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Empirical Tests of the Pricing of Nikkei Put Warrants

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Abstract

The purpose of this study is to empirically examine the pricing of Nikkei put warrants, which are long-term put options written on the Nikkei 225 index. Using warrants traded on the Toronto Stock Exchange, this study performs various tests on the pricing models proposed by Dravid, Richardson, and Sun [11], Reiner [24], and Wei [31]. It is found that the models tend to overprice the warrants. The overpricing, possibly caused by the omission of the credit risk and the Extraordinary Event Clause, is found to be positively related to the degree to which the warrants are in the money, the volatility level, and the trading volume.

Introduction

Recently cross-currency derivative securities have gained increasing popularity. Major exchanges in North America have listed options on such foreign stock indexes as the Nikkei 225, FT-SE 100, and CAC 40. Among the listed foreign index warrants, the most popular is the Nikkei Put Warrant (NPW hereinafter). For instance, the American Stock Exchange (AMEX) listed six different issues of NPWs in early 1990. The Toronto Stock Exchange (TSE) also listed six different issues of NPWs (see the appendix for details). Despite the increasing popularity of the foreign index warrants, the literature has been lacking in formal treatments on

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these instruments in terms of both theoretical pricing and empirical testing. There are a few exceptions. Rumsey [26] discusses the estimation of volatilities for a particular class of cross-currency options.[1] Reiner [24] shows in a heuristic fashion how certain cross-currency options can be priced. Wei [31] discusses specific issues for the pricing of NPWs. Wei [32] also examined the pricing of cross-currency options in a broader setting. A study that is closely related to the current paper is by Dravid, Richardson, and Sun [11]. Working with NPWs traded on the American Stock Exchange, they compare the trading prices with the model prices and conclude that the pricing models perform reasonably well. Other related papers are by Derman, Karasinski, and Wecker [10] and Gruca and Ritchken [16]. Except for Dravid, Richardson, and Sun [11] and Wei [32], the above cited papers concentrate on the description and theoretical pricing of cross-currency products. No attempt is made to empirically assess the pricing models.

The purpose of this study is to empirically examine the pricing models for NPWs developed in Dravid, Richardson, and Sun [11], Reiner [24], and Wei [31]. As an attempt in the literature to empirically investigate the NPW market, this study will shed light on the performance of the models and related issues. This should constitute the major contribution of the study. NPWs traded on the Toronto Stock Exchange are used to perform the tests. This choice is dictated by the fact that there are more varieties on the TSE than on the AMEX.[2] The remainder of the paper first briefly describes the NPWs and introduces the pricing models. It then describes the data and the tests, followed by discussions on empirical results and the conclusion.

The Nikkei Put Warrants and Their Valuation

The Nikkei Put Warrants

An NPW is a put option written on the Nikkei 225 index. The key difference between an NPW and a conventional put option is that an NPW is traded in dollars while the underlying asset (i.e., the Nikkei 225 index) is denominated in yen. Depending on how the yen payoff is converted into dollars, there are four possible payoff specifications. Notation:

S = Nikkei 225 index level in yen;
X = current US$/Jap¥ (or Cdn$/Jap¥) exchange rate;
X_0 = pre-specified, fixed exchange rate;
K = exercise price in yen;
r = domestic riskfree interest rate (constant);
r_f = Japanese riskfree interest rate (constant);
σ_s = annual volatility of the Nikkei index;
σ_x = annual volatility of the exchange rate;
ρ = correlation coefficient between the index and the exchange rate;
q = continuous dividend yield on the Nikkei index;
t = current time;
T = maturity date.

The possible specifications can then be summarized as follows:[3]

Category	Payoff upon Exercise in Dollars
Category I NPWs:	$\text{Max}[0, X_T(K - S_T)]$
Category II NPWs:	$\text{Max}[0, X_0(K - S_T)]$
Category III NPWs:	$\text{Max}[0, X_TK - X_0S_T]$
Category IV NPWs:	$\text{Max}[0, X_0K - X_TS_T]$

For the first category, the yen payoff of the warrant is converted into dollars at the prevailing exchange rate, X_T. For the second category, a pre-specified exchange rate, X_0, is used to convert the yen payoff. The fixed change rate, X_0, is specified at the time of issue and remains the same throughout the life of the warrant. The payoff conversion is slightly complicated for the last two categories. When a Category III NPW is exercised, the exercise price is converted at the prevailing exchange rate X_T, while the terminal Nikkei index level is converted at a pre-specified exchange rate X_0. It is the opposite for a Category IV NPW. Here, the exercise price is converted up-front at the fixed exchange rate X_0, and hence stays constant in dollars, whereas the Nikkei index is converted at the spot exchange rate. At any point in time, the quantity XS is simply the dollar price of the Nikkei index. Except for Category III, all the other categories of NPWs have appeared on either the American Stock Exchange or the Toronto Stock Exchange, or both. All listed NPWs are American style options in that they can be exercised anytime before maturity.

Due to the time zone difference, the exercise of an NPW can not be settled immediately. A simultaneous quote for the Nikkei index does not exist when the North American markets are open. As a result, NPW issuers typically specify a one- or two- (business) day exercise delay. On the TSE, Trilon Financial Corporation NPWs have a two-day exercise delay, while the other warrants have a one-day exercise delay. For instance, if an exercise notice is delivered on Thursday for a Trilon Financial Corporation NPW, the issuer will wait and use the following Monday's close of the Nikkei and the exchange rate to settle the exercise. For other warrants, Friday's closing data will be used for the exercise settlement.

Valuation of Nikkei Put Warrants[4]

Assuming joint geometric Brownian motions for the Nikkei index and the exchange rate in a Black-Scholes environment, Dravid, Richardson, and Sun [11], Reiner [24], and Wei [31] independently show that all categories of ordinary cross-currency options can be priced within a one-state-variable framework despite the presence of two underlying variables (i.e. the foreign index and the exchange rate). Wei [31] specifically shows that the one-state-variable framework also applies to NPWs. The dimension reduction enables Wei [31] to derive closed-form pricing formulas for all categories of European NPWs. For American style NPWs, closed-form formulas do not exist. Numerical procedures such as a binomial tree must be used. Generally speaking, a three-dimension tree is required to handle a two-state-variable American option. However, Wei [31] shows that the dimension reduction with European NPWs also extends to American NPWs.[5] Therefore, only a two-dimension tree (e.g., [9]) is needed. Specifically, Wei [31] suggests using either a binomial [9] or a trinomial [4] lattice, the framework of which can be summarized as follows.[6]

	Current Value of the State Variable	Strike Price	Drift Rate	Variance	Dividend Yield	Discount Rate
Category I	SX	KX	r_f	σ_s^2	q	r_f
Category II	SX_0	KX_0	$r_f - 2\rho\sigma_s\sigma_x$	σ_s^2	q	r
Category IV	SX	KX_0	r	$\sigma_s^2 + 2\rho\sigma_s\sigma_x + \sigma_x^2$	q	r

When pricing an NPW using the above framework, one proceeds as if an ordinary lattice tree is being built to price an ordinary put option. The only difference is that different inputs are used for the tree construction. For example, to price a Category II NPW, a trinomial lattice is built for a state variable with SX_0 being its current value, σ_s its volatility, and $(r_f - 2\rho\sigma_s\sigma_x)$ its drift. (In contrast, in a standard lattice, the risk-neutral drift is r.) Then one works backwards along the tree to solve for the American NPWs value with exercise price KX_0. (See Wei [31] for a detailed example of implementing the above framework.)

It should be pointed out that two approximation assumptions are necessary in order to apply the pricing framework to American NPWs. First, it is assumed that the closing level of the index from the previous trading session in Tokyo is the current value of S (i.e., prices are assumed to be simultaneous). Second, the exercise delay can be ignored, and the exercise of a warrant will be settled immediately. Obviously, pricing errors are introduced when making these two assumptions. However, as shown in Wei [31], the pricing errors are quite small. Therefore, for practical purposes, the approximations are well warranted.

In this study the above pricing framework is tested. Specifically, a 100-step trinomial lattice [4] in conjunction with the control variate technique [17] is used to price American NPWs traded on the Toronto Stock Exchange. The model prices are then compared with the market prices. The European counterpart of each warrant is used as the control variable when implementing the control variate technique. Closed-form pricing formulas for European NPWs are given in Wei [31].

Test Design

The Data

The data used in this study come from four sources: the Nikkei Telecom—Japan News & Retrieval, RBC Dominion Securities Inc., Nomura Inc. Canada, and the financial newspapers. Specifically, the warrant data set was originally supplied by the RBC Dominion Securities Inc. and was updated/completed using daily quotations from the *Globe and Mail* daily newspaper. The data set contains daily high, low, close, and volume for the six warrants listed on the Toronto Stock Exchange. The AB Svensk Exp. Corp. warrant is deleted from the data set due to thin trading. The sample period ends on May 31, 1991, for all five remaining warrants. The starting date varies. The sample period starts on June 14, 1989, for BT Bank of Canada Series I and II NPWs, and April 10, 1990, for the other two series. For Trilon Financial Corporation NPWs, the data start on February 23, 1990.

The daily open, high, low and close quotes for the Nikkei 225 are retrieved from the Nikkei Telecom—Japan News & Retrieval. Daily quotations for the Cdn$/Jap¥ exchange rate in the same time period are taken from the *Globe and Mail* newspaper.

Yields on government bonds with maturity dates close to those of the warrants are used in lieu of the risk-free interest rates. Because the warrants expire on different dates, bonds with different maturities are used. Daily Canadian government bond prices are obtained from the *Financial Post*. Their Japanese counterparts are obtained from Nikkei Telecom—Japan News & retrieval. Continuously compounded yields are calculated from bond prices.[7]

Finally, monthly dividend yield data, starting in January 1980 and ending in December 1990, are obtained from Nomura Inc. Canada. The yields in this period are used to obtain a simple moving average forecast for the next two years, which is 0.43 percent per annum. It is assumed that a continuous dividend yield of 0.43 percent will prevail during the data period.[8]

The Tests

The major objective is to test the predicting power of the proposed valuation models reviewed earlier. Broadly speaking, there are two classes of tests in the literature that investigate the predicting power of option models. Tests in the first class make direct comparisons of actual prices and model prices. If the model is valid, then the predicted model price should not deviate systematically from the actual price (e.g., [3, 20, 21, 30]). Tests in the second class are based on volatilities implied from actual option/stock prices, using the tested pricing model. If the model is valid (and if the market is efficient, and the other parameter inputs are accurate), then the implied volatilities should behave as dictated by the model. For example, if the Black-Scholes model is valid, then the implied volatility should be stationary over time, across maturities and exercise prices (e.g., [1, 5, 6, 25, 28]). The methodology employed in this study falls into the first class.

Two questions can be asked: 1) Are the market prices of the NPWs significantly different from those predicted by the pricing models? 2) If the answer to question 1) is "yes", are there any systematic relationships between the deviations and the model parameters, such as time to maturity? A deviation test will be carried out to answer the first question, and regression tests to answer the second question. Because model prices are needed to complete the tests, it is necessary to illustrate how the unobservable parameters are estimated.

Estimation of the Unobservable Parameter Values. It can be seen in the previous section that the unobservable parameters include the index volatility σ_s, the exchange rate volatility σ_x, and the correlation coefficient between the index and the exchange rate ρ. For Category I NPWs, only the index volatility enters the pricing model. For Category IV NPWs, the three parameters enter the pricing model as a single input $\upsilon^2 = \sigma_s^2 + 2\sigma_{sx} + \sigma_x^2$ ($\sigma_{sx} \equiv \rho\sigma_s\sigma_x$). For Category II NPWs, the three parameters enter the pricing model as two inputs, σ_s and σ_{sx}. There are two alternative approaches to estimating these parameters. The first approach involves calculating historical volatilities and correlation coefficient and using them as the required parameter estimates. The second approach involves computing the parameter values from the pricing models using values of the observable variables/parameters. The well known drawback of the first approach is its backward-looking nature; i.e., it can not incorporate investors' expectations about the future volatilities (and the correlation). Many researchers have used this approach for conventional options.[9] Because all NPWs have a relatively longer time to maturity, there is a unique problem which is less of a concern for ordinary options with shorter maturities: How far back should the data go in order to calculate a historical parameter value that will be used as a forward-looking estimate applicable to the next two (or three) years? If the data do not go back far enough (say, using data for the past three months), then there is the risk of getting a totally wrong estimate;[10] if, however, the data go back too far (say, using data for the past three years), there is the problem of putting too much weight on the remote observations that are less relevant for predicting the future. In light of this dilemma, the second approach is used in this study, whose major advantage is the ability to incorporate investors' expectations. The drawback, of course, is its reliance on a particular pricing model. If the model is the one under study, then this approach is subject to the error of "using the model to test the model." The commonly adopted technique to overcome this problem is to use the previous day's implied volatility as the current day's volatility estimate. Moreover, multiple options are used so that a weighted average of implied volatility can be calculated as the volatility forecast.

In this study, the implied volatilities and covariance are calculated in the following steps.

a) Compute the volatilities and covariances for day $t-1$ based on the observable variables on day $t-1$. Specifically, for Category I NPWs, σ_s is imputed; for Category IV NPWs, $v = \sqrt{\sigma_s^2 + 2\sigma_{sx} + \sigma_x^2}$ is imputed as a single parameter; for Category II NPWs, σ_s is imputed given a historical estimate of $\rho\sigma_x$.[11,12]

b) Calculate a weighted average of the implied index volatilities from Category I and Category II NPWs, with the weight being the degree to which the NPWs are in the money.[13] Specifically, the weighted average, denoted by $WGISD_{t-1}$, is calculated as,

$$WGISD_{t-1} = \left(\sum_{i=1}^{4} \sigma_s^{t-1}(i) / |K_t - S_{t-1}| \right) \Bigg/ \left(\sum_{i=1}^{4} 1/|K_i - S_{t-1}| \right)$$

where i denotes the NPWs used to compute the individual σ_s, and K_i is the strike price of warrant i. (There are one Category I and three Category II NPWs). The vertical bars stand for an absolute value operator. In the above procedures the warrant prices are the midpoint averages of high and low prices. The main rationale for using the high-low average in favor of the closing price is to achieve a better match between the index level and the warrant price. Recall that the Nikkei close from the previous trading session in Tokyo will be used as the model input. The averaging will hopefully even out the impact of inaccurate index input.

Deviation Tests. The first question posed before can be answered by calculating the absolute and percentage deviations between the market prices and the model prices. If the models are correct, then the deviations should not be significantly different from zero. The testing procedures are as follows.

a) Using the implied volatility and other model inputs, calculate the model price, P_t^{mod}, for each warrant on day t. A time series of model prices is obtained for each warrant.

b) For each warrant on day t, calculate the absolute deviation $P_t^{mkt} - P_t^{mod}$ and the percentage deviation $100\%*(P_t^{mkt} - P_t^{mod}) / P_t^{mod}$. The warrant market price, P_t^{mkt}, is the daily closing price.

c) Repeat b) with P_t^{mkt} being replaced by the midpoint average of the high and low prices.

Regression Tests, Bias Analysis. The second question concerns the relationship between the prediction errors and the (potential) systematic effects of the model inputs. Many authors have tried to relate pricing biases to such model inputs as time to maturity, the degree to which the option is in the money, and the volatility.[14] In this study the relative deviations are regressed on similar model inputs. Specifically, the following time-series regressions are run for each warrant:

$$PD_t = \gamma_0 + \gamma_1 Y_t + \varepsilon_t$$

where $PD_t = (P_t^{mkt} - P_t^{mod}) / P_t^{mod}$ (P_t^{mkt} is the midpoint average of high and low prices), and Y_t is the independent variable. For NPWs in categories I and II, Y_t is $(K - S)/K$ (the degree to which the warrants are in-the-money), τ (time to maturity), σ_s (volatility), and *ln(volume)* (log of the trading volume). Therefore, there are four separate regressions for each warrant. For Category IV NPWs, Y_t is $(KX_0 - SX)/KX_0$, τ, υ (volatility), and *ln(volume)*. There are also four separate regressions for each warrant.

If the models perfectly predict the market prices, then the two coefficients, γ_0 and γ_1, should not be significantly different from zero for each regression. On the other hand, if the models produce consistent prediction biases, but the biases are not systematically related to the model inputs, then for each regression the intercept γ_0 should be a non-zero number, while the coefficient γ_1 should not be statistically different from zero.

The Empirical Results

Deviation Tests

The results for deviation tests are shown in Table 1 and Table 2. By examining the average deviations (both absolute and percentage), it can be seen that the models tend to overprice NPWs. (The BT Bank of Canada Series III NPW is an exception.) This is true for both sets of market prices (i.e., the closing prices and high-low average prices). The average overpricing ranges from 7.5¢ to 42.1¢ per warrant, and the *t*-values for the absolute deviations are all statistically significant. Although the models seem to underprice BT Bank of Canada Series III NPWs, the *t*-values are not significant for the absolute average deviations.

The average deviations can not tell the whole story. In a particular time-series of pricing deviations, if the number and magnitude of the negative and positive deviations are such that the two types of deviations cancel each other, then the overall average deviation is zero, indicating that the model values the warrants correctly. This is, of course, a misleading inference. To address this issue, deciles are provided for the deviations for each warrant. In addition, the proportions and averages of negative and positive deviations are calculated for each warrant. If, for example, the proportion of negative deviations is high and their absolute average is bigger than that of the positive deviations, then it can be concluded that the model tends to overprice warrants most of the time. The numbers in Table 1 and Table 2 generally support this conclusion. It can be seen that, when closing prices are used to conduct the tests, all median deviations are negative. When the high-low average prices are used, all median deviations are again negative, with only one exception: the BT Bank of Canada Series III NPWs. Moreover, for each warrant, the average size of negative (absolute) deviations is bigger than that of the positive deviations. This should come as no surprise, since the overall averages are negative with significant *t*-values.

TABLE 1

Results of Absolute Deviation Tests

This table contains the absolute deviations measured in dollars. The numbers with the percentage symbol are the proportions of negative and positive deviations. The numbers in the parentheses are averages of the negative or positive category. For example, in Panel A, for BTI NPWs, 58 percent (285/490 ≈ 0.58) of the observed deviations are negative, while 42 percent (205/490) are positive deviations. The averages of the negative and positive deviations are –0.245 and 0.156, respectively.

	BTI	BTII	BTIII	BTIV	TFC
Panel A: Absolute Deviations Based on Closing Prices of NPWs[a]					
Minimum	–1.734	–1.744	–1.610	–2.206	–1.350
10th Percentile	–0.484	–0.568	–0.659	–1.007	–0.889
20th Percentile	–0.251	–0.329	–0.413	–0.765	–0.531
30th Percentile	–0.138	–0.164	–0.243	–0.572	–0.450
40th Percentile	–0.066	–0.087	–0.163	–0.390	–0.320
Median	–0.030	–0.045	–0.005	–0.281	–0.159
60th Percentile	0.007	0.000	0.194	–0.216	–0.035
70th Percentile	0.050	0.025	0.417	–0.133	0.062
80th Percentile	0.103	0.068	0.553	–0.053	0.308
90th Percentile	0.233	0.183	0.860	0.097	0.596
Maximum	0.959	1.207	1.559	0.507	1.673
Average	–0.075	–0.120	0.067	–0.400	–0.154
t-value[b]	–5.463***	–8.084***	1.168	–14.407***	–3.643***
(–) Deviation	58% (–0.245)	60% (–0.285)	50% (–0.408)	84% (–0.511)	65% (–0.463)
(+) Deviation	42% (0.156)	40% (0.125)	50% (0.541)	16% (0.173)	35% (0.410)
# of obs.	490	488	102	285	161

Panel B: Absolute Deviations Based on Averages of High-Low Prices of NPWs[a]

Minimum	–1.734	–1.595	–1.360	–2.605	–1.350
10th Percentile	–0.412	–0.497	–0.668	–1.004	–0.803
20th Percentile	–0.258	–0.297	–0.397	–0.794	–0.611
30th Percentile	–0.136	–0.185	–0.288	–0.557	–0.465
40th Percentile	–0.068	–0.097	–0.163	–0.437	–0.334
Median	–0.028	–0.048	0.019	–0.323	–0.177
60th Percentile	0.003	–0.011	0.176	–0.252	–0.083
70th Percentile	0.029	0.016	0.419	–0.176	0.039
80th Percentile	0.071	0.049	0.536	–0.087	0.212
90th Percentile	0.160	0.099	0.860	0.029	0.439
Maximum	0.885	1.020	1.059	0.444	1.173
Average	–0.076	–0.121	0.049	–0.421	–0.180
t-value [b]	–6.747***	–9.522***	0.894	–16.881***	–4.673***
(–) Deviation	59% (–0.212)	63% (–0.254)	49% (–0.423)	88% (–0.495)	65% (–0.467)
(+) Deviation	41% (0.121)	37% (0.103)	51% (0.505)	12% (0.120)	35% (0.343)
# of obs.	490	488	102	285	161

[a] Column headings: BTI, BTII, BTIII, and BTIV stand for BT Bank of Canada Series I, Series II, Series III, and Series IV NPWs, respectively; TFC stands for Trilon Financial Corp. NPWs.

[b] *: significant at 10 percent level for a two-tail test; **: significant at 5 percent level for a two-tail test; ***: significant at 1 percent level for a two-tail test.

TABLE 2

Results of Percentage Deviation Tests

This table contains deviations measured in percentage. The lines headed by "(–) Deviation" and "(+) Deviation" report the proportions of negative and positive deviations. The numbers in the parentheses are averages of the negative or positive category. For example, in Panel A, for BTI NPWs, 58 percent (285/490 ≈ 0.58) of the observed deviations are negative, while 42 percent (205/490) are positive deviations. The averages of the negative and positive percentage deviations are –4.28 percent and 4.98 percent, respectively.

	BTI	BTII	BTIII	BTIV	TFC
Panel A: Percentage Deviations Based on Closing Prices of NPWs[a]					
Minimum	–25.64%	–22.43	–12.28	–32.60	–21.82
10th Percentile	–7.34%	–9.30	–6.74	–16.82	–9.25
20th Percentile	–4.68%	–6.51	–4.80	–14.00	–6.04
30th Percentile	–3.04%	–4.42	–2.92	–11.32	–4.97
40th Percentile	–1.89%	–2.80	–1.70	–8.89	–3.34
Median	–0.86%	–1.36	–0.07	–6.71	–1.65
60th Percentile	0.24%	–0.11	1.98	–5.26	–0.52
70th Percentile	1.42%	0.86	4.30	–3.46	0.92
80th Percentile	3.21%	2.34	9.30	–1.20	2.77
90th Percentile	6.80%	5.26	13.25	1.89	6.96
Maximum	37.75%	27.19	21.63	11.04	26.44
Average	–0.41%	–1.81	1.92	–7.52	–1.39
t-value[b]	–1.386	–6.439***	2.611***	–17.100***	–2.525**
(–) Deviation	58% (–4.28%)	60% (–5.42%)	50% (–4.04%)	84% (–9.54%)	65% (–5.23%)
(+) Deviation	42% (4.98%)	40% (3.77%)	50% (7.71%)	16% (3.00%)	35% (5.63%)
# of obs.	490	488	102	285	161

Panel B: Percentage Deviations Based on Averages of High-Low Prices of NPWs[a]

Minimum	–18.33%	–17.84	–10.38	–28.70	–19.91
10th Percentile	–6.11%	–8.67	–6.74	–16.52	–8.86
20th Percentile	–4.33%	–5.57	–4.33	–13.88	–6.48
30th Percentile	–2.98%	–4.16	–2.86	–10.83	–4.90
40th Percentile	–1.98%	–2.67	–2.05	–9.41	–3.56
Median	–0.90%	–1.61	0.16	–7.08	–1.89
60th Percentile	0.09%	–0.48	2.10	–6.03	–0.90
70th Percentile	0.96%	0.52	4.24	–4.34	0.51
80th Percentile	2.06%	1.90	7.96	–2.41	2.77
90th Percentile	5.08%	3.82	13.25	0.46	6.36
Maximum	22.17%	17.60	19.23	7.49	18.54
Average	–0.54%	–1.84	1.77	–7.99	–1.86
t-value[b]	–2.237**	–7.851***	2.510**	–20.308***	–3.591***
(–) Deviation	59% (–7.98%)	63% (–4.76%)	49% (–4.12%)	88% (–9.36%)	65% (–5.49%)
(+) Deviation	41% (4.53%)	37% (3.07%)	51% (7.44%)	12% (2.15%)	35% (4.76%)
# of obs.	490	488	102	285	161

[a] Column headings: BTI, BTII, BTIII, and BTIV stand for BT Bank of Canada Series I, Series II, Series III, and Series IV NPWs, respectively; TFC stands for Trilon Financial Corp. NPWs.

[b] *: significant at 10 percent level for a two-tail test; **: significant at 5 percent level for a two-tail test; ***: significant at 1 percent level for a two-tail test.

Finally, although almost all the percentage deviations for all warrants are statistically significantly different from zero, the average size is not extremely large. The biggest average percentage deviation is –7.99 percent associated with BT Bank of Canada Series IV NPWs (Table 2, Panel B). Most average deviations are negative and smaller than 2 percent (in absolute terms). In terms of dispersion, it can be seen that within the twentieth and eightieth percentiles, most deviations are smaller than 7 percent. Note that the t-values for the positive percentage deviations for BT Bank of Canada Series III NPWs are significant, which means overall underpricing. However, relative to other warrants, this particular warrant is the most thinly traded in terms of both the trading volume and the number of trading days, and hence the results should be taken with a grain of salt. Overall, it can be concluded that the models generally tend to overprice warrants.

Regression Tests, Bias Analysis

Since the answer to the first question is "yes," the second question naturally follows: Is the overpricing systematically related to the model inputs? The regression test results are summarized in Table 3 through Table 7. The effects of each independent variable is examined separately.

$(K - S)/K$ (or $(KX_0 - SX)/KX_0$), The Degree to Which the Warrants are in the Money. As shown in Panel A of Tables 3 through 7, the F-values are all significant (except for Trilon Financial Corporation NPW), which means that the relative deviations are significant. Examining F-values alone can not reveal whether the deviations are systematically related to the degree to which the warrants are in the money. It is necessary to look at the regression coefficient of the independent variable. It can be seen that all coefficients (a_1) are negative. (BT Bank of Canada Series IV NPW is an exception.) The above findings seem to indicate that the models tend to overprice (underprice) in-the-money (out-of-the-money) warrants. The conclusion of underpricing out-of-

TABLE 3

Results of Regression Tests—Bias Analysis
Bankers Trust Bank of Canada Series I NPWs (Number of Observations: 490)

This table reports regression results for Bankers Trust Bank of Canada Series I NPWs. Percentage deviations are regressed on A) degree of being in the money, B) time to maturity, C) Nikkei index volatility, and D) natural log of trading volume. The numbers in parentheses are t-values. *: significant at 10 percent level for a two-tail test; **: significant at 5 percent level for a two-tail test; ***: significant at 1 percent level for a two-tail test. For all regressions, the dependent variable PD is defined as $PD = (P^{mkt} - P^{mod})/P^{mod}$, where P^{mkt} and P^{mod} are market and model prices of NPWs. The degree of freedom for the F-test is (1.488). #: significant at 5 percent level; ##: significant at 1 percent level.

Panel A: $PD = a_0 + a_1 \frac{K-S}{K} + \varepsilon$

$\hat{a}_0$	$\hat{a}_1$	R^2	F-value
0.00026	−0.09663	0.080	42.338##
(0.005)	(−6.507)***		

Panel B: $PD = b_0 + b_1\tau + \varepsilon$

$\hat{b}_0$	$\hat{b}_1$	R^2	F-value
−0.04505	0.02323	0.059	30.862##
(−0.863)	(5.555)***		

Panel C: $PD = c_0 + c_1\sigma_s + \varepsilon$

$\hat{c}_0$	$\hat{c}_1$	R^2	F-value
0.07392	−0.29147	0.083	44.369##
(1.434)	(−6.661)***		

Panel D: $PD = e_0 + e_1 \ln(volume) + \varepsilon$

$\hat{e}_0$	$\hat{e}_1$	R^2	F-value
0.01053	−0.00148	0.001	0.647
(0.196)	(−0.804)		

TABLE 4

Results of Regression Tests—Bias Analysis
Bankers Trust Bank of Canada Series II NPWs (Number of Observations: 488)

This table reports regression results for Bankers Trust Bank of Canada Series II NPWs. Percentage deviations are regressed on A) degree of being in the money, B) time to maturity, C) aggregate volatility of the Nikkei index and the exchange rate, and D) natural log of trading volume. The numbers in parentheses are *t*-values. *: significant at 10 percent level for a two-tail test; **: significant at 5 percent level for a two-tail test; ***: significant at 1 percent level for a two-tail test. For all regressions, the dependent variable *PD* is defined as $PD = (P^{mkt} - P^{mod})/P^{mod}$, where P^{mkt} and P^{mod} are market and model prices of NPWs. The degree of freedom for the *F*-test is (1.488). #: significant at 5 percent level; ##: significant at 1 percent level.

Panel A: $PD = a_0 + a_1 \frac{KX_0 - SX}{KX_0} + \varepsilon$				Panel B: $PD = b_0 + b_1\tau + \varepsilon$			
$\hat{a}_0$	$\hat{a}_1$	R^2	*F*-value	$\hat{b}_0$	$\hat{b}_1$	R^2	*F*-value
–0.0071	–0.1362	0.137	76.996##	–0.0647	0.0228	0.062	32.267##
(–0.148)	(–8.775)***			(–1.287)	(5.680)***		
Panel C: $PD = c_0 + c_1 v_s + \varepsilon$				**Panel D: $PD = e_0 + e_1 \ln(volume) + \varepsilon$**			
$\hat{c}_0$	$\hat{c}_1$	R^2	*F*-value	$\hat{e}_0$	$\hat{e}_1$	R^2	*F*-value
0.0276	–0.1537	0.037	18.661##	0.0149	–0.0031	0.010	4.664#
(1.434)	(–6.661)***			(0.288)	(–2.160)**		

TABLE 5

Results of Regression Tests—Bias Analysis
Bankers Trust Bank of Canada Series III NPWs (Number of Observations: 102)

This table reports regression results for Bankers Trust Bank of Canada Series III NPWs. Percentage deviations are regressed on A) degree of being in the money, B) time to maturity, C) volatility of the Nikkei index, and D) natural log of trading volume. The numbers in parentheses are t-values. *: significant at 10 percent level for a two-tail test; **: significant at 5 percent level for a two-tail test; ***: significant at 1 percent level for a two-tail test. For all regressions, the dependent variable PD is defined as $PD = (P^{mkt} - P^{mod})/P^{mod}$, where P^{mkt} and P^{mod} are market and model prices of NPWs. The degree of freedom for the F-test is (1.488). #: significant at 5 percent level; ##: significant at 1 percent level.

Panel A: $PD = a_0 + a_1 \frac{K-S}{K} + \varepsilon$				Panel B: $PD = b_0 + b_1\tau + \varepsilon$			
$\hat{a}_0$	$\hat{a}_1$	R^2	F-value	$\hat{b}_0$	$\hat{b}_1$	R^2	F-value
0.1178	−0.3855	0.268	36.670##	−0.0414	0.0228	0.005	0.524
(1.910)*	(−6.056)***			(−0.576)	(0.724)		
Panel C: $PD = c_0 + c_1\sigma_s + \varepsilon$				Panel D: $PD = e_0 + e_1 \ln(volume) + \varepsilon$			
$\hat{c}_0$	$\hat{c}_1$	R^2	F-value	$\hat{e}_0$	$\hat{e}_1$	R^2	F-value
0.0480	−0.1176	0.011	1.145	0.0571	−0.0047	0.008	0.811
(0.670)	(−1.070)			(0.794)	(−0.901)		

TABLE 6

Results of Regression Tests—Bias Analysis
Bankers Trust Bank of Canada Series IV NPWs (Number of Observations: 285)

This table reports regression results for Bankers Trust Bank of Canada Series IV NPWs. Percentage deviations are regressed on A) degree of being in the money, B) time to maturity, C) volatility of the Nikkei index, and D) natural log of trading volume. The numbers in parentheses are *t*-values. *: significant at 10 percent level for a two-tail test; **: significant at 5 percent level for a two-tail test; ***: significant at 1 percent level for a two-tail test. For all regressions, the dependent variable *PD* is defined as $PD = (P^{mkt} - P^{mod})/P^{mod}$, where P^{mkt} and P^{mod} are market and model prices of NPWs. The degree of freedom for the *F*-test is (1.488). #: significant at 5 percent level; ##: significant at 1 percent level.

Panel A: $PD = a_0 + a_1 \frac{K-S}{K} + \varepsilon$				Panel B: $PD = b_0 + b_1\tau + \varepsilon$			
$\hat{a}_0$	$\hat{a}_1$	R^2	F-value	$\hat{b}_0$	$\hat{b}_1$	R^2	F-value
−0.0933	0.1282	0.047	13.874##	0.1109	−0.0783	0.153	51.217##
(−1.434)	(3.725)***			(1.809)*	(−7.157)***		
Panel C: $PD = c_0 + c_1\sigma_s + \varepsilon$				Panel D: $PD = e_0 + e_1 \ln(volume) + \varepsilon$			
$\hat{c}_0$	$\hat{c}_1$	R^2	F-value	$\hat{e}_0$	$\hat{e}_1$	R^2	F-value
−0.0793	−0.0021	0.000	0.001	0.0457	−0.0109	0.046	13.628##
(−1.190)	(−0.033)			(0.702)	(−3.692)***		

TABLE 7

Results of Regression Tests—Bias Analysis
Trilon Financial Corp. NPWs (Number of Observations: 161)

This table reports regression results for Trilon Financial Corp. NPWs. Percentage deviations are regressed on A) degree of being in the money, B) time to maturity, C) volatility of the Nikkei index, and D) natural log of trading volume. The numbers in parentheses are t-values. *: significant at 10 percent level for a two-tail test; **: significant at 5 percent level for a two-tail test; ***: significant at 1 percent level for a two-tail test. For all regressions, the dependent variable PD is defined as $PD = (P^{mkt} - P^{mod})/P^{mod}$, where P^{mkt} and P^{mod} are market and model prices of NPWs. The degree of freedom for the F-test is (1.488). #: significant at 5 percent level; ##: significant at 1 percent level.

Panel A: $PD = a_0 + a_1 \frac{K-S}{K} + \varepsilon$				Panel B: $PD = b_0 + b_1\tau + \varepsilon$			
$\hat{a}_0$	$\hat{a}_1$	R^2	F-value	$\hat{b}_0$	$\hat{b}_1$	R^2	F-value
–0.0156	–0.0129	0.000	0.064	0.2329	–0.0954	0.116	20.772##
(–0.236)	(–0.252)			(3.747)***	(–4.558)***		
Panel C: $PD = c_0 + c_1\sigma_s + \varepsilon$				**Panel D: $PD = e_0 + e_1\ln(volume) + \varepsilon$**			
$\hat{c}_0$	$\hat{c}_1$	R^2	F-value	$\hat{e}_0$	$\hat{e}_1$	R^2	F-value
–0.0425	0.0956	0.009	1.489	0.1396	–0.0178	0.279	61.379##
(–0.646)	(1.220)			(2.486)	(–7.835)***		

the-money warrants should be taken with a grain of salt, since the warrants are in the money most of the time within the sample period. Until further empirical results are presented, the conclusion has to be treated as tentative. However, given the negative sign of the regression coefficient, it is safe to infer that the overpricing tends to be severe for deep in-the-money warrants.

τ, Time to Maturity. It is well known that when a new security is introduced it takes some time for investors to "learn," or for the security's price to become "well behaved." This "market learning" would be reflected in the decreasing of pricing deviations over time. This market learning hypothesis can be captured by the regression coefficient. Specifically, if there is a market learning effect, the intercept should be close to zero while the slope coefficient should be non-zero. As shown in Panel B of Tables 3 through 7, the *F*-values are all significant except for BT Bank of Canada Series III NPWs (Table 5). The null hypothesis of market learning is confirmed in Tables 3 and 4 (BT Bank of Canada Series I and Series II NPWs), where the intercept is not statistically different from zero but the slope is. For the last two warrants (in Tables 6 and 7) the intercept is also statistically different from zero. This is somewhat disturbing. A non-zero intercept means that even if the warrant is approaching its maturity, there still is a pricing deviation. Finally, no systematic relation between relative deviations and the time to maturity has been found for BT Bank of Canada Series III NPWs (Table 5).

σ_s (or $v = \sqrt{\sigma_s^2 + 2\sigma_{sx} + \sigma_x^2}$), the Volatility. As shown in Panel C of Tables 3 through 7, the regression coefficient is all negative, except for Trilon Financial Corporation NPWs (Table 7). For Series I and II BT Bank of Canada NPWs, the negative coefficient is significantly different from zero. Thus it could be inferred that the models tend to overprice the warrants and the overpricing is more manifest when the volatility is high.

***ln(volume)*, (log of) the Trading Volume.** It can be seen from Panel D of Tables 3 through 7 that the regression intercepts are all close to zero (statistically) but the regression coefficients are all negative, with three (out of five) having statistically significant t-values. This implies that the models over-price the NPWs, and a bigger overpricing tends to be related to high trading volumes. Although the coefficients in Tables 3 and 5 are not significant, the signs of the coefficients are in agreement with those of other regressions.

Overall, the deviation tests reveal that the models tend to overprice Nikkei put warrants. But the average size of the mispricing is generally small (less than 2 percent). The regression tests detect some systematic relationships between the mispricing and various model inputs. Specifically, the overpricing is more manifest when a) the warrants are deep in the money, b) the volatility is high, and c) the trading volume is high.

So far, it has been found that the models overprice warrants and the overpricing is systematically related to some model inputs. A natural question that follows is: What causes the overpricing? The systematic links between the mispricing and the model parameter inputs do not necessarily imply that the model inputs actually cause the mispricing. Instead, it is likely that the mispricing is due to some other unmeasurable factors which are reflected in the model parameters. There are many possible factors. A straightforward one is the omission of credit risk. Unlike conventional options that are guaranteed by the exchanges, Nikkei put warrants are guaranteed only by the issuers. This will put a downward pressure on the warrant prices. Another possible factor is the so-called "Extraordinary Event Clause" applicable to all Nikkei put warrants. An Extraordinary Event Clause is specified in the prospectus of an NPW that would prevent the exercise of NPWs upon the occurrence of certain abnormal events. It is therefore a protector for the issuer against undesirable market conditions. The detailed specifications of an Extraordinary Event Clause vary across warrants. But the common "events" generally include the following:

a) suspension or material limitation of trading in securities on the Tokyo Stock Exchange;

b) suspension or material limitation of trading in Nikkei 225 futures contracts on both the Singapore International Monetary Exchange and the Osaka Stock Exchange; and
c) any outbreak or escalation of national or international calamity or crisis.

Upon the occurrence of an extraordinary event, the issuer will either prevent any exercise of the warrants or settle an exercise at a lower value.

It is obvious that an Extraordinary Event Clause would lower the market price of NPWs in order for warrant holders to be compensated for the commensurate risk. The pricing models being tested above do not incorporate the effect of this Extraordinary Event Clause, so the model prices are biased upwards. This is exactly what has been observed. It is also observed that the overpricing is more severe when the warrants are deep in the money or when the volatility is high. This can also be explained by the omission of the Extraordinary Event Clause. When the warrants are deep in the money, the early exercise possibility increases, which makes the Extraordinary Event Clause more relevant. (When the warrants are deep out of the money, warrant holders would care (relatively) less about the Extraordinary Event Clause.) A high volatility makes trading suspension or limitation more possible; therefore, investors would require a higher risk premium (hence lower price), ceteris paribus. It is easy to see that the above reasoning also applies to the credit risk. (When the warrants are out of the money or when the volatility is low, credit risk is less of an issue.) It is therefore apparent that the true factors responsible for the overpricing are the credit risk and the Extraordinary Event Clause, which are reflected in the related model inputs.[15]

Exactly how investors price the Extraordinary Event Clause is a difficult question to answer. Theoretically, the model prices could be adjusted downwards by incorporating the probabilities of the extraordinary events occurring. But the probability estimation will inevitably be subjective. A more serious difficulty is that the specifications of the clause are not uniform across warrants. Also, it is believed that issuers, notwithstanding their desire to protect themselves, are reluctant to exercise the clause. The main reason is the concern for goodwill. If an issuer (a bank, e.g.) strives to exercise the Extraordinary Event Clause, then it may find making further issues very difficult. This is especially true when there are many issuers and some of them are lenient on the clause.

It should be noted that the discussions here are only speculative and suggestive. Although credit risk and the provision of the Extraordinary Event Clause would intuitively justify positive risk premium, the exactly amount of that premium is unknown. Moreover, there are other potential factors that could cause the observed pricing errors. For instance, the tested models assume constant volatilities and interest rates. To the author's best knowledge, no studies exist in the literature that incorporate stochastic volatilities into the pricing of cross-currency options. Therefore, it is difficult to precisely assess the effect in our context. However, the implied volatilities, which are updated daily, are used in the tests. To the extent that the stochastic nature of volatilities is partly reflected in daily changes, the strategy of updating the implied volatilities should mitigate any potential pricing biases (due to assuming constant volatilities). Nevertheless, until formal empirical results come into existence, it is not known for sure what effect stochastic volatilities will have on the warrant prices.

As for the interest rates, it should be realized that, in general, a constant interest rate pricing model omits two effects of a stochastic interest rate. The first can be called the "yield curve effect." When the interest rate is stochastic, a pricing model using the spot rate as the input for the constant interest rate will miss the effect of the non-flat term structure. The second effect is the volatility of the interest rate.[16] Choi and Hauser [7] have shown that the yield curve effect is very strong for currency options.[17] On the other hand, Wei [33] introduces stochastic interest rates into the pricing of long-term cross-currency options, and examines the pricing errors caused by assuming constant interest rates. Specifically, Wei [33] first corrects for the yield curve effect by using the discount bond yields (rather than spot rates) as the constant interest rates, and then tests the volatility effect. It is found that the constant interest rate models (with bond yields as interest rate inputs) tend to underprice cross-currency options, but the pricing errors are generally very small. In this study, as noted earlier, the bond yields (as opposed to the spot rates) are used as proxies for long term interest rates. Therefore, the results are not subject to the yield curve effect. The only effect that is omitted is the volatility effect, which is small anyway. Therefore it is unlikely that the assumption of constant interest rates is responsible for the observed overpricing.

Conclusions

One of the recent financial innovations in the market place is the formal listing/trading of foreign index warrants. Many exchanges have listed long-term options written on foreign stock indexes. The most popular is the Nikkei Put Warrant. Despite the ever increasing popularity of foreign index warrants, the literature has been lacking in formal treatment on these instruments in terms of both theoretical pricing and empirical testing. This study is an attempt to empirically examine the pricing of Nikkei put warrants.

Using data of Nikkei put warrants traded on the Toronto Stock Exchange, this paper empirically tests the pricing models developed in Dravid, Richardson, and Sun [11], Reiner [24], and Wei [31]. It is found that the models tend to overprice Nikkei put warrants. The overpricing becomes more severe in the following situations: 1) the warrants are deep in the money; 2) the index/exchange rate volatilities are high; and 3) the trading volume is high. It is suggested that the major reason for the overpricing is the omission of credit risk and the existence of Extraordinary Event Clauses. Exactly how much risk premium is attached to the warrant price is unknown, and it is the subject of further research.

Notes

1. In this paper, "cross-currency options" is a general term for options on foreign assets. An NPW is a particular type of cross-currency option.

2. See the following section for a classification of NPWs, NPWs on the AMEX cover only Categories I and II, while those on the TSE Cover categories I, II, and IV.

3. Most of the materials in this section are from Wei [31], which contains more detailed descriptions.

4. Since a Category III NPW does not exist, only the pricing of the remaining three categories will be discussed.

5. Dravid, Richardson, and Sun [11] are Reiner [24] make the same argument in a similar setting.

6. The intuition behind the dimension reduction lies in the nature of the second state-variable, the exchange rate. It is a special variable in the sense that it serves only as a "medium" between the domestic and the foreign economies. For example, *SX* can be treated as a single variable because it is simply the dollar price of the foreign asset.

7. Unlike the case in the US, long term discount bonds do not exist in Japan or Canada. The calculated yields are only approximations of the true discount bond yields. Factors such as taxation may cause coupon-bearing bond yields to be different from the pure discount bond yields. But the difference, if any, should be small as far as warrant pricing is concerned.

8. See Wei [31] for a detailed discussion about this assumption.

9. See, for example, [18] and [27].

10. An extreme example would be to use three-month data around the 1987 market crash to estimate a forward-looking three-year volatility.

11. $\rho\sigma_x$ is estimated using the past 250 observations. The choice of 250, the number of trading days in a year, hopefully will balance the two sides of the aforementioned dilemma. Alternatively, $\rho\sigma_x$ could also be imputed from warrant prices. This is not done in this study, because the magnitude of $\rho\sigma_x$ is small, and any potential pricing bias caused by an inaccurate historic estimate of $\rho\sigma_x$ is likely to be negligible. The computing costs do not justify the marginal gain. Of course, one may correctly argue that $\rho\sigma_x$ can be ignored altogether since it is small. Obviously, the choice here is suboptimal.

12. A secant method [23] is employed for the iterative procedures when calculating implied volatilities.

13. In the Black-Scholes model context, many different ad hoc weighing schemes have been used in estimating the implied volatility. Latane and Rendleman [19] use as the weights the partial derivatives of the option price with respect to the standard deviation. Whaley [34] employs a procedure that minimizes the residual sum of squares between the model and the market option prices when imputing an implied volatility. Choi and Hauser [7] take as the weights the partial derivatives of the option price with respect to the time to maturity. The scheme used in this study is similar to that in Sterk [29], which in turn is based on the findings in Black [2], MacBeth and Merville [20], and MacBeth and Merville [21].

14. See, for example, [7], [15], [25], [27], and [34].

15. The trading volume can be considered as an indirect identifier of the two factors. More specifically, an empirical check reveals that trading volumes are positively correlated with the implied volatilities.

16. In Meton's stochastic interest rate model [22], the "yield curve effect" is reflected in the discount bond price, and the volatility effect is captured in the overall volatility term.

17. As explicitly noted in their paper, Choi and Hauser did not study the volatility effect.

APPENDIX

Nikkei Put Warrants Listed on The Toronto Stock Exchange (TSE)

Warrants	Issue Date	Expiration Date	Issue Size—#wts	Exercise Price	Exercise Multiple[b]	Fixed Exchange Rate	Category
AB Svensk Exp. Corp. I (SEK.WT)[a]	Dec. 1, 1989	Nov. 16, 1992	2,366,181	¥35963.74	0.11680	N/A	Category I
AB Svensk Exp. Corp. II (SEK.WT)[a]	Feb. 7, 1990	Nov. 16, 1992	1,726,651	¥35963.74	0.11680	N/A	Category I
BT Bank of Canada Series I (NKP.WT)	Feb. 17, 1989	Feb. 17, 1992	9,100,000	¥32174.00	0.11680	N/A	Category I
BT Bank of Canada Series II (NKP.WT.A)	Jun. 15, 1989	Jun. 15, 1992	12,375,000	¥33403.00	0.10311	$X_0 = 1/123.47$	Category IV
BT Bank of Canada Series III (NKP.WT.B)	Feb. 16, 1990	Mar. 16, 1993	4,800,000	¥37460.32	0.00092	N/A[c]	Category II
BT Bank of Canada Series IV (NKP.WT.C)	Mar. 22, 1990	Apr. 12, 1993	6,000,000	¥29843.34	0.00116	N/A[c]	Category II
Trilon Financial Corp. (TFC.WT.N)	Feb. 22, 1990	Feb. 22, 1993	3,734,900	¥37460.32	0.00105	N/A[c]	Category II

[a] Although these two series are issued at different times, they are traded on the AMEX as a single issue, due to the same specifications of terms.

[b] The exercise multiple is used by issuers to rescale the payoff so that the warrants can be traded with small denomination. For instance, if a BT Bank of Canada Series II NPW is exercised when the index and the exchange rate are at ¥25000 and 0.008 Cdn$/¥ respectively, then the payoff will be 0.10311 * (33403/123.47 – 25000 * 0.008) = $7.27 (Cdn).

[c] The fixed change rate is not independently specified. It is reflected in the exercise multiple.

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