PRICING OPTIONS ON FOREIGN ASSETS WHEN INTEREST RATES ARE STOCHASTIC

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ABSTRACT

Options on foreign assets, or cross-currency options, are options whose value depends on both foreign asset prices and the exchange rate. The objective of this paper is to investigate whether we could use constant interest rate models to price cross-currency options in a stochastic interest rate environment. Stochastic interest rate models are developed using techniques similar to those in Merton (1973) and Grabbe (1983). Then, using the Ornstein-Uhlenbeck interest rate processes, the paper examines the pricing errors of constant interest rate models compared against their stochastic counterparts. It is found that pricing errors are on average small when the interest rate volatilities are not high. Therefore a key conclusion of the paper is that constant interest rate models may be used for pricing cross-currency options, even though interest rates are stochastic.

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INTRODUCTION

Recently options on foreign assets (cross-currency options) have gained increasing popularity. For example, many exchanges have listed foreign index warrants, which essentially are options on foreign stock market indexes. These cross-currency options have different exchange rate/payoff specifications and have created a rich array of investment and hedging opportunities. Some cross-currency options appear simple but have subtle exchange rate/interest rate complications.[1] Therefore a straightforward extension of the Black-Scholes model (Black and Scholes, 1973) will not apply.

The pricing of financial products based on the exchange rate alone has been well studied.[2] Although many of the studies have examined the effects of interest rate uncertainty on currency option prices, two studies are especially noticeable. Relying on equivalent martingale techniques, Amin and Jarrow (1991) derive closed form formulas for European options on currencies and currency futures in an environment of stochastic interest rates. Using a combination of hedging and risk-neutral valuation techniques, Ritchken and Sankarasubramanian (1991) develop a series of valuation models for contingent claims in a stochastic interest rate environment with multiple risky assets. The two studies share a common advantage, which is the ability to match the entire domestic and foreign term structures with the market data. With proper parameter choices, both models can be used to price cross-currency options.

Although a few papers exist in the literature that specifically deal with cross-currency options,[3] they all share a common drawback: the assumption of constant interest rates (both domestic and foreign). Given that most cross-currency instruments have a longer time to maturity,[4] the assumption of constant interest rates may not be realistic. It is therefore of interest to examine the pricing of cross-currency options when the aforementioned assumption is relaxed.

The current paper concerns the pricing of European style cross-currency options when the domestic and foreign interest rates are stochastic. Specifically, we investigate if constant interest rate models perform reasonably well in a stochastic interest rate environment. This investigation is important because the constant interest rate pricing models require substantially fewer parameters. If we could establish that the constant and stochastic interest rate models will give sufficiently close prices, then we will be able to justify the use of the former models. The use of constant interest rate models will not only simplify the model implementation, but also avoid the estimation of extra parameters, which in turn will reduce the potential mispricing due to errors in parameter estimation.

The remainder of the paper is organized as follows. In the next section, we develop pricing models that incorporate stochastic interest rates. Then we investigate the potential pricing errors caused by using constant interest rate models in a stochastic interest rate environment. We conclude the study in the last section.

PRICING CROSS-CURRENCY OPTIONS

Notation:

t = current time;
T = time at maturity;
S = current price of the underlying foreign asset denominated in foreign currency;
S_T = foreign asset price at time T;
X = current exchange rate: the value in units of domestic currency of one unit of the foreign currency;
X_T = spot exchange rate at time T;
X_0 = prespecified exchange rate;
K = exercise price in foreign currency;
q = continuous dividend yield on the asset;[5]
r = short-term domestic risk-free interest rate;
r_f = short-term foreign risk-free interest rate.

A cross-currency option is a put or call option on a foreign asset. Depending on the terminal payoff specifications, cross-currency options can be classified into four categories. For example, cross-currency put options can be classified as follows in terms of terminal payoffs in domestic currency:

Category I: $\text{Max}\,[0, X_T(K - S_T)]$
Category II: $\text{Max}\,[0, X_0(K - S_T)]$
Category III: $\text{Max}\,[0, X_TK - X_0S_T]$
Category IV: $\text{Max}\,[0, X_0K - X_TS_T]$

Note that the exercise price and the underlying asset's price are both denominated in foreign currency. The difference among the categories lies in how the payoff is converted to domestic currency. A category I cross-currency put option entitles a holder to the difference between the strike price and the asset price upon exercise. This difference will then be converted to domestic currency at the prevailing exchange rate on the exercise day. A category II cross-currency put option also entitles the holder to the difference between the strike price and the asset price upon exercise. However, this difference will be converted to domestic currency at a prespecified exchange rate. The payoff of a category III cross-currency put option is calculated such that the strike price is converted at the spot exchange rate upon exercise, but the asset price is converted at a prespecified exchange rate. Finally, for a category IV cross-currency put option, the strike price is converted to domestic currency at a prespecified exchange rate, and the foreign asset price at the spot exchange rate on the exercise day.

A similar categorization also applies to cross-currency call options. Note that the exchange traded foreign index warrants on various exchanges cover all categories except category III. To the author's knowledge, no instruments of category III have been formally traded on exchanges. We discuss the pricing of this category simply for completeness.

In a world of stochastic interest rates, there are four factors that directly affect a cross-currency option's value: the foreign asset price, the exchange rate, the domestic risk-free interest rate, and the foreign risk-free interest rate. As in Merton (1973), we use discount bond prices to represent the stochastic interest rates. Let B_d denote the price of a domestic risk-free discount bond which has the same maturity date as the option and pays one dollar at maturity. B_f is the foreign counterpart of B_d. We assume that the four factors follow joint geometric Brownian motions:

$$\frac{dS}{S} = \mu_s dt + \sigma_s dz_1 \tag{1}$$

$$\frac{dX}{X} = \mu_x dt + \sigma_x dz_2 \tag{2}$$

$$\frac{dB_d}{B_d} = \mu_{B_d} dt + \sigma_{B_d} dz_3 \tag{3}$$

$$\frac{dB_f}{B_f} = \mu_{B_f} dt + \sigma_{B_f} dz_4 \tag{4}$$

where $z_i(t)$ ($i = 1, 2, 3$, and 4) are four standard Wiener processes; μ_s, μ_x, μ_{Bd}, and μ_{Bf} are instantaneous drift rates of S, X, B_d, and B_f, respectively; σ_s, σ_x, σ_{Bd} and σ_{Bf} are instantaneous standard deviations of S, X, B_d, and B_f and at most are a function of time. The instantaneous correlations, which are constants by assumption, are as follows:

	dz_1	dz_2	dz_3	dz_4
dz_1	1	ρ_{sx}	ρ_{sBd}	ρ_{sBf}
dz_2	..	1	ρ_{xBd}	ρ_{xBf}
dz_3	..	..	1	ρ_{BdBf}
dz_4	..	..	..	1

Based on the stochastic processes for S, X, B_d, and B_f, we follow Merton (1973) and Grabbe (1983) to derive a general partial differential equation (PDE) which must be satisfied by all cross-currency options.[6] This PDE will then be solved for each category of cross-currency options subject to proper boundary conditions. Since this technique is now well known in the literature, we will omit the detailed derivation and simply state the results for each category of cross-currency options.[7]

Pricing Category I Cross-Currency Options

Category I cross-currency options are special in that they can be priced simply by arbitrage arguments. Notice that a category I cross-currency option gives exactly the same payoff as that of an ordinary foreign option purchased by a domestic investor. For example, a domestic holder of an ordinary foreign put option will receive $X_T \max[0, K - S_T]$ at maturity, which is the same as the payoff of a category I cross-currency option. Since the ordinary foreign put is worth XP (P denotes the price of the foreign put) for the domestic investor, a category I cross-currency put must have the same value, XP. But P is immediately available via Merton's result (Merton, 1973, p. 162–167). Therefore using the previous notation, the values of cross-currency European put and call options are:

$$P_I(S, X, B_f, t) = X[KB_f N(-\alpha_2) - Se^{-q(T-t)} N(-\alpha_1)] \tag{5}$$

$$C_I(S, X, B_f, t) = X[Se^{-q(T-t)} N(\alpha_1) - KB_f N(\alpha_2)] \tag{6}$$

where N(·) is a standard normal probability function, and:

$$\alpha_1 = \frac{\ln(\frac{S}{KB_f}) + (-q + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}, \quad \alpha_2 = \alpha_1 - \sigma\sqrt{T-t}$$

$$\sigma^2 = \frac{1}{T-t}\int_t^T \left(\sigma_s^2(\tau) - 2\sigma_{sB_f}(\tau) + \sigma^2_{B_f}(\tau)\right)d\tau$$

If σ_s and the foreign interest rate are constant, all variance and covariance terms involving B_f become zero, and the pricing formulas in (5) and (6) reduce to:[8]

$$P_I(S, X, t) = X[Ke^{-r_f(T-t)} N(-a_2) - Se^{-q(T-t)} N(-a_1)] \tag{7}$$

$$C_I(S, X, t) = X[Se^{-q(T-t)} N(a_1) - Ke^{-r_f(T-t)} N(a_2)] \tag{8}$$

where:

$$a_1 = \frac{\ln(\frac{S}{K}) + (r_f - q + \frac{\sigma_s^2}{2})(T-t)}{\sigma_s\sqrt{T-t}}, \quad a_2 = a_1 - \sigma_s\sqrt{T-t}$$

Pricing Category II Cross-Currency Options

For category II cross-currency options, Merton's results can not be applied directly, since both the domestic and the foreign risk-free interest rates enter the pricing model. We must solve the PDE directly. The pricing formulas for category II put and call options are as follows:

$$P_{II}(S, B_d, B_f, t) = X_0 B_d \left[KN(-\beta_2) - \frac{S}{B_f} e^{-(\lambda+q)(T-t)} N(-\beta_1) \right] \quad (9)$$

$$C_{II}(S, B_d, B_f, t) = X_0 B_d \left[\frac{S}{B_f} e^{-(\lambda+q)(T-t)} N(\beta_1) - KN(\beta_2) \right] \quad (10)$$

where:

$$\beta_1 = \frac{\ln(\frac{S}{KB_f}) + (-\lambda - q + \frac{\Psi^2}{2})(T-t)}{\Psi\sqrt{T-t}}, \quad \beta_2 = \beta_1 - \Psi\sqrt{T-t}$$

and:

$$\Psi^2 = \frac{1}{T-t}\int_t^T [\sigma_s^2(\tau) - 2\sigma_{sB_f}(\tau) + \sigma^2{}_{B_f}(\tau)]d\tau$$

$$\lambda = \frac{1}{T-t}\int_t^T [\sigma_{sx}(\tau) + \sigma_{B_dB_f}(\tau) + \sigma_{sB_f}(\tau) - \sigma_{sB_d}(\tau) - \sigma_{xB_f}(\tau) - \sigma^2{}_{B_f}(\tau)]d\tau$$

When σ_s, σ_x and the interest rates are constant, all variance and covariance terms involving B_d and B_f become zero, and the pricing formulas in (9) and (10) reduce to the following:

$$P_{II}(S, X, t) = X_0 \left[Ke^{-r(T-t)}N(-b_2) - Se^{-(r+q+\sigma_{sx}-r_f)(T-t)}N(-b_1) \right] \quad (11)$$

$$C_{II}(S, X, t) = X_0 \left[Se^{-(r+q+\sigma_{sx}-r_f)(T-t)}N(b_1) - Ke^{-r(T-t)}N(b_2) \right] \quad (12)$$

where:

$$b_1 = \frac{\ln(\frac{S}{K}) + (r_f - q - \sigma_{sx} + \frac{\sigma_s^2}{2})(T-t)}{\sigma_s\sqrt{T-t}}, \quad b_2 = b_1 - \sigma_s\sqrt{T-t}$$

Pricing Category III Cross-Currency Options

Again, solving the PDE directly, we obtain pricing formulas for category III cross-currency put and call options:

$$P_{III}(S, X, B_d, B_f, t) = XB_fKN(-\gamma_2) - \frac{X_0 S}{B_f} B_d e^{-(\lambda+q)(T-t)} N(-\gamma_1) \quad (13)$$

$$C_{III}(S, X, B_d, B_f, t) = \frac{X_0 S}{B_f} B_d e^{-(\lambda+q)(T-t)} N(\gamma_1) - XB_fKN(\gamma_2) \quad (14)$$

where:

$$\gamma_1 = \frac{\ln(\frac{X_0 S B_d}{XKB_f^2}) + (-\lambda - q + \frac{\xi^2}{2})(T-t)}{\xi\sqrt{T-t}}, \quad \gamma_2 = \gamma_1 - \xi\sqrt{T-t}$$

and:

$$\xi^2 = \frac{1}{T-t}\int_t^T [\sigma_s^2(\tau) - 2\sigma_{sx}(\tau) + \sigma_x^2(\tau) + \sigma_{B_d}^2(\tau) - 4\sigma_{sB_f}(\tau) + 4\sigma^2{}_{B_f}(\tau)$$
$$+ 4\sigma_{xB_f}(\tau) + 2\sigma_{sB_d}(\tau) - 2\sigma_{sB_d}(\tau) - 4\sigma_{B_dB_f}(\tau)]d\tau$$

λ is defined as before. When interest rates and the volatilities of the foreign asset price and the exchange rate are constant, (13) and (14) reduce to:

$$P_{III}(S, X, t) = X \cdot Ke^{-r_f(T-t)}N(-c_2) - X_0 Se^{-(r+q+\sigma_{sx}-r_f)(T-t)}N(-c_1) \quad (15)$$

$$C_{III}(S, X, t) = X_0 Se^{-(r+q+\sigma_{sx}-r_f)(T-t)}N(c_1) - XKe^{-r_f(T-t)}N(c_2) \quad (16)$$

where:

$$c_1 = \frac{\ln(\frac{X_0 S}{XK}) + (2r_f - r - q - \sigma_{sx} + \frac{u^2}{2})(T-t)}{u\sqrt{T-t}},$$

$$c_2 = c_1 - u\sqrt{T-t}, \quad u = \sqrt{\sigma_s^2 - 2\sigma_s\sigma_x\rho + \sigma_x^2}$$

Pricing Category IV Cross-Currency Options

Note that the payoff of a category IV cross-currency option is such that the value of the option depends on the product of X and S. But SX is simply the foreign asset price measured in domestic currency. Using this fact and directly solving the PDE, we get the following formulas for category IV cross-currency put and call options:

$$P_{IV}(S, X, B_d, t) = X_0 KB_d N(-\delta_2) - SXe^{-q(T-t)}N(-\delta_1) \quad (17)$$

$$C_{IV}(S, X, B_d, t) = SXe^{-q(T-t)}N(\delta_1) - X_0 KB_d N(\delta_2) \quad (18)$$

where:

$$\delta_1 = \frac{\ln(\frac{SX}{B_d KX_0}) + (-q + \frac{\omega^2}{2})(T-t)}{\omega\sqrt{T-t}}, \quad \delta_2 = \delta_1 - \omega\sqrt{T-t}$$

$$\omega^2 = \frac{1}{T-t}\int_t^T [\sigma_s^2(\tau) + \sigma_x^2(\tau) + \sigma_{B_d}^2(\tau) + 2\sigma_{sx}(\tau) - 2\sigma_{sB_d}(\tau) - 2\sigma_{xB_d}(\tau)]d\tau$$

If the domestic interest rate and the volatilities for S and X are constant, then the formulas in (17) and (18) simplify to:

$$P_{IV}(S, X, t) = X_0Ke^{-r(T-t)}N(-d_2) - SXe^{-q(T-t)}N(-d_1) \quad (19)$$

$$C_{IV}(S, X, t) = SXe^{-q(T-t)}N(d_1) - KX_0e^{-r(T-t)}N(d_2) \quad (20)$$

where:

$$d_1 = \frac{\ln(\frac{SX}{KX_0}) + (r - q + \frac{v^2}{2})(T-t)}{v\sqrt{T-t}}, \quad d_2 = d_1 - v\sqrt{T-t}$$

$$v = \sqrt{\sigma_s^2 + 2\rho\sigma_s\sigma_x + \sigma_x^2}$$

It can be seen that the pricing formulas for European cross-currency options with stochastic interest rates (and time dependent variances) are in the same form as those for European cross-currency options with constant interest rates and variances. However, there are three adjustments in the new formulas. First, the interest rates now are the yields on risk-free discount bonds (both domestic and foreign), and are changing over time. Second, the volatility terms (σ^2, Ψ^2, ξ^2, and ω^2) are time-weighted averages of the component variances and covariances. Finally, for category II and category III cross-currency options, the term λ is a time-weighted average of variance and covariances. In the case of constant interest rates and volatilities, λ simply takes the value of σ_{sx}.

The above results make intuitive sense. When stochastic interest rates are introduced to European style cross-currency options, as far as the time value of money is concerned, we are only interested in the yields-to-maturity of the (maturity matching) discount bonds. However, the presence of non-constant interest rates brings about extra risk, which is captured in the form of variances (of the discount bonds) and covariances.

INVESTIGATING PERFORMANCE OF CONSTANT INTEREST RATE MODELS

The previous section shows that when interest rates are assumed to be constant, pricing formulas are dramatically simplified for all categories of cross-currency options. At least two advantages can be identified for the constant-interest-rate models: fewer parameters and easy implementation.[9] Regarding the former, it is conceivable that any potential pricing accuracy gained by using the stochastic

interest rate model can easily be swamped by the estimation errors associated with the extra parameters.[10] For these reasons, there is a natural tendency to use constant interest rate models to price cross-currency options. Therefore one has to ask an important question: Do constant interest rate models perform reasonably well in a stochastic interest rate environment?[11] If the answer is yes, then it is advantageous to use them in lieu of their stochastic counterparts. The rest of the paper will address this question.

To answer the previously stated question, we need to compare the option prices generated from two sets of models. It can be seen in the second section that the constant-interest-rate models leave out two effects: non-flat yield curves and fluctuations of the interest rate level over time. In a stochastic interest rate model, the former is captured by the bond prices, and the latter by the extra variance/covariance terms. Theoretically speaking, we should examine both effects when comparing the two sets of models. For example, to examine the yield curve effect, we could use the current short-term rates for the constant interest rate models. But so doing will unfairly favor the stochastic interest rate models, because in practice people would use the yield (on a discount bond) to option's maturity rather than the spot rate as the constant interest rate. In other words, the omission of yield curve effect of constant interest rate models is corrected when the models are actually used.[12] Therefore a more meaningful comparison should employ the yields (of discount bonds) to the expiration of the option as the risk-free rates for constant interest rate models, so that we could concentrate merely on the second effect, the uncertainty of the interest rates.

We will use the following methodology. We assume that the true world is a stochastic interest rate environment, and hence, the stochastic interest rate models developed in the second section will be the correct/precise models to use. To see if we could use the constant rate models (as approximations) in this environment, we will compare model prices under different parameter inputs. What we hope to establish is that the model prices are sufficiently close under normal circumstances (i.e., with normal ranges of parameter inputs), and thus, for simplicity, constant interest rate models can be used to price cross-currency options even though we are in a stochastic environment.

To implement the stochastic interest rate models, we will use an Ornstein-Uhlenbeck process to proxy the interest rates so that the term structure of interest rates (represented by the bond prices) and the term structure of bond volatilities can be derived.[13] Specifically, we assume that the domestic and the foreign risk-free interest rates following joint Ornstein-Uhlenbeck processes:[14]

$$dr_d = k_d(\mu_d - r_d) + \sigma_d dz_d \quad (21)$$

$$dr_f = k_f(\mu_f - r_f) + \sigma_f dz_f \quad (22)$$

where z_d and z_f are standard Wiener processes with a constant instantaneous correlation ρ_{df}, k_d and k_f are measures of reversion speed, μ_d and μ_f are reversion targets, and σ_d and σ_f are instantaneous standard deviations. These parameters are all assumed to be constant. The discount bond prices and variance/covariance terms in the stochastic interest rate models are given in the appendix. The pricing formulas with stochastic interest rates can now be restated. Specifically, we substitute the bond prices and variances/covariances in the appendix into the pricing formulas in the second section, and obtain new formulas stated in the parameters of the interest rate processes.

We are now ready to investigate the performance of constant interest rate models. As mentioned early, the effect of interest rate risk is reflected in the extra variance/covariance terms. It can be seen from the appendix that the long-term reversion targets, μ_d and μ_f, do not affect the variance/covariance terms. Only the interest rate volatility and the reversion speed parameters play a role. Here, we will carry out the performance analyses by experimenting with different volatility values while keeping the reversion speed parameters constant. In addition, to make the calculations more realistic, we take the Nikkei 225 index as the underlying foreign asset. The model parameters are roughly in line with the market conditions between 1989 and 1991.[15] To reveal the interest rate risk effect on long-term options, we assume a time-to-maturity of three years for all options. Let P_S and P_C be the option price when the interest rates are stochastic and constant, respectively. We calculate a percentage difference in the two prices: $100*(P_C - P_S)/P_S$. For ease of exposition, we will call this percentage difference the "pricing error," the error caused by using a constant interest rate model in a stochastic interest rate environment.

Category I Cross-Currency Options

For category I cross-currency options, only the foreign (Japanese) risk-free interest rate enters the pricing model. We assume that the Japanese spot interest rate is 0.065. We calculate pricing biases by varying σ_f between 0.005 and 0.065.[16] We looked at both put and call options with different levels of the index (but fixed exercise price and exchange rates). The results are reported in Table 1. There are several interesting observations. First, for the set of assumed parameter values, the constant interest rate model underprices cross-currency call and put options. The underpricing is entirely due to the smaller volatility input in the constant interest rate model. Second, when σ_f is below 0.02 (corresponding to a volatility of 31%), the pricing bias is within 2 percent (in absolute terms) with only a few exceptions. Third, out-of-the-money options tend to have the biggest pricing biases. This is to be expected since (deep) in-the-money options tend to be insensitive to interest rate changes. Finally, the size of percentage bias is positively related to the level of interest rate volatility. This is true for both call and put options, no matter whether the options are out-of-the-money, at-the-money, or in-the-money. But the relation-

Table 1. Percentage Pricing Errors Caused by Assuming Constant Interest Rates[a]
Category I Cross-Currency Options
—Varying the Foreign Interest Rate Standard Deviation, σ_f—

The Foreign Interest Rate Standard Deviation, σ_f	Cross-Currency Calls Current Asset Price, S^b					Cross-Currency Puts Current Asset Price, S^b				
	21,000	23,000	25,000	27,000	29,000	21,000	23,000	25,000	27,000	29,000
0.005	−0.22%	−0.17%	−0.13%	−0.10%	−0.07%	−0.26%	−0.34%	−0.42%	−0.50%	−0.59%
0.010	−0.48	−0.36	−0.27	−0.21	−0.16	−0.57	−0.73	−0.90	−1.08	−1.27
0.015	−0.77	−0.58	−0.44	−0.34	−0.26	−0.92	−1.18	−1.45	−1.75	−2.05
0.020	−1.10	−0.84	−0.63	−0.48	−0.37	−1.31	−1.68	−2.07	−2.48	−2.92
0.025	−1.47	−1.12	−0.85	−0.65	−0.50	−1.74	−2.23	−2.75	−3.29	−3.86
0.030	−1.88	−1.42	−1.08	−0.83	−0.64	−2.21	−2.83	−3.48	−4.17	−4.89
0.035	−2.31	−1.76	−1.34	−1.03	−0.79	−2.72	−3.47	−4.27	−5.11	−5.98
0.040	−2.79	−2.12	−1.62	−1.24	−0.95	−3.26	−4.15	−5.10	−6.10	−7.13
0.045	−3.29	−2.51	−1.92	−1.47	−1.13	−3.83	−4.87	−5.98	−7.14	−8.34
0.050	−3.82	−2.92	−2.24	−1.72	−1.33	−4.43	−5.63	−6.90	−8.22	−9.59
0.055	−4.39	−3.36	−2.57	−1.98	−1.53	−5.05	−6.41	−7.85	−9.35	−10.89
0.060	−4.98	−3.82	−2.93	−2.26	−1.75	−5.70	−7.22	−8.83	−10.50	−12.21
0.065	−5.60	−4.30	−3.31	−2.56	−1.98	−6.37	−9.06	−9.84	−11.68	−13.57

Notes: a. The percentage differences are calculated as $100*(P_C - P_S)/P_S$, where P_S and P_C are option prices with stochastic interest rates and constant interest rates, respectively. In the constant interest rate model, the yield on the foreign discount bond is taken as the constant interest rate.

b. The foreign asset is assumed to be the Nikkei 225 index. All option prices are calculated based on a single exercise price K = ¥25,000. Other parameter values are as follows: $X = 0.008$ \$/¥, $T-t = 3$ years, $q = 0.004$, $\sigma_s = 0.25$, $r_f = 0.065$, $k_f = 0.5$, $\mu_f = 0.08$, $\rho_{sf} = -0.1$, $\pi_f = 0.0$.

ship is relatively stronger for out-of-the-money options. For example, when σ_f increases from 0.005 to 0.065, the pricing bias for a deep in-the-money put (S = 21,000) increases from −0.26 percent to −6.37 percent. But the corresponding bias for a deep out-of-the-money put (S = 29,000) increases from −0.59 percent to −13.57 percent.

Category II Cross-Currency Options

Since the value of a category II cross-currency option depends on two interest rates (domestic and foreign), we will examine two cases: (1) constant foreign interest rate standard deviation σ_f with varying domestic interest rate standard deviation σ_d, and (2) constant domestic interest rate standard deviation σ_d with varying foreign interest rate standard deviation σ_f. The results are summarized in Tables 2 and 3.

Case 1. It is seen in Table 2 that the constant interest rate model underprices call and overprices put options, which implies that the extra drift term λ takes on larger values in the stochastic rate model.[17] The pricing biases are very insensitive to the domestic interest rate volatility. When the domestic interest rate standard deviation is not very high (within 0.02, e.g.), the pricing bias is generally within 3 percent (in absolute terms).

Case 2. Table 3 reports pricing biases for the case where we keep the domestic interest rate standard deviation constant while varying its foreign counterpart, σ_f. Similar to case 1, the constant interest rate model also tends to underprice calls and overprice puts. When the foreign interest rate standard deviation is below 0.02, the pricing bias is generally within 1 percent (in absolute terms). Unlike case 1, the pricing biases are no longer insensitive to the level of interest rate standard deviation. Finally, in both cases, the pricing biases are higher for out-of-the-money option.

Categories III and IV Cross-Currency Options

For category III cross-currency options, we also examine two cases: (1) constant foreign interest rate standard deviation σ_f with varying domestic interest rate standard deviation σ_d, and (2) the opposite of (1). The results are summarized in Tables 4 and 5. The value of a category IV cross-currency option is independent of the foreign interest rate. Therefore, we calculate the pricing errors by varying the domestic interest rate standard deviation only. The results are reported in Table 6. It can be seen that the performance of constant interest rate models for these two categories are very similar to that for the first two categories. For brevity, we will not reiterate the results here.

Table 2. Percentage Pricing Errors Caused by Assuming Constant Interest Rates[a]
Category II Cross-Currency Options
—Varying the Domestic Interest Rate Standard Deviation, σ_d—

The Domestic Interest Rate Standard Deviation, σ_d	*Cross-Currency Calls Current Asset Price,* S^b					*Cross-Currency Puts Current Asset Price,* S^b				
	21,000	*23,000*	*25,000*	*27,000*	*29,000*	*21,000*	*23,000*	*25,000*	*27,000*	*29,000*
0.005	−3.07%	−2.78%	−2.55%	−2.36%	−2.19%	1.70%	1.84%	1.97%	2.08%	2.18%
0.010	−3.05	−2.77	−2.54	−2.35	−2.18	1.69	1.83	1.95	2.07	2.17
0.015	−3.04	−2.76	−2.53	−2.34	−2.17	1.68	1.82	1.94	2.05	2.16
0.020	−3.03	−2.75	−2.52	−2.33	−2.16	1.67	1.81	1.93	2.04	2.14
0.025	−3.02	−2.74	−2.51	−2.31	−2.15	1.66	1.80	1.92	2.03	2.13
0.030	−3.00	−2.72	−2.50	−2.30	−2.14	1.65	1.79	1.91	2.02	2.11
0.035	−2.99	−2.71	−2.49	−2.29	−2.13	1.64	1.78	1.90	2.00	2.10
0.040	−2.98	−2.70	−2.47	−2.28	−2.12	1.63	1.77	1.88	1.99	2.09
0.045	−2.97	−2.69	−2.46	−2.27	−2.11	1.62	1.76	1.87	1.98	2.07
0.050	−2.95	−2.68	−2.45	−2.26	−2.10	1.61	1.75	1.86	1.97	2.06
0.055	−2.94	−2.67	−2.44	−2.25	−2.09	1.60	1.74	1.85	1.95	2.04
0.060	−2.93	−2.65	−2.43	−2.24	−2.09	1.59	1.73	1.84	1.94	2.03
0.065	−2.91	−2.64	−2.42	−2.23	−2.08	1.58	1.71	1.83	1.93	2.02

Notes: a. The percentage differences are calculated as $100*(P_C - P_S)/P_S$, where P_S and P_c are option prices with stochastic interest rates and constant interest rates, respectively. In the constant interest rate model, the yield on the foreign discount bond is taken as the constant interest rate.

b. The foreign asset is assumed to be the Nikkei 225 index. All option prices are calculated based on a single exercise price K = ¥25,000. Other parameter values are as follows: $X_0 = 0.0085$ \$/¥, $T-t = 3$ years, $q = 0.004$, $\sigma_s = 0.25$, $\sigma_x = 0.12$, $r_d = 0.1$, $k_d = 0.5$, $r_f = 0.065$, $k_f = 0.5$, $\sigma_f = 0.05$, $\rho_{sx} = 0.3$, $\rho_{sd} = 0.02$, $\rho_{sf} = -0.10$, $\rho_{xd} = 0.04$, $\rho_{xf} = -0.20$, $\rho_{df} = -0.04$, $\mu_d = 0.12$, $\mu_f = 0.08$, $\pi_d = 0.0$, $\pi_f = 0.0$.

Table 3. Percentage Pricing Errors Caused by Assuming Constant Interest Rates[a]
Category II Cross-Currency Options
—Varying the Foreign Interest Rate Standard Deviation, σ_f—

The Foreign Interest Rate Standard Deviation, σ_f	*Cross-Currency Calls Current Asset Price, S^b*					*Cross-Currency Puts Current Asset Price, S^b*				
	21,000	*23,000*	*25,000*	*27,000*	*29,000*	*21,000*	*23,000*	*25,000*	*27,000*	*29,000*
0.005	0.38%	0.32%	0.28%	0.24%	0.22%	0.02%	0.05%	0.09%	0.13%	0.17%
0.010	0.41	0.33	0.27	0.22	0.19	0.21	0.29	0.37	0.46	0.56
0.015	0.34	0.25	0.19	0.14	0.10	0.40	0.51	0.63	0.76	0.90
0.020	0.16	0.09	0.03	–0.01	–0.04	0.58	0.72	0.87	1.03	1.20
0.025	–0.11	–0.17	–0.20	–0.23	–0.25	0.76	0.93	1.09	1.27	1.45
0.030	–0.49	–0.50	–0.51	–0.51	–0.51	0.94	1.11	1.29	1.47	1.66
0.035	–0.96	–0.92	–0.89	–0.86	–0.82	1.11	1.29	1.47	1.65	1.83
0.040	–1.53	–1.43	–1.34	–1.26	–1.19	1.28	1.46	1.62	1.79	1.95
0.045	–2.20	–2.01	–1.86	–1.73	–1.62	1.45	1.61	1.75	1.89	2.03
0.050	–2.95	–2.68	–2.45	–2.26	–2.10	1.61	1.75	1.86	1.97	2.06
0.055	–3.80	–3.42	–3.11	–2.85	–2.64	1.77	1.87	1.95	2.01	2.05
0.060	–4.73	–4.23	–3.83	–3.50	–3.23	1.92	1.99	2.02	2.02	2.00
0.065	–5.74	–5.12	–4.62	–4.21	–3.87	2.07	2.10	2.07	2.00	1.91

Notes: a. The percentage differences are calculated as $100*(P_C - P_S)/P_S$, where P_S and P_C are option prices with stochastic interest rates and constant interest rates, respectively. In the constant interest rate model, the yield on the foreign discount bond is taken as the constant interest rate.

b. The foreign asset is assumed to be the Nikkei 225 index. All option prices are calculated based on a single exercise price K = ¥25,000. Other parameter values are as follows: $X_0 = 0.0085$ \$/¥, $T-t = 3$ years, $q = 0.004$, $\sigma_s = 0.25$, $\sigma_x = 0.12$, $r_d = 0.1$, $k_d = 0.5$, $\sigma_d = 0.05$, $r_f = 0.065$, $k_f = 0.5$, $\rho_{sx} = 0.3$, $\rho_{sd} = 0.02$, $\rho_{sf} = -0.10$, $\rho_{xd} = 0.04$, $\rho_{xf} = -0.20$, $\rho_{df} = -0.04$, $\mu_d = 0.12$, $\mu_f = 0.08$, $\pi_d = 0.0$, $\pi_f = 0.0$.

Table 4. Percentage Pricing Errors Caused by Assuming Constant Interest Rates[a]
Category III Cross-Currency Options
—Varying the Domestic Interest Rate Standard Deviation, σ_d—

The Domestic Interest Rate Standard Deviation, σ_d	*Cross-Currency Calls Current Asset Price, S^b*					*Cross-Currency Puts Current Asset Price, S^b*				
	21,000	*23,000*	*25,000*	*27,000*	*29,000*	*21,000*	*23,000*	*25,000*	*27,000*	*29,000*
0.005	–12.00%	–9.78%	–8.05%	–6.68%	–5.61%	–5.31%	–7.16%	–9.17%	–11.29%	–13.48
0.010	–12.08	–9.84	–8.10	–6.72	–5.63	–5.39	–7.26	–9.30	–11.44	–13.65
0.015	–12.19	–9.93	–8.16	–6.77	–5.67	–5.51	–7.41	–9.48	–11.65	–13.89
0.020	–12.34	–10.05	–8.26	–6.84	–5.73	–5.66	–7.60	–9.71	–11.91	–14.20
0.025	–12.52	–10.19	–8.37	–6.93	–5.80	–5.85	–7.83	–9.99	–12.24	–14.57
0.030	–12.74	–10.36	–8.51	–7.04	–5.88	–6.07	–8.11	–10.31	–12.62	–15.01
0.035	–12.99	–10.56	–8.66	–7.16	–5.98	–6.32	–8.42	–10.69	–13.06	–15.51
0.040	–13.28	–10.79	–8.84	–7.30	–6.09	–6.60	–8.77	–11.11	–13.56	–16.08
0.045	–13.59	–11.04	–9.04	–7.46	–6.21	–6.92	–9.16	–11.58	–14.11	–16.70
0.050	–13.93	–11.31	–9.26	–7.63	–6.35	–7.26	–9.59	–12.10	–14.70	–17.38
0.055	–14.30	–11.61	–9.49	–7.82	–6.50	–7.64	–10.06	–12.65	–15.35	–18.11
0.060	–14.70	–11.93	–9.74	–8.02	–6.66	–8.04	–10.56	–13.25	–16.04	–18.89
0.065	–15.12	–12.26	–10.01	–8.23	–6.83	–8.48	–11.10	–13.89	–16.78	–19.72

Notes: a. The percentage differences are calculated as $100*(P_C - P_S)/P_S$, where P_S and P_C are option prices with stochastic interest rates and constant interest rates, respectively. In the constant interest rate model, the yield on the foreign discount bond is taken as the constant interest rate.

b. The foreign asset is assumed to be the Nikkei 225 index. All option prices are calculated based on a single exercise price K = ¥25,000. Other parameter values are as follows: $X_0 = 0.0085$ \$/¥, $T-t = 3$ years, $q = 0.004$, $\sigma_s = 0.25$, $\sigma_x = 0.12$, $r_d = 0.1$, $k_d = 0.5$, $r_f = 0.065$, $k_f = 0.5$, $\sigma_f = 0.05$, $\rho_{sx} = 0.3$, $\rho_{sd} = 0.02$, $\rho_{sf} = -0.10$, $\rho_{xd} = 0.04$, $\rho_{xf} = -0.20$, $\rho_{df} = -0.04$, $\mu_d = 0.12$, $\mu_f = 0.08$, $\pi_d = 0.0$, $\pi_f = 0.0$.

Table 5. Percentage Pricing Errors Caused by Assuming Constant Interest Rates[a]
Category III Cross-Currency Options
—Varying the Foreign Interest Rate Standard Deviation, σ_f—

The Foreign Interest Rate Standard Deviation, σ_f	*Cross-Currency Calls Current Asset Price,* S^b					*Cross-Currency Puts Current Asset Price,* S^b				
	21,000	*23,000*	*25,000*	*27,000*	*29,000*	*21,000*	*23,000*	*25,000*	*27,000*	*29,000*
0.005	−2.48%	−1.87%	−1.40%	−1.05%	−0.78%	−2.22%	−2.87%	−3.58%	−4.33%	−5.12%
0.010	−2.90	−2.20	−1.67	−1.27	−0.97	−2.41	−3.12	−3.90	−4.73	−5.60
0.015	−3.56	−2.74	−2.11	−1.64	−1.27	−2.70	−3.52	−4.42	−5.36	−6.35
0.020	−4.47	−3.47	−2.71	−2.13	−1.69	−3.10	−4.06	−5.11	−6.21	−7.37
0.025	−5.59	−4.39	−3.47	−2.76	−2.21	−3.60	−4.73	−5.96	−7.27	−8.63
0.030	−6.93	−5.48	−4.37	−3.51	−2.84	−4.18	−5.52	−6.97	−8.50	−10.09
0.035	−8.45	−6.73	−5.41	−4.38	−3.58	−4.85	−6.42	−8.11	−9.88	−11.73
0.040	−10.14	−8.13	−6.58	−5.36	−4.41	−5.60	−7.40	−9.35	−11.40	−13.51
0.045	−11.97	−9.67	−7.86	−6.45	−5.34	−6.40	−8.47	−10.69	−13.01	−15.41
0.050	−13.93	−11.31	−9.26	−7.63	−6.35	−7.26	−9.59	−12.10	−14.70	−17.38
0.055	−16.00	−13.06	−10.74	−8.90	−7.45	−8.16	−10.77	−13.56	−16.44	−19.40
0.060	−18.14	−14.90	−13.32	−10.26	−8.62	−9.10	−11.98	−15.05	−18.22	−21.44
0.065	−20.36	−16.80	−13.96	−11.68	−9.86	−10.05	−13.21	−16.56	−20.00	−23.48

Notes: a. The percentage differences are calculated as $100*(P_C - P_S)/P_S$, where P_S and P_C are option prices with stochastic interest rates and constant interest rates, respectively. In the constant interest rate model, the yield on the foreign discount bond is taken as the constant interest rate.

b. The foreign asset is assumed to be the Nikkei 225 index. All option prices are calculated based on a single exercise price $K =$ ¥25,000. Other parameter values are as follows: $X_0 = 0.0085$ \$/¥, $T-t = 3$ years, $q = 0.004$, $\sigma_s = 0.25$, $\sigma_x = 0.12$, $r_d = 0.1$, $k_d = 0.5$, $\sigma_d = 0.05$, $r_f = 0.065$, $k_f = 0.5$, $\rho_{sx} = 0.3$, $\rho_{sd} = 0.02$, $\rho_{sf} = -0.10$, $\rho_{xd} = 0.04$, $\rho_{xf} = -0.20$, $\rho_{df} = -0.04$, $\mu_d = 0.12$, $\mu_f = 0.08$, $\pi_d = 0.0$, $\pi_f = 0.0$.

Table 6. Percentage Pricing Errors Caused by Assuming Constant Interest Rates[a]
Category IV Cross-Currency Options
—Varying the Domestic Interest Rate Standard Deviation, σ_d—

The Domestic Interest Rate Standard Deviation, σ_d	*Cross-Currency Calls Current Asset Price,* S^b					*Cross-Currency Puts Current Asset Price,* S^b				
	21,000	*23,000*	*25,000*	*27,000*	*29,000*	*21,000*	*23,000*	*25,000*	*27,000*	*29,000*
0.005	−0.05%	−0.04%	−0.03%	−0.03%	−0.02%	−0.08%	−0.09%	−0.11%	−0.13%	−0.15%
0.010	−0.13	−0.10	−0.08	−0.06	−0.05	−0.19	−0.23	−0.27	−0.32	−0.36
0.015	−0.23	−0.18	−0.14	−0.11	−0.09	−0.33	−0.40	−0.48	−0.56	−0.64
0.020	−0.35	−0.27	−0.22	−0.18	−0.14	−0.51	−0.62	−0.74	−0.86	−0.98
0.025	−0.49	−0.39	−0.31	−0.25	−0.20	−0.72	−0.88	−1.04	−1.21	−1.39
0.030	−0.66	−0.52	−0.42	−0.33	−0.27	−0.96	−1.18	−1.39	−1.62	−1.85
0.035	−0.85	−0.67	−0.54	−0.43	−0.35	−1.24	−1.51	−1.79	−2.08	−2.37
0.040	−1.06	−0.84	−0.67	−0.54	−0.44	−1.54	−1.88	−2.23	−2.59	−2.95
0.045	−1.30	−1.03	−0.82	−0.66	−0.53	−1.88	−2.28	−2.71	−3.14	−3.58
0.050	−1.56	−1.24	−0.99	−0.79	−0.64	−2.24	−2.72	−3.22	−3.74	−4.26
0.055	−1.84	−1.46	−1.17	−0.94	−0.76	−2.63	−3.19	−3.78	−4.38	−4.99
0.060	−2.14	−1.70	−1.36	−1.10	−0.89	−3.04	−3.69	−4.36	−5.05	−5.76
0.065	−2.46	−1.96	−1.57	−1.26	−1.02	−3.47	−4.22	−4.98	−5.77	−6.56

Notes: a. The percentage differences are calculated as $100*(P_C - P_S)/P_S$, where P_S and P_C are option prices with stochastic interest rates and constant interest rates, respectively. In the constant interest rate model, the yield on the foreign discount bond is taken as the constant interest rate.

b. The foreign asset is assumed to be the Nikkei 225 index. All option prices are calculated based on a single exercise price $K =$ ¥25,000. Other parameter values are as follows: $X_0 = 0.0085$ \$/¥, $T-t = 3$ years, $q = 0.004$, $\sigma_s = 0.25$, $\sigma_x = 0.12$, $r_d = 0.1$, $k_d = 0.5$, $\sigma_d = 0.08$, $\rho_{sx} = 0.3$, $\rho_{sd} = 0.02$, $\rho_{xd} = 0.04$, $\mu_d = 0.12$, $\pi_d = 0.0$.

Discussions

Overall, the results in Table 1 through Table 6 suggest that ignoring interest rate risk can cause some downward pricing errors (i.e., underpricing). The significance of the mispricing depends on the category of the cross-currency option and the level of the interest rate volatility. Except for category III cross-currency options, the pricing errors are generally within three percent when the interest rate volatility is not very high (i.e., below 30%). Given that the existing cross-currency options are of categories I, II, and IV only, we can thus conclude that the constant interest rate models can be used in lieu of the complex stochastic rate models. This conclusion is further strengthened by the fact that the stochastic rate models require more parameter inputs. For example, for a category II cross-currency option, the stochastic rate model requires eight extra parameters: σ_d, σ_f, k_d, k_f, ρ_{df}, ρ_{sf}, ρ_{sd}, and ρ_{xf}.[18] The effects of potential estimation errors associated with these extra parameters will easily overwhelm the marginal gains in accuracy from using the stochastic rate models.

In this analysis, we have used the discount bond yields as the interest rate inputs for the constant interest rate models. Similar analyses (not reported here) are also carried out using spot rates as the interest rate inputs. The results are very similar except that the magnitude of mispricing tends to be larger. This is to be expected because the use of spot rates will ignore not only the volatility effect but also the yield curve effect. As mentioned earlier, spot rates are seldom used as model inputs in a stochastic interest rate environment.[19] Therefore our conclusions remain valid.

Finally, two caveats are in order. First, for these findings the size and direction of pricing errors depend, among other things, on the parameter values of the interest rate processes. Second, the comparison of the constant interest rate model (which employs discount bond yields as the interest rate inputs) and the stochastic interest rate model bears mainly practical implications. Theoretically speaking, fitting a discount bond yield (generated from a stochastic world) into a constant interest rate model is not warranted, since it is not compatible with the model's underlying assumptions. It is tempting to argue that a constant interest rate model (which uses yields as the interest rate inputs) *is* a stochastic interest rate model assuming that the extra variance and covariance terms in the latter are negligible. Whether the "negligibility" assumption is valid is a pure empirical question. Our limited simulation results seem to support the assumption.

CONCLUSIONS

This study investigates the pricing of options on foreign assets when interest rates are stochastic. Our major objective has been to examine the performance of constant interest rate pricing models for cross-currency options in a stochastic interest rate environment. To this end, stochastic interest rate models are developed using standard techniques as in Merton (1973) and Grabbe (1983). Pricing errors of constant interest rate models are then investigated using Ornstein-Uhlenbeck interest rate processes. It is shown that the use of constant interest rate models (after adjusting for the yield curve effect) is justified when the interest rate volatilities are not very high. This conclusion is based on two facts: (1) the magnitude of the biases is generally small (within 3%) for reasonable ranges of parameter values; and (2) the stochastic interest rate models require many extra parameters whose estimation errors will swamp the limited gain in pricing accuracy. One important practical implication of the analyses is that constant interest rate models can be used to price different categories of options on foreign assets when interest rates are actually stochastic. This holds even if the options have a time to maturity as long as three years.

APPENDIX

Calculation of Bond Prices and Variance/Covariance Terms

Let $B_d(r_d, t, T)$ and $B_f(r_f, t, T)$ denote the domestic and foreign discount bond prices respectively. Then according to Vasicek (1977), $B_d(r_d, t, T)$ and $B_f(r_f, t, T)$ can be solved as:

$$B_i(r_i, t, T) = D_i\, e^{-r_i E_i}$$

where:

$$E_i \equiv E_i(t,T) = \frac{1 - e^{-k_i(T-t)}}{k_i}$$

$$D_i \equiv D_i(t, T) = e^{m_i(E_i - (T-t)) - \frac{\sigma_i^2 E_i^2}{4k_i}}$$

$$m_i = \mu_i + \frac{\sigma_i \pi_i}{k_i} - \frac{\sigma_i^2}{2k_i^2}$$

$$(i = d, f)$$

In the above, π_d (π_f) denotes the market price of the domestic (foreign) interest rate risk. In addition, applying Ito's lemma to $B_d(r_d, t, T)$ and $B_f(r_f, t, T)$ leads to:

$$\sigma_{B_d}(t, T) = E_d(t, T)\sigma_d$$

$$\sigma_{B_f}(t, T) = E_f(t, T)\sigma_f$$

and

$$dz_3 = -\,dz_d, \quad dz_4 = -\,dz_f$$

The correlation matrix in the second section can be re-written as:

	dz_1	dz_2	dz_d	dz_f
dz_1	1	ρ_{sx}	ρ_{sd}	ρ_{sf}
dz_2	..	1	ρ_{xd}	ρ_{xf}
dz_d	..	..	1	ρ_{df}
dz_f	..	..	..	1

and

$$\rho_{sB_d} = -\rho_{sd}, \quad \rho_{sB_f} = -\rho_{sf}, \quad \rho_{xB_d} = -\rho_{xd}$$

$$\rho_{xB_f} = -\rho_{xf}, \quad \rho_{B_dB_f} = \rho_{df}$$

Finally, using the results stated in the text, we can restate the volatility parameters and the λ factor in terms of the interest rate process parameters. For the pricing formulas for category I cross-currency options, the volatility is

$$\sigma^2 = \sigma_s^2 + \frac{\sigma_f^2}{k_f^2} + \frac{2\sigma_{sf}}{k_f} - \left[\frac{\sigma_f^2}{(T-t)k_f^2} + \frac{2\sigma_{sf}}{(T-t)k_f}\right]E_f - \frac{\sigma_f^2 E_f^2}{2(T-t)k_f}$$

Similarly, the time-weighted parameters in the pricing formulas for the other three categories of cross-currency options can be calculated as follows:

$$\lambda = \sigma_{sx} + \frac{(T-t)\sigma_{df} - (E_f + E_d)\sigma_{df}}{k_d k_f (T-t)} + \frac{[1 - e^{-(k_d+k_f)(T-t)}]\sigma_{df}}{k_d k_f (k_d + k_f)(T-t)}$$

$$+ \frac{(2\sigma_{sf} - 2\sigma_{xf})E_f + \sigma_f^2 E_f^2}{2(T-t)k_f} + \frac{\sigma_{xf} - \sigma_{sf}}{k_f} + \frac{\sigma_{sd}(T-t-E_d)}{(T-t)k_d} + \frac{\sigma_f^2[E_f - (T-t)]}{(T-t)k_f^2}$$

$$\psi^2 = \sigma^2$$

$$\xi^2 = \sigma_s^2 - 2\sigma_{sx} + \sigma_x^2 + \frac{\sigma_d^2}{k_d^2} - \frac{\sigma_d^2 E_d}{(T-t)k_d^2} + \frac{4\sigma_f^2}{k_f^2} - \frac{4\sigma_f^2 E_f}{(T-t)k_f^2}$$

$$+ \frac{(\sigma_{sd} - \sigma_{xd})[4E_d - 2(T-t)] - \sigma_d^2 E_d^2}{2(T-t)k_d} + \frac{4(\sigma_{sf} - \sigma_{xf})(T-t-E_f) - 2\sigma_f^2 E_f^2}{(T-t)k_f}$$

$$- \frac{4\sigma_{df}}{T-t}\left[\frac{(T-t) - (E_f + E_d)}{k_d k_f} + \frac{1 - e^{-(k_d+k_f)(T-t)}}{k_d k_f (k_d + k_f)}\right]$$

$$\omega^2 = \sigma_s^2 + 2\sigma_{sx} + \sigma_x^2 + \frac{\sigma_d^2}{k_d^2} + \frac{2(\sigma_{sd} + \sigma_{xd})}{k_d}$$

$$- \frac{\sigma_d^2 + 2k_d(\sigma_{sd} + \sigma_{xd})}{(T-t)k_d^2}E_d - \frac{\sigma_d^2 E_d^2}{2(T-t)k_d}$$

In the above formulas, a σ with two subscripts denotes a covariance.

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NOTES

1. One example is the so-called "quantos" (Reiner, 1992). A quanto is a foreign index warrant whose payoff is converted at a guaranteed exchange rate. Most of the listed Nikkei put warrants are quantos. They are American put options on the Nikkei 225 index, whose yen payoffs are converted to dollars at a prespecified exchange rate. See Rubinstein (1991) for a description of the guaranteed warrants.
2. Typical work includes Adams and Wyatt (1987), Amin and Jarrow (1991), Biger and Hull (1983), Bodurtha and Courtadon (1987), Choi and Hauser (1990), Garman and Kohlhagen (1983), Grabbe (1983), Hull and White (1987), Ritchken and Sankarasubramanian (1991), Shastri and Tandon (1986), and Shastri and Tandon (1987).
3. These papers include Derman, Karasinski, and Wecker (1990), Dravid, Richardson, and Sun (1991), Gruca and Ritchken (1991), Eric Reiner (1992), Rumsey (1991), Shaw, Thorp, and Ziemba (1991), Ziemba (1991), and Wei (1992a).
4. The Nikkei put warrants mentioned were all issued with three-years to maturity. See Wei (1992b) for details.
5. A continuous dividend yield is assumed because most cross-currency instruments are based on foreign stock indexes. The dividend streams on most indexes are approximately continuous.
6. Other papers that incorporate stochastic interest rates into the pricing of conventional options include: Choi and Hauser (1990), Grabbe (1983), Hilliard, Madura, and Tucker (1991), Rabinovitch (1989), and Ramaswamy and Sundaresan (1985).
7. Detailed derivations can be obtained from the author upon request.
8. In this case, $B_f = e^{-r_f(T-t)}$. Similarly, when the domestic interest rate is constant, $B_d = e^{-r(T-t)}$.
9. "Easy implementation" refers to the fact that the integrals appearing in the stochastic-interest-rate models have been eliminated.
10. I thank an anonymous referee for pointing this out.
11. "Reasonably well" is to be interpreted as: option prices generated from the constant interest rate models are sufficiently close to those from the stochastic rate models.
12. I thank John Hull and Alan White for pointing this out to me.
13. Several authors have derived discount bond and bond option prices based on stochastic interest rates. Vasicek (1977) pioneered an Ornstein-Uhlenbeck process for the short-term interest rate and derived a formula for discount bond prices. Cox, Ingersoll, and Ross (1985) assumed a more complicated interest rate process (with a constant elasticity variance, CEV) and derived closed-form formulas for discount bond and European bond option prices. European bond option pricing formulas based on an Ornstein-Uhlenbeck interest rate process are later independently derived by Chaplin (1987), Jamshidian (1989), and Rabinovitch (1989). Using the same interest rate process, Chen (1992) derived closed-form pricing formulas for futures and European futures options on discount bonds.
14. Our choice here is basically model-driven. As correctly pointed out by Rabinovitch (1989), a CEV process is incompatible with the bond processes we have assumed. Specifically, a CEV process

implies a stochastic bond volatility, whereas we require that the bond volatility is at most time-dependent. This fact reveals the limitations of Merton's stochastic interest rate model, and hence of the models developed here.

15. This is a period during which all major foreign index warrants, especially Nikkei put warrants, were actively traded.

16. Given a spot interest rate of 0.065, the range of σ_f from 0.005 to 0.065 translates to an interest rate volatility range from 7.7 percent (0.005/0.065) to 100.0 percent (0.065/0.065). Normally, the interest rate volatility is below 20 percent.

17. Notice that the effect of the drift term λ is analogous to that of the dividend yield. It is well known that a call (put) option's value is negatively (positively) related to the dividend yield.

18. Note that we used the discount bond yields as the interest rate inputs in the constant interest rate models. We would need σ_d, σ_f, k_d, k_f, and other parameters if we must actually *calculate* the yields. But in practice yields can be directly calculated from discount bond prices which are generally available as market data.

19. An anonymous referee also pointed this out.

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