

# **The Risk of Tranches Created from Residential Mortgages**

**John Hull and Alan White**

**Joseph L. Rotman School of Management  
University of Toronto**

**First Draft: May 2009**

**This Version: November 2009**

## **Abstract**

This paper examines, ex-ante, the risk in the tranches of ABSs and ABS CDOs that were created from residential mortgages between 2000 and 2007. Using the criteria of the rating agencies, it tests how wide the AAA tranches can be under different assumptions about the correlation model and recovery rates. It concludes that the AAA ratings assigned to the senior tranches of ABSs were not totally unreasonable. However, the AAA ratings assigned to tranches of Mezz ABS CDOs cannot be justified. The risk of a Mezz ABS CDO tranche depends critically on the correlation between mortgage pools as well as on the correlation model and the thickness of the underlying BBB tranches. The BBB tranches of ABSs cannot be considered equivalent to BBB bonds for the purposes of subsequent securitizations.

## The Risk of Tranches Created from Residential Mortgages

The rating agencies have come under a great deal of criticism since the subprime crisis started in July 2007. It is argued that the AAA ratings assigned to the structured products created from residential mortgages convinced investors that the products were almost completely free of risk. As a result, the investors were lulled into a false sense of confidence and did not evaluate the products for themselves.

The traditional business of rating agencies is the rating of corporate and sovereign bonds. This is based on a combination of analysis and judgment. The rating of structured products was a departure from this traditional business. Instead of analysis and judgment, it involved the application of a model. The rating agencies were quite open about the models they used. Standard and Poor's (S&P) and Fitch based their ratings on the probability of loss given by their models. If the calculated probability of loss on a structured product corresponded to the probability of loss on a AAA-rated bond, the structured product was rated AAA. Moody's, by contrast, based its ratings on expected loss. If the expected loss on a structured product corresponded to the expected loss from default on a Aaa-rated bond, the structured product was rated Aaa. When a structured product was designed, creators wanted to achieve their target ratings for tranches by meeting the model requirements of rating agencies.<sup>1</sup> Their objective was usually to make the total principal of the AAA tranches that they created as large as possible. Usually, they got advance rulings on ratings before finalizing product design.

In this paper we evaluate whether the ratings assigned to structured products by rating agencies were reasonable. We look at both asset-backed securities (ABSs), which were products created from pools of mortgages and ABS collateralized debt obligations (ABS CDOs), which were products created from the tranches of several ABSs. Coval *et al* (2008) also evaluate ratings for these types of products. However, they assume that the asset pools underlying ABS CDOs have zero default correlation with each other. We do not make this assumption. Indeed, we find that the risks of the tranches in an ABS CDO are critically dependent on the correlation between different asset pools.

---

<sup>1</sup> See Brennan, Hein, and Poon (2008) for a discussion of this.

## **1. The Products That Were Created**

During the 1999 to 2007 period, financial institutions found the originate-to-distribute model increasingly attractive. Rather than keeping assets they originated on their balance sheets, they securitized them so that the credit risk could be passed to investors. Many different types of assets were securitized: corporate debt, credit card receivables, car loans, etc. In this paper we focus on the securitization of subprime residential mortgages.

### **ABSs**

The nature of the ABSs that were created from subprime residential mortgages is discussed in some detail by Gorton (2007). A typical structure is illustrated in Figure 1. The subprime mortgage portfolio might consist of a total of 1000 mortgages. This underlying collateral is tranching into one or more senior tranches rated AAA, a number of mezzanine tranches rated AA, A, and BBB, and subordinated tranches which are either rated BB or unrated. A principal is allocated to each tranche. In a relatively simple structure, the total principal of the mortgages equals the total principal of the tranches. In more complicated structures, there is some overcollateralization where the total principal of the mortgage portfolio exceeds the total principal of the tranches.

A key aspect of the design of the structure is the amount of principal allocated to each rating category. Prior to 2006, typically 75% to 85% of the mortgage principal was allocated to AAA tranches. The principals allocated to other rating categories were much smaller. For example, the BBB tranches taken together typically accounted for 3% or less of the mortgage principal. The goal of the creator of the ABS is to maximize the quality of the tranches that were created. First, the AAA tranches were made as wide as possible; the AA tranches were then made as wide as possible; and so on.

The rules for allocating cash flows from the mortgages to tranches are defined by what is known as a “waterfall.” The interest payments promised to tranches are typically LIBOR plus a spread with the spread decreasing with seniority. Interest payments from mortgages are typically allocated to tranches in order of seniority so that the AAA-rated tranches get promised interest payments on their outstanding principal first; after that, the AA-rated tranches get their promised interest payments on their outstanding principal; and so on. The interest payments on subprime

mortgages are sometimes fixed for the first two or three years and then become floating. There is therefore a risk that, even when all scheduled mortgage payments are made, the floating interest payments on tranches cannot be made. For this reason, the rate of interest payable to tranches may be capped.

The principal payments (both scheduled and prepayments) are handled separately from interest payments and the rules are relatively complicated. There is typically a lockout period during which principal payments are sequential. This means that all principal payments go first to the most senior tranche. When that tranche has been completely amortized, they go to the next-most-senior tranche, and so on. After the lock out period, if certain performance targets are met, principal payments are allocated to tranches in proportion to their outstanding principals. However, if there is a “cumulative loss trigger event” (where cumulative losses on the mortgages are higher than certain levels) or a “delinquency event” (where the rate of delinquency over a three-month period is above a certain level), principal payments become sequential again.

As an approximation, it can be assumed that, when the default rate proves to be high, the repayment of principal is entirely sequential. The effect of this is that tranches bear losses in order of reverse seniority. The unrated tranches absorb losses first. Once their principal has been lost, the BB-rated tranches bear losses, and so on.

There were usually several tranches corresponding to each rating category. For example, in the Structured Asset Investment Loan Trust (SAIL), which was issued in 2006 and is described by Gorton (2008, p33), there were four AAA tranches (with equal seniority) accounting for 83.25% of the collateral; two AA-rated tranches (with unequal seniority) accounting for 8.2% of the collateral; three A-rated tranches (with unequal seniority) accounting for 4.1% of the principal; and three BBB tranches (with unequal seniority) accounting for 2.8% of the principal.

A point relevant to our analysis is that BBB tranches were often very thin. Although the total of all BBB tranches might be about 3% of the collateral, each individual BBB tranche was often about 1% of the principal. If the macroeconomic environment is relatively benign so that there are large repayments of principal, the AAA tranche can be expected to shrink and the proportion of the remaining mortgage principal accounted for by the BBB tranches can be expected to increase. But, if default rates are high, a thin BBB tranche can easily lose its entire principal. In

the SAIL structure just mentioned, the most senior BBB tranche (rated Baa1/BBB+) was 1.1% wide with 3.25% subordination; the next BBB tranche (rated Baa2/BBB) was 0.85% wide with 2.40% subordination, and the most junior BBB tranche (rated Baa3/BBB-) tranche was 0.8% wide with 1.60% subordination. Assuming principal payments are sequential, the three tranches will lose their entire principal if losses on the subprime mortgage portfolio are greater than 4.35%, 3.25%, and 2.40%, respectively.

### **ABS CDOs**

In the second level of securitization, tranches are created from tranches. The products created are known as ABS CDOs. As Gorton (2008) explains, two types of ABS CDOs were common. These were a “High Grade ABS CDO” created from the AAA, AA, and A tranches of ABSs and a “Mezz ABS CDO” created from the BBB tranches of ABSs.

We will focus on the Mezz ABS CDO. Its creation is illustrated in Figure 2. The AAA-rated tranche in Figure 2 is typically less wide than that in Figure 1. However, it still usually accounted for more than 50% of the ABS CDO principal. (In an example given by Gorton on page 35, which is taken from a UBS publication, the AAA-rated tranche of the ABS CDO accounts for 76% of the principal.)

Many of the ABS CDOs are managed. This means that the tranches forming the collateral do not remain fixed over time. A portfolio manager is allowed to trade a certain percentage of the underlying collateral each year. However there are restrictions relating to measures involving the ratings of the collateral, correlation, and the weighted average life of the underlying assets.

ABS CDOs are like ABSs in that the waterfall is complex. Losses tend to be allocated to the most junior tranches first. There are coverage tests and triggers which cause amortizations to be sequential and divert cash flows from junior to senior tranches. In certain circumstances, the senior tranche holders may be able to liquidate the assets

## 2. Modeling Losses From Defaults

The models that we will use focus on the losses of principal that are incurred by tranches during the whole life of the structures. We do not concern ourselves with the timing of losses. We assume no overcollateralization. However, this is not too limiting. In high-default-rate situations,  $x\%$  of overcollateralization can be thought of as a “dummy” junior tranche that absorbs the first  $x\%$  of losses.

We assume that principal payments are allocated to tranches sequentially so that losses are allocated in reverse order of seniority. As mentioned earlier, this corresponds to the way ABS CDOs usually work. It also corresponds to the way ABSs usually work for the first few years and to the way they usually work in subsequent years when the default rate is high. In assigning ratings we are interested in observing how tranches fare in high-default-rate situations. The assumption that principal is always allocated sequentially is therefore reasonable for ABSs as well as ABS CDOs.

The mortgages in the pool are assumed to have equal principal and to have the same probability of default. A mortgage pool is assumed to be sufficiently large that a “large portfolio assumption” applies so that the actual proportion of mortgages defaulting in the portfolio equals the probability of each mortgage defaulting. (We refer to this as the “default rate.”) In practice, there are about 1000 mortgages in a pool. Tests we have carried out show that the large portfolio assumption (which considerably reduces computation time) has only a small effect on our results.

### Single Pool Correlation Model

Suppose that  $Q$  is the fraction of mortgages in the pool that are expected to default within  $T$  years. If all the mortgages are of similar risk then  $Q$  is the probability of default for any individual mortgage. A natural model to assume is the one-factor Gaussian copula model that has become the standard market model for valuing synthetic CDOs. In this model, there is a factor that is common to all mortgages, which we will denote by  $M$ , and a factor specific to mortgage  $i$  which we will denote by  $Z_i$ . The factors  $M$  and  $Z_i$  are assumed to have independent standard normal distributions. In the model, mortgage  $i$  defaults within  $T$  years if

$$\sqrt{\rho}M + \sqrt{1-\rho}Z_i < K$$

where  $\rho$  is the correlation between the transformed times to default of any two mortgages. Under the assumptions of standard normal distributions the probability of default is  $N(K)$  where  $N$  is the cumulative normal distribution function. The model is calibrated to the expected default rate by setting  $N(K) = Q$ .

The  $i$ th mortgage therefore defaults if

$$\sqrt{\rho}M + \sqrt{1-\rho}Z_i < N^{-1}(Q)$$

or

$$Z_i < \frac{N^{-1}(Q) - \sqrt{\rho}M}{\sqrt{1-\rho}}$$

The realized default rate,  $P$ , conditional on  $M$  is therefore

$$P = N\left(\frac{N^{-1}(Q) - \sqrt{\rho}M}{\sqrt{1-\rho}}\right) \quad (1)$$

Hull and White (2004) show that any zero mean unit variance distributions can be chosen for  $M$  and  $Z_i$ . They find that the “double  $t$ ” copula model where both  $M$  and  $Z_i$  have  $t$ -distributions with 4 degrees of freedom (scaled so that the variance is one) fits market data on synthetic CDOs well. It has considerably more tail default correlation (i.e., it has a higher probability of extreme clustering of defaults) than the Gaussian copula model.

In the double  $t$  copula model, the  $i$ th mortgage defaults if

$$\sqrt{\rho}M + \sqrt{1-\rho}Z_i < F^{-1}(Q)$$

where  $F$  is the cumulative probability distribution of:<sup>2</sup>

$$\sqrt{\rho}M + \sqrt{1-\rho}Z_i$$

---

<sup>2</sup> In general, this distribution has to be determined numerically. The procedure we adopt is to determine a look up table in advance of the main analysis.

The realized default rate, conditional on the factor  $M$ , is

$$P = H\left(\frac{F^{-1}(Q) - \sqrt{\rho}M}{\sqrt{1-\rho}}\right) \quad (2)$$

where  $H$  is the cumulative probability distribution of a scaled  $t$ -distribution with four degrees of freedom.

We will present results for tests assuming both the Gaussian copula model and the double  $t$  copula model.

### The Multi-Pool Correlation Model

When several pools are considered simultaneously it is necessary define a “between-pool” factor,  $M_{bp}$ , and within pool factors,  $M_{wp,j}$ . The factor  $M_{bp}$  affects the probability of default for all mortgages while  $M_{wp,j}$  affects the probability of default only for mortgages in pool  $j$ . In this model, when distributions are assumed to be normal, the  $i$ th mortgage in the  $j$ th pool defaults if

$$\sqrt{\alpha\rho}M_{bp} + \sqrt{(1-\alpha)\rho}M_{wp,j} + \sqrt{1-\rho}Z_{ij} < \Psi^{-1}(Q)$$

where  $Z_{ij}$  is a variable affecting only the  $i$ th mortgage in the  $j$ th pool and  $\Psi$  is the cumulative probability distribution of

$$\sqrt{\alpha\rho}M_{bp} + \sqrt{(1-\alpha)\rho}M_{wp,j} + \sqrt{1-\rho}Z_{ij}$$

The factors and the variables  $Z_{ij}$  are independent of each other.

As before, the parameter  $\rho$  is the total within pool correlation. The parameter  $\alpha$  indicates the proportion of the default correlation that comes from a factor common to all pools. When  $\alpha = 0$  the default rates of different pools are independent of each other. (As noted earlier, this is the model assumed by Coval *et al* (2008).) At the other extreme, when  $\alpha=1$ , there is a single factor affecting all mortgage defaults and the default rates in all mortgage pools are the same. The realized default rate for pool  $j$  conditional on  $M_{bp}$  and  $M_{wp,j}$  is

$$\Phi\left(\frac{\Psi^{-1}(Q) - \sqrt{\alpha\rho}M_{bp} - \sqrt{(1-\alpha)\rho}M_{wp,j}}{\sqrt{1-\rho}}\right) \quad (3)$$

where  $\Phi$  the cumulative probability distribution of  $Z_{ij}$ . The simplest version of the model is the case in which the  $M$ 's and  $Z$ 's have standard normal distributions. We will also consider the case where they all have  $t$  distribution with four degrees of freedom (scaled so that the variance is one). We refer to this as the “triple  $t$  copula model.”

### Recovery Rate Model

Credit derivatives models often assume that the recovery rate realized when there is a default is constant. This is less than ideal. As the default rate increases, the recovery rate for a particular asset class can be expected to decline. This is because a high default rate leads to more of the assets coming on the market and a reduction in price.<sup>3</sup>

As is now well known, this argument is particularly true for residential mortgages. In a normal market, a recovery rate of about 75% is often assumed for this asset class. If this is assumed to be the recovery rate in all situations, the worst possible loss on a portfolio of residential mortgages given by the model would be 25%, and the 25% to 100% senior tranche of an ABS created from the mortgages could reasonably be assumed to be safe. In fact, recovery rates on mortgages have declined in the high default rate environment experienced since 2007.

Define the recovery rate applicable to average market conditions as  $R_{\text{avg}}$ , the maximum recovery rate (occurring when the default rate is very low) as  $R_{\text{max}}$  and the minimum recovery rate (occurring when the default rate is very high) as  $R_{\text{min}}$ . We use the following simple recovery rate model for the actual recovery rate  $R$

$$R = R_{\text{min}} + (R_{\text{max}} - R_{\text{min}}) \exp(-aP) \quad (4)$$

where

$$a = - \frac{\ln \left[ \frac{(R_{\text{avg}} - R_{\text{min}})}{(R_{\text{max}} - R_{\text{min}})} \right]}{Q} \quad (5)$$

---

<sup>3</sup>The negative relationship between recovery rates and default rates has been documented for bonds by Altman *et al* (2005) and Moody's Investors Service (2008).

As before,  $P$  is the actual default rate and  $Q$  is the expected default rate. As  $P$  increases from zero to 100%, the recovery rate decreases from  $R_{\max}$  to close to  $R_{\min}$  in such a way that, when  $P = Q$ ,  $R = R_{\text{avg}}$ . Using equation (1) or (2),  $R$  can be expressed as a function of  $M$ .

### 3. Subprime Default Experience

Subprime first mortgages became common in the United States in 1999. This means that in 2006 and 2007 rating agencies had relatively little experience of the performance of these mortgages.

Figure 3 shows statistics collected by Moody's in March 2007.<sup>4</sup> The charts show, for subprime mortgages originated in a certain year, the cumulative percentage that was "delinquent" after a certain number of months. For this purpose, delinquent mortgages are defined as the total of those where payments are more than 60 days overdue, those in foreclosure, and those where the properties are being sold by the lender. Moody's had over five years of experience for mortgages originated between 1999 and 2003. The cumulative default rate for mortgages originated some time ago was between 2% and 4%. Note that the percentage of delinquent loans in the charts does not increase monotonically with time. This is because borrowers who become delinquent sometimes subsequently catch up on their late payments, refinance, or sell the house.

Figure 3 shows that there were signs that mortgages originated in 2006 were performing worse than mortgages originated in the four previous years (first chart). However, in March 2007 they appeared to be performing similarly to mortgages originated between 1999 and 2001 (second chart). The percentage of mortgages in the delinquent category after 11 months for the 1999, 2000, and 2001 vintages mortgages were 6.10%, 7.63%, and 7.15%, respectively. The percentage for the 2006 vintage was similar.<sup>5</sup>

In March 2007, investors in the AAA tranches of ABSs could draw some comfort from the AAA ABX indices which indicated no serious impairment. The TABX index, which aims to track the value of AAA tranches formed from the BBB (BBB-) tranches of ABSs, stood at 92.75 (84.00) at the end of March 2007.

Of course, there were a number of warning signals. The S&P Case-Shiller Composite 10 house price index, which was set at 100 in January 2000, reached over 225 in mid-2006, but had started to decline by the beginning of 2007. Although few people anticipated the full extent of the fall in house prices that took place in over the next two years, there was general agreement that some

---

<sup>4</sup> See Moody's Investors Services (2007).

<sup>5</sup> However, the 11-month percentage calculated in March 2007 reflects only loans originated early in 2006. The percentage of all loans originated in 2006 that became delinquent loans after 11 months (calculated at the end of 2007) was 12.13%

decline would take place. For obvious reasons, home owners are much more likely to default when house prices are falling than when they are rising. Mortgage default experience during the 1999 to 2006 period should therefore have been treated with caution.

The evaluation of ABSs depends on a) the expected default rate,  $Q$ , for mortgages in the underlying pool, b) the default correlation,  $\rho$ , for mortgages in the pool, and c) the recovery rate,  $R$ . Data from the 1999 to 2006 period suggest a value of  $Q$  less than 5% assuming an average mortgage life of 5 years. But, as has been mentioned, a different macroeconomic environment could be anticipated over the next few years. It would seem to be more prudent to use an estimate of 10%, or even higher. We will present results for values of  $Q$  equal to 5%, 10%, and 20%. The Basel II capital requirements are based on a copula correlation of 0.15 for residential mortgages.<sup>6</sup> We will present results for values of  $\rho$  between 0.05 and 0.30. As already mentioned, a recovery rate of 75% is often assumed for residential mortgages, but this is probably optimistic in a high default rate environment. We will present results for the situation where the recovery rate is fixed at 75% and for the situation where the recovery rate model in the previous section is used with  $R_{\min}=50\%$  and  $R_{\max}=100\%$ .

ABS CDOs also depend on the parameter,  $\alpha$ . Loosely speaking, this measures the proportion of the default correlation that comes from a factor common to all pools. A value of  $\alpha$  close to zero indicates that investors obtain good diversification benefits from the ABS CDO structure. In adverse market conditions some mezzanine tranches can be expected to suffer 100% losses while others incur no losses. However, a value of  $\alpha$  close to one indicates that all mezzanine tranches will tend to sink or swim together. We do not know what estimates rating agencies made for  $\alpha$ . (Ex post of course, we know that it was high.) We will therefore present results based on a wide range of values for this parameter.

---

<sup>6</sup> See Bank for International Settlements (2006, p77) and Hull (2009). Basel II uses essentially the same copula model that we do with  $M$  and the  $Z_i$  normally distributed.

#### 4. Results

Although mortgages are amortized over many years, repayments lead to a weighted average life of about five years. When determining the ratings of instruments created from mortgages, it is therefore appropriate to compare their losses with the losses on bonds over a five year period. Statistics published by Moody's for the period 1970 to 2007 show that the cumulative five-year probability of default for AAA and BBB bonds are as shown in Table 1. The expected loss in the table is calculated from the probability of default assuming a recovery rate of 40% (which is a typical recovery rate for a corporate bond).

##### The Probability of Loss Criterion for ABSs

Suppose that the attachment point for the AAA tranche of an ABS is  $X\%$  so that the tranche is responsible for losses between  $X\%$  and 100%. The probability of the tranche experiencing losses is the probability that losses on the underlying portfolio are greater than  $X\%$ . Given our large portfolio assumption that the proportion of mortgages defaulting equals the default rate, the tranche experiences losses when the default rate is greater than

$$\frac{X}{1-R}$$

where  $R$  is the recovery rate on the mortgages. Equation (1) shows that this happens in the case of the Gaussian copula model when

$$(1-R)N\left(\frac{N^{-1}(Q) - \sqrt{\rho}M}{\sqrt{1-\rho}}\right) > X$$

From Table 1, the minimum attachment point is the value of  $X$  for which the probability of this is 0.1%. It follows that the minimum attachment point is

$$(1-R)N\left(\frac{N^{-1}(Q) - \sqrt{\rho}N^{-1}(0.001)}{\sqrt{1-\rho}}\right)$$

The variable  $R$  is the recovery rate when  $M = N^{-1}(0.001)$ .

Similarly, for the double  $t$  copula model the minimum attachment point is

$$(1 - R)H\left(\frac{F^{-1}(Q) - \sqrt{\rho}H^{-1}(0.001)}{\sqrt{1 - \rho}}\right)$$

where, as before,  $H$  is the cumulative probability distribution for a  $t$ -distribution with four degrees of freedom (scaled so that the variance is one). In this case,  $R$  is the recovery rate when the  $M = H^{-1}(0.001)$ .

Table 2 shows results for different values of the expected default rate,  $Q$ , and the copula correlation,  $\rho$ . Four different models are considered:

- i. The Gaussian copula model with a recovery rate of 75% on the underlying mortgages
- ii. The double  $t$ -copula with a recovery rate of 75% on the underlying mortgages
- iii. The Gaussian copula model with the stochastic recovery rate model in equations (4) and (5) with  $R_{\max}=100\%$  and  $R_{\min}=50\%$
- iv. The double  $t$  copula model with the stochastic recovery rate model in equations (4) and (5) with  $R_{\max}=100\%$  and  $R_{\min}=50\%$

As might be expected, the minimum value of  $X$  increases as we move from the Gaussian copula to the double  $t$ -copula and from the constant recovery rate model to the stochastic recovery rate model. As mentioned, the attachment point for AAA-rated tranches was typically 15% to 25% prior to 2006. There are some indications that attachment points were raised in 2006. To quote from Moody's Investment Services (2007) "Moody's Aaa-rated bonds issued in 2006 were designed to withstand a total loss on the underlying mortgage pool of approximately 26% to 30% without defaulting."

Table 2 shows that when a 20% default rate is combined with a high default correlation, and a stochastic recovery rate model, the AAA ratings that were made seem a little high. Also, the ratings are difficult to justify when the most extreme model (double  $t$  copula, stochastic recovery rate) is used. But overall the results in Table 2 indicate that the AAA ratings that were assigned were not totally unreasonable.

## The Expected Loss Criterion for ABSs

If  $L(M)$  is the proportional loss on the mortgage portfolio for a particular value of  $M$ , the expected proportional loss on the ABS when the attachment point for the senior tranche is  $X$  is

$$\int_{M^*}^{\infty} [L(M) - X] \theta(M) dM$$

where  $M^*$  is the value of  $M$  that leads to a loss on the portfolio equal to  $X$  and  $\theta$  is the probability density of  $M$ . Because  $L(M)$  is always less than  $1 - R_{\min}$ ,  $L(M) - X$  is also less than  $1 - R_{\min}$ . It follows that the expected loss is always less than  $1 - R_{\min}$  times the probability of a loss.

Assuming that  $R_{\min}$ , the minimum recovery rate on mortgages, is greater than the recovery rate assumed on bonds, it follows that a value of  $X$  that satisfies the probability of loss criterion must also satisfy the expected loss criterion.

To put this another way, the minimum attachment point when the expected loss criterion is used must be less than the minimum attachment point when the probability of loss criterion is used. This is confirmed by Table 3 for the case where the model is a double  $t$  copula with stochastic recovery. It can be seen that, even when this exacting model is used, the expected loss criterion would lead to a 70% to 75% wide AAA-rated senior tranche being judged to be reasonable when  $\rho = 0.1$ .

## The Creation of BBB Tranches

BBB tranches must satisfy both Moody's and S&P/Fitch. As we have seen, for a senior tranche covering losses from  $X\%$  to 100%, the S&P/Fitch criterion dominates the Moody's criterion in determining the minimum  $X$ . This is not true for other tranches. The minimum attachment point is determined using the S&P/Fitch criterion in the same way as for the senior tranche. However, the minimum width of the tranche is determined by the Moody's criterion.

When forming a BBB tranche, to satisfy the S&P/Fitch criterion the probability of default must be less than 1.8% (see Table 1). If the Gaussian copula constant recovery rate model is used and the expected default rate, the copula correlation, and the recovery rate are 10%, 0.1 and 75%, respectively, the minimum attachment point is 6.4%. Any higher attachment point will produce a lower probability of default. If 6.4% is chosen as the attachment point, the Moody's criterion,

which requires the expected loss to be less than 1.08%, results in a minimum tranche width of 1.2% (the minimum detachment point is 7.6%). A higher detachment points results in lower expected loss. As the attachment point increases, the minimum tranche width decreases. In theory, an infinitesimally narrow tranche where both the attachment and detachment points are equal to (or greater than) 6.98% satisfies both the Moody's and S&P/Fitch criteria.

As noted earlier, BBB tranches were frequently about 1% wide. In what follows, we assume that an ABS CDO is created from BBB tranches, each of which covers losses on a mortgage portfolio from 6.50% to 7.54%. This tranche is about 1% wide, satisfies the S&P/Fitch criterion and just satisfies the Moody's criterion – at least for a constant recovery Gaussian copula model.

### **The Probability of Loss Criterion for ABS CDOs**

The probability distribution of losses for an ABS CDO can be determined using Monte Carlo simulation.<sup>7</sup> Values for  $M_{bp}$  and  $M_{bpj}$  are simulated to determine the default rate and the loss rate the mortgages in each pool. If the average loss rate is less than the attachment point, the loss on the ABS CDO tranche is zero. If it is greater than the detachment point the loss on the ABS CDO is 100%. When the average loss rate is between the attachment point and the detachment point, there is a partial loss on the ABS CDO tranche.

Results for the situation where the ABS CDO is created from 100 BBB tranches of CDSs, each tranche being responsible for losses in the range 6.50% to 7.54% of the underlying portfolio, are shown in Table 4.<sup>8</sup> A number of different values for the  $\alpha$  and  $\rho$  parameters are considered. The expected default rate on the underlying mortgages is assumed to be 10%. Analogously to before, the models considered are:

---

<sup>7</sup> We find that the following analytic approximate approach gives good results. Calculate the mean and standard deviation of the loss on one BBB tranche of an ABS conditional on  $M_{bp}$ . Use the central limit theorem to estimate the conditional probability distribution of the average loss across all tranches. Integrate over  $M_{bp}$  to calculate the unconditional distribution.

<sup>8</sup> Finding the AAA tranche attachment point is equivalent to determining the value at risk for a portfolio. In both cases we are seeking the level of loss that is exceeded only 0.1% of the time. Our estimates are based on 2.5 million simulations. The standard errors are fairly small, usually less than 0.5%.

- i. The two-factor Gaussian copula model with a recovery rate of 75% on the underlying mortgages
- ii. The two factor triple  $t$ -copula with a recovery rate of 75% on the underlying mortgages
- iii. The two-factor Gaussian copula model with the stochastic recovery rate model in equations (4) and (5) with  $R_{\max}=100\%$  and  $R_{\min}=50\%$
- iv. The two-factor triple  $t$  copula model with the stochastic recovery rate model in equations (4) and (5) with  $R_{\max}=100\%$  and  $R_{\min}=50\%$

The pattern of results in Table 4 is different from that in Table 2. It is clear that under a wide range of assumptions the attachment point for a AAA-rated tranche must be much higher than 30%. In many cases the attachment point is so high that a AAA-rating for even a very thin senior tranche is not warranted.

It should be noted that a CDO created from the triple BBB tranches of ABSs is quite different from a CDO created from BBB bonds. This is true even when the BBB tranches have been chosen so that their probabilities of default and expected losses are consistent with their BBB rating. The reason is that the probability distribution of the loss from a BBB tranche is quite different from the probability distribution of the loss from a BBB bond.

An insight into the characteristics of the loss distribution of BBB-rated tranches can be obtained by considering an extreme case. Suppose tranches are infinitesimally thin and  $\alpha=1$  so that the losses on tranches are perfectly correlated with each other. It is then the case that either a) the BBB tranches lose none of their principal or b) each BBB tranche loses its entire principal. An ABS CDO consisting of a portfolio of these tranches suffers either zero loss or 100% loss. It follows that every tranche of the ABS CDO are also in the situation where they either lose everything or nothing. There means that there should be no differences between the ratings of the tranches. (Indeed, they should all be rated BBB.)

As explained earlier the BBB tranches that were created were often very thin. Furthermore, inspecting publicly available data on ABSs we find that the underlying mortgages are often from all parts of the United States rather than being concentrated in one geographical area, suggesting that  $\alpha$  is quite high.

### **The Expected Loss Criterion for ABS CDOs**

It is not always the case that the expected loss criterion leads to lower minimum attachment points than the probability of loss criterion for ABS CDOs. But our numerical results indicate that this is true in most circumstances. Table 5 provides an example of our results. For the parameters considered, it shows that the minimum attachment point when expected loss is used as the criterion is always less than that when probability of loss is used as the criterion.

## **5. Conclusions**

Contrary to many of the opinions that have been expressed, the AAA ratings for the senior tranches of ABSs were not unreasonable. The weighted average life of mortgages is about five years. The probability of loss and expected loss of the AAA-rated tranches that were created were similar to or better than those of AAA-rated five-year bonds.

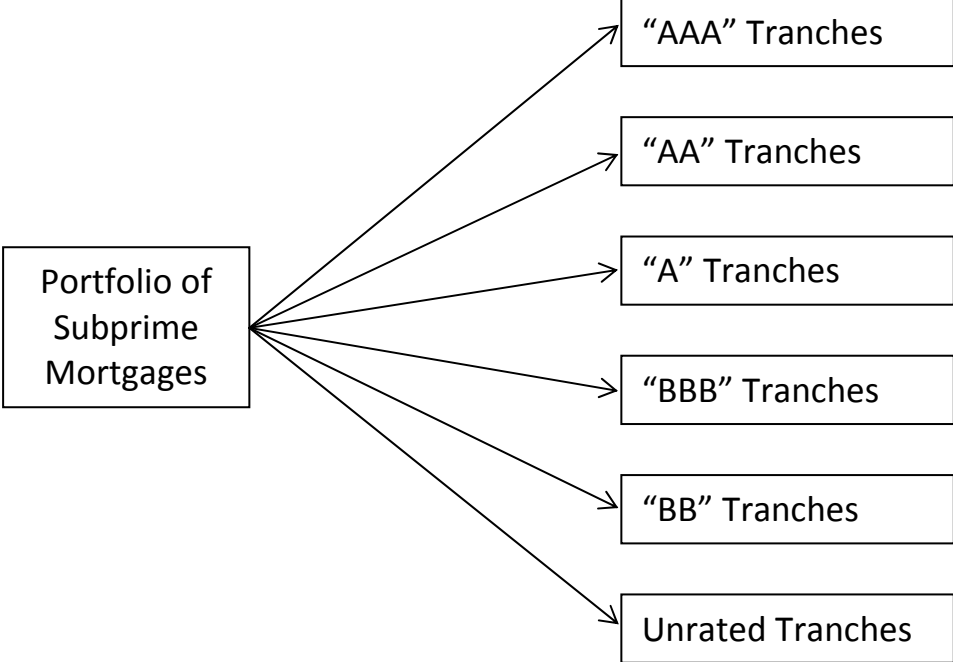
The AAA ratings for Mezz ABS CDOs are much less defensible. Scenarios where all the underlying BBB tranches lose virtually all their principal are sufficiently probable that it is not reasonable to assign a AAA rating to even a quite thin senior tranche. The risks in Mezz ABS CDOs depend critically on a) the width of the underlying BBB tranches, b) the correlation between pools, c) the tail default correlation, and d) the relationship between the recovery rate and the default rate. An important point is that the BBB tranche of an ABS cannot be assumed to be similar to a BBB bond for the purposes of determining the risks in ABS CDO tranches.

In practice Mezz ABS CDOs accounted for about 3% of all mortgage securitizations. Our conclusion is therefore that the vast majority of the AAA ratings assigned to tranches created from mortgages were reasonable, but in a small minority of the cases they cannot be justified.

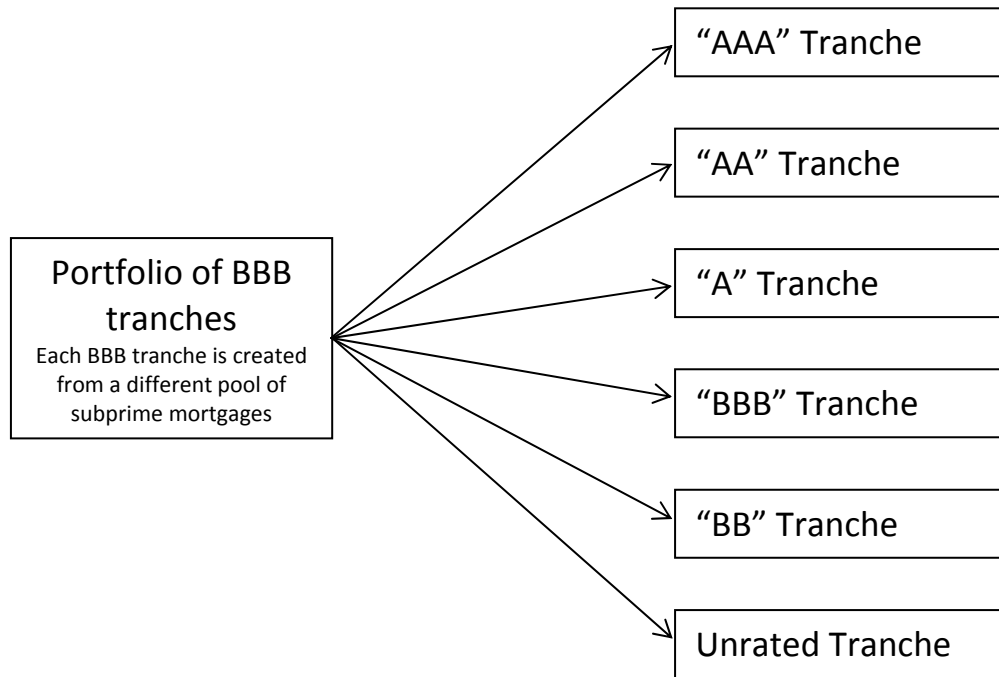
## References

- Altman, Edward I., Brooks Brady, Andrea Resti, and Andrea Sironi. 2005. The link between default and recovery rates: implications for credit risk models and procyclicality. *Journal of Business*, 78, 6: 2203-2228.
- Bank for International Settlements, 2006, "International Convergence of Capital Measurement and Capital Standards."
- Brennan, Michael J., Julia Hein, and Ser-Huang Poon, 2008, "Tranching and Rating," Working paper, Anderson School, UCLA.
- Coval, Joshua D., Jakub Jurek, and Erik Stafford, Working Paper, Harvard Business School, 2008
- Gorton, Gary. 2008. "The Panic of 2007," Working Paper, Yale School of Management.
- Hull, John. 2009. *Risk Management and Financial Institutions*. 2<sup>nd</sup> edition, Upper Saddle River, NJ: Pearson.
- Hull, John, and Alan White, "Valuation of a CDO and  $n$ th to Default CDS without Monte Carlo Simulation," *Journal of Derivatives*, 12, 2 (Winter 2004) pp 8-23.
- Moody's Investors Service. 2008. "Corporate Default and Recovery Rates, 1920-2007".
- Moody's Investors Service. 2007. "Challenging Times for the US Subprime Market," Special report, March 7.

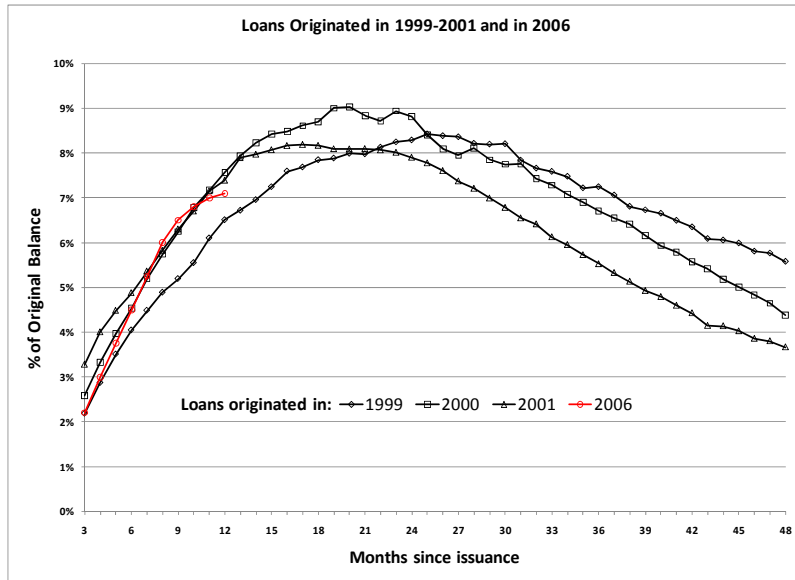
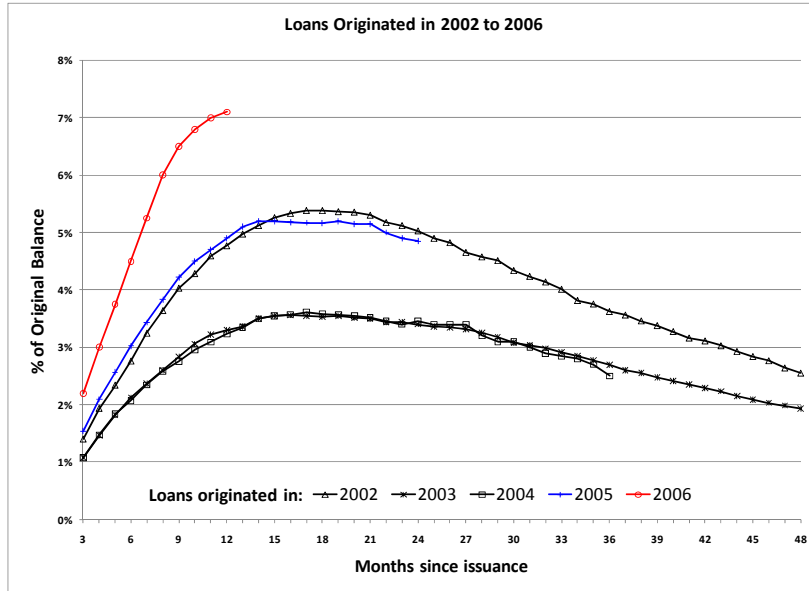
**Figure 1: Creation of Tranches from a Portfolio of Subprime Mortgages**



**Figure 2: A “Mezz” ABS CDO Created from the BBB tranches of an ABS**



**Figure 3: Data from March 2007 on Subprime Loans, 60 or More Days Delinquent, In Foreclosure or Held for Sale**



**Table 1**

Cumulative probability of default over 5 years for bonds initially rated AAA and BBB taken from Moody's statistics for the 1970 to 2007 period. Expected losses are calculated by assuming a recovery rate of 40%

	Probability of Loss	Expected Loss
AAA	0.1%	0.06%
BBB	1.8%	1.08%

**Table 2**

Minimum attachment point for the AAA rated tranche of an ABS to achieve a probability of loss less than 0.001. In the constant recovery rate model the recovery rate is 75%. In the stochastic recovery rate model the recovery rate depends on the default rate and ranges from a high of 100% to a low of 50%.

		Expected Default Rate		
		5%	10%	20%
Gaussian Copula	$\rho = 0.05$	4.1%	6.8%	11.0%
Constant Recovery	$\rho = 0.10$	6.0%	9.4%	13.9%
	$\rho = 0.20$	9.6%	13.6%	18.2%
	$\rho = 0.30$	13.1%	17.2%	21.1%
Double $t$ Copula	$\rho = 0.05$	7.6%	13.0%	18.2%
Constant Recovery	$\rho = 0.10$	13.6%	18.7%	21.9%
	$\rho = 0.20$	21.1%	23.2%	24.1%
	$\rho = 0.30$	23.7%	24.4%	24.7%
Gaussian Copula	$\rho = 0.05$	7.3%	11.6%	17.1%
Stochastic Recovery	$\rho = 0.10$	11.6%	17.3%	23.8%
	$\rho = 0.20$	19.1%	26.6%	33.4%
	$\rho = 0.30$	26.1%	34.1%	40.0%
Double $t$ Copula	$\rho = 0.05$	15.0%	25.3%	33.4%
Stochastic Recovery	$\rho = 0.10$	27.2%	37.2%	41.8%
	$\rho = 0.20$	42.2%	46.3%	46.6%
	$\rho = 0.30$	47.4%	48.7%	47.8%

**Table 3**

Comparison of minimum attachment point for a AAA-rated tranche of an ABS when a) the expected loss criterion is used so that a AAA tranche is chosen to achieve an expected loss less than 0.06% and b) the probability of loss criterion is used so that a AAA tranche is chosen to achieve a probability of loss less than 0.1%.

The model is the double  $t$  copula model with a stochastic recovery rate. The recovery rate depends on the default rate and ranges from a high of 100% to a low of 50%.

		Expected Default Rate		
		5%	10%	20%
Expected Loss Criterion	$\rho = 0.05$	3.9%	10.9%	19.7%
	$\rho = 0.10$	10.5%	21.2%	28.9%
	$\rho = 0.20$	24.7%	33.2%	37.3%
	$\rho = 0.30$	33.4%	39.0%	41.1%
Probability of Loss Criterion	$\rho = 0.05$	15.0%	25.3%	33.4%
	$\rho = 0.10$	27.2%	37.2%	41.8%
	$\rho = 0.20$	42.2%	46.3%	46.6%
	$\rho = 0.30$	47.4%	48.7%	47.8%

**Table 4**

Minimum attachment points for the AAA senior tranche of an ABS CDO. The ABS CDO is created from 100 BBB tranches of ABS tranches. The attachment point for each BBB tranche is 6.50% and the detachment point is 7.54%. The expected default rate for all underlying mortgages is 10%. The model determining the actual default rate is given in Section 2. The parameters  $\alpha$  and  $\rho$  are defined so that the between pool copula correlation is  $\alpha\rho$  and the within pool correlation is  $(1-\alpha)\rho$ . The parameter  $\rho$  is a measure of the overall correlation. As the parameter  $\alpha$  increases the pools become more similar to each other.

		$\alpha = 0.05$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.95$
Gaussian Copula	$\rho = 0.05$	1.3%	3.7%	8.6%	15.8%	21.8%
Constant Recovery	$\rho = 0.10$	6.8%	20.3%	43.8%	76.1%	99.8%
	$\rho = 0.20$	17.8%	44.3%	75.9%	97.6%	99.9%
	$\rho = 0.30$	24.9%	55.8%	85.8%	99.6%	99.9%
Triple $t$ Copula	$\rho = 0.05$	30.7%	91.3%	99.4%	100.0%	100.0%
Constant Recovery	$\rho = 0.10$	49.0%	96.1%	100.0%	100.0%	100.0%
	$\rho = 0.20$	65.6%	98.0%	100.0%	100.0%	100.0%
	$\rho = 0.30$	69.0%	98.4%	100.0%	100.0%	100.0%
Gaussian Copula	$\rho = 0.05$	12.8%	34.6%	65.2%	93.4%	99.9%
Stochastic Recovery	$\rho = 0.10$	25.6%	56.8%	86.4%	99.6%	99.9%
	$\rho = 0.20$	36.6%	70.4%	94.2%	99.9%	99.9%
	$\rho = 0.30$	41.0%	74.9%	96.0%	99.9%	99.9%
Triple $t$ Copula	$\rho = 0.05$	81.0%	99.0%	99.9%	100.0%	100.0%
Stochastic Recovery	$\rho = 0.10$	82.7%	99.6%	100.0%	100.0%	100.0%
	$\rho = 0.20$	84.4%	99.9%	100.0%	100.0%	100.0%
	$\rho = 0.30$	84.5%	100.0%	100.0%	100.0%	100.0%

**Table 5**

Comparison of minimum attachment point for a AAA-rated tranche of an ABS CDO when a) the expected loss criterion is used so that a AAA tranche is chosen to achieve an expected loss less than 0.06% and b) the probability of loss criterion is used so that a AAA tranche is chosen to achieve a probability of loss less than 0.1%.

The model is the Gaussian copula model with a stochastic recovery rate. The recovery rate depends on the default rate and ranges from a high of 100% to a low of 50%. The attachment point for each BBB tranche is 6.50% and the detachment point is 7.54%. The expected default rate for all underlying mortgages is 10%. The model determining the actual default rate is given in Section 2. The parameters  $\alpha$  and  $\rho$  are defined so that the between pool copula correlation is  $\alpha\rho$  and the within pool correlation is  $(1-\alpha)\rho$ . The parameter  $\rho$  is a measure of the overall correlation. As the parameter  $\alpha$  increases the pools become more similar to each other.

		$\alpha = 0.05$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.95$
Expected Loss Criterion	$\rho = 0.05$	10.0%	20.5%	44.5%	72.9%	92.3%
	$\rho = 0.10$	17.8%	40.8%	68.2%	88.9%	98.1%
	$\rho = 0.20$	27.8%	55.0%	80.1%	93.7%	98.6%
	$\rho = 0.30$	31.6%	59.7%	83.1%	94.8%	98.9%
Probability of Loss Criterion	$\rho = 0.05$	12.8%	34.6%	65.2%	93.4%	99.9%
	$\rho = 0.10$	25.6%	56.8%	86.4%	99.6%	99.9%
	$\rho = 0.20$	36.6%	70.4%	94.2%	99.9%	99.9%
	$\rho = 0.30$	41.0%	74.9%	96.0%	99.9%	99.9%