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Asset Pricing with Conditioning Information: A New Test

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ABSTRACT

This paper presents a new test of conditional versions of the Sharpe–Lintner CAPM, the Jagannathan and Wang (1996) extension of the CAPM, and the Fama and French (1993) three-factor model. The test is based on a general nonparametric methodology that avoids functional form misspecification of betas, risk premia, and the stochastic discount factor. Our results provide a novel view of empirical performance of these models. In particular, we find that a nonparametric version of the Fama and French model performs well, even when challenged by momentum portfolios.

ASSET PRICING MODELS have been the cornerstone in finance. In particular, the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) and its multifactor extensions are the most widely used tools in empirical studies. The CAPM has been carefully examined by numerous authors. Recently, a number of anomalies against the unconditional version of the CAPM have been identified. The evidence against this constant beta model is so forceful that some argue that the CAPM is dead (see Fama and French (1992, 1996b)). However, as Dybvig and Ross (1985) and Hansen and Richard (1987) show theoretically, the conditional version of the CAPM can hold perfectly even when the unconditional CAPM exhibits serious pricing errors. In general, dynamic models that allow for time-varying betas and risk premia can perform substantially better than static models. On the other hand, researchers have uncovered evidence of time variation in betas and expected returns (as well as return volatilities) over the past two dec-

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ades. These findings have motivated a growing literature on testing conditional asset pricing models.[1]

A challenging issue arises in studies of conditional beta-pricing models. These models imply that the conditional expected return on an asset is a linear function of one or more conditional betas that measure the asset's sensitivity to sources of undiversifiable risk. While this trade-off between time-varying risk and expected returns makes such models intuitively appealing, it is empirically challenging, since there is no theoretical guidance on how betas and risk premia vary with variables that represent conditioning information. To conduct tests, previous studies assume statistical models (typically linear models) relating betas to conditioning variables. Consequently, their empirical results can be shaped by the modeling assumptions. Recently, Ghysels (1998) discussed the problem in detail and stressed the impact of misspecification of beta risk dynamics on inference and estimation. He shows that several well-known time-varying beta models are seriously misspecified such that they are outperformed by constant beta models in many cases.[2]

In this paper, we provide a new test of conditional versions of the Sharpe–Lintner CAPM and two of the most influential extensions: the Jagannathan and Wang (1996) model and the Fama and French (1993) three factor model. Our empirical study is based on a testing methodology that completely avoids specification of time-varying betas. This can be critically important if the betas are not well approximated by linear models and if it is difficult to capture nonlinear beta functions. Our testing method is a flexible nonparametric approach to incorporate conditioning information. It is based on a nonparametric presentation of restrictions on the stochastic discount factor implied by a conditional linear factor pricing model. This methodology allows us to conduct tests in a way that is free from functional form misspecification about dynamics of conditional betas, risk premia, and the stochastic discount factor.

Our results provide a novel view about how these asset pricing models perform empirically. For the conditional CAPM, we find that our nonparametric version performs substantially better than the unconditional CAPM, in contrast to Ghysels' (1998) results on some parametric versions of the conditional CAPM.[3] The conditional model is still statistically rejected. Our tests reveal some interesting patterns in the pricing errors, or the Jensen's alphas, of the conditional CAPM. On one hand, they are positively correlated with the stock market. The pricing errors have a strong size pattern in volatility but not in time-series average. Such a dynamic size pattern implies that abnormal returns can be generated with sim-

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[1] A partial list includes Ferson, Kandel, and Stambaugh (1987), Bollerslev, Engle, and Wooldridge (1988), Harvey (1989), Shanken (1990), Cochrane (1996), He et al. (1996), Jagannathan and Wang (1996), Ferson and Siegel (1998), and Ferson and Harvey (1999).

[2] This issue is recognized by many authors. For example, see Harvey (1991) and He et al. (1996). More recently, Brandt (1999) has stressed importance of this issue in estimation of portfolio and consumption choice.

[3] This confirms that without misspecification of beta risk, time-varying beta models are sure to beat constant beta models. This point is explained and shown explicitly by Ghysels (1998).

ple strategies that increase holdings of small stocks and reduce holdings of large stocks when the stock market goes up, and vice versa. On the other hand, these pricing errors have a clear book-to-market pattern in time-series average but not in volatility. In other words, the size and book-to-market effects seem to appear in different dimensions.

The extension of the CAPM by Jagannathan and Wang (1996) has attracted considerable attention. Jagannathan and Wang argue that it is important to include labor income risk when pricing the cross section of stock returns. While the model is well motivated, our results indicate that the labor income risk factor is, surprisingly, not significant in capturing dynamics of the deviations from the conditional CAPM. The factor can hardly reduce the pricing error volatility, giving rise to strong rejections in our tests. However, we find that the labor income factor is significant in reducing the average pricing errors, which is consistent with the results of Jagannathan and Wang.

The Fama and French (1993) three factor model has now become a focal point in empirical asset pricing. The empirical success of the model has received multiple competing interpretations (e.g., see Fama and French (1996a) and MacKinlay (1995)). While the issue is far from closed, many begin to use the three factor model in applications. However, recent tests of the conditional Fama and French model by He et al. (1996), Ferson and Siegel (1998), and Ferson and Harvey (1999) cast doubt on empirical performance of this widely applied model. The tests strongly reject conditional versions of the model and generate the impression that the conditional Fama and French model fails miserably at dynamics of asset returns.

These test results seem surprising, as one would expect that by allowing for time-varying betas and risk premia, dynamic versions of the Fama and French model should perform even better than the unconditional version. One explanation is that the Ghysels' (1998) critique applies here. That is, the results against the three-factor model may arise from serious misspecification of the betas or the stochastic discount factor. Consistent with this argument, our tests of a more flexible nonparametric version of the model lead to a totally different conclusion. We find that when cast in the nonparametric form, the conditional Fama and French model performs well. It captures the most significant features of deviations from the conditional CAPM.

The continuation of short-term returns or the momentum effect documented by Jegadeesh and Titman (1993) has been one of the most serious challenges to the CAPM. The momentum effect has received special attention since Fama and French (1996a) find that their three factor model in the unconditional form does not explain the anomaly. Recently, several authors have debated about the source of momentum profits. Conrad and Kaul (1998) and Chordia and Shivakumar (2002) argue that momentum profits are due to cross-sectional dispersion in expected returns or persistence in expected returns. In contrast, Jegadeesh and Titman (1999) and Grundy and Martin (2001) conclude that the momentum profits are due to stock-specific returns.[4] A critical issue in this debate is how to

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[4] Meanwhile, Moskowitz and Grinblatt (1999) find that the momentum profits may be explained by industries.

measure expected returns that vary over time. Different ways to model systematic risk, conditional versus unconditional models in particular, can produce very different expected returns, giving rise to different views about whether profits to momentum strategies are consistent with time-varying expected returns. Surprisingly, none of the above studies has applied a conditional asset pricing model that allows risk and risk premia to vary over time in flexible ways.[5] In general, little is known about whether conditional models can help explain the momentum puzzle.

We challenge the conditional CAPM and the conditional Fama and French model with momentum portfolios. We find that while it is rejected by the tests, the conditional CAPM implies that winners tend to have higher conditional expected returns than losers, consistent with the explanation of Conrad and Kaul (1998) and Chordia and Shivakumar (2002).[6] For the conditional Fama and French model in the nonparametric form, we get more favorable test results. The model is not rejected, and it implies that the conditional expected returns of the winners are on average about one percent (per month) higher than those of the losers. This suggests that a major cause of the momentum profits is that winners tend to have higher conditional expected returns. The result disagrees with those of Jegadeesh and Titman (1999) and Grundy and Martin (2001) that rely on more restrictive models to measure expected returns.

For any nonparametric tests, one of the most critical questions is whether the test has power in finite sample applications. Nonparametric tests can avoid effects of misspecification, but typically the underlying nonparametric estimators converge at rates slower than parametric estimators. This problem that plagues nonparametric methods in general is known as the "curse of dimensionality."[7] In contrast, our approach has a very appealing property. Although this approach is based on a nonparametric pricing kernel, the estimator underlying the test converges at the standard parametric rate, no matter how many conditioning variables are used. We have conducted Monte Carlo experiments to study finite sample performance of the nonparametric testing approach and to back up our empirical applications.[8] We find that the nonparametric testing approach performs well. The simulation results are consistent with the fast convergence rate property.

The paper proceeds as follows. Section I presents the nonparametric presentation of the conditional CAPM, describes the testing approach for the conditional

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[5] Grundy and Martin (2001) allow betas to vary, but their model is more restrictive than general conditional models. They assume that each beta can take at most three different values determined only by factor performance.

[6] This is also consistent with Karolyi and Kho (1996). They find in a simulation experiment that the conditional CAPM is capable of producing momentum profits as large as observed in actual data.

[7] The problem is more serious for higher dimensional applications. For example, Silverman (1986) explains the curse of dimensionality in details. See Pritsker (1998) and Chapman and Pearson (2000) for discussions of the problems associated with some nonparametric methods in finance.

[8] These results are not reported but available upon request. For one factor models, Wang (2002) reports a simulation experiment for the nonparametric method.

CAPM, and outlines two extensions for conditional multifactor models. Section II provides empirical results. Section III concludes.

I. Methodology

In this section, we present a nonparametric approach to testing conditional asset pricing models. This inference methodology is constructed in the stochastic discount factor (SDF) framework. The simplicity of the SDF to present a general theory of asset pricing is now well recognized.[9] The SDF framework is universal, as every modern asset pricing model delivers a basic pricing equation

$$E_t(m_{t+1}R_{i,t+1}) = 1,$$

where E_t denotes the conditional expectation, m_{t+1} is the SDF, and $R_{i,t+1}$ is the return on the i-th asset.[10] Different models impose different restrictions on the SDF. When m_{t+1} is a fully specified parametric function of data and parameters, the model is usually estimated and tested by the generalized method of moments (GMM) of Hansen (1982).

Bansal, Hsiesh, and Viswanathan (1993) and Bansal and Viswanathan (1993) are the first to advocate the idea of a flexible SDF in empirical asset pricing. They focus on nonlinear APT models that assume that the SDF is a nonlinear function of a few state variables. To cope with the issue that the SDF functional form is unknown, they propose a series expansion GMM approach. That is, one first approximates the SDF by a series expansion (such as a polynomial expansion) and then applies the standard GMM for estimation and testing.[11] This approach is intuitive and very general. The drawback is that it is difficult to obtain the distribution theory and an effective assessment of finite sample performance.

Our approach focuses on the conditional CAPM and its multifactor extensions. Like the nonlinear APT models, the conditional CAPM does not yield a fully specified parametric SDF; however, it does imply an SDF of certain structure. Our approach is designed to test the model's restriction on the SDF, without imposing any auxiliary functional form assumptions. To ease exposition, we first show the SDF restriction implied by the conditional CAPM and describe how to capture it by a nonparametric SDF. Then, we outline the test method and extensions for multifactor models.

A. A Nonparametric Presentation of the Conditional CAPM

The conditional version of the Sharpe–Lintner CAPM implies that conditional expected excess returns are linearly related to conditional betas. In this section,

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[9] See Cochrane (2001) for a recent comprehensive discussion and references of studies about the SDF approach.

[10] For models about excess returns such as the Sharpe–Lintner CAPM, the basic equation is

$$E_t(m_{t+1}r_{i,t+1}) = 0$$

for $i = 1,\dots, n$, where $r_{i,t+1}$ is the excess return on the ith of n assets.

[11] Chapman (1997) extends this approach to evaluating consumption-based models.

we show that the return-beta relation can be translated into a restriction on the SDF m_{t+1} in the equation

$$E_t(m_{t+1} r_{i,t+1}) = 0$$

for $i = 1, \ldots, n$, where n is the number of assets, and $r_{i,t+1}$ is the excess return on the ith test asset. This SDF can be easily presented in a nonparametric way, providing the basis for constructing our inference approach.

The conditional CAPM states that the market portfolio is conditionally mean-variance efficient, and thus satisfies the following equation

$$E(r_{i,t+1}|I_t) = E(r_{p,t+1}|I_t)\frac{\operatorname{cov}(r_{i,t+1}, r_{p,t+1}|I_t)}{\operatorname{var}(r_{p,t+1}|I_t)}, \tag{1}$$

for $t = 1, \ldots, N$, where N is the sample size, I_t is the time t information set of investors, $r_{p,t+1}$ is the return on the market portfolio in excess of the riskless rate, and $r_{i,t+1}$ is the excess return on the ith test asset. The CAPM beta-pricing equation (1) is equivalent to

$$E(r_{i,t+1}|I_t) = E(r_{p,t+1}|I_t)\frac{E(r_{i,t+1} r_{p,t+1}|I_t)}{E(r^2_{p,t+1}|I_t)}, \tag{2}$$

which is the so called 'cross-moment' representation.[12] Our test aims at equation (2).[13]

Let x_t be a $k \times 1$ vector of conditioning variables such that[14]

$$E(r_{p,t+1}|I_t) = E(r_{p,t+1}|x_t) \tag{3}$$

$$E(r^2_{p,t+1}|I_t) = E(r^2_{p,t+1}|x_t). \tag{4}$$

The excess returns and the conditioning variables are assumed to be strictly stationary. Let $g_p(x_t) = E(r_{p,t+1}|x_t)$, $g_{pp}(x_t) = E(r^2_{p,t+1}|x_t)$, and $b(x_t) = g_p(x_t)/g_{pp}(x_t)$. Given that (3) and (4) hold, conditional pricing errors (or Jensen's alphas) from (2) can be expressed as

$$E(r_{i,t+1}|I_t) - E(r_{p,t+1}|I_t)\frac{E(r_{i,t+1} r_{p,t+1}|I_t)}{E(r^2_{p,t+1}|I_t)} = E(m_{t+1} r_{i,t+1}|I_t), \tag{5}$$

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[12] (1) implies that $E(r_{i,t+1}|I_t)\operatorname{var}(r_{p,t+1}|I_t) = E(r_{p,t+1}|I_t)\operatorname{cov}(r_{i,t+1}, r_{p,t+1}|I_t)$. Replace $\operatorname{var}(r_{p,t+1}|I_t)$ and $\operatorname{cov}(r_{i,t+1}, r_{p,t+1}|I_t)$ with $E(r^2_{p,t+1}|I_t) - (E(r_{p,t+1}|I_t))^2$ and $E(r_{i,t+1} r_{p,t+1}|I_t) - E(r_{i,t+1}|I_t)E(r_{p,t+1}|I_t)$, respectively, and cancel out the common term on both sides. This shows that (1) implies (2). Conversely, it is easy to see that (2) also implies (1).

[13] A nonparametric test that directly targets (1) is considered in a previous version. It is omitted since the test based on (2) is simpler and performs better in Monte Carlo simulation.

[14] Note that (3) and (4) are only for the market portfolio p. This is much weaker than requiring x_t to be a full characterization of the information set I_t.

where $m_{t+1} = 1 - b(x_t) r_{p,t+1}$. Thus (2) is equivalent to

$$E(m_{t+1} r_{i,t+1} | I_t) = 0. \tag{6}$$

The SDF m_{t+1} implied by the conditional CAPM is determined by the first two conditional moments of the market portfolio. Thus, we can estimate it nonparametrically, that is,

$$\hat{m}_{t+1} = 1 - \hat{b}(x_t) r_{p,t+1}, \tag{7}$$

where $\hat{b}(x) = \hat{g}_p(x)/\hat{g}_{pp}(x)$, and

$$\hat{f}(x) = N^{-1} h^{-k} \sum_{s=1}^{N} K\left(\frac{x - x_s}{h}\right), \tag{8}$$

$$\hat{g}_p(x) = N^{-1} h^{-k} \hat{f}(x)^{-1} \sum_{s=1}^{N} K\left(\frac{x - x_s}{h}\right) r_{p,s+1}, \tag{9}$$

$$\hat{g}_{pp}(x) = N^{-1} h^{-k} \hat{f}(x)^{-1} \sum_{s=1}^{N} K\left(\frac{x - x_s}{h}\right) r_{p,s+1}^2. \tag{10}$$

The nonparametric estimators in (8), (9), and (10) are standard. The estimator $\hat{f}(x)$ is the Rosenblatt–Parzen kernel density estimator, with kernel function $K(\cdot)$ and bandwidth parameter h. The estimators $\hat{g}_p(x)$ and $\hat{g}_{pp}(x)$ are the Nadaraya–Watson kernel regression function estimators.

The nonparametric presentation serves as the basis for our inference approach. Note that unlike the nonlinear APT models, the SDF derived above is not positive. However, this does not imply existence of arbitrage opportunities, since, in general, there exists at least a strictly positive SDF of the CAPM in a discrete time finite assets setting.[15] For the purpose of statistical inference about pricing errors, a positive SDF is often inconvenient in constructing tests and it generally does not offer any significant benefits in empirical analysis. As a result, the constraint of the SDF being positive is typically not imposed in tests of factor-based asset pricing models (e.g., see Cochrane (1996)).

B. Statistical Inference about Pricing Errors

Our testing approach is based on a simple idea. Let $e_{i,t+1} = m_{t+1} r_{i,t+1}$. Then, (6) is simply

$$E(e_{i,t+1} | I_t) = 0.$$

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[15] That is, the CAPM is consistent with nonexistence of arbitrage opportunity, but it is inconsistent with the complete markets paradigm. Dybvig and Ingersoll (1982) explain the issue in detail.

In other words, $e_{i,t+1}$ is not predictable by variables in I_t. Thus, a natural approach is to run a regression

$$e_{i,t+1} = z_t' \delta_i + u_{i,t+1}, \tag{11}$$

where z_t is a $q \times 1$ vector of observed variables in I_t,[16] and test if the regression coefficients are zero. Apparently, the moment condition (6) implies that $\delta = 0$, where $\delta = (\delta_1' \delta_2' \ldots \delta_n')'$.

For statistical inference, we replace m_{t+1} with the nonparametric discount factor $\hat{m}_{t+1}$ as defined in (7), and estimate the parameter vector δ_i by

$$\hat{\delta}_i = \left(\frac{1}{N} \sum_{t=1}^{N} \hat{w}_t z_t z_t' \right)^{-1} \left(\frac{1}{N} \sum_{t=1}^{N} \hat{w}_t z_t \hat{e}_{i,t+1} \right), \tag{12}$$

for $i = 1, \ldots, n$, where $\hat{e}_{i,t+1} = \hat{m}_{t+1} r_{i,t+1}$, and the weighting function is set to be $\hat{w}_t = \hat{f}(x_t) \hat{g}_{pp}(x_t)$. The weighting function is chosen because of a technical reason. This choice makes it feasible to establish the large sample theory, which is otherwise very complex.[17] For the same reason, Powell, Stock, and Stoker (1989), Robinson (1989), and Lee (1992) have employed density weighting in designs of various econometric methods.

The test that we propose is based on the weighted least squares estimator $\hat{\delta}_N$, where

$$\hat{\delta}_N = (\hat{\delta}_1' \hat{\delta}_2' \cdots \hat{\delta}_n')'.$$

Intuitively, $\hat{\delta}_N$ converges to zero if the market portfolio p is conditionally mean-variance efficient. Otherwise, the estimator converges to a nonzero limit in general. So we can conduct a test by checking how far $\hat{\delta}_N$ is away from zero, using asymptotic distribution theory to account for sampling errors. This regression approach also provides a simple way to look into pricing errors. By design, our test aims at detecting time variation in Jensen's alphas. A significant component of $\hat{\delta}_N$ indicates that the expected return errors are correlated with the corresponding forecast variable. Thus, the test may lead to a simple view of time variation in the pricing errors. In applications, we may use (11) as a model for pricing errors, that is, $z_t' \delta_i$ may serve to approximate $E(e_{i,t+1} I_t)$, the time-varying Jensen's alphas from (2).

Our test has an important property. Although we use nonparametric kernel density and regression estimators in construction, the estimator $\hat{\delta}_N$ behaves like a parametric estimator. It has a standard limiting multivariate normal distribu-

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[16] Note that lagged one period behind m_{t+1}, z_t can be any vector in I_t. In practice, a natural choice is to set $z_t = x_t$, as we do in our empirical analysis.

[17] Because of this choice, both $N^{-1} \sum_{t=1}^{N} \hat{w}_t z_t z_t'$ and $N^{-1} \sum_{t=1}^{N} \hat{w}_t z_t \hat{e}_{i,t+1}$ can be expressed as second-order generalized U-statistics, making it straightforward to analyze large sample properties of $\hat{\delta}_i$. In contrast, rather complex technical issues arise in developing distribution theory if we use weighting functions that do not give rise to simple U-statistic structures. (In particular, the choice of setting $\hat{w}_t = 1$ does not yield U-statistic structures. This is why it is not used here.) The weighting function chosen above is the simplest among those that yield U-statistic structures.

tion and, in particular, it converges at the fast parametric convergence rate.[18] In sum, under certain regularity conditions, and if $h \to 0$, $Nh^{2k} \to \infty$, and $Nh^{2k+2} \to 0$, then the weighted least squares estimator $\hat{\delta}_N$ is such that $\sqrt{N}(\hat{\delta}_N - \delta)$ has a limiting multivariate normal distribution with mean 0 and variance–covariance matrix Ω. Thus, with a consistent estimator $\hat{\Omega}_N$ of Ω, we propose to use the following test statistic

$$N\hat{\delta}_N'\hat{\Omega}_N^{-1}\hat{\delta}_N,$$

which has a limiting chi-squared distribution with qn degrees of freedom under the null hypothesis that the conditional CAPM holds. Details about both Ω and $\hat{\Omega}_N$ are given in Appendix A.

C. Extensions for Multifactor Models

In this section, we provide two extensions for testing multifactor models. First, we consider the case in which a parametric structure is proposed for excess returns on the benchmark portfolio p. That is, the benchmark excess returns are assumed to be a function of a $l \times 1$ parameter vector θ, denoted as $r_{p,t+1}(\theta)$. The hypothesis is that the benchmark is conditionally mean-variance efficient for some parameter value θ_0. Such parametric hypotheses are often of interest in empirical asset pricing. For example, a conditional version of the Fama and French (1993) three-factor model is such that a benchmark portfolio with time $(t+1)$ excess return

$$r_{p,t+1}(\theta) = \mathrm{MKT}_{t+1} + \theta_1 \mathrm{SMB}_{t+1} + \theta_2 \mathrm{HML}_{t+1} \tag{13}$$

is conditionally mean-variance efficient, where MKT_{t+1} is the excess return on the market portfolio, and SMB_{t+1} and HML_{t+1} are returns on the mimicking portfolios for the size and the book-to-market factors. Another example is the model of Jagannathan and Wang (1996), which may be viewed as a conditional two-factor model. This model predicts that a benchmark portfolio with the return equal to a linear combination of a stock market index and labor income growth rate is conditionally mean-variance efficient.

We propose in this case an approach that combines the nonparametric test with the GMM. For any given value of θ, with $r_{p,t+1}(\theta)$ replacing $r_{p,t+1}$, we can obtain an estimator $\hat{\delta}_N(\theta)$ from equation (12). In general, the parameter vector θ is unknown. We proceed with the standard GMM to estimate the parameters and perform the overidentification test, treating $\hat{\delta}_N(\theta)$ as a set of regular moments. Appendix B explains the method in detail. This procedure allows us to test the parametric hypothesis without imposing any auxiliary functional form assumptions.

Next, we consider a more general version of multifactor models, a case in which no parametric assumptions are made about the benchmark returns. To ease exposition, we focus on the conditional Fama and French three-factor model. The

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[18] See Wang (2002) for proofs and regularity conditions.

hypothesis is that a benchmark portfolio with time $(t+1)$ excess return

$$r_{p,t+1} = \mathrm{MKT}_{t+1} + \theta_{1,t}\mathrm{SMB}_{t+1} + \theta_{2,t}\mathrm{HML}_{t+1} \tag{14}$$

is conditionally mean-variance efficient, where the proportions of the size and book-to-market portfolios in the benchmark are not fixed constants, but instead they are allowed to vary over time. Moreover, $\theta_{1,t}$ and $\theta_{2,t}$ are fully nonparametric.[19]

In this nonparametric case, we propose a bootstrap-based inference approach. Again, we focus on the SDF. The conditional three-factor model implies an SDF of the form

$$m_{t+1} = 1 - b_{0,t}\mathrm{MKT}_{t+1} - b_{1,t}\mathrm{SMB}_{t+1} - b_{2,t}\mathrm{HML}_{t+1}, \tag{15}$$

given the maintained hypothesis that the benchmark with the return $r_{p,t+1}$ defined in (14) is conditionally mean-variance efficient.[20] We identify the time-varying coefficients $b_{0,t}$, $b_{1,t}$, and $b_{2,t}$ from the following set of three equations:

$$E_t(m_{t+1}\mathrm{MKT}_{t+1}) = 0$$

$$E_t(m_{t+1}\mathrm{SMB}_{t+1}) = 0$$

$$E_t(m_{t+1}\mathrm{HML}_{t+1}) = 0.$$

That is, we require the SDF m_{t+1} to correctly price the three factors.[21] In matrix form, these equations can be expressed as

$$A_t b_t = c_t,$$

where

$$A_t = \begin{pmatrix} E_t(\mathrm{MKT}_{t+1}^2) & E_t(\mathrm{MKT}_{t+1}\mathrm{SMB}_{t+1}) & E_t(\mathrm{MKT}_{t+1}\mathrm{HML}_{t+1}) \\ E_t(\mathrm{MKT}_{t+1}\mathrm{SMB}_{t+1}) & E_t(\mathrm{SMB}_{t+1}^2) & E_t(\mathrm{SMB}_{t+1}\mathrm{HML}_{t+1}) \\ E_t(\mathrm{MKT}_{t+1}\mathrm{HML}_{t+1}) & E_t(\mathrm{SMB}_{t+1}\mathrm{HML}_{t+1}) & E_t(\mathrm{HML}_{t+1}^2) \end{pmatrix}$$

$b_t = (b_{0,t}\ b_{1,t}\ b_{2,t})'$, and $c_t = (E_t(\mathrm{MKT}_{t+1})\ E_t(\mathrm{SMB}_{t+1})\ E_t(\mathrm{HML}_{t+1}))'$.

The SDF can be estimated nonparametrically

$$\hat{m}_{t+1} = 1 - \hat{b}_{0,t}\mathrm{MKT}_{t+1} - \hat{b}_{1,t}\mathrm{SMB}_{t+1} - \hat{b}_{2,t}\mathrm{HML}_{t+1}$$

using the following estimate of b_t

$$\hat{b}_t = \hat{A}_t^{-1}\hat{c}_t,$$

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[19] The three-factor model in such a general form has never been examined in the literature.

[20] A simple way to see this is to start with $E_t(m_{t+1}r_{i,t+1}) = 0$ and show that $E_t(r_{i,t+1})$ is linearly related to the conditional betas with the three factors. See Cochrane (2001) for discussions on general equivalence between SDF and beta representations.

[21] Such an SDF is perfectly consistent with beta-pricing equations. For example, in the case of the one-factor CAPM, requiring the SDF (i.e., $m_{t+1} = 1 - b(x_t)r_{p,t+1}$) to correctly price the factor (i.e., $E(m_{t+1}r_{p,t+1}|x_t) = 0$) yields the same $b(x_t)$ as obtained in Section I.B above. Also note that all of the Fama–French three factors are in the form of difference between two returns.

where $\hat{A}_t$ and $\hat{c}_t$ are obtained by replacing each element in A_t and c_t with the kernel regression function estimate (as in (9)).[22]

With the nonparametric SDF estimate $\hat{m}_{t+1}$, we repeat the procedure described in Section I.B above to obtain

$$T_\delta = N\hat{\delta}'_N\hat{\Omega}_N^{-1}\hat{\delta}_N.$$

Based on this statistic, we test the conditional Fama and French model by applying the stationary bootstrap of Politis and Romano (1994). Appendix C explains details of our application of the bootstrap. Resampling the data yields N_b bootstrapped values of $\hat{\delta}_N$, denoted as $\hat{\delta}^*_{N,j}$, where j indexes the N_b bootstrap samples. We then construct the following statistics:

$$T^*_{\delta,j} = N(\hat{\delta}^*_{N,j} - \hat{\delta}_N)'(\hat{\Omega}^*_{N,j})^{-1}(\hat{\delta}^*_{N,j} - \hat{\delta}_N)$$

for $j = 1,\ldots, N_b$. We compare T_δ to the quantiles of $T^*_{\delta,j}$ to obtain the p-value for statistical inference.[23] Though the test statistic is constructed as that for the one-factor conditional CAPM, the bootstrap approach takes into account extra noises in estimating nonparametrically the SDF of the conditional three-factor model, without making any auxiliary functional form assumptions.[24]

Finally, we end this section with a remark on simulation experiments that we have conducted. Our inference approach is based upon asymptotic distribution results. A critical question is whether it works in finite samples. For conditional single factor models, Wang (2002) presents a Monte Carlo experiment which shows that the nonparametric approach performs well. We have used a more complicated data-generating process and get very similar results. In our simulation study, we have included the nonparametric tests of the Fama and French (1993) three-factor model and the Jagannathan and Wang (1996) two-factor model. We have also considered a set of GMM tests to serve as the performance benchmark. We find that the nonparametric tests of the multifactor models also perform well.[25]

II. Empirical Results

In this section, we present test results for the conditional Sharpe–Lintner CAPM, the conditional Jagannathan and Wang (1996) model, and the conditional Fama and French (1993) three-factor model. We also present results on testing if

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[22] For example, to get the estimate for $E_t(\mathrm{MKT}_{t+1}\mathrm{SMB}_{t+1})$, simply replace $r_{p,s+1}$ with $\mathrm{MKT}_{s+1}\mathrm{SMB}_{s+1}$ in (9).

[23] That is, we use $T^*_{\delta,j}$, $j = 1,\ldots, N_b$, to approximate the distribution of $N(\hat{\delta}_N - \delta)'\hat{\Omega}_N^{-1}(\hat{\delta}_N - \delta)$, which is the statistic T_δ under the null hypothesis that $\delta = 0$. This is a popular way to conduct bootstrap-based tests. See Sullivan, Timmermann, and White (1999) for a recent application in finance.

[24] With the bootstrap, we avoid rather complex asymptotics in the case of multifactor models as we need to cope with generalized U-statistics of order $k+1$ for a conditional k factor model. We leave this possibility to construct a procedure based on the limiting distribution theory for future research.

[25] The results are omitted, but available upon request.

either the CAPM or the Fama and French model, in the flexible nonparametric form, can explain the momentum profits.

We start with two sets of test portfolios. The first set consists of five NYSE stock portfolios, which are the value-weighted NYSE size decile one, three, five, seven, and nine portfolios (SZ1, SZ3, SZ5, SZ7, and SZ9, for short). With this set of returns, we use the value-weighted portfolio of NYSE stocks as the market portfolio. In the second set are five of the Fama and French (FF; 1993) size and book-to-market portfolios that have the size-BE/ME quintile combinations SZ1/BM1, SZ1/BM5, SZ3/BM3, SZ5/BM1, and SZ5/BM5.[26] The variables SZ1 through SZ5 and BM1 through BM5 stand for the Fama–French quintiles on size and BE/ME. Table I provides details. SZ1/BM1 refers to, for example, the portfolio of stocks in the smallest size quintile (SZ1) and the lowest book-to-market equity quintile (BM1).

The conditioning variables in our tests are the dividend price ratio (DPR), the default premium (DEF), the one-month Treasury bill rate (RTB), and the excess return on the NYSE equally weighted portfolio (EWR). These variables are selected out of a larger set of 10 variables including the term premium (TERM), industry growth rate, inflation rate, short-end term structure slope, January dummy, and excess return on the NYSE value-weighted index. The 10 variables are lagged one month behind the stock returns. We run ordinary least squares regressions and find that joint use of the three popular forecasters DPR, DEF, and RTB drives out other variables in predicting the market, and EWR shows up strongly in the second moment.[27]

A. Evidence for Nonlinearity

To set the stage for our tests, we first present some evidence for significance of nonlinearity. First, we start with plots of conditional beta functions that are estimated nonparametrically. Next, we test for the presence of nonlinearity by applying the Sup LM method of Andrews (1993). Finally, we compare cross-sectional forecasts generated by the nonparametric SDF and an SDF based on linear conditional moments.

We estimate conditional betas of the size-BE/ME portfolios with respect to the Fama and French three factors. For brevity, we only report plots of the betas of SZ3/BM3 (i.e., a portfolio of medium size and medium book-to-market value) in Figures 1, 2, and 3. The beta estimates are obtained from the standard kernel regression estimates of the first two moments of returns, while confidence intervals are constructed using the stationary bootstrap method of Politis and

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[26] When using all 25 size-BE/ME portfolios, the test depends on the inverse of a 125×125 matrix, which causes numerical problems in our simulation experiments. To reduce the dimension of the matrix while keeping a good representation of the size-BE/ME portfolios, we decide to use these five portfolios.

[27] We have also performed a set of nonparametric tests. The nonparametric tests do not reject that the four variables DPR, DEF, RTB, and EWR are sufficient to characterize the first two moments of the market, but produce rejections against dropping any one of the four. We have used TERM as an additional conditioning variable to check robustness of our results.

Table I

Summary Statistics of Data

Panel A presents summary statistics of monthly observations from January 1947 to December 1995 for portfolio returns and conditioning variables defined below. The returns are arithmetic nominal rates of return in excess of the one-month Treasury bill rate, measured in percent (multiplied by 100). The variable VWR denotes the excess return on the value-weighted portfolio of NYSE common stocks. The variable SZN refers to the excess return on the value-weighted portfolio of the *N*th size decile NYSE stocks. The variable LBR refers to the growth rate (in percent) on per capita personal labor income, which is constructed as $1+LBR_t = (L_{t-1}+L_{t-2})/(L_{t-2}+L_{t-3})$, where L_{t-1} denotes the per capita labor income for month $t-1$. DPR is the dividend yield (in percent) on the NYSE value-weighted index, measured as the sum of previous 12 months' dividend payments divided by the level of the index. DEF is the Baa-rated corporate bond yield minus that of the Aaa-rated bond. RTB is the one-month T-bill yield. EWR is the excess return on the NYSE equally-weighted index. TERM is the Aaa-rated corporate bond yield minus the 1-month T-bill yield. All the bond or bill yields are annualized and measured in percent. Presented in Panel B are means and standard deviations of excess returns on the 25 size and book-to-market portfolios of Fama and French (1993). The sample period is from July 1963 to December 1995. The variables SZ1 through SZ5 stand for the five size quintiles (from small to large), while BM1 through BM5 stand for the five book-to-market equity quintiles (from low to high). Panel C is for the Fama-French mimicking portfolios for three stock market factors. MKT is excess return on a value-weighted portfolio of stocks listed on NYSE, AMEX, and NASDAQ that Fama and French use to proxy for the market portfolio. SMB is return on a zero-investment portfolio constructed to mimic the size factor. HML is return on a zero-investment portfolio constructed to mimic the book-to-market factor. For more details on the 25 size-BE/ME portfolios and the factor mimicking portfolios, see Fama and French (1993).

Panel A: NYSE Size Portfolios and Conditioning Variables

Variable	Mean	Std. dev.	First auto.	Cross correlations				
SZ1	1.01	6.44	0.12					
SZ3	0.82	5.39	0.14	0.94				
SZ5	0.77	4.94	0.13	0.89	0.97			
SZ7	0.75	4.66	0.12	0.84	0.94	0.97		
SZ9	0.71	4.33	0.07	0.77	0.88	0.92	0.97	
VWR	0.65	4.06	0.04	0.73	0.85	0.90	0.95	0.97
DPR	4.03	1.06	0.98					
DEF	0.92	0.43	0.97	0.19				
RTB	4.64	2.99	0.97	0.00	0.64			
EWR	0.77	4.84	0.14	– 0.04	0.12	– 0.11		
TERM	2.22	1.47	0.91	– 0.12	0.39	– 0.10	0.11	

Panel B: The Fama and French Size-BE/ME Portfolios

	Means					Standard Deviations				
	BM1	BM2	BM3	BM4	BM5	BM1	BM2	BM3	BM4	BM5
SZ1	0.29	0.72	0.80	0.95	1.08	7.48	6.58	5.98	5.69	5.99
SZ2	0.51	0.71	0.89	0.93	1.06	6.98	6.09	5.57	5.12	5.82
SZ3	0.46	0.65	0.74	0.86	1.06	6.37	5.38	4.99	4.68	5.37
SZ4	0.53	0.42	0.64	0.80	1.03	5.74	5.15	4.86	4.71	5.52
SZ5	0.42	0.43	0.41	0.59	0.74	4.73	4.52	4.20	4.12	4.81

Panel C: The Fama and French Three Factors

Variable	Mean	Std. dev.	First auto.	Correlations	
MKT	0.48	4.32	0.05		
SMB	0.25	2.84	0.17	0.31	
HML	0.42	2.53	0.20	– 0.37	– 0.10

Romano (1994).[28] To cope with the curse of dimensionality, we focus on "one-dimensional snapshots." While betas are estimated as multivariate functions, we look at univariate functions, that is, relations between one conditioning variable and betas, keeping all the other conditioning variables at their means.[29] By doing so, we focus on areas where we have more data (as opposed to areas where there are few data points) and also avoid the problem of how to plot four-dimensional functions.

The plots of Figures 1 through 3 provide suggestive evidence for nonlinear conditional betas. This should not be surprising as there is no theoretical reason to expect betas to be linear in the conditioning variables. Visually, none of the betas with respect to the Fama and French three factors is close to a linear function. In contrast, many of the panels, such as Panel B of Figure 1, Panels A and C of Figure 2, and Panels B and D of Figure 3, suggest that the betas are highly nonlinear. We have also inspected plots of betas for the other size-BE/ME portfolios and the NYSE size portfolios. In sum, these plots generally suggest the presence of nonlinearity in conditional betas, as well as that the beta function differs significantly across test assets.

While the beta function plots are suggestive, the estimates are too noisy in many cases, in view of the confidence intervals.[30] To draw a clear conclusion, we perform the Sup LM tests of Andrews (1993). We test a set of moment conditions associated with the explicit beta model of Ghysels (1998) using our data.[31] Like Ghysels, we also find extremely strong rejections. Panel A of Table II shows that rejections are obtained at one percent in most cases. The results indicate that the convenient linear specifications are totally inadequate.

Finally, we compare the nonparametric SDF and an SDF based on linear moments in terms of their cross-sectional forecasts. We divide the sample into two parts (of equal length). For each time point τ in the second half, we use the data up to time $\tau-1$ to estimate the SDF

$$m_t = 1 - \left(\frac{g_p(x_{t-1})}{g_{pp}(x_{t-1})}\right) r_{p,t},$$

where $r_{p,t}$ is the market excess return, $g_p(x_{t-1}) = E(r_{p,t}|x_{t-1})$, and $g_{pp}(x_{t-1}) = E(r^2_{p,t}|x_{t-1})$. We first use standard linear regressions to estimate g_p and g_{pp} to obtain a parametric SDF, and next use the nonparametric regressions

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[28] To estimate betas, the moments $E_t(r_{i,t+1})$, $E_t(r_{p,t+1})$, $E_t(r^2{}_{p,t+1})$, and $E_t(r_{i,t+1}r_{p,t+1})$ are estimated by standard kernel regressions, which are used to compute $\text{cov}_t(r_{i,t+1},r_{p,t+1})$, $\text{var}_t(r_{p,t+1})$, and hence $\beta_{i,t}$.

[29] In each panel, we consider only the interval of the conditioning variable that ranges from two standard deviations below its mean to two standard deviations above the mean. To present more consistently, we rescale the interval to range from 0 to 200 (so that 100 represents the mean of the variable in each case).

[30] This illustrates the curse of dimensionality and shows that it is difficult to get precise nonparametric estimates of the nonlinear beta functions.

[31] Table II provides the set of moment conditions. See Ghysels (1998) and Andrews (1993) for more details.

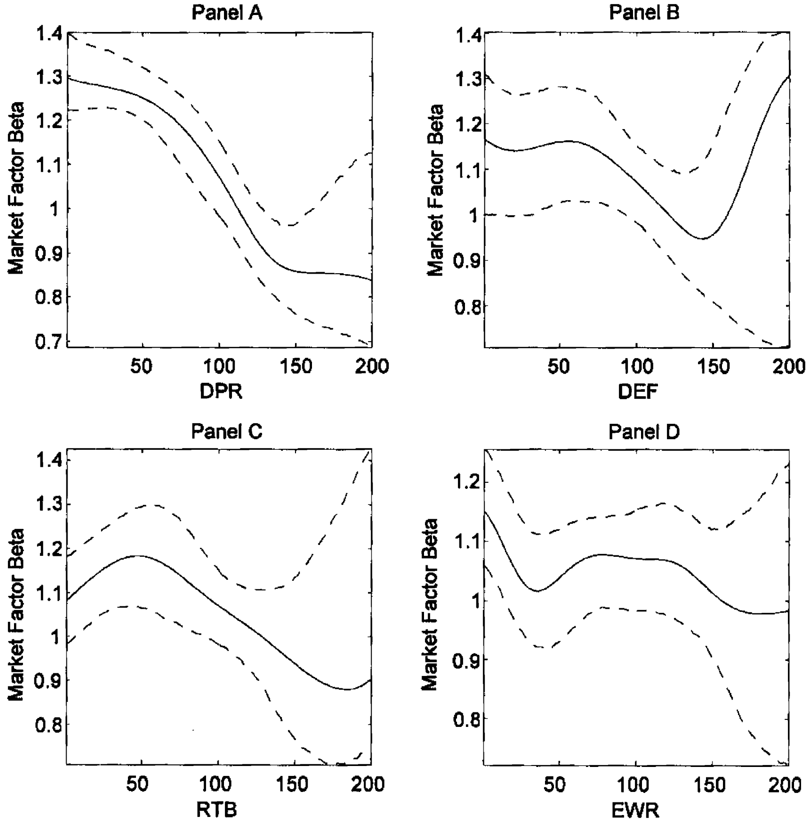

Figure 1. Plots of the market factor beta. The multivariate beta function is estimated nonparametrically. The solid lines are plots of one-dimensional snapshots for the relation between the beta and one conditioning variable, while the other variables are fixed at their means. The pairs of dashed lines represent 95 percent confidence intervals. The plots correspond to an interval for the variable that ranges from two standard deviations below the mean to two standard deviations above. The intervals are rescaled to range from 0 to 200 (so 100 represents the location of the mean in each case).

to estimate g_p and g_{pp} to get the nonparametric SDF. Then, we compare performances, or pricing errors, of the one-step ahead forecasts of the two SDFs.

Panel B of Table II reports the average forecast errors of the two SDFs over time for each portfolio in the cross section.[32] For the NYSE size portfolios, the nonparametric SDF does a significantly better job in every case. For the 25 FF portfolios, the nonparametric SDF also performs better. It outperforms the para-

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[32] Following Cochrane (1996, 2001), the average forecast error of a SDF $\hat{m}_\tau$ for the ith asset is

$$\frac{\frac{1}{N-N_0}\sum_{\tau=N_0+1}^{N}\hat{m}_\tau r_{i,\tau}}{\frac{1}{N-N_0}\sum_{\tau=N_0+1}^{N}\hat{m}_\tau},$$

where N_0 is the end point of the first half of the sample.

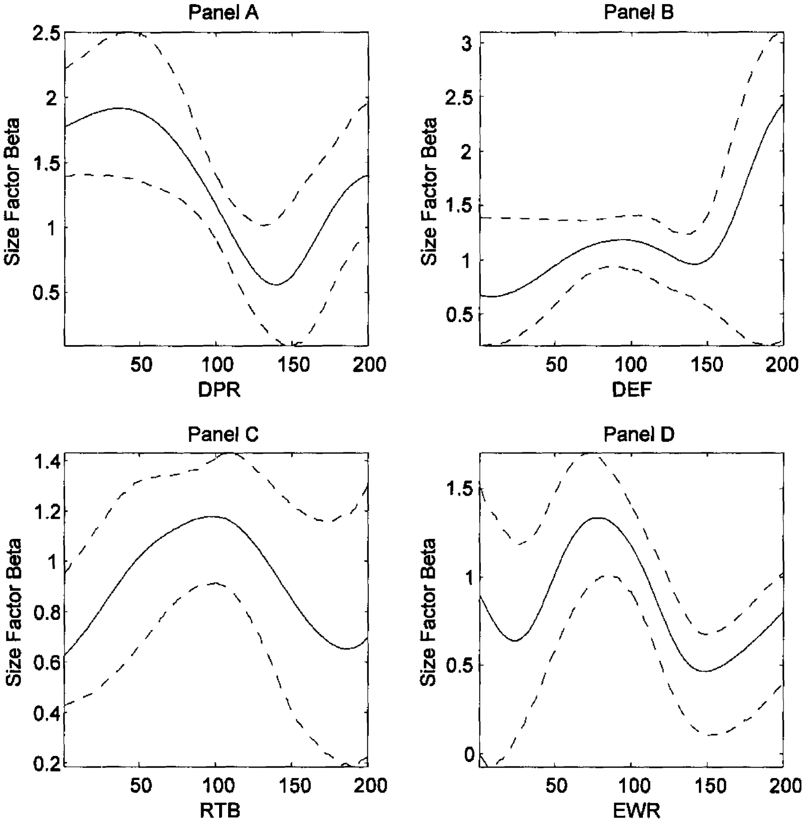

Figure 2. Plots of the size factor beta. The multivariate beta function is estimated nonparametrically. The solid lines are plots of one-dimensional snapshots for the relation between the beta and one conditioning variable, while the other variables are fixed at their means. The pairs of dashed lines represent 95 percent confidence intervals. The plots correspond to an interval for the variable that ranges from two standard deviations below the mean to two standard deviations above. The intervals are rescaled to range from 0 to 200 (so 100 represents the location of the mean in each case).

metric SDF in almost all the cases (except one), and the margin of the outperformance is substantial for most portfolios.[33] These results suggest that nonlinearity in the first two conditional moments of the market return is important, and the nonparametric SDF is a solution that may have significant value.

B. Testing the Conditional CAPM

We use both the NYSE size portfolios and the size-BE/ME portfolios to test the conditional CAPM. We report results based on the normal kernel and the set of

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[33] We have also compared the forecasts using industry portfolios. The results give the same conclusion.

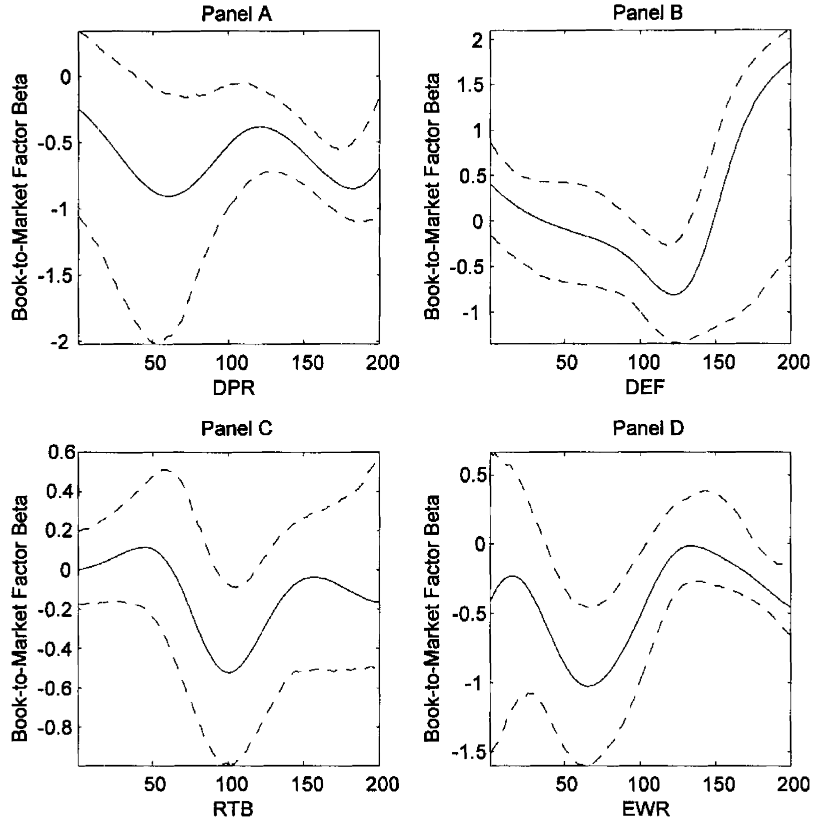

Figure 3. Plots of the book-to-market factor beta. The multivariate beta function is estimated nonparametrically. The solid lines are plots of one-dimensional snapshots for the relation between the beta and one conditioning variable, while the other variables are fixed at their means. The pairs of dashed lines represent 95 percent confidence intervals. The plots correspond to an interval for the variable that ranges from two standard deviations below the mean to two standard deviations above. The intervals are rescaled to range from 0 to 200 (so 100 represents the location of the mean in each case).

the four conditioning variables DPR, DEF, RTB, and EWR.[34] In sensitivity analyses, we have used the higher order kernel. We have considered small changes in the bandwidth. We have also checked results when adding the term premium (TERM) to the conditioning information set. The test results are qualitatively robust. For brevity, these robustness checking results are omitted.

Our tests strongly reject the conditional Sharpe–Lintner CAPM. In Table III, Panel A shows that the conditional mean-variance efficiency of the NYSE market proxy is rejected at p-value of 0.008, using the NYSE size portfolios as test assets.

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[34] We utilize the simulation results to back up the empirical analysis. Our setup for the analysis with historical data is identical to that for the simulation experiment. With the actual data, we go through exactly the same test procedures that we have carried out in the simulations.

Table II

Evidence for Nonlinearity

The vector of conditioning variables is $x_t = (\text{DPR}_t\ \text{DEF}_t\ \text{RTB}_t\ \text{EWR}_t)'$ and the vector of regressors is $z_t = (1\ x'_t)'$ in our tests. Panel A presents the Sup LM tests of Andrews (1993) against a set of moment conditions (which is the explicit beta model of Ghysels (1998))

$$E\begin{pmatrix} r_{i,t+1} - z_t'\delta_1 \\ \vdots \\ r_{n,t+1} - z_t'\delta_n \\ r_{M,t+1} - z_t'\delta_M \\ z_t'\delta_1 - z_t'\beta_1 z_t'\delta_M \\ \vdots \\ z_t'\delta_n - z_t'\beta_n z_t'\delta_M \end{pmatrix} \otimes z_t = 0.$$

These moment conditions are implied by assumptions that expected excess returns of the n test assets and the market portfolio, as well as the conditional betas are all linear in z. That is, $E_t(r_{i,t+1}) = z_t'\delta_i$ for $i = 1,\ldots, n$; $E_t(r_{M,t+1}) = z_t'\delta_M$; $\beta_{i,t} = z_t'\beta_i$ for $i = 1, \cdots, n$. Nine of the Fama and French size and book-to-market portfolios are used as test assets. Rejections at ten percent appear in the table with *, at five percent with **, and at one percent with ***. Panel B presents cross-sectional forecasts. First, the sample is split into two parts of equal lengths. For each time point τ in the second half, we use the data up to time $\tau - 1$ to estimate the SDF $m_t = 1 - (g_p(x_{t-1})/g_{pp}(x_{t-1}))r_{p,t}$, where $r_{p,t}$ is the market excess return, $g_p(x_{t-1}) = E(r_{p,t}|x_{t-1})$ and $g_{pp}(x_{t-1}) = E(r_{p,t}^2|x_{t-1})$. We first use standard linear regressions to estimate g_p and g_{pp} to obtain a parametric SDF, and next use the nonparametric regressions to estimate g_p and g_{pp} to get a nonparametric SDF. Then the average forecast errors, or pricing errors, of the two SDFs are reported for all the portfolios in the cross section. Two cross sections are considered: the 5 NYSE size portfolios and the 25 size and book-to-market portfolios.

Panel A: Sup LM Test Results

Size-BE/ME Portfolio	Constant	DPR	DEF	RTB	EWR
SZ1/BM1	24.129***	24.552***	22.753***	24.083***	13.046*
SZ1/BM3	20.299***	19.081***	18.293***	17.612**	10.744
SZ1/BM5	25.675***	23.034***	21.394***	22.347***	15.976**
SZ3/BM1	22.302***	25.052***	20.351***	24.351***	17.028**
SZ3/BM3	33.010***	31.305***	28.066***	34.403***	20.563***
SZ3/BM5	40.885***	38.555***	33.041***	42.386***	15.581**
SZ5/BM1	38.722***	34.585***	32.253***	38.078***	12.626*
SZ5/BM3	39.710***	27.570***	34.413***	35.796***	17.993***
SZ5/BM5	41.109***	37.699***	35.675***	36.828***	15.796**

Panel B: Cross-Sectional Forecasts

NYSE Size Portfolios

	The Parametric SDF					The Nonparametric SDF				
	SZ1	SZ3	SZ5	SZ7	SZ9	SZ1	SZ3	SZ5	SZ7	SZ9
	0.75	0.51	0.53	0.48	0.46	0.44	0.25	0.32	0.20	0.23

Size-BE/ME Portfolios

	The Parametric SDF					The Nonparametric SDF				
	BM1	BM2	BM3	BM4	BM5	BM1	BM2	BM3	BM4	BM5
SZ1	−0.01	0.66	0.69	0.78	0.84	−0.46	0.35	0.38	0.62	0.63
SZ2	0.38	0.75	0.93	1.04	0.91	−0.07	0.37	0.65	0.73	0.61
SZ3	0.52	0.66	0.76	0.77	1.12	0.13	0.32	0.44	0.51	0.87
SZ4	0.64	0.62	0.64	0.61	0.97	0.23	0.24	0.30	0.36	0.74
SZ5	0.60	0.67	0.57	0.68	0.83	0.30	0.36	0.30	0.44	0.69

Table III

Testing the Conditional CAPM

The vector of conditioning variables is $x_t = (\text{DPR}_t\ \text{DEF}_t\ \text{RTB}_t\ \text{EWR}_t)'$ and the vector of regressors is $z_t = (1\ x'_t)'$ in our tests. Two sets of test portfolios are used. One consists of the five NYSE size portfolios SZ1, SZ3, SZ5, SZ7, and SZ9. The sample period is from February 1947 to December 1995. The other consists of the five size-BE/ME portfolios SZ1/BM1, SZ1/BM5, SZ3/BM3, SZ5/BM1, and SZ5/BM5. The sample is from July 1963 to December 1995. Panel A presents results from tests for the significance of each regressor as well as the joint tests. The tests of significance (across equations) of each regressor are constructed as follows. To test $H\delta = 0$, where H is a $n \times qn$ matrix, the test statistic is $N(H\hat{\delta}_N)'(H\hat{\Omega}_N H')^{-1}(H\hat{\delta}_N)$, which has a limiting $\chi^2(n)$ distribution. The p-value is the probability that a draw from the chi-squared distribution exceeds the test statistic. In Panel B are the WLS regression coefficient estimates, with the estimated standard errors in parentheses. In Panel A and Panel B, the variable regressors are the conditioning variables minus their means. Panel C presents means and standard deviations of estimated pricing errors of the conditional CAPM for the 25 size and book-to-market portfolios. The pricing errors are estimated by the WLS regression method described in Appendix E.

Panel A: Test Statistics and p-Values

	Joint Test	Significance of Individual Regressors				
	(All Regressors)	Intercept	DPR	DEF	RTB	EWR
NYSE size portfolios						
χ^2-stat	45.0	2.4	2.6	11.1	8.6	29.6
p-value	0.008	0.793	0.758	0.049	0.127	0.000
Size-BE/ME portfolios						
χ^2-stat	62.1	14.6	7.6	8.3	8.5	22.7
p-value	0.000	0.012	0.178	0.142	0.130	0.000

Panel B: The WLS Regressions

	Intercept	DPR	DEF	RTB	EWR
NYSE size portfolios					
SZ1	0.08 (0.19)	– 0.18 (0.16)	– 0.05 (0.75)	– 0.04 (0.14)	0.21 (0.05)
SZ3	– 0.02 (0.13)	– 0.08 (0.11)	0.42 (0.54)	– 0.10 (0.10)	0.11 (0.03)
SZ5	0.00 (0.11)	– 0.03 (0.09)	0.60 (0.47)	– 0.09 (0.09)	0.09 (0.03)
SZ7	0.00 (0.08)	– 0.01 (0.08)	1.01 (0.42)	– 0.16 (0.07)	0.04 (0.03)
SZ9	0.03 (0.06)	0.03 (0.07)	0.77 (0.39)	– 0.13 (0.06)	– 0.01 (0.02)
Size-BE/ME Portfolios					
SZ1/BM1	– 0.26 (0.27)	1.46 (0.77)	– 1.68 (0.92)	– 0.24 (0.28)	0.23 (0.07)
SZ1/BM5	0.34 (0.22)	1.11 (0.52)	– 1.22 (0.74)	– 0.28 (0.21)	0.23 (0.06)
SZ3/BM3	0.17 (0.15)	1.02 (0.39)	– 0.04 (0.59)	– 0.33 (0.14)	0.09 (0.04)
SZ5/BM1	– 0.01 (0.11)	0.30 (0.30)	– 0.18 (0.40)	– 0.15 (0.11)	– 0.05 (0.03)
SZ5/BM5	0.15 (0.14)	0.47 (0.40)	0.33 (0.65)	– 0.29 (0.13)	– 0.03 (0.05)

Panel C: Pricing Errors of the Conditional CAPM

	Average					Standard Deviation				
	BM1	BM2	BM3	BM4	BM5	BM1	BM2	BM3	BM4	BM5
SZ1	– 0.26	0.11	0.17	0.39	0.34	1.31	1.19	1.23	1.21	1.32
SZ2	– 0.18	0.05	0.34	0.32	0.31	0.95	1.04	0.88	0.84	0.84
SZ3	– 0.20	0.01	0.17	0.35	0.41	0.83	0.89	0.80	0.77	0.64
SZ4	– 0.09	– 0.08	0.06	0.23	0.44	0.54	0.75	0.58	0.59	0.56
SZ5	– 0.01	0.04	0.02	0.12	0.15	0.38	0.43	0.54	0.43	0.52

The significance tests of individual regressors show that this rejection comes largely from the conditioning variable EWR, which produces a huge test statistic (around 30 for $\chi^2(5)$). For the size and book-to-market portfolios, our test rejects with a p-value of 0.0 percent that the market portfolio is conditionally mean-variance efficient. Still, the lagged market index EWR generates the largest test statistic in the regressor significance tests. The intercepts for the size-BE/ME portfolios are also highly significant with a p-value around one percent.[35]

The WLS regression results in Panel B provide more details about the Jensen's alphas. The conditional CAPM does not seem to have any difficulty pricing average returns on the NYSE size portfolios. The intercepts or average pricing errors are small, only between -0.02 percent and 0.08 percent, and are statistically insignificant. However, for small- to medium-size portfolios (SZ1, SZ3, SZ5), the pricing errors are positively correlated with the stock market index EWR, and the slopes on EWR are salient, about three or four standard errors above zero. These slopes are also large in economic terms, in the sense that they indicate very volatile pricing errors. Note that the pricing error components 0.21EWR, 0.11EWR, and 0.09EWR in the three regressions have standard deviation of 1.02 percent, 0.53 percent, and 0.44 percent, respectively. The slopes on EWR are clearly related to size. The slope falls strictly with size from 0.21 to -0.01. To a great extent, this determines a negative size pattern in the pricing error volatilities. The regressions with the size and BE/ME portfolios confirm the pattern of loadings on EWR. We also find the same pattern when replacing EWR with the value-weighted index VWR.

Are there significant size and book-to-market effects in the pricing errors of the conditional CAPM? In Panel C, we present the time-series means and standard deviations of the pricing errors using the 25 size-BE/ME portfolios. The panel shows that there are strong size and book-to-market effects, and interestingly, the effects appear to be in different dimensions. The average pricing errors are obviously related to book-to-market equity. In every size quintile, the means tend to increase with BE/ME. On average, they increase strictly from -0.16 percent in the lowest BE/ME quintile to 0.33 percent in the highest. There is, however, no clear size pattern in the average pricing errors, which is consistent with the regression results for the NYSE size portfolios. On the other hand, the standard deviations of the pricing errors show a strong negative relation to size, but no clear BE/ME pattern.[36]

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[35] We let the four regressors be the conditioning variables minus their means, so that the intercepts are just average pricing errors. This transformation does not affect estimation and inference for slopes on the nonconstant regressors. Nor does it alter the joint statistic and pricing error estimates. For inference about the intercepts (which is affected), we ignore estimation noise in the sample means of the regressors. This is equivalent to assuming that the regressors are observed in deviation form.

[36] The absence of size effect in average pricing errors seems consistent with results of Knez and Ready (1997), who questioned the robustness of the size factor in explaining average returns. Our pricing error estimates are based upon the WLS method. We also used the kernel regression method. The conclusions are identical. Appendix E provides details of the two estimation methods.

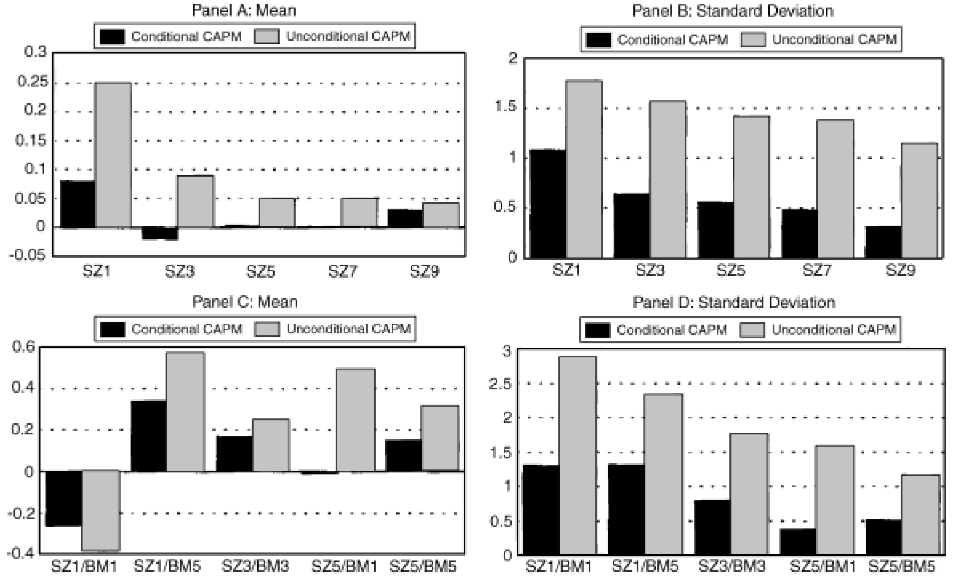

Figure 4. Pricing errors of the conditional CAPM and the unconditional CAPM. The pricing errors (Jensen's alphas) of the conditional CAPM are computed by the WLS regression method described in Appendix E. The pricing errors of the unconditional CAPM, which has the pricing kernel $m_t = 1 - b_0\mathrm{MKT_t}$, are obtained using the same regression approach. The summary statistics in Panels A and B are for the NYSE size portfolios. Those in Panels C and D are for the size and book-to-market portfolios.

Ghysels (1998) points out that if we correctly specify the dynamics of the beta risk, time-varying beta models are sure to outperform constant beta models. Yet he argues that if beta risk is misspecified, we may commit serious pricing errors that could be even bigger than a constant beta model, and he shows that this has indeed happened in many cases. Does the nonparametric version of the conditional CAPM outperform the unconditional CAPM? We find that the nonparametric conditional CAPM performs much better than the constant beta version, for each and every one of our test portfolios! In Figure 4, we plot the means and standard deviations of the pricing errors. Clearly, the conditional CAPM is a substantial improvement over the static CAPM, in terms of both the average and the volatility of the Jensen's alphas. To some extent, Figure 4 confirms that our nonparametric approach works well.[37] It also illustrates that there are significant payoffs in studying conditional asset pricing models.

C. Labor Income Risk

Jagannathan and Wang (1996) propose an appealing model that has become one of the most influential models in empirical asset pricing. Jagannathan and

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[37] See also Panel B of Table II.

Wang argue that it is important to include labor income risk. They find that a labor income risk factor plays a significant role in explaining cross-sectional variation in average stock returns.[38] Their labor income risk factor is motivated as a proxy for return on human capital. Alternatively, their model may be viewed as a conditional two-factor model.

We test the two-factor model assuming that the benchmark portfolio has return of the form $(1-\theta)\mathrm{MKT}_t + \theta\mathrm{LBR}_t$, where LBR_t is the labor income risk factor, measured as the per capita labor income growth rate. The variable MKT_t is a stock market index. Our tests strongly reject that the benchmark is conditionally mean-variance efficient. Panel A of Table IV shows that the model is rejected with p-values below one percent for both the NYSE size portfolios and the size-BE/ME portfolios. Estimates of the parameter θ are small and statistically insignificant, showing no support for significance of the labor income risk factor.

The labor income risk factor does not help much in capturing dynamics of asset returns. Panel B presents minimums of three summary measures of the pricing errors that can be achieved with the free parameter θ. The measures are average absolute bias (AAB), average standard deviation (ASD), and average root mean squared error (ARMSE). Detailed definitions are given in Appendix E. As the panel shows, the ARMSE cannot achieve any significant reduction with the free parameter θ. The minimums of ARMSE in the two cases are 0.59 percent and 0.82 percent, respectively, compared to 0.61 percent and 0.84 percent for the conditional CAPM without the labor income risk factor.

Why are our results different from those of Jagannathan and Wang (1996)? Panel A of Figure 5 points to an explanation. Plotted in this panel are the two summary measures AAB and ASD of the pricing errors for the 25 size and BE/ME portfolios. It shows that the labor income factor can hardly reduce the pricing error volatility to any significant extent. This is why it receives strong rejections in our tests. Consistent with the finding of Jagannathan and Wang, however, the labor income factor can significantly reduce average pricing errors. The AAB has the minimum value of 0.13 percent, obtained at $\theta = 0.94$, which is more than a 30 percent reduction of the bias 0.19 percent at $\theta = 0$.[39] This suggests that Jagannathan and Wang obtain favorable results because their tests are focused on average pricing errors. In fact, their target is to consider only pricing average returns and they do not challenge the model with dynamics of asset returns.[40]

D. Size and Book-to-Market

We perform tests of the conditional Fama and French three-factor model using the size and book-to-market portfolios. In the most general form, the model pre-

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[38] Using Japanese stock return data but an unconditional four-factor model, Jagannathan, Kubota, and Takehara (1998) also find that a labor income risk factor is significant in explaining average stock returns.

[39] Similarly, Jagannathan and Wang get estimates of the fraction parameter θ that are close to 1.0.

[40] In the GMM tests, they do not use any conditioning instrument to scale returns on test portfolios.

Table IV

Labor Income Risk Factor

This table presents results on testing the Jagannathan and Wang (1996) model. The model predicts the conditional mean-variance efficiency of a benchmark portfolio that has return of the form $(1-\theta)\mathrm{MKT}_t+\theta\,\mathrm{LBR}_t$, where MKT_t is a stock market index and LBR_t is the per capita labor income growth rate. We let MKT_t be the NYSE value-weighted index when using the five NYSE size portfolios as test assets. The sample is from February 1947 to December 1995. In the second set of test assets are the five size-BE/ME portfolios (SZ1/BM1, SZ1/BM5, SZ3/BM3, SZ5/BM1, SZ5/BM5), and we let MKT_t be the market factor of Fama and French. The sample is from July 1963 to December 1995. The vector of conditioning variables is $x_t = (\mathrm{DPR}_t\ \mathrm{DEF}_t\ \mathrm{RTB}_t\ \mathrm{EWR}_t)'$ and the vector of regressors is $z_t = (1x_t')'$ in our tests. Panel A reports test results. The weighting matrix is the inverse of $\hat{\Omega}_N(\theta)$ that is being updated through iteration. The matrix is evaluated at zero ($\theta = 0$) for the first stage estimation and evaluated at the first-stage estimate of θ for the second stage estimation. The results at the end of the second round are reported. The p-value is the probability that a draw from the chi-squared distribution exceeds the test statistic. Panel B reports three summary measures of pricing errors detailed in Appendix E. The measures are average absolute bias (AAB), average standard deviation (ASD), and average root mean squared error (ARMSE). This panel also includes the minimum of the test statistic (min T). The pricing errors are estimated by the WLS regression method described in Appendix E.

Panel A: Test Results

Test Assets	χ^2-statistic	p-value	estimate of θ	std. err.
NYSE Size Portfolios	45.0	0.006	0.04	0.27
Size-BE/ME Portfolios	61.5	0.000	0.35	0.24

Panel B: Pricing Errors

	5 NYSE Size Portfolios					25 Size-BE/ME Portfolios			
	AAB	ASD	ARMSE	min T		AAB	ASD	ARMSE	min T
$\theta = 0.0$	0.03	0.61	0.61	45.0		0.19	0.80	0.84	204.1
Min	0.03	0.56	0.59	44.9	Min	0.13	0.80	0.82	203.1
$\theta*$	0.01	0.77	0.52	0.08	$\theta*$	0.94	0.42	0.50	0.24

dicts the conditional mean-variance efficiency of a benchmark portfolio with time $(t+1)$ excess return

$$\mathrm{MKT}_{t+1} + \theta_{1,t}\mathrm{SMB}_{t+1} + \theta_{2,t}\mathrm{HML}_{t+1},$$

where MKT_{t+1} is excess return on the market portfolio, and SMB_{t+1} and HML_{t+1} are returns on the mimicking portfolios for the size factor and the book-to-market factor, respectively.[41] We consider two versions of the model:

FF1: $\theta_{1,t} = \theta_1$ and $\theta_{2,t} = \theta_2$,
FF2: $\theta_{1,t}$ and $\theta_{2,t}$ are nonparametric.

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[41] Note that $\mathrm{MKT}_{t+1}+\theta_{1,t}\mathrm{SMB}_{t+1}+\theta_{2,t}\,\mathrm{HML}_{t+1}$ indeed represents excess return on a risky portfolio, no matter whether there are any restrictions on $\theta_{1,t}$ and $\theta_{2,t}$ or not.

Table V

Size and Book-to-Market

This table presents results on testing the conditional Fama and French (1993) three-factor model. The hypothesis is that a benchmark portfolio with time (t+1) excess return

$$MKT_{t+1} + \theta_{1,t} SMB_{t+1} + \theta_{2,t} HML_{t+1}$$

is conditionally mean-variance efficient. MKT_{t+1} represents excess return on the market portfolio. The variables SMB_{t+1} and HML_{t+1} are returns on the mimicking portfolios for the size factor and the book-to-market factor, respectively. Two versions are being tested: FF1: $\theta_{1,t} = \theta_1$ and $\theta_{2,t} = \theta_2$, and FF2: both $\theta_{1,t}$ and $\theta_{2,t}$ are nonparametric. The tests use the five size-BE/ME portfolios (SZ1/BM1, SZ1/BM5, SZ3/BM3, SZ5/BM1, SZ5/BM5). The sample is from July 1963 to December 1995. The vector of conditioning variables is $x_t = (\mathrm{DPR}_t\ \mathrm{DEF}_t\ \mathrm{RTB}_t\ \mathrm{EWR}_t)'$ and the vector of regressors is $z_t = (1\ x_t')'$ in our tests. The p-value is the probability that a draw from the chi-squared distribution exceeds the test statistic. The table also reports three summary measures of pricing errors estimated by the WLS regression method described in Appendix E. The measures are average absolute bias (AAB), average standard deviation (ASD), and average root mean squared error (ARMSE). The pricing errors are evaluated using all 25 size and book-to-market portfolios.

	χ^2-statistic	p-value	AAB	ASD	ARMSE
Conditional CAPM					
$\theta_{1,t} = \theta_{2,t} = 0$	62.1	0.000	0.19	0.80	0.84
Conditional Fama-French Model					
FF1: $\theta_{1,t} = \theta_1$, $\theta_{2,t} = \theta_2$	48.8	0.001	0.17	0.74	0.77
FF2: $\theta_{1,t}$, $\theta_{2,t}$ nonparametric	38.4	0.210	0.13	0.28	0.32

In FF1, the proportions $\theta_{1,t}$ and $\theta_{2,t}$ are assumed to be constant over time. This version is convenient to provide a simple illustration about the size and book-to-market effects, which may play different roles. In the case of FF2, the benchmark is in its most general form: No parametric restrictions are made on $\theta_{1,t}$ and $\theta_{2,t}$. A conditional form of the three-factor model as general as FF2 has not been examined before in the literature.[42]

Table V presents the test results using the size and BE/ME portfolios as test assets. The version FF1 is strongly rejected with a p-value as low as 0.1 percent. The estimates of θ_1 and θ_2 are above two standard errors from zero, showing some support for the two factors.[43] The version FF2 is not rejected by the bootstrap test, with the p-value around 21 percent. In terms of the pricing errors, FF1 has some limited improvement over the conditional CAPM. All the three pricing error measures of FF1 are smaller than those of the CAPM. The nonparametric version FF2 does a much better job, showing that the conditional FF model captures the prominent features of the conditional CAPM pricing errors. Used together,

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[42] Even though it is much simpler than FF2, FF1 avoids auxiliary restrictions on betas and conditional moments of the benchmark returns.

[43] FF1 and FF2 are tested using the two methods explained in Section I.C. For FF1, the estimates (standard errors) of θ_1 and θ_2 are 2.67 (0.72) and 1.03 (0.38), respectively. They are not reported in the table.

the size and book-to-market factors can significantly reduce either the bias measure (AAB) or the volatility measure (ASD) of the pricing errors. In terms of the joint measure, the ARMSE of FF2 drops to 0.32 percent, which is less than a half of the ARMSE value 0.84 percent of the CAPM. Plots of AAB and ASD of FF1 show that when used alone, either the size factor or the book-to-mark factor is effective in some way. The size factor can significantly reduce the pricing error volatility, as suggested by Panel B of Figure 5 (in which θ_1 is free but $\theta_2 = 0$). On the other hand, the book-to-market factor is effective on bias reduction, as shown in Panel C of Figure 5 (in which θ_2 is free but $\theta_1 = 0$).[44]

Our finding that the nonparametric version FF2 performs well differs from those of recent studies by He et al. (1996), Ferson and Siegel (1998), and Ferson and Harvey (1999). As in our case of FF1, these tests strongly reject some dynamic specifications of the Fama and French three-factor model. The difference suggests that modeling assumptions of betas, risk premia, and/or the SDF play a critical role in evaluation of conditional asset pricing models. In particular, the evidence provided by Ghysels (1998) and the evidence for nonlinearity that we find suggest that the test results may be seriously affected by the convenient linearity assumptions.[45] Consistent with this explanation, we find favorable evidence for the general nonparametric version of the conditional Fama and French three-factor model.

E. Momentum

Now we challenge the conditional CAPM and the conditional Fama and French three-factor model with momentum portfolios. It is well known that the momentum effect documented by Jegadeesh and Titman (1993) has been one of the most serious anomalies to the unconditional versions of the CAPM and the Fama and French model. Surprisingly, however, little is known about abilities of conditional models to explain the momentum. Here we evaluate the time-varying expected returns and momentum profits using the nonparametric versions of the conditional CAPM and the conditional Fama and French model.

We focus on momentum in industry portfolios and the size-BE/ME portfolios. First, we sort the industry portfolios of Moskowitz and Grinblatt (1999) in each month according to their returns of the previous month. Table VI reports results of two tests. Test 1 uses the winner and loser portfolios: The winner is the industry portfolio that has the highest return in the previous month, while the loser is the one that has the lowest return in the preceding month. Test 2 uses five portfolios which are the 1st (winner), 5th, 10th, 15th, and 20th (loser) in the return performance ranking of the preceding month. The 20 industry returns are from July

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[44] As in the CAPM case, the conditional Fama and French model significantly outperforms its unconditional version. The plots for the Fama and French model (like those of Figure 4) are omitted for brevity.

[45] Ferson and Harvey (1999) assume linear forms for time-varying betas. Ferson and Siegel (1998) assume a linear form for the SDF. He et al. (1996) also impose some auxiliary linearity assumptions.

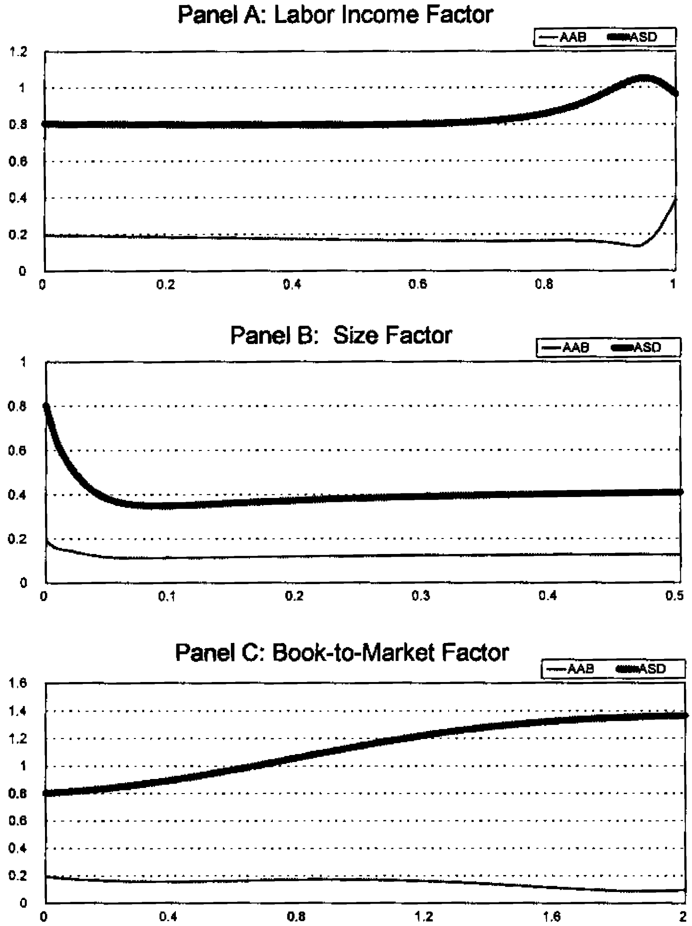

Figure 5. Effects on pricing errors of the conditional CAPM when including an additional factor. Two pricing error measures are plotted. The variable AAB represents the average absolute bias and ASD represents the average standard deviation (defined in Appendix E). On the horizontal axis is the fraction of the factor in the benchmark portfolio. Panels A, B, and C correspond to the addition of the labor income risk factor, the size factor, and the book-to-market factor, respectively.

1963 to July 1995. The nonparametric test statistics and associated p-values are reported. Next we report results for the 25 size-BE/ME portfolios in Panel B.[46] Here Test 2 uses five portfolios which are the 1st (winner), 5th, 13th, 20th, and 25th (loser) in the return performance ranking of the preceding month. The two panels are otherwise identical in construction.

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[46] We start with the industry portfolios as Moskowitz and Grinblatt (1999) find that momentum profits may be explained by industries. We use the size-BE/ME portfolios as a robustness check.

Table VI

Momentum

This table presents results on testing the conditional CAPM and the conditional Fama and French three-factor model using momentum portfolios. The nonparametric version of the Fama-French model (i.e., FF2 in Table V) is being considered in this table. For Panel A, the industry portfolios of Moskowitz and Grinblatt (1999) are sorted for each month according to their returns of the previous month. Two tests are performed. Test 1 uses the winner and loser portfolios: the winner is the industry portfolio that has the highest return in the previous month, while the loser is the one that has the lowest return in the preceding month. Test 2 uses five portfolios, which are the 1st (winner), 5th, 10th, 15th, and 20th (loser) in the return performance ranking of the preceding month. The 20 industry returns are from July 1963 to July 1995. The nonparametric test statistics and associated *p*-values are reported. The table also reports the averages of returns (r_{t+1}) of winners, losers, and the difference of average returns (W − L), as well as the fraction of times that the winner outperforms the loser (P(W > L)). The next row replaces the time t+1 winner or loser returns with the conditional expected returns (i.e., $E_t(r_{t+1})$) of time t winner or loser obtained using either the conditional CAPM or the conditional Fama-French model, respectively. For panel B, instead of the 20 industry portfolios, we use the 25 size and book-to-market portfolios of Fama and French, and the 5 portfolios for Test 2 are the 1st (winner), 5th, 13th, 20th, and 25th (loser) in the return performance ranking of the preceding month. Otherwise, the two panels A and B are identical in construction.

Panel A: Industry Momentum

	Conditional CAPM				Conditional Fama-French			
	test 1		test 2		test 1		test 2	
	stat	*p*-value	stat	*p*-value	stat	*p*-value	stat	*p*-value
	21.2	0.020	31.6	0.170	14.7	0.236	25.4	0.560
	Winner	Loser	W − L	P(W > L)	Winner	Loser	W − L	P(W > L)
r_{t+1}	0.81	− 0.36	1.16	0.57	0.81	− 0.36	1.16	0.57
$E_t(r_{t+1})$	0.71	0.16	0.54	0.63	1.05	0.09	0.95	0.70

Panel B: Size-BE/ME Momentum

	Conditional CAPM				Conditional Fama-French			
	test 1		test 2		test 1		test 2	
	stat	*p*-value	stat	*p*-value	stat	*p*-value	stat	*p*-value
	34.7	0.000	63.3	0.000	15.6	0.120	33.9	0.268
	Winner	Loser	W − L	P(W > L)	Winner	Loser	W − L	P(W > L)
r_{t+1}	1.03	− 0.41	1.45	0.66	1.03	− 0.41	1.45	0.66
$E_t(r_{t+1})$	0.78	0.30	0.48	0.63	1.20	0.19	1.01	0.70

Table VI also reports the average returns of winners and losers and the difference between the average returns (W − L), as well as the fraction of times that the winner outperforms the loser (P(W > L)). The next row replaces time t+1 realized returns of time t winner or loser with the conditional expected returns of the time t winner or loser obtained using either the conditional CAPM or the conditional Fama and French model, respectively.[47] The goal is to see if winners tend to have higher conditional expected returns than losers, a hypothesis forwarded by Conrad and Kaul (1998) and Chordia and Shivakumar (2002), but disputed by Jegadeesh and Titman (1999) and Grundy and Martin (2001).

The tests do not support the nonparametric conditional CAPM. In particular, the strong rejections from Test 1 indicate that the model is inadequate to explain return dynamics of winners and losers. Interestingly, however, the conditional expected returns of winners implied by the CAPM are indeed higher on average than those of the losers. In Panel A, for example, the winner portfolio's conditional expected returns have an average of 0.71 percent versus an average of 0.16 percent for the loser portfolio. But the difference of 0.54 percent is still significantly smaller than the 1.16 percent difference between the average returns of the winner and loser portfolios. The results from Panel B are quite similar.

The test results are more favorable for the conditional Fama and French model. The p-values do not produce rejections at conventional significance levels. In Panel A, according to the three-factor model, on average, the winner portfolio's conditional expected returns are higher than those of the loser in 7 out of every 10 months. The conditional expected return difference is 0.95 percent on average, close to the 1.16 percent return difference between the winner and the loser. In Panel B, the results are similar (though somewhat weaker). Finally, note that Table VI shows a common weakness of the two conditional models. In all the cases, the conditional expected returns of the loser have an average significantly higher than the loser's average return. That is, both models are not doing a good job for the loser portfolio.

These results shed light on the debate about the source of momentum profits. Using more restrictive models, Jegadeesh and Titman (1999) and Grundy and Martin (2001) argue that the momentum profits are due to nonsystematic returns. Our results show that it is critical how the time-varying expected returns are measured. With the nonparametric conditional models, the winners do indeed have higher conditional expected returns on average than the losers. Therefore, our results support the arguments of Conrad and Kaul (1998) and Chordia and Shivakumar (2002) that momentum profits are due to cross-sectional dispersion in expected returns or persistence in expected returns.

III. Conclusion

The Sharpe–Lintner CAPM, the Jagannathan and Wang (1996) extension, and the Fama and French (1993) three-factor model are three of the most influential

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[47] That is, replace r_{t+1} by $E_t(r_{t+1})$ in every entry in the previous row, where $E_t(r_{t+1})$ is computed using either the conditional CAPM or the conditional Fama and French model. For the conditional CAPM, for example, the implied expected return is given in the right-hand side of (2) of Section I. By (5) of Section I, the expected return on the ith industry portfolio implied by the model can be expressed as $E_t[(1 - m_{t+1})r_{i,t+1}]$, where m_{t+1} is the SDF of the CAPM. Thus, we simply regress $(1 - \hat{m}_{t+1})r_{i,t+1}$ on z_t to estimate the expected return implied by the model, where $\hat{m}_{t+1}$ is the nonparametric SDF defined in (7). Similarly, we use the nonparametric SDF defined in Section I.C, for the conditional three-factor model.

and controversial models in empirical asset pricing. Using the nonparametric stochastic discount factor approach, we present a new test of conditional versions of these models. The new methodology allows us to draw inferences in a way that is free from functional form misspecification of beta risk, risk premia, and the stochastic discount factor. Consistent with Ghysels (1998), we find that such misspecification tends to play a critical role in the estimation and testing of conditional asset pricing models. Our results lead to a novel view about how these three models perform empirically.

The new testing methodology can be applied to examine anomalies. In the evaluation of anomalies, a critical issue is how to measure risk and risk premia (e.g., see Fama (1998)). In this paper, we provide a brief application to the momentum puzzle.[48] We find that it makes a difference whether we use conditional or unconditional models to explain momentum profits. In particular, the momentum effect does not seem to be a serious anomaly to the nonparametric conditional version of the Fama and French model. According to the model, the winners tend to have conditional expected returns that are significantly higher than the losers. On average, the difference between expected returns on the winners and the losers is about one percent (per month), which can account for a large portion of the observed profits to momentum strategies.

Performance evaluation is another example of potential applications. Motivated by the evidence of time-varying risk and expected returns, several recent papers have advocated conditional performance evaluation measures for managed portfolios. For example, Ferson and Schadt (1996) and Christopherson, Ferson, and Glassman (1998) propose and apply conditional measures constructed with time-varying betas that are linear functions of predetermined variables such as dividend yield. However, it is known that performance evaluation results are in general sensitive to choice of the performance measure being used. Moreover, as Ghysels' (1998) results and ours imply, the commonly adopted models of time-varying betas for performance evaluation may be problematic. This suggests that functional form specification may also be important in performance evaluation for mutual funds and other managed portfolios. A detailed analysis of the issue seems to be an interesting project for further research.

Appendix

A. *Details of* Ω *and* $\hat{\Omega}_N$

Wang (2002) provides asymptotic results for the nonparametric test in the context of conditional single-beta models. For the convenience of the reader, we replicate details of the asymptotic variance–covariance matrix Ω of $\hat{\delta}_N$ and its estimator $\hat{\Omega}_N$.

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[48] Other examples include potential applications to anomalies associated with initial public offerings and seasoned public offerings, since multifactor models play a key role in studies of IPO and SPO, as shown by Eckbo and Norli (2000) and Eckbo, Masulis, and Norli (2000).

Let $r_{t+1} = (r_{1,t+1} \cdots r_{n,t+1})' \otimes z_t$ and $y_{t+1} = (x_t' \, z_t' \, r_{p,t+1} \, r_{t+1}')'$, where $\otimes$ is the Kronecker operator. Denote $w_t = f(x_t)g_{pp}(x_t)$, $A = \imath_n \otimes E[w_t z_t z_t']$, and $\hat{A}_N = \imath_n \otimes N^{-1}\sum_{t=1}^{N} \hat{w}_t z_t z_t'$, where $\imath_n$ is the $n \times n$ identity matrix. Let $\delta = (\delta_1' \cdots \delta_n')'$ with $\delta_i = [E(w_t z_t z_t')]^{-1}E[w_t z_t e_{i,t+1}]$. Define

$$\begin{aligned}\gamma(y_{t+1}) &= \eta(y_{t+1}) - [\imath_n \otimes a(y_{t+1})]\delta,\\ \eta(y_{t+1}) &= f(x_t)[g_{pp}(x_t)r_{t+1} - g_p(x_t)r_{p,t+1}r_{t+1}\\ &\qquad + g_r(x_t)r_{p,t+1}^2 - g_{pr}(x_t)r_{p,t+1}],\\ a(y_{t+1}) &= f(x_t)[g_{pp}(x_t)z_t z_t' + r_{p,t+1}^2 g_{zz}(x_t)],\end{aligned}$$

where $g_r(x_t) = E(r_{t+1}|x_t), g_{pr}(x_t) = E(r_{p,t+1}r_{t+1}|x_t)$, and $g_{zz}(x_t) = E(z_t z_t'|x_t)$.

The variance–covariance matrix Ω is given by

$$\Omega = A^{-1}\Gamma A^{-1},$$

where $\Gamma = \sum_{-\infty}^{\infty}\Gamma_j$, and $\Gamma_{\mathrm{j}} = E[\gamma(y_{t+1})\gamma(y_{t+j+1})']$.

To estimate the covariance matrix Ω, consider first estimation of $\gamma(y_{t+1})$. Replacing the functions $f(x)$, $g_p(x)$, $g_{pp}(x)$, $g_r(x)$, $g_{pr}(x)$, and $g_{zz}(x)$ by standard kernel estimators,[49] and replacing δ by $\hat{\delta}_N$, we obtain an estimator of $\gamma(y_{t+1})$:

$$\begin{aligned}\hat{\gamma}_N(y_{t+1}) &= \hat{\eta}_N(y_{t+1}) - [\imath_n \otimes \hat{a}_N(y_{t+1})]\hat{\delta}_N,\\ \hat{\eta}_N(y_{t+1}) &= \hat{f}(x_t)[\hat{g}_{pp}(x_t)r_{t+1} - \hat{g}_p(x_t)r_{p,t+1}r_{t+1}\\ &\qquad + \hat{g}_r(x_t)r_{p,t+1}^2 - \hat{g}_{pr}(x_t)r_{p,t+1}],\\ \hat{a}_N(y_{t+1}) &= \hat{f}(x_t)[\hat{g}_{pp}(x_t)z_t z_t' + r_{p,t+1}^2\hat{g}_{zz}(x_t)],\end{aligned}$$

where $\hat{f}$, $\hat{g}_p$, and $\hat{g}_{pp}$ are defined as in (8), (9), (10), and

$$\begin{aligned}\hat{g}_r(x) &= N^{-1}h^{-k}\hat{f}(x)^{-1}\sum_{s=1}^{N}K\left(\frac{x - x_s}{h}\right)r_{s+1},\\ \hat{g}_{pr}(x) &= N^{-1}h^{-k}\hat{f}(x)^{-1}\sum_{s=1}^{N}K\left(\frac{x - x_s}{h}\right)r_{p,s+1}r_{s+1},\\ \hat{g}_{zz}(x) &= N^{-1}h^{-k}\hat{f}(x)^{-1}\sum_{s=1}^{N}K\left(\frac{x - x_s}{h}\right)z_s z_s'.\end{aligned}$$

One can show that the estimator

$$\hat{\Gamma}_j = N^{-1}\sum_{t=1}^{N-j}\hat{\gamma}_N(y_{t+1})\hat{\gamma}_N(y_{t+j+1})'$$

is consistent for Γ_j. The covariance matrix estimator that we use is $\hat{\Omega}_N = \hat{A}_N^{-1}\hat{\Gamma}_0\hat{A}_N^{-1}$. Note that we have necessary inputs ($\hat{\Gamma}_j$) to obtain covariance matrix estimators that are consistent under general circumstances. Whether there exists any advantage to using such estimators in applications remains to be studied.

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[49] Note that $g_{zz}(x_t) = z_t z_t'$ when z_t is a fixed transformation of x_t. For example, when $z_t = (1\, x_t')'$, simply replace $g_{zz}(x_t)$ by $z_t z_t'$, which gives $\hat{a}_N(y_{t+1}) = \hat{f}(x_t)[\hat{g}_{pp}(x_t) + r_{p,t+1}^2]z_t z_t'$.

B. Parametric Benchmark Returns

In this Appendix, we provide details for the case in which the benchmark portfolio has excess returns of the form $r_{p,t+1}(\theta)$, that is, a function of a $l \times 1$ parameter vector θ.

For any given value of θ, one can obtain an estimator $\hat{\delta}_N(\theta)$ as from (12) of Section I, with $r_{p,t+1}(\theta)$ replacing $r_{p,t+1}$. Let $\Omega(\theta)$ and $\hat{\Omega}_N(\theta)$ denote the covariance matrix of $\hat{\delta}_N(\theta)$ and the estimator for this matrix, respectively, with $r_{p,t+1}(\theta)$ replacing $r_{p,t+1}$. Let $\hat{\theta}_N$ be the parameter value that minimizes

$$\hat{\delta}_N(\theta)'\hat{W}_N\hat{\delta}_N(\theta),$$

where the weighting matrix $\hat{W}_N$ is regarded as fixed with respect to θ, $\hat{W}_N \xrightarrow{p} \Omega_0^{-1}$, and $\Omega_0 \equiv \Omega(\theta_0)$. Let $D_N(\theta)$ denote the $l \times qn$ matrix of partial derivatives of $\hat{\delta}_N(\theta)$ with respect to the parameter vector θ: $D_N(\theta) \equiv \partial\hat{\delta}_N(\theta)'/\partial\theta$. The estimator $\hat{\theta}_N$ is assumed to satisfy the following first-order condition:

$$D_N(\hat{\theta}_N)\hat{W}_N\hat{\delta}_N(\hat{\theta}_N) = 0.$$

Then one can show that

$$\sqrt{N}(\hat{\theta}_N - \theta_0) \xrightarrow{L} \mathcal{N}(0, (D_0\Omega_0^{-1}D_0')^{-1}),$$

and the test statistic $N\hat{\delta}_N(\hat{\theta}_N)'\hat{W}_N\hat{\delta}_N(\hat{\theta}_N)$ has a limiting chi-squared distribution with $qn - l$ degrees of freedom.

In applications, we set the weighting matrix $\hat{W}_N$ to be the inverse of the covariance matrix estimate $\hat{\Omega}_N(\theta)$ and update the value of θ through iteration. The weighting matrix is evaluated at zero $(\theta = 0)$ for the initial round estimation. Then the first-stage estimate of θ is used to update the weighting matrix at the second stage.

C. The Stationary Bootstrap

There are several block bootstrap methods for stationary and weakly dependent data, which can be implemented by dividing the data into blocks and sampling the blocks randomly with replacement. The stationary bootstrap of Politis and Romano (1994) uses overlapping blocks with lengths that are sampled randomly from the geometric distribution. The advantage of random block lengths is that the resulting bootstrap data series is stationary, while it is not with blocks of fixed (nonrandom) lengths. In this section, we describe our application of the stationary bootstrap. The notations correspond to those of the text and Appendix A.

Given the data $\{y_{t+1}\}$, $t = 1, \ldots, N$, we use a resampled time series $\{y^*_{t+1,j}\}$, $t = 1, \ldots, N$ to obtain $\hat{\delta}^*_{N,j}$ and $\hat{\Omega}^*_{N,j}$, a resampled version of $\hat{\delta}_N$ and $\hat{\Omega}_N$, respectively, where j indexes one of N_b bootstrapped samples. The following statistics are then used to compute the p-value associated with T_δ:

$$T^*_{\delta,j} = N(\hat{\delta}^*_{N,j} - \hat{\delta}_N)'(\hat{\Omega}^*_{N,j})^{-1}(\hat{\delta}^*_{N,j} - \hat{\delta}_N)$$

for $j = 1, \ldots, N_b$. The series $\{y^*_{t+1,j}\}$, is given by

$$y^*_{t+1,j} = y_{\xi_j(t)+1} \quad t = 1, \ldots, N,$$

where $\xi_j(t)$ is a random index chosen according to the stationary bootstrap algorithm of Politis and Romano. For this, one should choose a priori a "smoothing parameter" $q = q_N$, such that $0 < q_N \leqslant 1$, $q_N \to 0$, and $Nq_N \to \infty$, as $N \to \infty$. Then proceed in three steps as follows:

- For $t = 1$, draw $\xi_j(1)$ as a random variable, uniformly distributed over $\{1, \ldots, N\}$, independently from other variables.
- Increase t by 1. If $t > N$, stop. Otherwise, draw a standard uniform random variable u, independently from other variables.
  - If $u < q$, draw $\xi_j(t)$ as a random variable, uniformly distributed over $\{1, \ldots, N\}$, independently from other variables.
  - If $u \geq q$, set $\xi_j(t) = \xi_j(t-1)+1$; if $\xi_j(t) > N$, set $\xi_j(t) = 1$.
- Repeat the second step.

To choose q, we follow Sullivan et al. (1999) to set $q = 0.1$. This corresponds to an average block length of 10. We also check various values of q ranging from 0.01 to 0.5. We find that the results are not sensitive to the choice of q.[50]

D. Kernel and Bandwidth

We use two kernel functions. The first kernel is an independent multivariate normal density function

$$K(u) = \prod_{i=1}^{k} \phi_i(u_i),$$

where ϕ_i is the density of a univariate normal with mean zero and variance σ_i^2, and σ_i is the standard deviation of the ith state variable. In computation, σ_i is replaced by the sample standard deviation estimate. Note that scale adjustment of the state variables is already made in the above kernels through the standard deviations σ_i. An independent multivariate normal kernel is a popular choice in practice. This kernel is referred to as the normal kernel in our discussion.

The second one is a bias-corrected higher order kernel, which is constructed from the normal kernel as follows (see Powell et al. (1989)):

$$K^*(u) = \frac{K(u) - \sum_{j=1}^{k} a_j b_j^{-k} K(u/b_j)}{1 - \sum_{j=1}^{k} a_j},$$

where $a = B^{-1}e$. Here $a = (a_1, \ldots, a_k)'$, B is a $k \times k$ matrix with the (i,j)th component $B_{ij} = b_j^i$, e is a $k \times 1$ vector of ones, and $b_j = k+j$ for $j = 1, \ldots, k$.

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[50] A large value of q is appropriate when there is little dependence in the data, while a smaller value is appropriate when there is strong dependence.

Optimal bandwidth selection is an unresolved issue. We set

$$h = cN^{-\frac{1}{2k+1}}.$$

This takes into account the bandwidth convergence rate conditions. However, the limiting distribution of our test statistic does not depend on the constant c. Thus, commonly used cross-validation procedures cannot be justified in our context. For a practical choice, we set $c = 1$. This simple bandwidth rule is regarded as an objective starting point and adopted by many authors (e.g., Silverman (1986), Pagan and Schwert (1990), and Harvey (1991), among others).[51] In simulation experiments, we find that this simple rule gives rise to good test size when using the NYSE size portfolios and momentum portfolios. For the size and BE/ME portfolios, the test size is off by a few percent. We adjust the rule to correct the test size by searching over the interval (0.9,1.1). The test has satisfactory size at $c = 0.93$. In our empirical applications, we use exactly the same bandwidths and kernel functions.[52]

E. Pricing Error Measures

Three summary measures of conditional expected return errors are used in applications. They are average absolute bias (AAB), average standard deviation (ASD), and average root mean squared error (ARMSE). Let $\varepsilon_{i,t}$ be time-t conditional Jensen's alpha associated with $E_t(r_{i,t+1})$, for $t = 1,\ldots, N$, and for $i = 1,\ldots, n$. For every i, let B_i, SD_i, and RMSE_i be the sample mean, the sample standard deviation, and the sample root mean squared error of the series $\varepsilon_{i,1}, \varepsilon_{i,2}, \ldots, \varepsilon_{i,N}$. Then the three summary measures for the cross section of the n pricing error series are defined as

$$\mathrm{AAB} = \frac{1}{n}\sum_{i=1}^{n} |\mathrm{B}_i|$$

$$\mathrm{ASD} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{SD}_i$$

$$\mathrm{ARMSE} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{RMSE}_i.$$

AAB is a measure for average pricing error or bias of the model. ASD is for pricing error volatility. ARMSE is a joint measure for both bias and volatility of the pricing errors.

These summary measures are plain and simple. Yet we do not directly observe $\varepsilon_{i,t}$. In applications, we use two methods to estimate the pricing errors. The first

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[51] We tried a cross-validation method to determine the constant c in $h = cN^{\frac{-1}{2k+1}}$. For the postwar period (January 1947 to December 1995), $c = 1.04$.

[52] Consistent with Härdle (1990), we find that the higher order kernel produces more variable estimates, and that in terms of bias reduction, it does not seem effective in this finite sample context. As a result, we focus on the normal kernel in our empirical tests. We report only the results based on the normal kernel.

approach estimates $\varepsilon_{i,t}$ by

$$\hat{\varepsilon}_{i,t} = z_t' \hat{\delta}_i,$$

where $z_1 = (1_t\ x_t')'$, and $\hat{\delta}_i$ is defined by (12) of Section I. This approach is referred to as the WLS regression method.

We have applied another method as a robustness check. In this approach, we estimate the error as follows:

$$\hat{\varepsilon}_{i,t} = \hat{g}_i(x_t) - \hat{g}_p(x_t)\hat{g}_{ip}(x_t)/\hat{g}_{pp}(x_t),$$

where $\hat{g}_p(x)$ and $\hat{g}_{pp}(x)$ are defined as in Section I, and

$$\hat{g}_i(x) = N^{-1}h^{-k}\hat{f}(x)^{-1}\sum_{s=1}^{N} K\left(\frac{x - x_s}{h}\right) r_{i,s+1},$$

$$\hat{g}_{ip}(x) = N^{-1}h^{-k}\hat{f}(x)^{-1}\sum_{s=1}^{N} K\left(\frac{x - x_s}{h}\right) r_{p,s+1} r_{i,s+1}.$$

For this method, we use the normal kernel defined in Appendix D, with bandwidth $h = N^{\frac{-1}{k+4}}$. This is referred to as the kernel regression method.

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