b-2)(b+1)(ua_h-c+bc)^2}{u(b-1)(b+2)^2(b-4)^2},$$

from which after some algebra (24) follows. ■

Note that the problem of choosing an optimal channel strategy when  $b > \frac{2}{3}$  is a classical prisoners' dilemma game.

The result of Theorem 1 differs from a large stream of existing works (McGuire and Staelin (1983) and Moorthy (1988)) in terms of equilibrium solutions. The reason for the difference is that they use Cournot equilibria while we use Bertrand equilibria. We emphasize that this difference is not the result of uncertainty, as can be seen by setting  $u = 1$ . (The certain demand case in Theorem 1.)

#### 4.2.1.2 The VI versus BW( $\alpha$ ) competition

We now analyze VI and BW( $\alpha$ ) in channel selection decisions using numerical solutions. For the case  $a_l = 0.5$ ,  $a_h = 2$ ,  $u = 0.5$ ,  $b = 0.5$ , and  $\alpha = 0.5$ . Tables 5 and 6 give the optimal decisions and profits for the three possible industry structures where  $c = 0.3$  and  $c = 0$ , respectively. Tables 7 and 8 show the strategic games for this case when  $c = 0.3$  and  $c = 0$ , respectively. (VIVI) is the unique Nash Equilibrium solution of the games in both Tables 7 and 8. In the strategic game for two SCs in Table 7, numerical analysis demonstrates that both SCs are better off with BW(0.5)BW(0.5) than with VIVI when  $b$  is greater than approximately 0.37. For this numerical analysis (and similar ones to follow), we compared the profit of the supply chains for the VIVI and BW(0.5,0.5) cases as expressed in Tables 3 and 4, and solved for the  $b$  that equates them. Thus, the problem of choosing an optimal channel strategy when  $b > 0.37$  is a classical prisoners' dilemma game. In the strategic game in Table 8, numerical analysis demonstrates that both SCs are better off with pure BW(0.5)BW(0.5) than with VIVI when  $b$  is greater than approximately 0.31. Thus, the problem of choosing an optimal channel strategy when  $b > 0.31$  is a classical prisoners' dilemma game.

We note that the cases analyzed in Tables 7 and 8 compare a specific choice of  $\alpha = 0.5$ . In contrast, one can think of a SC that coordinates itself by choosing some  $\alpha \in [0, 1]$  optimally as a best response to the other SC structure. This approach, which was taken in Baron et al. 2007, is not further pursued here.

## 4.2.2 Competition over many periods with discounting

### 4.2.2.1 The VI, MS competition

Consider the following **Strategy 1**: use MS from period 0 and on, unless in any past period the other SC used VI. In such case use VI forever. Then we have:

		SC2	
SC1		VI	BW(0.5)
	VI	.749,.749	.844, .681
	BW(0.5)	.681,.844	.775,.775

Table 7: Strategic(table) game in uncertain case when  $c = 0.3$

		SC2	
SC1		VI	BW(0.5)
	VI	1.015,1.015	1.145, .925
	BW(0.5)	.925,1.145	1.053,1.053

Table 8: Strategic(table) game in uncertain case when  $c = 0$

**Theorem 2** *In the infinitely repeated discounted strategic game with VI and MS with the uncertain demand structure considered, Strategy 1 is a Nash Equilibrium if  $\frac{2}{3} < b \leq 1$  and*

$$\beta > \frac{b^4 + 64}{b(b + 8)(b - 4)^2}. \quad (25)$$

*In other cases the VIVI strategy is a Nash equilibrium.*

**Proof.** If both SCs choose MS in each period then the net present value (NPV) of the profit at any period is:

$$\sum_{k=0}^{\infty} \beta^k \pi^{SC}(MSMS) = \frac{\pi^{SC}(MSMS)}{1 - \beta}. \quad (26)$$

However, if the first SC deviates from MS to VI, then the NPV of its profits in this period is

$$\pi_1^{SC}(VIMS) + \sum_{i=1}^{\infty} \beta^i \pi_1^{SC}(VIVI) = \pi^{SC}(VIMS) + \frac{\beta \pi_1^{SC}(VIVI)}{1 - \beta}. \quad (27)$$

Subtracting (27) from (26) yields:

$$(b + 1)(ua_h - c + bc)^2 \frac{128b\beta - 48b^2\beta + b^4\beta - b^4 - 64}{16u(b - 1)(b + 2)^2(b - 4)^2(\beta - 1)}. \quad (28)$$

Thus, if  $128b\beta - 48b^2\beta + b^4\beta - b^4 - 64 > 0$ , which is equivalent to (25), the right hand side in (28) is positive, completing the proof. ■

#### 4.2.2.2 The VI versus BW( $\alpha$ ) competition

$\alpha$	.1	.2	.3	.4	.5	.6	.7
$\pi_1^{SC} (VIBW)$	1.04	1.06	1.09	1.12	1.15	1.17	1.19
$\pi_1^{SC} (BWBW)$	1.03	1.04	1.05	1.06	1.05	1.04	1.03
$\hat{\beta}$	.03	.06	.12	.18	.23	.31	.36

Table 9: Conditions for Strategy 2 to be optimal for  $\alpha = .1, \dots, .7$

To demonstrate the analysis for the VI versus BW( $\alpha$ ) case, consider the same example discussed in Table 8, i.e.  $\alpha = 0.5, a_l = 0.5, a_h = 2, u = 0.5$ , and  $c = 0$ . Then for  $b > 0.31$  we noted that the single period game is a prisoner's dilemma. Note that when  $b > \frac{2}{3}, \frac{b^4+64}{b(b+8)(b-4)^2} < 1$ .

Consider the following **Strategy 2**: Use BW(0.5) from period 0 and on unless in any past period the other SC used VI. In such case use VI forever.

We now search for the discount factor,  $\beta$ , that would lead this strategy to be a Nash equilibrium for our infinite horizon strategic game.

When  $b = 0.31$ , if both SCs adopt BW(0.5) they each get  $\frac{\pi^{SC}(BW(\alpha)BW(\alpha))}{1-\beta} = \frac{0.486}{1-\beta}$ . If the first SC deviates from BW( $\alpha$ ) to choose VI, even just once, then it gets  $\pi_1^{SC}(VIBW(0.5)) + \frac{\beta\pi^{SC}(VIVI)}{1-\beta} = 0.52 + \frac{0.482\beta}{1-\beta}$ . It can be seen that the first of these is larger if  $\beta \geq 0.9$ . Thus, for  $b = 0.31$ , any  $\beta \in [0.9, 1]$  allows Strategy 2 to be Nash Equilibrium. A similar analysis when  $b = 0.96$  shows that Strategy 2 is a Nash Equilibrium for any  $\beta \in [0.8, 1]$ .

We investigated the conditions for Strategy 2 to be optimal for different values of  $\alpha$  for the case where  $b = 0.5, c = 0, u = 0.5, a_h = 2$ , and  $a_l = .5$ . In this case,  $\pi_1^{SC}(VIVI) = 0.749$ . Also for  $\alpha \geq 0.8$   $\pi_1^{SC}(BWBW) \leq .745$ , thus Strategy 2 cannot be optimal for any discount factor  $\beta$ . In Table 9 we let  $\alpha = 0.1, 0.2, \dots, 0.7$ . For each  $\alpha$  value, we present,  $\pi_1^{SC}(VIBW)$ , the supply chain profit of the VI chain in the VIBW case,  $\pi_1^{SC}(BWBW)$ , the supply chain profit in the BWBW case, and  $\hat{\beta}$ , the minimal discount factor that makes Strategy 2 optimal. (That is for any  $\beta \geq \hat{\beta}$ , Strategy 2 is optimal in these settings.)

## 5 Summary and Future Research

We studied the Nash Equilibrium of an industry with two supply chains, each using Nash Bargaining on the wholesale price (BW) strategy in the uncertain demand case. We showed that VI (subject to a condition) and MS strategies are both special cases of BW. We demonstrated

that using VIVI in both chains is the unique Nash Equilibrium over one period while MSMS or  $BW(\alpha)BW(\alpha)$  in both chains can be Nash Equilibrium over infinitely many periods.

We suggest a few directions for future research. The first is to obtain empirical evidence for the dependence and shifts in supply chains structure due to the level of substitutability. The second is to investigate the Nash Equilibrium when one supply chain is a leader and other supply chain chooses its position later. A third one would be to consider similar models when there is information asymmetry both within and between the chains. A last direction is to extend the uncertainty structure, e.g., consider a newsvendor demand model in each period.

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## Appendix 1: Informal Definitions

- a. Vertically integrated chain is a chain with a single owner.
- b. Manufacturer Stackelberg game: a game where the manufacturer first chooses a price and then the retailer chooses its order quantity.
- c. Nash Bargaining on the wholesale price is defined in page 5, paragraph 1.
- d. Cournot competition: There are a few firms producing a homogeneous product. These firms compete in the market by choosing quantities simultaneously to maximize their profit while considering the other firms actions. The price is chosen to clear the total produced quantity
- e. Bertrand competition: There are a few firms producing homogeneous products and facing the same constant marginal cost and a linear demand. Firms compete on price choosing prices simultaneously to maximize profit, while considering the other firms actions. Consumers buy everything from the cheaper firm or half at each, if the price is equal. Given prices firms supply the actual quantity demanded.

- f. Nash Equilibrium: Informally, a set of strategies is a Nash equilibrium in a game if no player can improve his or her payoff by changing strategy unilaterally.
- g. First order conditions: These conditions are necessary for an objective function of an unconstrained optimization problem to be a local minimum or maximum.
- h. Discount factor: the number by which a cash flow to be received at the next period must be multiplied in order to obtain its value in this period.

## Appendix 2: Proof of Proposition 2

The proof follows the idea in Padmanabhan and Png (1997). For two symmetric supply chains, we have  $a_{il} = a_{jl} = a_l, a_{ih} = a_{jh} = a_h$  and all the marginal cost to the retailer is  $w$ . It is easy to see that  $s_i$  equals  $q_i$  at the high demand state. Otherwise, it could incur more costs without increasing the revenues  $p_i s_i$  if it stocked more than  $q_i$ . Accordingly, the retailer prefers to decrease stock till  $s_i = q_i$ . The retailer price at high demand status determined by solving:

$$a_h - p_{ih} + bp_{jh} = q_i, \quad i = 1, 2; \quad j = 3 - i.$$

The solution gives the optimal retailer price at high demand state:

$$p_{ih} = \frac{1}{1 - b^2} (a_h - q_i + ba_h - bq_j), \quad i = 1, 2; \quad j = 3 - i.$$

When demand is low, there are two possibilities: to set the price to clear all stocks or to leave some stock unsold.

- Suppose that for low demand, sales are restricted by stock.

Then, we have  $a_l - p_{il} + bp_{jl} = q_i$  and  $p_{il} = -\frac{1}{b^2 - 1} (a_l - q_i + ba_l - bq_j)$ .

The retailer's expected profit is

$$\tilde{\pi}_i^R = u \left( -\frac{1}{b^2 - 1} (a_h - q_i + ba_h - bq_j) \right) q_i + (1 - u) \left( -\frac{1}{b^2 - 1} (a_l - q_i + ba_l - bq_j) \right) q_i - wq_i, \quad i = 1, 2; \quad j = 3 - i.$$

FOC with respect to  $q_i$  leads to the reaction function,

$$-w + a_l - 2q_i + ba_l - bq_j + ua_h - ua_l + wb^2 + bua_h - bua_l = 0$$

By solving the above equation, we get the optimal  $q_i$ :

$$q_i = \frac{b+1}{b+2} (\bar{a} - w + wb)$$

where  $\bar{a} = ua_h + (1-u)a_l$  is the average demand.

Thus,  $\tilde{\pi}_i^R = \frac{1}{1-b} \frac{b+1}{(b+2)^2} (\bar{a} - w + wb)^2$ .

- Alternatively, suppose that for low demand, sales are not restricted by stock.

Then,  $p_{il}$  is chosen to maximize the revenue  $p_{il}(a_l - p_{il} + bp_{jl})$ . FOC leads to a reaction function with a solution for the optimal price  $p_{il} = \frac{a_l}{2-b}$ , and  $s_i = \frac{a_l}{2-b}$ .

In this case, the retailer's expected profit is

$$\bar{\pi}_i^R = u \left( -\frac{1}{b^2-1} (a_h - q_i + ba_h - bq_j) \right) q_i + (1-u) \frac{a_l^2}{(b-2)^2} - wq_i.$$

FOC with respect to  $q_i$  leads to the reaction function,

$$\frac{1}{b^2-1} (w - ua_h + 2uq_i - wb^2 - bua_h + buq_j) = 0$$

with a solution:

$$q_i = \frac{1}{u} \frac{b+1}{b+2} (ua_h - w + wb).$$

The stock will be sufficient for sales  $s_i = \frac{a_l}{2-b}$  if  $\frac{1}{u} \frac{b+1}{b+2} (ua_h - w + wb) > \frac{a_l}{2-b}$ , which is equivalent to (7).

Substituting the expression for the stock into the retailer's profit results in:

$$\begin{aligned} \bar{\pi}_i^R = & \frac{1}{u(b-2)^2(b+2)^2(1-b)} (4u^2a_h^2 - 4u^2a_l^2 - 8w^2b + 4ua_l^2 + w^2b^2 + 7w^2b^3 - 5w^2b^4 + w^2b^5 + \\ & 4w^2 - 3b^2ua_l^2 - b^3ua_l^2 - 8wua_h - 3b^2u^2a_h^2 + b^3u^2a_h^2 + 3b^2u^2a_l^2 + b^3u^2a_l^2 + 8wbuah + \\ & 6wb^2ua_h - 8wb^3ua_h + 2wb^4ua_h) \end{aligned}$$

- In order to compare  $\tilde{\pi}_{il}^R$  and  $\bar{\pi}_{il}^R$ , we construct the function:

$$\begin{aligned} f(\beta) = & \frac{-u}{b+1} (-2b^3a_l^2 - 2b^4a_l^2 + 16u\beta^2 + 4ua_l^2 - 8b^2u\beta^2 + b^4u\beta^2 - 3b^2ua_l^2 - 2b^3ua_l^2 + \\ & b^4ua_l^2 - 16u\beta a_l + 4bua_l^2 - 8bu\beta a_l + 12b^2u\beta a_l + 2b^3u\beta a_l - 2b^4u\beta a_l), \end{aligned}$$

and note that  $\tilde{\pi}_{il}^R - \bar{\pi}_{il}^R = f\left(\frac{1}{u} \frac{b+1}{b+2} (ua_h - w + wb)\right)$ .

Now, since  $\frac{df(\beta)}{d\beta} = 2\frac{u^2}{b+1} (b-2)^2 (b+2) (a_l - 2\beta - b\beta + ba_l)$ , for  $\beta > \frac{a_l(b+1)}{b+2}$  we have  $\frac{df(\beta)}{d\beta} < 0$  and  $f(\beta)$  is a monotonic decreasing function in  $\beta$  over this range.

Moreover, as  $\frac{a_l}{2-b} > \frac{a_l(b+1)}{b+2} \quad \forall b \neq 0$  we have  $\tilde{\pi}_{il}^R - \bar{\pi}_{il}^R = f\left(\frac{1}{u} \frac{b+1}{b+2} (ua_h - w + wb)\right) \leq f\left(\frac{a_l}{2-b}\right) = -b^3u \frac{a_l^2}{b+1} (2b - bu + 2) < 0$  for each  $b > 0$ , we have  $\tilde{\pi}_{il}^R - \bar{\pi}_{il}^R < 0$ . This completes our proof of Proposition 2.

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