Single Beta Models and Currency Futures Prices

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The conditional capital asset pricing model is applied to foreign currency futures prices, covariance risk being measured relative to excess returns from a broadly diversified international portfolio of equities. Positive time-varying risk premia are found in all five currencies tested when the difference between the US and the average foreign interest rates is used as an instrumental variable for the expected excess return from the common stock portfolio.

I Introduction

(i) Scope of the Work

In recent work (McCurdy and Morgan, 1991) we examined weekly data for foreign currency futures prices and found evidence of risk premia related to covariation with the expected excess returns from two benchmark portfolios, representing wealth and consumption, in a trivariate empirical model related to non-expected utility explanations of asset prices. The wealth portfolio was an internationally diversified equity portfolio and the consumption portfolio was constructed to have returns maximally correlated with US consumption.

We now concentrate our investigation of risk premia in the pricing of foreign currency futures contracts on a single-factor conditional capital asset pricing model. Since the stock markets around the world close at different times of the global day we continue to use an observation interval of a week, even though an interval of a day has attractive features in the analysis of futures prices. By using a weekly interval we hope to avoid most of the awkward problems associated with measurement of covariation between two series for which the recording of prices is not perfectly synchronized. The futures prices are from the Chicago Mercantile Exchange, which opens several hours after the Tokyo Stock Exchange has closed and closes before the Tokyo market reopens. The world index we use contains a substantial Japanese component but stock prices of this component will not reflect the influence of new information arriving during those hours. European market prices reflect some but not all of this information. Use of weekly intervals does not eliminate the problem but must reduce its importance.

In the conditional capital asset pricing model the risk premium is the product of the conditional covariance between the rate of change of futures price and the ratio of the conditional expected return to the conditional variance of the benchmark portfolio. We find that the strength of the evidence about risk premia in the futures prices depends critically on the form of the equation used to specify the expected excess return from the world stock market portfolio as benchmark. The evidence is strong when this equation includes a term for the difference between the US and a simple average of the foreign interest rates and much weaker when this term is excluded and the excess returns are specified as following either a simple first-order moving average process or an ARCH-M process (Engle, Lilien and Robins, 1987).

We assume the world market for equities is integrated, there being no barriers to equity investment by the US representative consumer; this consumer measures investment returns in excess of the US short-term rate.

* Associate Professor and Professor of Economics respectively. The Social Sciences and Humanities Research Council of Canada provided financial support. The paper was written for the International Conference on Futures Markets, Melbourne, December 1990. We thank the referees, seminar participants at Nuffield College, Oxford, the Australian Graduate School of Management and the Australian National University for their discussion. We are particularly grateful for written comments from Tim Gokey. Information about the Swiss reserve requirement rules was provided by Frank Lieber.

The average interest rate differential is a device for forecasting the returns from the world equity market portfolio and we will give a brief summary of the evidence obtained from our data in support of this. In this view the model has a single beta formulation and the inclusion of the average interest rate differential improves the specification of the conditionally expected excess return from the world equity index as benchmark portfolio. Other interpretations are possible; a second view being that the average interest rate differential is a direct predictor of a risk premium component in the futures price. One possibility, as in Giovannini and Jorion (1987), is that it is related to the conditional covariance of the rate of change of futures price with the return from the benchmark portfolio. Another is that it represents risk not accountable in terms of the risk premium associated with the conditional capital asset pricing model. This view replaces the single beta formulation of the model with a two-factor model in which the second factor is related to the sensitivity of the futures price to the interest rate differential. We explore ways of distinguishing between these interpretations.

(ii) Relation to Previous Research

The intertemporal asset pricing model was extended to foreign currencies by Hodrick (1981), Stulz (1981), Lucas (1982), and Hansen and Hodrick (1983). At first, most empirical analyses were in forward markets. Later, the theories of Cox, Ingersoll and Ross (1981), and Richard and Sundaresan (1981) were applied to foreign currency futures data by Hodrick and Srivastava (1987), McCurdy and Morgan (1987, 1988). These analyses and those of Taylor (1986, Chapter 6), rejected the martingale hypothesis for futures prices but did not test any specific model of equilibrium as the alternative hypothesis.

One specific alternative version, the single-period capital asset pricing model (CAPM), was first adapted to futures contracts by Dusak (1973) and Black (1976). The link between the intertemporal asset pricing model and the conditional CAPM has been described by Hansen and Hodrick (1983), Campbell (1987), Hansen and Richard (1987), and Mark (1988).

Empirical application of the conditional CAPM to prices of assets or contracts in various markets has taken one of two paths. In the first, the expected return from the benchmark portfolio is treated as unobservable but a testable condition is imposed on the model through an additional assumption that becomes part of the joint hypothesis under test. The assumption is that for any two assets the ratio of the covariances between the asset return and the benchmark return is constant over time. This was the path followed by Campbell (1987), Campbell and Clarida (1987), Giovannini and Jorion (1987), Cumby (1990) and Lewis (1990). The second path, followed by Mark (1988), Harvey (1989), McCurdy and Morgan (1991), assumes that a close enough approximation to the set of benchmark portfolio returns is available. Empirical testing proceeds with observable quantities without further restrictions being imposed on the model. The assumption here can be regarded as one of conditional mean variance efficiency of the benchmark portfolio, as in Roll (1977), and this becomes part of the joint hypothesis.

Also related to the work on this second path are the papers by Bollerslev, Engle and Wooldridge (1988), Engel and Rodrigues (1989) and Giovannini and Jorion (1989), which assume that the ratio of conditionally expected return, in excess of the riskless rate of interest, to the conditional variance of return of the benchmark is constant over time.

The role of the difference between interest rates across countries has been discussed by Korajczyk (1985), who inferred that risk premia in forward markets for foreign currency are correlated with the differences between the real interest rates across countries, and by Giovannini and Jorion (1987). For the US market in isolation, Fama and Schwert (1977), and Keim and Stambaugh (1986), Campbell (1987, 1990), Fama and French (1989), have identified various functions of *ex ante* nominal interest rates such as the rate itself, term premia (the spread in promised yields between a longer term and a short-term US government bond) and default risk premia (differences between the promised yields of low and high grade bonds) as variables that predict a small proportion of the variation of stock market monthly returns.

For the time-series structure of the conditional variances and covariances in our bivariate systems we use the parameterization giving positive definiteness (Baba, Engle, Kraft and Kroner, 1989) in the generalized ARCH model of Engle (1982) and Bollerslev (1986). Our analysis differs from that of Mark (1988), for forward markets, by its use of the GARCH specification in place of the simplest forms of ARCH processes he used for the conditional covariances and conditional variance of the benchmark portfolio. It is also different in its use of the average interest rate differential as a predictor of the return from the benchmark portfolio instead of the reliance of an autoregressive process for prediction and in its use of quasi-maximum likelihood instead of generalized methods of moments estimation.

II Model and Notation

Let

$M_{t-1,T}$ be the intertemporal marginal rate of substitution of domestic currency between time t-1 and time T,

F_t be the price at t of a futures contract to deliver one unit of the foreign currency at T,

R_{t-1} be one plus the US riskless rate of interest from t-1 to t,

Z_{t-1} be one plus the foreign riskless rate of interest from t-1 to t, and $\bar{Z}_{t-1}$ the average of these,

R_{mt} be one plus the rate of return from a benchmark portfolio from t-1 to t.

Dunn and Singleton (1986) listed several sources for intertemporal models from which first-order conditions for maximum expected utility lead to a stochastic Euler condition,

$$1 = E_{t-1}[M_{t-1,T} \prod_{k=t-1}^{T-1} R_k], \tag{1}$$

for the present value of the random cash flows generated by one dollar rolled over in one-period bonds. Since (1) applies to any horizon and asset, if we set $T = t$ and substitute the benchmark portfolio return for the riskless rate we have

$$1 = E_{t-1}[M_{t-1,t} R_{m,t}], \tag{2}$$

so that equating (1) and (2) yields, for the excess returns on the benchmark portfolio,

$$0 = E_{t-1}[M_{t-1,t}(R_{mt} - R_{t-1})]. \tag{3}$$

This says that the present value of the total return from a position with zero net investment must be zero.

A futures contract also has zero present value when it is initiated at t-1 because the initial investment outlay is zero. Under the institutional practice of marking to market, the settlement price F_t is the equilibrium price that resets the present value of the contract to zero again at t. No cash flow beyond that at t need be considered and

$$0 = E_{t-1}[M_{t-1,t}(F_t - F_{t-1})]. \tag{4}$$

Then, from (1) with $T = t$ and from the definition of covariance, (4) gives

$$F_{t-1} = E_{t-1}F_t + R_{t-1}cov_{t-1}[M_{t-1,t}, F_t], \tag{5}$$

or, with scaling by the price F_{t-1} known at t-1,

$$E_{t-1}(F_t/F_{t-1}) - 1 = -R_{t-1}cov_{t-1}[M_{t-1,t}, F_t/F_{t-1}]. \tag{6}$$

Equation (6) defines the conditional risk premium, or expected rate of change of the futures price. The martingale property for futures prices, $E_{t-1}F_t = F_{t-1}$, does not hold in this model unless the conditional covariance in (6) is zero.

Let $R^*_{mt} \equiv R_{mt} - R_{t-1}$. From (1), (3) and the definition of covariance,

$$E_{t-1}R^*_{mt} = -R_{t-1}cov_{t-1}[M_{t-1,t}, R_{mt}]. \tag{7}$$

We now simplify, for purposes of exposition, an analysis in Campbell (1987) by assuming that R_{mt} is perfectly conditionally correlated with $M_{t-1,t}$, so that we may write

$$M_{t-1,t} = a_{t-1} + b_{t-1}R_{mt}, \tag{8}$$

and

$$cov_{t-1}[M_{t-1,t}, R_{mt}] = b_{t-1}var_{t-1}[R_{mt}]. \tag{9}$$

It follows directly from (7) that

$$E_{t-1}R^*_{mt} = -b_{t-1}R_{t-1}var_{t-1}[R_{mt}]. \tag{10}$$

Similarly, (6) becomes

$$E_{t-1}[F_t/F_{t-1}]-1 = -b_{t-1}R_{t-1}cov_{t-1}[R_{mt}, F_t/F_{t-1}]. \tag{11}$$

Together, (10) and (11) imply

$$E_{t-1}[F_t/F_{t-1}]-1 = \frac{cov_{t-1}[F_t/F_{t-1}, R_{mt}]}{var_{t-1}[R_{mt}]} E_{t-1}R^*_{mt}, \tag{12}$$

which is the conditional capital asset pricing model applied to the rate of change of futures price.

At first glance, it might appear surprising that a futures contract with zero present value can satisfy the conditions of an equilibrium model that was originally developed to explain asset prices, which, by definition, are positive. Proposition 7 of Cox, Ingersoll and Ross (1981) [CIR-7] is sufficient to establish the validity of this result. CIR-7 equates the futures price, which is not by itself the value of an asset, with a series of payments that corresponds to an asset. The payments at times $t, t+1 \ldots T-1$, consist of the products of the prevailing one-period interest rate and futures price at each of these times; the final payment is the prevailing spot price at time T. Since the present value of the payments is the futures price at time t-1, an equilibrium model applied directly to the asset defined in CIR-7 also extends to the futures price itself.

III The Test Equation System

Since the right-hand side of (12) or risk premium is the product of the conditional beta of the rate of change of the futures price and the conditional expected excess return from the benchmark portfolio, the test equation system should incorporate all these time-varying moments as determinants of the rate of change of futures price. We pair the time series of the rate of change of futures price for a given currency with the excess returns from the benchmark portfolio in a bivariate version of the GARCH model using the Baba, Engle, Kraft and Kroner (1989) positive definite parameterization for the conditional covariance matrix.

Under rational expectations, the realized value of the scaled payoff in (12) is equal to the conditional expectation plus an error term uncorrelated with past information. Let ϵ_{ft} be this error term and ϵ_{mt} the corresponding term for the benchmark portfolio. The vector of these error terms, ϵ_t, is assumed to have a conditional bivariate normal distribution with mean zero and covariance matrix H_t. In H_t, let h_{mt} be the conditional variance of the return from the benchmark portfolio, h_{ft} the conditional variance of $(F_t/F_{t-1}) - 1$, and h_{fmt} the covariance between the two. Let μ be a multiplicative parameter that can be chosen to have a value of zero to exclude the risk premium term from the model. Let $x_{f,t-1}, x_{m,t-1}, g_{f,t-1}, g_{m,t-1}$ and $g_{fm,t-1}$ be vectors of explanatory variables known at time t-1. Using the positive definite parameterization for the ARCH components only, we can write the system of test equations for the rate of change of futures prices as

$$(F_t/F_{t-1})-1 = \gamma_f' x_{f,t-1} + \mu \frac{h_{fmt}}{h_{mt}}(\gamma_m' x_{m,t-1}) + \epsilon_{ft}, \qquad (13)$$

$$R^*_{mt} = \gamma_m' x_{m,t-1} + \epsilon_{mt}, \qquad (14)$$

$$\begin{bmatrix} h_{ft} & h_{fmt} \\ h_{fmt} & h_{mt} \end{bmatrix} = \begin{bmatrix} c_f & 0 \\ c_{fm} & c_m \end{bmatrix} \begin{bmatrix} c_f & c_{fm} \\ 0 & c_m \end{bmatrix} + \begin{bmatrix} a_f & a_{fm} \\ a_{fm} & a_m \end{bmatrix} \begin{bmatrix} \epsilon^2_{f,t-1} & \epsilon_{f,t-1}\epsilon_{m,t-1} \\ \epsilon_{f,t-1}\epsilon_{m,t-1} & \epsilon^2_{m,t-1} \end{bmatrix} \begin{bmatrix} a_f & a_{fm} \\ a_{fm} & a_m \end{bmatrix} + \begin{bmatrix} b_f & b_{fm} \\ b_{fm} & b_m \end{bmatrix} \begin{bmatrix} h_{f,t-1} & h_{fm,t-1} \\ h_{fm,t-1} & h_{m,t-1} \end{bmatrix} \begin{bmatrix} b_f & b_{fm} \\ b_{fm} & b_m \end{bmatrix} + \begin{bmatrix} \phi_f' g_{f,t-1} & \phi_{fm}' g_{fm,t-1} \\ \phi_{fm}' g_{fm,t-1} & \phi_m' g_{m,t-1} \end{bmatrix} \qquad (15)$$

The dependent variable in (13) is the scaled *ex post* payoff from the futures position, and also, by the choice of scaling factor, the rate of change of futures price. Under the null hypothesis of the model this is equal to a conditional risk premium plus a rational expectations forecast error. The conditional risk premium is the product of the conditional expected excess return from the benchmark portfolio and the conditional covariance of the rate of change in futures price with the excess return from the benchmark portfolio divided by the conditional variance of the latter. Ordinarily, the risk premium is part of the estimated model but it can be suppressed for purposes of testing hypotheses by setting the parameter μ to zero.

IV Data

Futures prices for the British pound, Canadian dollar, Deutsche mark, Japanese yen, and Swiss franc (BP, CD, DM, JY, SF, respectively), were taken from the 1985 version of the file provided by the Center for Research in Futures Markets (CRFM) of the University of Chicago and from Reuters for subsequent years. We used futures settlement prices for the contracts with the shortest maturity available at any time up to and including the last Friday before the end of the life of the contract. Wednesday to Wednesday rates of change of price were computed for all weeks for which both parties were available. If the Wednesday price was missing in a given week the Thursday price was taken to be the final price for the week and the initial price for the following week. No adjustment was made when a price was one resulting from a limit move, as happened on four occasions in the CD, eight in the DM, five in the JY and three in the SF. No adjustments were made to reflect the marking to market within the week, it being assumed that all marking to market occurred at the end. Returns from the benchmark portfolio were calculated from the Morgan Stanley Capital International world equity index expressed in US dollar levels. No attempt was made to incorporate dividends in the returns because the available data do not appear to be suitable for the task unless the interval of observation of the data is one month or longer. The returns were computed for the intervals matching those used for the futures data. The data series start on 1980 01 02 and end on 1988 12 28. The effective sample size is 469 because the first observation is used in the start of the estimation.

To compute the excess returns from the benchmark portfolio and the interest rate differential we used the seven-day Eurocurrency rates, converting the annualized figures to rates applicable to seven calendar days. When either the foreign or the US interest rate was unavailable because of holidays we substituted both rates for the previous day.

The maximum of the joint log likelihood function of the bivariate system was found numerically. All rates of change of futures price were scaled by multiplication by 100 except for the CD, for which the data were scaled by 400. The world index returns and the indicator variable for the week including the 19 October 1987 market crash were scaled by ten. The average interest rate differential variable $R_{t-1}/\bar{Z}_{t-1} - 1$ was scaled by 1000. The purpose of this scaling was to keep the estimated values of most of the coefficients within the range -1 to +1 and to aim to have the optimal objective function value in the same range, as recommended for their routines by Numerical Algorithms Group.

V *Empirical Analysis*

(i) *Outline*

In equation (13) the vector $x_{f,t-1}$ consists of only a constant in the standard model. It is expanded to include other variables for testing purposes only. In (14), $x_{m,t-1}$ consists of a constant and $R_{t-1}/\bar{Z}_{t-1} - 1$. The vectors $g_{f,t-1}$ and $g_{fm,t-1}$ in (15) are normally used only for testing for omitted variables but for the SF $g_{f,t-1}$ consists of $Z_{t-1}/\bar{Z}_{t-1} - 1$. Also, in (15), $g_{m,t-1}$ consists of the indicator for the week of the market crash.

With one exception, the same model is fitted to all currencies. The exception is the SF, for which a local interest rate differential was added to the variance function. Without this variable, many of the diagnostic test statistics were large. The Euro-currency rates for the SF were more variable and, in annual terms, about 3 per cent higher in the last week of the calendar month than in other weeks. It is probably that these unusual patterns in interest rates were induced by the procedure of measuring short term, liquid, reserves on the last business day of the month. On 1 January 1988, new procedures measuring these reserves as an average over a period equal to one month were introduced.

Table 1 presents the detailed estimates of this standard model, Table 2 some implications of these estimates, Table 3 the diagnostic tests, Table 4 the tests for omitted variables and Table 5 the evidence concerning risk premia. Included in Table 4 is the evidence used in the choice not to follow the full theoretical development of (10) in the specification of the expected excess return from the benchmark portfolio in equation (14), for which we use the average interest rate differential as an instrument but exclude the term involving the conditional variance.

Since the futures time series showed evidence that the hypothesis of conditionally normal error terms did not hold, all standard errors were computed to be robust. To do this, we follow a procedure similar to that of Bollerslev and Wooldridge (1988). Let J be the numerical approximation to the matrix of second derivatives with respect to the free variables. Let K be formed by

TABLE 1

Equation Estimates for Futures and Benchmark

	futures		benchmark	
	γ_{0f}	μ	γ_{0m}	γ_{1m}
BP	-0.108 (0.072)	1.55 (0.53)	0.047 (0.010)	-0.074 (0.017)
CD	-0.063 (0.125)	2.17 (0.91)	0.052 (0.010)	-0.086 (0.019)
DM	-0.193 (0.068)	1.29 (0.46)	0.048 (0.009)	-0.076 (0.018)
JY	-0.097 (0.075)	1.96 (0.57)	0.050 (0.009)	-0.079 (0.019)
SF	-0.213 (0.077)	1.08 (0.43)	0.048 (0.009)	-0.074 (0.018)

Note: The equations estimated are (13) and (14).

TABLE 1 CONTINUED

Covariance Matrix Parameter Estimates

	BP	DC	DM	JY	SF
c_f	0.35 (0.07)	0.71 (0.21)	0.49 (0.19)	0.55 (0.13)	-0.00 (0.01)
c_{fm}	0.12 (0.04)	0.06 (0.14)	0.06 (0.08)	0.16 (0.06)	-0.03 (0.07)
c_m	0.07 (0.02)	0.11 (0.05)	0.06 (0.01)	0.07 (0.02)	0.06 (0.02)
a_f	0.27 (0.04)	0.38 (0.06)	0.28 (0.11)	0.25 (0.06)	0.22 (0.07)
a_{fm}	-0.00 (0.01)	-0.00 (0.01)	-0.01 (0.01)	-0.01 (0.01)	-0.01 (0.01)
a_m	0.37 (0.05)	0.33 (0.08)	0.42 (0.05)	0.40 (0.05)	0.42 (0.05)
b_f	0.94 (0.01)	0.88 (0.03)	0.91 (0.06)	0.89 (0.04)	0.93 (0.02)
b_{fm}	0.00 (0.01)	0.01 (0.02)	0.01 (0.01)	0.01 (0.02)	0.01 (0.02)
b_m	0.83 (0.05)	0.67 (0.27)	0.83 (0.04)	0.82 (0.04)	0.82 (0.04)
$\phi_{f,t-1}$					0.31 (0.11)
$\phi_{m,t-1}$	0.07 (0.04)	0.14 (0.16)	0.07 (0.03)	0.08 (0.04)	0.07 (0.03)

Note: The equation estimated is (15).

taking the average of the period by period outer products of the gradient. The standard errors are computed from the diagonal elements of the matrix $J^{-1}KJ^{-1}$. Engle and Gonzalez-Rivera (1990) evaluated the potential loss of efficiency arising from this quasi-maximum likelihood estimation and proposed a more efficient semiparametric estimator. Another approach is to deal with departures from conditional normality directly, as in the exponential GARCH model of Nelson (1991). This model, designed for and successfully applied to the stock market, incorporates an asymmetric relationship between the conditional mean and the conditional variance. However, there is no reason to suppose that such a relationship should hold in currency markets.

(ii) Results for the Standard Model

Detailed estimates of the equation system (13)–(15) are given in Table 1. Joint estimates of this system were obtained for each currency individually.

In the equation for the conditional mean for the futures, the main results are the values obtained for μ, the multiplicative parameter for inclusion of the term for the risk premium of the conditional CAPM. In the conditional CAPM this parameter should have a value of unity and its estimates, according to t-tests, are consistent with the theory in all currencies. In this respect, the conditional CAPM provides a useful description of the data.

The important details in the conditional mean of the benchmark portfolio returns are the estimates

TABLE 2

Implied Estimates

Panel A

	BP	CD	DM	JY	SF
Proportion of variation explained					
$F_t/F_{t-1} - 1$	0.019	0.019	0.011	0.026	0.008
R^*_{mt}	0.030	0.032	0.031	0.032	0.032
risk premia annual percent					
mean	-0.3	1.1	-5.7	2.4	-7.0
standard deviation	9.6	4.2	9.6	12.3	8.8

Panel B

frequencies of negative values by week

number of currencies	h_{fmt}	$h_{fmt}E_{t-1}R^*_{mt}/h_{mt}$
0	402	315
1	44	32
2	10	10
3	3	6
4	9	26
5	1	80
total	469	469

of the coefficient γ_{1m} for the average interest rate differential. These estimates are all significantly negative. We will postpone further discussion of these results until Table 5, which presents more evidence potentially useful in interpreting the role of the interest rate differential.

The parameter estimates for the conditional covariance matrix are shown as a group. Included is the estimate of ϕ_{1m}, the coefficient for the indicator variable for the week of the market crash. The estimates of this parameter are generally about two standard errors above zero. In the SF, the estimate of ϕ_{1f} for the local interest rate differential in the futures equation is about two and a half standard errors above zero.

Some of the implications of these estimates are shown in Table 2. The first row of Table 2 gives the proportion of unconditional variation explained by the equation for the conditional mean of the rate of change of futures price. Since the only explanatory variable in the equation is the risk premium term, consisting of the product of the conditional covariance and the ratio of conditional expected excess return on the benchmark portfolio to its conditional variance, it is unlikely that the proportion of variation that can be explained will exceed the equivalent values for the excess returns from the benchmark portfolio. These values are shown in the second row. The third row expresses the estimated risk premia as annual percentages. These numbers were obtained from the fitted values for the rate of change of futures price in equation (13), rescaled to give values corresponding to 5200 times the weekly rates and then averaged over the period. The largest risk premium found is for the JY, the currency for which Tables 4 and 5 will later reveal that the model applies most closely.

Negative estimates of risk premia can occur if the conditional covariance or the conditional expected excess return from the benchmark portfolio is negative. Panel B of Table 2 shows the frequency of negative estimates.

TABLE 3

Diagnostic Checks on Models in Table 1

	BP	CD	DM	JY	SF
R_f	-1.94 (0.05)	0.79 (0.43)	-1.71 (0.09)	0.12 (0.90)	-2.13 (0.03)
$Q_f(10)$	7.53 (0.67)	2.67 (0.99)	9.25 (0.51)	14.56 (0.15)	5.98 (0.82)
$Q^2_f(10)$	4.99 (0.89)	10.50 (0.40)	10.03 (0.44)	5.59 (0.85)	11.33 (0.33)
S_f	4.94 (0.03)	0.63 (0.43)	16.23 (0.00)	22.62 (0.00)	23.18 (0.00)
K_f	10.97 (0.00)	13.59 (0.00)	3.38 (0.07)	6.46 (0.01)	2.30 (0.13)
P_f	-0.07 (0.94)	-2.33 (0.04)	-1.08 (0.28)	0.06 (0.95)	-0.23 (0.82)
$Q_{fm}(10)$	11.66 (0.31)	6.87 (0.74)	11.94 (0.29)	12.70 (0.24)	15.93 (0.10)
P_{fm}	-1.54 (0.12)	-2.30 (0.02)	-2.20 (0.03)	-1.26 (0.23)	-1.55 (0.12)
R_m	0.14 (0.89)	-0.79 (0.43)	0.14 (0.89)	0.51 (0.61)	-0.57 (0.57)
$Q_m(10)$	4.64 (0.91)	6.94 (0.73)	7.66 (0.66)	9.71 (0.47)	10.56 (0.39)
$Q^2_m(10)$	17.47 (0.06)	36.55 (0.00)	20.11 (0.03)	26.36 (0.00)	26.98 (0.00)
S_m	0.01 (0.92)	0.08 (0.78)	0.18 (0.67)	0.06 (0.81)	0.92 (0.34)
K_m	1.47 (0.23)	0.78 (0.38)	1.44 (0.23)	1.25 (0.26)	2.10 (0.15)
P_m	0.32 (0.75)	0.89 (0.37)	-0.11 (0.91)	0.02 (0.98)	-0.04 (0.97)

R is the test statistic for runs above the mean, *Q(10)* and $Q^2(10)$ the Ljung-Box (1978) form of the portmanteau statistic for the first 10 lags of the autocorrelation functions of the standardized residuals and their squares, $Q_{fm}(10)$ the same for their cross products, *S* and *K* the Newey (1985) or Tauchen (1985) conditional moment statistics for skewness and kurtosis, *P* the Pagan-Sabau (1987) test statistics computed from robust standard errors. The *p*-values in parenthesis are for the chi-square distribution, except for *R* and *P*, where they are for the unit normal distribution.

TABLE 4

OPG-LM Tests for Omitted Variables

	BP	CD	DM	JY	SF
conditional mean: futures					
$\epsilon_{f,t-1}$	0.53	0.01	0.49	0.05	0.05
	(0.47)	(0.92)	(0.48)	(0.82)	(0.82)
$h^{1/2}_{ft}$	0.64	1.29	4.89	2.48	4.30
	(0.42)	(0.26)	(0.03)	(0.12)	(0.04)
$T-t$	0.02	3.39	0.10	0.05	1.56
	(0.89)	(0.07)	(0.75)	(0.82)	(0.21)
$R_{t-1}/\bar{Z}_{t-1} - 1$	0.01	0.05	0.82	0.87	1.57
	(0.92)	(0.82)	(0.37)	(0.35)	(0.21)
$Z_{t-1}/\bar{Z}_{t-1} - 1$	15.85	12.26	0.43	0.28	0.01
	(0.00	(0.00)	(0.51)	(0.60)	(0.92)
conditional mean: benchmark					
$\epsilon_{m,t-1}$	0.74	0.26	2.16	2.81	1.45
	(0.39)	(0.61)	(0.14)	(0.09)	(0.22)
h_{mt}	3.42	0.88	2.57	3.14	1.96
	(0.06)	(0.35)	(0.11)	(0.08)	(0.16)
conditional variance: futures					
C_t	0.86	0.27	0.89	1.41	1.35
	(0.35)	(0.60)	(0.35)	(0.24)	(0.25)
$/Z_{t-1}/\bar{Z}_{t-1} - 1/$	0.27	3.16	2.71	0.89	
	(0.60)	(0.08)	(0.10)	(0.35)	
$T-t$	0.50	1.21	0.23	3.23	0.39
	(0.48)	(0.27)	(0.63)	(0.07)	(0.53)
conditional covariance					
C_t	1.18	0.17	1.23	0.39	1.34
	(0.28)	(0.68)	(0.27)	(0.63)	(0.25)
$Z_{t-1}/\bar{Z}_{t-1} - 1$	0.30	0.01	0.54	2.34	0.24
	(0.58)	(0.92)	(0.46)	(0.13)	(0.62)

References: Godfrey and Wickens (1982), Davidson and MacKinnon (1990). *p*-values for the chi-square distribution with one degree of freedom are in parenthesis.

TABLE 5

Risk Premia Restrictions

	$c_{fm} = a_{fm} = b_{fm} = 0$	$\mu = 0$	$\gamma_{0f} = 0, \mu = 1$	$\gamma_{1m} = 0$
BP	27.35 (0.00)	5.44 (0.02)	2.37 (0.30)	14.14 (0.00)
CD	16.76 (0.00)	8.51 (0.00)	2.61 (0.27)	19.67 (0.00)
DM	27.69 (0.00)	5.17 (0.02)	7.59 (0.02)	11.24 (0.00)
JY	34.83 (0.00)	11.71 (0.00)	3.73 (0.15)	20.00 (0.00)
SF	30.63 (0.00)	3.73 (0.05)	6.28 (0.04)	14.55 (0.00)

Note: The restriction indicated at the top of each column is the null hypothesis in tests against the alternative model (the model in Table 1), in which these parameters are free. Log likelihood ratio test statistics are shown and the p-values in parenthesis are for the chi-square distribution.

Diagnostic tests are shown in Table 3. Included are tests for autocorrelation, heteroscedasticity, and skewness and excess kurtosis. There are also specification tests of the conditional variances and covariances (Pagan and Sabau, 1987), and for remaining interaction between the futures series and the benchmark portfolio series. Except for the Pagan-Sabau tests, standardized residuals are examined in these analyses. Let $H_t^{-1/2}$ be the 2 × 2 matrix such that $H_t^{-1/2}H_t^{-1/2} = H_t^{-1}$, obtained by orthonormal transformation of H_t, the estimate of the conditional covariance matrix. Then the standardized residuals for period t are obtained from the vector of raw residuals ϵ_t as $u_t = H_t^{-1/2}\epsilon_t$. Some of the tests designed to detect one form of deviation from the hypothesized properties of the error terms, such as autocorrelation, may be sensitive to the presence of another form, such as any heteroscedasicity remaining after the standardization.

From the portmanteau statistics for the heteroscedasticity tests in Table 3, there may be evidence against the null hypothesis of homoscedasticity in the benchmark portfolio series. From the autocorrelation function (not shown in detail) for the squared standardized residuals, relatively large contributions to the portmanteau test statistic occurred at lags five and ten.

The conditional moment test statistics for the futures show evidence of skewness and excess kurtosis as a general rule, but for the benchmark portfolio no such evidence is found.

The Pagan-Sabau test statistics reveal no problem for the benchmark portfolio. With the possible exception of the CD, the null hypothesis of the test is also retained for the futures equations. The test statistics are t-tests for the slope coefficient in a regression of the difference between the squared residuals and the estimated conditional variance on the latter. However, the corresponding test based on the regression of the difference between the cross products of the residuals and the estimated conditional covariances yields test statistics with p-values of 0.02 in the CD and 0.03 for the DM.

Table 3 has detected relatively few shortcomings in the estimated models. Table 4 focuses on specific elements that may have been omitted inappropriately. A factor that may tend to exaggerate the size of the Lagrange multiplier statistics in Table 4 is that, unlike the standard errors reported in the paper, the values are obtained from a non-robust formulation of the Lagrange multiplier test.

The first two groups of rows in Table 4 examine the conditional means of the futures equation and the benchmark equation, respectively, for missing variables. The first row of each group tests for the need for a first-order moving average term and no evidence of a need for the added term is found.

The second row in each of the first two groups in Table 4 examines the data for evidence of the ARCH-M characteristic that the conditional mean is an *ex ante* function of the conditional standard deviation. In the DM and SF futures, the test statistics with p-values of 0.03 and 0.04 may indicate a time-varying risk factor that the specification of the conditional CAPM, or our implementation of it, failed to capture. For the benchmark portfolio analysis there is a specific reason for testing the conditional variance as opposed to the conditional standard deviation: in equation (10) the conditional mean is proportional to the conditional variance of this portfolio. The evidence from Table 4 on this aspect of the model is clear; insufficient benefit would be obtained from the inclusion of the conditional variance as an ARCH-M term in the conditional mean equation, the lowest p-value found being 0.06 in the BP.

The last two rows of the group dealing with the conditional mean of the futures equations are important because they provide evidence about a possible second factor in foreign currency futures pricing. Two versions of the interest rate differential are tested. First, the interest rate differential variable for the US and the average foreign rate is not found to be directly relevant to the futures equation for any currency. Its effect has already been obtained, indirectly, through the risk premium term. Second, the local interest rate differential, calculated for the particular foreign currency relative to the average of all such rates, suggests some shortcomings of the model we have used. For the BP and CD, the test statistics for the local interest rate differential in the conditional mean have p-values of 0.00. The next subsection discusses this further, including the evidence that this variable is not relevant to the conditional covariance between the futures and the benchmark portfolio in any currency.

(iii) Evidence about Risk Premia and Interpretation

The final evidence we present consists of a series of likelihood ratio tests designed to reveal more about the nature and role of the risk premium in the system. Table 5 shows four such tests for each currency. The first column of the table shows the effect of the interaction between the series for the futures and the benchmark through the covariance parameters. Suppression of these three parameters simplifies the formulation of the covariance matrix but greatly reduces the likelihood function value in all currencies. Variation in the two series is linked but by itself this is not sufficient for it to be concluded that a meaningful risk premium in the futures equation can be related to the conditional CAPM. The second column tests for this risk premium by examining the effect of eliminating the risk premium term by setting the parameter μ to zero. The results obtained by this restriction are stronger in four currencies, the BP, CD, DM and JY, than in the SF but the p-values are small in all five. The restricted model is rejected in favour of the risk premium model based on the conditional CAPM.

The third column of Table 5 is a more stringent test of the implications of the conditional CAPM. The restricted model, implied by equation (12), has values of zero for the intercept and unity for the multiplicative parameter in equation (13). The restrictions corresponding to equation (12) are tested against the standard model reported in Table 1 where neither of these parameters is restricted. For three currencies, the BP, CD and JY, the restrictions appear to be compatible with the data but they give p-values of 0.02 in the DM and 0.04 in the SF.

The evidence presented so far is that the formulation of the risk premium in the single-factor conditional CAPM appears to be useful for interpreting the evolution of the futures series. However, evidence that there may be risk premia not captured by this model has also been obtained in the form of large Lagrange multiplier test statistics for the local interest rate differential in the BP and CD, and for the ARCH-M term in the DM and SF futures equations. The apparent failure of the single-factor formulation to capture these predictable components needs to be examined further.

Two views about the role of the interest rate differential were stated in Section 1 of the paper. In the first view, the interest rate differential predicts the benchmark portfolio excess returns and this accounts for its usefulness in a single-factor model. This may be tested by comparing the fit of the standard model, augmented by an ARCH-M term, with one in which the ARCH-M term replaces the average interest rate differential as predictor of the conditional excess return for the benchmark portfolio. As may be seen from the statistics in the final column of Table 5 in every currency, likelihood ratio tests reject the null hypothesis that the interest rate differential is irrelevant for the conditional mean of the benchmark portfolio.

In the second view, the added explanatory power of the interest rate differential depends on an association with risk, arising in two possible ways; indirectly through a link with the covariance term in (12) or directly through the futures equation (although the theory in Section II provides no direct foundation for this view). The Lagrange multiplier tests shed some light on the merits of these two possibilities, which are essentially arguments for a single-factor and two-factor model, respectively. If the two-factor model was useful, the test statistics for the coefficient for the interest rate differential in the conditional mean of the futures equations in Table 4 would be large and its p-value small. The results depend very much on which of the two interest rate variables is considered. Only the variable computed from the local rate is relevant and then only in the BP and CD, where the results are particularly strong. The results seem to indicate an additional source of risk for these two currencies. The other possible interpretation, that these results indicate some power of the local interest rate differential to predict the conditional covariance in a one-factor model, is not supported by the results in the final row of Table 4. In no currency does the local interest rate differential improve the estimation of the conditional covariance in the equation system (15).

The overall assessment of the risk premium evidence in this paper is that the single beta model accounts for an important part of the behaviour of the futures prices in all five currencies but the estimated parameter values are not consistent with the model in the DM and SF. Similarly, other sources of risk premia may have been indicated by the Lagrange multiplier statistics for the ARCH-M term in the futures equation in these currencies. However, the strongest and most direct evidence of a potential role for a second factor is obtained from the local interest rate differential in the futures equations for the BP and CD.

VI Conclusion

The single-factor formulation of the risk premium in the conditional capital asset pricing model has been found to be useful in describing the behaviour of foreign currency futures prices observed at weekly intervals with a world equity index as benchmark portfolio. Our analysis ignores the day-to-day cash flows resulting from marking to market within the weekly observation interval and it emphasizes the role of the difference between foreign and US interest rates as an instrumental variable for predicting the benchmark portfolio returns. The strength of the evidence for the existence of risk premia related to the conditional capital asset pricing model is linked to the important role played by the interest rate differential, apparently as a predictor of the benchmark portfolio excess returns.

Other results, particularly those of tests of restrictions on the theoretical values of the parameters of the futures equation, as well as certain diagnostic test statistics and tests for missing variables, reject the joint hypothesis of the paper. One interpretation of this rejection is that the single-factor model does not capture all of the systematic variation in risk premia in futures prices; a second factor appears to be required. Another interpretation is that the world equity index we have used as a proxy for the market portfolio is not mean variance efficient. As is usual with rejection of joint hypotheses, many questions remain unanswered. We will mention only the most important of these. If the conditional CAPM is appropriate and the rejection merely indicates an inadequate choice of the proxy representing world wealth, why would the local interest rates for the BP and CD be relevant but not those of the other three countries? If more than one factor is priced, how does the second factor relate to the local interest rates in these two countries?

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