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Executive stock options and incentive effects due to systematic risk

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Abstract

Existing research on executive stock options mainly focuses on total risk when studying risk incentives. In this study, we use a GARCH option pricing framework to show that the incentive effects of executive stock options depend on the composition of risk. Controlling for total risk, the value of executive stock options increases with systematic risk and this effect is stronger when the total risk is low. Thus, when firms grant standard or non-indexed options, CEOs will have incentives to increase systematic risk even when the total risk remains constant. In contrast, indexed options will provide CEOs with incentives to reduce systematic risk. We therefore conclude that an optimal mix of indexed and non-indexed option grants will provide CEOs with incentives to take the desired level of systematic risk.

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1. Introduction

Executive stock options have become an important compensation vehicle for most companies. They are used to align the CEO's interest with shareholders', i.e., to alleviate the agency problem in the sense of Jensen and Meckling (1976). Typically, stock options are issued at-the-money with a time to maturity of 10 years, and a

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certain vesting period (please see Appendix A for examples of option grants). Empirical evidence seems to support the general rationale of stock options. For example, Hall and Liebman (1998) found a strong link between a firm's performance and its CEO's compensation.

Broadly speaking, the existing literature on executive stock options can be classified into four strands. The first focuses on valuation and incentive effects from the CEO's perspective in light of the CEO's risk-aversion and hedging restrictions. A partial list of studies includes Lambert et al. (1991), Hall and Murphy (2001), Meulbroek (2001a,b), Tian (2001, 2002), and Jin (2002). The second examines the repricing of stock options. Studies on this topic include Brenner et al. (2000), Chance et al. (2000), and Emm and Ince (2000). The third investigates alternative designs of stock options for the purpose of better aligning performance and pay. Johnson and Tian (2000), Meulbroek (2001b), and Tian (2001) are representative studies in this area. The fourth strand of literature, such as Hall and Liebman (1998), and William and Rao (2000), examines the empirical efficacy of stock options. When considering CEOs' risk-aversion, researchers typically assume a certain form of utility function and then find the so-called certainty equivalent value of the stock option. Otherwise, a Black–Scholes setup is generally used to study valuation, incentive effects and repricing.

When studying risk incentives, most researchers only focus on the total risk or the return volatility. Very little attention has been paid to the breakdown of the total risk. One exception is the study by Meulbroek (2001a). From a diversification perspective, Meulbroek (2001a) demonstrated that managers would value the company's stocks and options much less than the market would, simply due to the fact that a major portion of their wealth is tied to the company's fortune. Therefore, CEOs will have an incentive to increase the systematic risk of the company's stock in order to reduce the exposure to the company specific risk.¹ Another exception is Tian (2002). Using a utility and certainty equivalent framework, he showed that stock options create incentives for CEOs to reduce idiosyncratic risk and, under certain conditions, the systematic risk as well. The direction of risk incentive concerning the systematic risk is not always clear due to the trade-off between risk aversion and higher option payoffs. When the systematic risk is in the modest range, the benefit of potential higher payoffs due to higher expected returns more than offsets the cost of higher risk. Both authors relied on the CEO's trading constraint to arrive at their respective conclusions.

This paper examines the risk incentive effects of executive stock options by assuming a GARCH process for the stock returns. We employ the GARCH option pricing theory to break down the total risk into systematic and non-systematic components, and show explicitly how option values depend on the amount of systematic risk. Our use of the GARCH option pricing theory is motivated by two facts. First, this pricing theory offers a constructive answer to a conceptual issue that is difficult to rec-

¹ The discussions of systematic risk versus non-systematic risk also appear in Meulbroek (2001b), with a focus on indexed options. Jin (2002) also studied the relation between CEOs' incentives and their firms' risk characteristics.

oncile with the two largely incompatible pricing paradigms – the capital asset pricing model and the Black–Scholes option pricing model. The former paradigm concludes that only the systematic risk of a capital asset should be priced in equilibrium whereas the latter renders the issue of risk composition irrelevant for derivative contracts.² Second, many empirical studies documented the GARCH effect on exchange-traded options; for example, Duan (1996), Heston and Nandi (2000), Hsieh and Ritchken (2000), Hardle and Hafner (2000), Duan and Zhang (2001) and Christoffersen and Jacobs (2002).

We find that, with non-indexed options, CEOs have an incentive to increase the systematic risk, consistent with the findings by Meulbroek (2001a) but for a different reason. A higher systematic risk will enhance the value of the stock option, even when the total risk remains constant. The incentive to increase the systematic risk is stronger when the total risk is lower. The Black–Scholes model tends to underestimate the value of the stock option when the systematic risk is high, since it is incapable of distinguishing the separate effects of systematic and non-systematic risks. With indexed options, CEOs generally have an incentive to reduce systematic risk. The opposite incentives induced by the two types of options imply that firms could grant a mixture of indexed and non-indexed options to target a desirable composition of systematic and non-systematic risks.

In analyzing the risk-incentive issue, we do not distinguish a stock option's value to the CEO from its cost to the firm as many authors have done (e.g., Lambert et al., 1991; Hall and Murphy, 2001; Meulbroek, 2001a,b; Tian, 2001, 2002). The gap between the two values arises from the fact that the hedging restriction forces the CEO to live with a suboptimal portfolio. Although it is difficult to determine precisely how binding the trading constraint is in reality, some studies have provided empirical evidence indicating that trading restrictions may have a material impact on valuations.³ In this study, we do not explore the asymmetry between the CEO's private value and the firm's cost. However, we argue that our qualitative conclusions would remain valid when trading restrictions are explicitly considered.

Alternatively, our model can be viewed as providing a new and practical approach to determining the cost of executive compensation to the issuing firm with an explicit recognition of the role played by systematic risk. Arguably, the cost issue is of more significant interest to the accounting profession. Even though the incentive aspect is more suitably cast in a principal–agent framework, the cost to the issuing firm is what market-value accounting attempts to capture. In the current literature of

² Dybvig and Ingersoll (1982) demonstrated that applying the CAPM to derivative contracts will lead to arbitrage opportunities.

³ One consequence of trading constraint is for CEOs to exercise options earlier in order to diversify their investment portfolios. Huddart and Lang (1996) examined exercise behavior of over 50,000 employees at eight corporations, and found that employees typically exercise options long before expiration. Hemmer et al. (1996) studied 110 option exercises by 74 top executives, and found that the extent of early exercise is reduced by firm's hedging of the stock options' returns. While focusing on the cost of stock options, Carpenter (1998) also documented the early exercise patterns for CEOs at 40 firms. Finally, Bettis et al., (2003) examined over 100,000 option exercises at over 3000 firms and found that employees exercise their options nearly five years prior to expiration.

executive compensation, the principal–agent analysis is limited to addressing the qualitative implications of the current compensation practice or to contemplating a better contract design; for example, Garen (1994), Aggarwal and Samwick (1999) and Jin (2002). The existing principal–agent models appear to have limited applicability as to providing a practical tool, comparable to the Black–Scholes model, of computing a fair market value for accounting purposes.

The rest of the paper is organized as follows. Section 2 presents the valuation framework which allows the systematic risk to be an explicit factor. Section 3 reports numerical results pertaining to valuation and risk incentives for both non-indexed and indexed stock options. Section 4 provides a brief summary and some concluding remarks.

2. The theoretical framework

The seminal work of Black and Scholes (1973) on option pricing lays the foundation for the modern option pricing theory. One of the most significant theoretical implications of the Black–Scholes theory is the irrelevancy of the underlying asset’s risk premium. Specifically, it stipulates that an option’s value is a direct function of the underlying asset’s volatility but is not a function of the expected return. This implies that an option’s value is independent of the relative proportions of systematic and non-systematic risks as long as the total asset risk remains the same. In short, CEOs do not have any incentive, other than the distortion induced by the hedging restriction imposed on them, to change the proportions of systematic and idiosyncratic risks because the value of their compensation (i.e., call option value) depends only on the total risk level. Although this conclusion logically follows from a continuous hedging argument within the Black–Scholes framework, it seems at odds with a simple economic intuition: a higher expected return (due to a bigger systematic risk) should increase the chance for a call option to finish in the money keeping other variables fixed.⁴

The recently developed GARCH option pricing theory, however, offers a drastically different theoretical predication on the effect of systematic risk. In what follows, we will lay out this valuation framework, with a particular focus on the role of systematic risk. We consider a specific version of the GARCH process, the exponential GARCH(1, 1)-in-mean process for the stock return with respect to the physical probability measure P :⁵

$$\begin{aligned} \ln \frac{S_{t+1}}{S_t} &= r_{t+1} + \lambda_{t+1} \sigma_{t+1} - \frac{1}{2} \sigma_{t+1}^2 + \sigma_{t+1} \varepsilon_{t+1}, \\ \ln \sigma_{t+1}^2 &= \alpha_0 + \alpha_1 \ln \sigma_t^2 + \alpha_2 (|\varepsilon_t| - \theta \varepsilon_t), \\ \varepsilon_{t+1} | \phi_t &\overset{P}{\sim} \mathcal{N}(0, 1), \end{aligned} \tag{2.1}$$

⁴ A higher expected return can be a result of (1) a lower current asset price in conjunction with the same level of future cash flows, or (2) the same current asset price in conjunction with a higher level of future cash flows, or both. Our discussion on the effect of the expected return is concerned with the second case.

⁵ Please note that our general conclusions remain valid under other GARCH specifications. We choose to use EGARCH because it has a better numerical property for the current application.

where ϕ_t is the information set containing all information up to and including time t ; r_{t+1} is the risk-free interest rate; λ_{t+1} is the risk premium per unit of standard deviation; $\alpha_0, \alpha_1, \alpha_2$, and θ are the EGARCH parameters governing the variance process; and $N(0, 1)$ denotes a standard normal distribution. The term $\lambda_{t+1}\sigma_{t+1}$ captures the total risk premium in the expected return. We use time subscript for the interest rate and the risk premium to allow them to be potentially stochastic. For the conditional volatility process to be stationary under P , parameter α_1 must be strictly between one and minus one.

According to Duan (1995), under the locally risk-neutralized pricing measure Q , the system in (2.1) becomes

$$\begin{aligned} \ln \frac{S_{t+1}}{S_t} &= r_{t+1} - \frac{1}{2}\sigma_{t+1}^2 + \sigma_{t+1}\xi_{t+1}, \\ \ln \sigma_{t+1}^2 &= \alpha_0 + \alpha_1 \ln \sigma_t^2 + \alpha_2 [|\xi_t - \lambda_t| - \theta(\xi_t - \lambda_t)], \\ \xi_{t+1} | \phi_t &\stackrel{Q}{\sim} N(0, 1), \end{aligned} \tag{2.2}$$

where $\xi_t = \varepsilon_t + \lambda_t$. It is seen that the risk premium, λ_t plays a critical role in the GARCH option pricing model. In contrast to the Black–Scholes framework, an option’s value is a direct function of the stock’s expected return (via the risk premium λ_t).

In order to explicitly characterize the risk premium λ_t in terms of systematic risk, we define $Y_t = \kappa - \ln(U'(C_t)/U'(C_{t-1}))$ with $U(C_t)$ being the utility function of time- t consumption C_t and κ the impatience parameter for exponential discounting over time. In words, Y_t is the negative of the logarithmic one-period marginal rate of substitution (or the logarithmic one-period stochastic discount factor). As shown in Appendix B,

$$\lambda_t = q_t \delta_t, \tag{2.3}$$

where $q_t = \text{Corr}^P(\ln \frac{S_t}{S_{t-1}}, Y_t | \phi_{t-1})$ and $\delta_t^2 = \text{Var}^P(Y_t | \phi_{t-1})$. Next, in order to relate systematic risk to the market portfolio, we adopt the following one-factor model for the stochastic discount factor:

$$Y_t = a + b \ln \frac{I_t}{I_{t-1}} + \epsilon_t, \tag{2.4}$$

where I_t is the value of the market portfolio at time t and ϵ_t is the regression residual. We further assume that the residual ϵ_t has a constant variance and is independent of any asset returns. With the assumption of (2.4),

$$\delta_t = \sqrt{b^2 \sigma_{1,t}^2 + c}, \tag{2.5}$$

where $\sigma_{1,t}^2 = \text{Var}^P(\ln \frac{I_t}{I_{t-1}} | \phi_{t-1})$ and $c = \text{Var}^P(\epsilon_t)$. Moreover, it is straightforward to derive

$$\lambda_t = \frac{b\beta_t \sigma_{1,t}^2}{\sigma_t \sqrt{b^2 \sigma_{1,t}^2 + c}} \delta_t = \frac{b\beta_t \sigma_{1,t}^2}{\sigma_t}, \tag{2.6}$$

where $\beta_t = \text{Cov}^P(\ln \frac{S_t}{S_{t-1}}, \ln \frac{I_t}{I_{t-1}} | \phi_{t-1}) / \sigma_{I,t}^2$. The risk premium per unit of total asset risk (λ_t) is thus completely characterized relative to the market portfolio. Note that b is constant across different assets and can be identified using the market portfolio because its own systematic risk equals one by definition. The difference across assets only comes from the difference in the systematic risk as measured relative to the market portfolio. Since the total risk premium is $\lambda_t \sigma_t = b \beta_t \sigma_{I,t}^2$, it is seen that an asset with a larger systematic risk will have a higher total risk premium, a result that is consistent with the standard asset pricing theory.

To actually implement the model, we adopt an EGARCH(1, 1) process for the market portfolio return. Specifically,

$$\begin{aligned} \ln \frac{I_{t+1}}{I_t} &= r_{t+1} - d_{1,t+1} + b \sigma_{I,t+1}^2 - \frac{1}{2} \sigma_{I,t+1}^2 + \sigma_{1,t+1} \varepsilon_{1,t+1}, \\ \ln \sigma_{I,t+1}^2 &= \alpha_{1,0} + \alpha_{1,1} \ln \sigma_{I,t}^2 + \alpha_{1,2} (|\varepsilon_{1,t}| - \theta_1 \varepsilon_{1,t}), \\ \varepsilon_{1,t+1} | \phi_t &\stackrel{P}{\sim} N(0, 1), \end{aligned} \tag{2.7}$$

where $d_{1,t+1}$ is the dividend yield on the market portfolio and $\text{Corr}^P(\varepsilon_{t+1}, \varepsilon_{1,t+1} | \phi_t) = \rho$.⁶ Note that the risk premium term results from (2.1) and (2.6) by using the fact that the systematic risk of the market portfolio equals one. Again, according to Duan (1995), the pricing system under the locally risk-neutralized measure Q is

$$\begin{aligned} \ln \frac{I_{t+1}}{I_t} &= r_{t+1} - d_{1,t+1} - \frac{1}{2} \sigma_{I,t+1}^2 + \sigma_{1,t+1} \xi_{1,t+1}, \\ \ln \sigma_{I,t+1}^2 &= \alpha_{1,0} + \alpha_{1,1} \ln \sigma_{I,t}^2 + \alpha_{1,2} [|\xi_{1,t} - b \sigma_{1,t}| - \theta_1 (\xi_{1,t} - b \sigma_{1,t})], \\ \xi_{1,t+1} | \phi_t &\stackrel{Q}{\sim} N(0, 1). \end{aligned} \tag{2.8}$$

Notice that $\text{Corr}^Q(\xi_{t+1}, \xi_{1,t+1} | \phi_t) = \rho$. In addition, it can be seen that $q_t = \rho$ and λ_t in (2.6) can be simplified to $\rho b \sigma_{1,t}$.

The systems in (2.2) and (2.8) allow the valuation of any options on the stock and the market portfolio. For instance, with a constant risk-free interest rate r , an executive stock option of European style with an exercise price X and maturity T can be valued as

$$C_t = e^{-r(T-t)} E^Q[\max(S_T - X, 0) | \phi_t], \tag{2.9}$$

where S_T is the stock price at the option's maturity, and $E^Q(\cdot)$ is the expectation operator with respect to the risk-neutral measure Q . An indexed option as proposed by Johnson and Tian (2000) can be valued in a similar fashion:

$$C_t^{\text{indexed}} = e^{-r(T-t)} E^Q[\max(S_T - H(S_0, I_0, I_T), 0) | \phi_t], \tag{2.10}$$

where $H(S_0, I_0, I_T)$ in general is a function that sets the exercise price in relation to the future index level, the current index level and the current stock price. There is no closed-form solution to either (2.9) or (2.10) but numerical procedures such as

⁶ The dividend yield term in (2.7) is used to account for the fact that many component stocks pay cash dividends, which can be approximated as a continuous rate. For individual stocks, we do not incorporate a similar term in (2.1) because periodic cash payments are typically treated by an adjustment to the stock price.

Monte Carlo simulation, the lattice method of Ritchken and Trevor (1999) and the Markov chain method of Duan and Simonato (2001) can be used to price European executive options or American style options subject to a vesting period restriction.

3. Valuation and risk incentives of executive stock options

In studying the value and risk incentives of executive stock options, we follow previous authors by assuming away the complications associated with vesting and the early exercise feature. We treat the stock option as a European call option. Monte Carlo simulations are used to value the options.

3.1. Parameter estimation and design of numerical analyses

To obtain the value of a stock option, we simulate the systems in (2.2) and (2.8), and evaluate (2.9) or (2.10) numerically.⁷ Specifically, we generate 10,000 correlated paths of the stock and the index via (2.2) and (2.8) to obtain 10,000 stock and index prices at the option's maturity. We then calculate the option's value by averaging the 10,000 discounted payoffs in (2.9) or (2.10).

GARCH process parameters are obtained by empirically estimating (2.1) and (2.7) using the daily closing prices for Philip Morris' stock and the S&P500 index for the period of 2 January 1992 to 29 December 2000. To guide the choice of parameter values for comparative analysis, we also estimate (2.1) for the other 29 component stocks of the Dow Jones industrial average (DJIA). (Philip Morris is a component stock of the index, and has been renamed to Altria since 27 January 2003.) Table 1 contains the estimation results for Philip Morris and the S&P500 index.

For all cases, we keep the S&P500 parameters constant. For the stock, we vary the volatility asymmetry parameter θ and the correlation coefficient ρ to obtain comparative static results. For the 30 DJIA component stocks, θ ranges from -0.06 to 1.07 , with an average close to 0.6 . The parameter takes a negative value for only two stocks. We examine three values of θ : 0.2 , 0.6 , and 1.0 , representing respectively low, average and high levels of θ . Notice that the volatility asymmetry parameter, θ is crucial in determining the return distribution properties and is directly linked to the so-called leverage effect. This parameter is also related to the skew feature of the implied volatility smile. Roughly speaking, a positive θ induces a negative skewness in the cumulative stock return.

The range for the correlation coefficient, ρ for the 30 DJIA component stocks is from 0.20 to 0.49 . To ensure generality of the numerical results, we examine three values of ρ : 0.0 , 0.35 , and 0.70 . Given the market volatility and the overall level

⁷ Even for non-indexed options, we need to simulate the index return since the risk premium term, λ_t is a function of the market return variance.

Table 1
Maximum likelihood estimation results

| | | Parameter estimate | Standard error |
|---------------|----------------|--------------------|----------------|
| Philip Morris | α_0 | -0.104526 | 0.013253 |
| | α_1 | 0.991743 | 0.001417 |
| | α_2 | 0.057173 | 0.004759 |
| | θ | 0.831291 | 0.070553 |
| | ρ | 0.247279 | 0.018487 |
| S&P500 | $\alpha_{1,0}$ | -0.337300 | 0.036177 |
| | $\alpha_{1,1}$ | 0.976618 | 0.003122 |
| | $\alpha_{1,2}$ | 0.147977 | 0.015064 |
| | θ_1 | 0.664131 | 0.104156 |
| | b | 4.759124 | 2.208925 |

Note: Estimated systems are (2.1) and (2.7). We use daily closing prices for the period from 2 January 1992 to 29 December 2000. For simplicity, we set the dividend yields to zero. The risk-free rate is approximated by daily yields of the 90-day T-bills.

of stock volatility, the amount of systematic risk is purely determined by the correlation coefficient.

The annualized standard deviation for the 30 DJIA component stocks ranges from 0.27 to 0.51, and that for the S&P500 is 0.19. Unless otherwise specified, we set the stock's standard deviation at 0.40, and the market's at 0.2, both under measure P .

At this point, we need to discuss how to ensure comparability between the Black–Scholes and GARCH option prices. Traditionally, within the Black–Scholes framework, we estimate the constant volatility using total returns. Once we stipulate that GARCH properly describes the real world, we should use the total return generated via (2.1) to estimate the constant volatility. Although we typically refer to the average standard deviation of the residual in (2.1) as the stationary volatility for a GARCH process, in the current setting, we refer to the constant standard deviation of the total return as the stationary volatility, just to be consistent with the Black–Scholes assumption. Due to stochastic conditional mean, the typical stationary standard deviation of the EGARCH-in-mean model differs from the stationary volatility considered here. With this understanding, we then back out the value for $\alpha_{1,0}$ in (2.7) for the given level of the market index volatility, 0.2. The constant volatility is estimated by calculating the standard deviation of total returns in (2.7) for a very long time series (i.e., 50,000,000 days). The search algorithm will pinpoint a value of $\alpha_{1,0}$ which leads to the constant volatility of 0.2. Similar procedures can be used to back out the value of α_0 in (2.1) for a given level of the stock's constant volatility.

Since changes in θ lead to changes in the overall standard deviation under measure P , whenever we vary this parameter, we must re-adjust α_0 so that the total volatility remains at 0.4. Similarly, changes in the correlation coefficient ρ also affect the constant volatility of the total return in (2.1), although this impact is very minor. Nonetheless, to ensure comparability, we also re-adjust the value of α_0 in (2.1) so that the constant volatility remains at 0.4. Although changes in ρ only slightly affect the

total return standard deviation under measure P , they will greatly affect the corresponding standard deviation under measure Q . This is essentially how the impact of systematic risk is manifested in the GARCH framework. All else being equal, varying ρ amounts to varying the amount of systematic risk while keeping the total risk constant. Given the unconditional volatilities, $\rho = 0.0, 0.35$ and 0.70 correspond to a stationary beta of $0.0, 0.7$, and 1.4 .

Without loss of generality, we set the risk-free interest rate to 5%, and the current stock price to \$100. We examine nine moneyness situations: $PV(X)/S = 0.80, 0.85, 0.90, 0.95, 1.00, 1.05, 1.10, 1.15$, and 1.20 , where $PV(X)$ stands for the present value of the exercise price, i.e., the exercise price discounted at the risk-free rate. For brevity, we only report five moneyness situations: deep-in-the-money (DITM), in-the-money (ITM), at-the-money (ATM), out-of-the-money (OTM), and deep out-of-the-money (DOTM). For the ITM case, we average the two prices for $PV(X)/S = 0.95$ and 0.90 ; for the DITM case, we average the two prices for $PV(X)/S = 0.85$ and 0.80 . Similar calculations and reporting apply to the OTM and DOTM cases.⁸ The option's maturity ranges from 0.5 to 10 years. For brevity, we report only five maturities: 0.5, 1, 2, 5, and 10 years. As mentioned before, most of the executive stock options are granted with a 10-year maturity, so the longest maturity corresponds to the case of newly granted stock options.

Three variance reduction techniques are used to reduce simulation errors: anti-thetic variable, control variate and empirical martingale correction. The first two are well known and the third was proposed by Duan and Simonato (1998). The control variate is the Black–Scholes value with the aforementioned stationary standard deviation as the volatility input. The empirical martingale correction is important for long-term options because the stock price is typically modeled as a stochastic exponential. Without such a correction, the Monte Carlo option value tends to have a serious downward bias.

3.2. Risk incentive analysis of non-indexed stock options

Since most previous researchers used the Black–Scholes framework to analyze executive stock options, we use the Black–Scholes value as the benchmark value, and consider the GARCH option value as the true value for the executive stock option. Risk incentives are also derived from the GARCH option prices. Table 2 reports the numerical results. Here, for each parameter combination, we report the percentage difference between the GARCH option price and the Black–Scholes price. In addition, for the GARCH process, we set the initial volatility equal to

⁸ Note that our definition of moneyness deviates from the normal practice in the CEO stock option literature. Almost all stock options are granted with an exercise price equal to the stock price at the time of granting, and they are typically considered as at-the-money options. When studying long term options, with a non-zero interest rate, this type of “at-the-money” options effectively becomes in-the-money options. The advantage of our definition is that the option values are actually independent of the risk-free interest rate. In other words, it is equivalent to using the usual moneyness definition while setting the interest rate to zero.

Table 2
Impact of systematic risk on CEO stock option prices – Initial volatility equal to the stationary volatility

| Maturity (years) | Moneyness | B-S | Percentage difference between CEO option prices under GARCH and Black–Scholes | | | | | | | | |
|---------------------|-----------|-------|---|---------------|---------------|-------------------------|---------------|---------------|-------------------------|---------------|---------------|
| | | | Panel A, $\theta = 0.2$ | | | Panel B, $\theta = 0.6$ | | | Panel C, $\theta = 1.0$ | | |
| | | | $\rho = 0.00$ | $\rho = 0.35$ | $\rho = 0.70$ | $\rho = 0.00$ | $\rho = 0.35$ | $\rho = 0.70$ | $\rho = 0.00$ | $\rho = 0.35$ | $\rho = 0.70$ |
| 0.5 | DITM | 21.31 | 0.49 | 0.77 | 1.09 | 1.35 | 2.13 | 3.00 | 2.09 | 3.37 | 4.81 |
| 0.5 | ITM | 15.01 | 0.11 | 0.56 | 1.05 | 0.39 | 1.70 | 3.09 | 0.50 | 2.65 | 4.98 |
| 0.5 | ATM | 11.23 | -0.56 | 0.04 | 0.68 | -1.47 | 0.35 | 2.24 | -2.57 | 0.43 | 3.61 |
| 0.5 | OTM | 8.32 | -1.51 | -0.76 | 0.01 | -4.42 | -2.06 | 0.35 | -7.58 | -3.66 | 0.42 |
| 0.5 | DOTM | 5.41 | -3.25 | -2.31 | -1.37 | -10.24 | -7.12 | -3.98 | -17.53 | -12.36 | -7.04 |
| 1 | DITM | 24.86 | 0.34 | 0.84 | 1.40 | 0.83 | 2.25 | 3.81 | 1.14 | 3.45 | 6.05 |
| 1 | ITM | 19.33 | -0.12 | 0.58 | 1.34 | -0.40 | 1.64 | 3.82 | -0.89 | 2.45 | 6.09 |
| 1 | ATM | 15.85 | -0.64 | 0.23 | 1.15 | -1.99 | 0.59 | 3.32 | -3.58 | 0.67 | 5.24 |
| 1 | OTM | 12.99 | -1.24 | -0.21 | 0.86 | -3.99 | -0.86 | 2.41 | -7.05 | -1.89 | 3.60 |
| 1 | DOTM | 9.87 | -2.31 | -1.07 | 0.22 | -7.53 | -3.62 | 0.40 | -13.09 | -6.64 | 0.16 |
| 2 | DITM | 30.23 | 0.03 | 0.74 | 1.52 | 0.04 | 2.08 | 4.31 | -0.12 | 3.21 | 6.93 |
| 2 | ITM | 25.41 | -0.28 | 0.62 | 1.60 | -0.97 | 1.67 | 4.51 | -1.85 | 2.47 | 7.23 |
| 2 | ATM | 22.27 | -0.56 | 0.49 | 1.62 | -1.96 | 1.16 | 4.50 | -3.59 | 1.54 | 7.13 |
| 2 | OTM | 19.56 | -0.98 | 0.22 | 1.51 | -3.25 | 0.37 | 4.21 | -5.78 | 0.17 | 6.62 |
| 2 | DOTM | 16.43 | -1.65 | -0.25 | 1.24 | -5.34 | -1.06 | 3.46 | -9.32 | -2.26 | 5.36 |
| 5 | DITM | 40.95 | -0.19 | 0.69 | 1.65 | -0.80 | 1.77 | 4.55 | -1.51 | 2.70 | 7.38 |
| 5 | ITM | 37.12 | -0.40 | 0.63 | 1.75 | -1.46 | 1.59 | 4.87 | -2.62 | 2.38 | 7.89 |
| 5 | ATM | 34.53 | -0.58 | 0.56 | 1.81 | -2.03 | 1.39 | 5.05 | -3.60 | 2.02 | 8.16 |
| 5 | OTM | 32.19 | -0.77 | 0.48 | 1.84 | -2.64 | 1.12 | 5.15 | -4.67 | 1.54 | 8.32 |
| 5 | DOTM | 29.38 | -1.06 | 0.34 | 1.86 | -3.54 | 0.70 | 5.22 | -6.20 | 0.81 | 8.45 |
| 10 | DITM | 52.35 | -0.38 | 0.53 | 1.53 | -1.20 | 1.50 | 4.39 | -2.08 | 2.36 | 7.20 |
| 10 | ITM | 49.35 | -0.52 | 0.50 | 1.62 | -1.65 | 1.40 | 4.67 | -2.82 | 2.20 | 7.67 |
| 10 | ATM | 47.29 | -0.64 | 0.47 | 1.68 | -2.01 | 1.32 | 4.86 | -3.42 | 2.05 | 8.00 |
| 10 | OTM | 45.39 | -0.75 | 0.45 | 1.74 | -2.34 | 1.23 | 5.04 | -4.00 | 1.92 | 8.33 |
| 10 | DOTM | 43.05 | -0.90 | 0.41 | 1.82 | -2.80 | 1.11 | 5.28 | -4.75 | 1.73 | 8.75 |

Notes: (1) The annualized stationary volatility is 0.40 and is kept constant for all cases. (2) The column with the heading B-S contains Black–Scholes option prices in dollars. (3) The columns under different ρ values contain the percentage difference between the GARCH option price and the Black–Scholes option price. (4) DITM: deep in-the-money, ITM: in-the-money, ATM: at-the-money, OTM: out-of-the-money, DOTM: deep out-of-the-money.

the stationary volatility. Again, it should be kept in mind that the stationary volatility under measure P is kept constant at 0.4.

Several observations are in order. First, for all parameter combinations, the true value of executive stock options increases as the systematic risk increases (i.e., as ρ increases). An immediate implication is that CEOs will have an incentive to increase their companies' systematic risk in order to enhance their stock options value. Without proper monitoring, the company's systematic risk level may be above the desired level. Conceivably, the CEO could achieve a higher systematic risk by keeping the total risk constant. Within the Black–Scholes framework, this would have no effect on the option's value. Therefore, using the Black–Scholes framework to study stock options' risk incentives will overlook the incentive for the CEO to take on excessive systematic risk. It is interesting to note that our conclusion is in agreement with that of Meulbroek (2001a) who studied the value of a stock option from a diversification perspective.

Second, with several minor exceptions, the true stock option value is always lower than the Black–Scholes counterpart when the systematic risk is zero (i.e., when $\rho = 0.0$ corresponding to a zero stationary beta), and higher when the systematic risk is high (i.e., when $\rho = 0.7$ corresponding to a stationary beta of 1.4). The difference in value can be substantial in many cases, especially for options with shorter maturities. With a medium level of systematic risk (i.e., when $\rho = 0.35$ corresponding to a stationary beta of 0.7), it appears that the true option value is lower (higher) than its Black–Scholes counterpart for out-of-the-money (in-the-money) options. Overall, the results imply that the Black–Scholes formula can lead to substantial errors in estimating the cost of executive stock options to the firm. Whether it under- or over-estimates the true stock option's value depends on the stock's systematic risk and moneyness of the option. For instance, if the stock has a zero systematic risk, the Black–Scholes value will be upward biased most of the time; if the stock has a high systematic risk, the Black–Scholes value will be downward biased.⁹

Third, in order to see whether the impact of changing systematic risk is different across maturities, for each moneyness situation and under a particular level of the volatility asymmetry parameter, θ , we examine the difference between the percentage

⁹ There is a subtle reason for the GARCH option values to be lower than their Black–Scholes counterparts when the correlation coefficient is zero (i.e., no systematic risk). In practice, the standard way of estimating the constant volatility is based on one sample path. Implicit in such an estimation is the i.i.d. return assumption. For a time series model such as GARCH, it is equivalent to computing the average variance of one-period return. By the Central Limit Theorem, the standardized cumulative stock return (continuously compounded) will converge to a normal distribution, but the normalizing constant should be the standard deviation of the standardized cumulative return. Once a correlation exists among one-period returns such as the case of GARCH, the variance of the standardized cumulative return no longer equals the average variance of one-period return. In other words, although the Black–Scholes formula is applicable for long-term options due to the Central Limit Theorem, it must be evaluated at a volatility level different from the standard constant volatility estimate. We performed a separate investigation on this issue, and it turns out that the average variance of one-period return is always higher than the variance of the standardized cumulative return. As a result, one would expect the Black–Scholes value to be higher than the GARCH option value for long-term options when the systematic risk is zero.

value differences, since the latter are both with respect to the same Black–Scholes value. For example, with the volatility asymmetry parameter θ being 1.0, the value of a deep-in-the-money option with a maturity of five years would go up by $[7.38 - (-1.51)] / (100 - 1.51) = 9.03\%$ when the correlation coefficient goes from 0.0 to 0.7. This is approximately equal to $7.38 - (-1.51) = 8.89\%$. A quick look reveals no apparent pattern of the value change across different maturities for the same moneyness. This means that the CEO's incentive to increase systematic risk is uniform across options with different maturities, as long as the options have similar exercise prices. Since different maturities mean different granting dates, the above implies that CEOs will try to increase the systematic risk to equally benefit all existing stock options in his/her compensation portfolio.

Fourth, the percentage difference in price is generally bigger, the higher the volatility asymmetry parameter, θ . As mentioned before, this parameter determines the distribution property of the stock return. Specifically, it determines the skewness of the distribution. The results imply that, other things being equal, CEOs of those companies whose stock returns exhibit more skewness will have more incentive to increase the systematic risk.

So far, the current volatility of the stock is assumed to be equal to the stationary volatility. However, the model allows the volatility to evolve, and at a particular point in time, the actual volatility can be above or below the long run average. To see if the previous observations still obtain under those circumstances, we re-run the simulations by setting the initial volatility to, respectively, 50% and 150% of the long run average. To conserve space, we omit the tables and only discuss the key findings.

To begin with, the true stock option prices are all lower (higher) when the initial volatility is low (high). More important, it remains true that a higher systematic risk leads to a higher option value. Therefore, the incentive to increase systematic risk is present regardless of the current level of the stock's volatility.

We also find that the difference between the true stock option value and the Black–Scholes value is much more pronounced for shorter term options. To illustrate this finding, we plot in Fig. 1 the percentage difference in values against moneyness for two maturities: 1 and 10 years (the volatility asymmetry parameter θ is fixed at 0.6 and the correlation at 0.35). The percentage differences for longer term options are all within 3%, whereas those for shorter term options can be more than 30%.

The dramatic difference across maturities is due to the dampening effect. In a GARCH framework, the stochastic volatility reverts to the long-run stationary level when the time horizon is long. Over a shorter horizon, the current level of volatility dominates valuation. Our findings once again reveal the limitations of the Black–Scholes model in estimating the value of previously granted CEO stock options. When the stock's volatility changes, the value of the option can change significantly. In the Black–Scholes setting, when the long-run volatility is estimated using historical data, the option value does not respond to current fluctuations of the return volatility.

The above analyses focus on the varying levels of the current volatility relative to the long run volatility. Stocks can have inherently different levels of long run or aver-

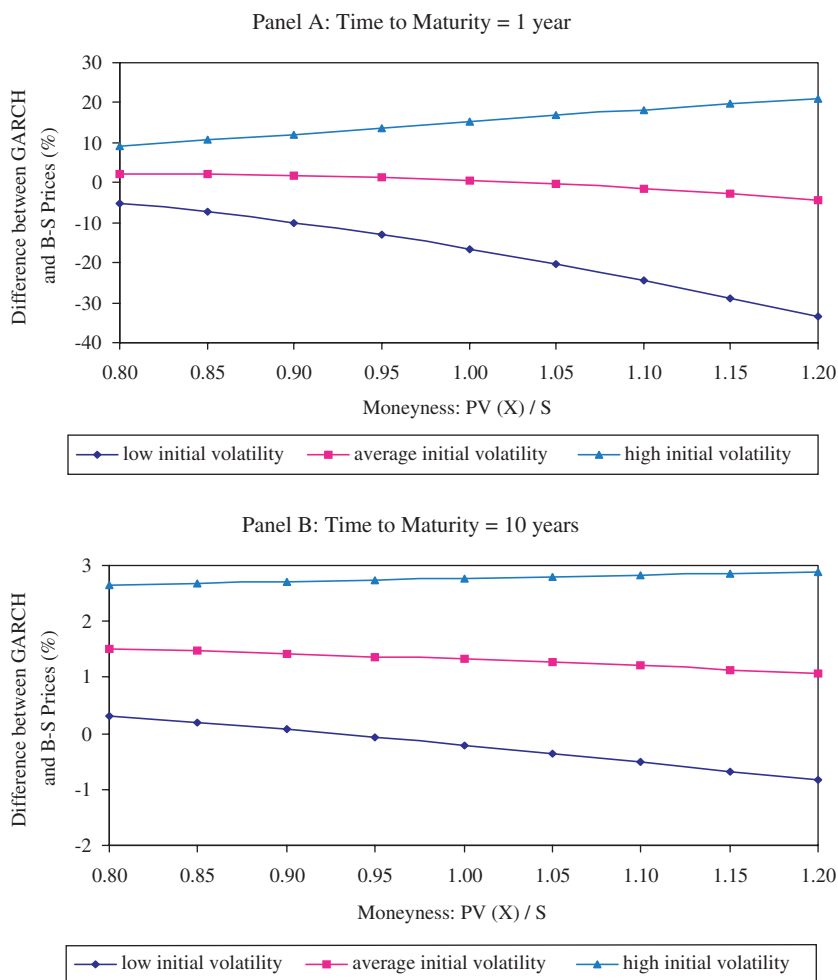


Fig. 1. Impact of systematic risk on CEO stock option prices – Different levels of initial volatility.

Notes: (1) In each panel, we plot the percentage difference between GARCH and Black–Scholes option prices against option’s moneyness which is the present value of the exercise price divided by the current stock price. For all plots, $\theta = 0.6$, $\rho = 0.35$, and the remaining parameters take their values. (2) The three settings for the initial volatility are: 50% of the stationary volatility (low), 100% of the stationary volatility (average), and 150% of the stationary volatility (high).

age volatilities due to different business lines. For instance, high tech stocks have a much higher average volatility than utility stocks. It is useful to know if risk incentives are different under different levels of long run volatilities. To this end, we repeat calculations similar to those in Table 2 by setting the initial volatility to the long run average. Here we vary the systematic risk parameter ρ under different levels of long run volatilities. Specifically, we examine three scenarios: the annualized stationary

standard deviation being 0.2, 0.4, and 0.6. The volatility asymmetry parameter θ is kept at the base level of 0.6. Table 3 contains the results.¹⁰

There are two interesting observations. First, it remains true that the higher the systematic risk, the higher the stock option's value, implying that the CEO always has the incentive to increase the proportion of systematic risk, regardless of the total risk level. Second, increasing the systematic risk has a bigger impact on the true option value when the long run volatility is low. This result is quite intuitive: since the option value is always lower when the long run volatility or total risk is low, any change to the systematic risk will likely bring about a bigger impact on the option value in a relative sense. The implication is that, for companies with a lower total risk, the board should be extra vigilant about their CEOs' actions.

Our analyses so far are for a single stock option. We examined how the option value changes with respect to a particular parameter. In other words, we assume that the CEO will modify the firm's risk profile after they receive the stock options. When comparing alternative compensation packages, it is sometimes meaningful to keep the cost constant. To this end, we calculate the option's sensitivities to the stock price (delta) and stock volatility (vega) per \$1 cost. Here we implicitly assume that the CEO cannot alter the risk profile and the firm can pick an "optimal" risk profile to maximize incentives. In Fig. 2, we plot option delta's and vega's against the total risk and the systematic risk (measured by correlation). For delta or value incentives (Panel A), lower correlation and lower volatility seem to bring about a higher incentive. Although not shown here, similar observations regarding volatility also obtain in the Black–Scholes setting. For vega or risk incentives (Panel B), the patterns are similar, except that the negative association between per \$1 vega and correlation is not visually apparent. Overall, on a fixed cost basis, Fig. 2 implies that the value incentive and the risk incentive are mostly driven by the total risk and the composition has very little bearing on them. Therefore, total risk plays an important role in deciding on the option package. However, after the option is granted, both total risk and its composition become important.

Taken together, the results in this section seem to suggest the following: (1) once the options are granted, CEOs have an incentive to increase their stocks' systematic risk to enhance their own wealth; (2) the risk incentive is stronger when the total risk level is lower; (3) when the systematic risk is high, the Black–Scholes formula tends to under-estimate the true option value because of its inability to incorporate the systematic risk into pricing; when the systematic risk is very low, the Black–Scholes formula tends to over-estimate the true option value due to its failure to properly account for return dependency in computing the constant volatility estimate.

¹⁰ The Black–Scholes prices are different under different stationary volatilities. The percentage differences are calculated using the proper Black–Scholes price under each stationary volatility. For brevity, we omit the Black–Scholes prices in the table. Calculations are also done for cases of lower and higher initial volatilities. Since they do not offer additional insights, we omit the tables for brevity.

Table 3
Impact of systematic risk on CEO stock option prices – Under different levels of stationary volatility

| Maturity (years) | Moneyness | Percentage difference between CEO option prices under GARCH and Black–Scholes | | | | | | | | |
|---------------------|-----------|---|---------------|---------------|---------------------------|---------------|---------------|---------------------------|---------------|---------------|
| | | Panel A, $\sigma_s = 0.2$ | | | Panel B, $\sigma_s = 0.4$ | | | Panel C, $\sigma_s = 0.6$ | | |
| | | $\rho = 0.00$ | $\rho = 0.35$ | $\rho = 0.70$ | $\rho = 0.00$ | $\rho = 0.35$ | $\rho = 0.70$ | $\rho = 0.00$ | $\rho = 0.35$ | $\rho = 0.70$ |
| 0.5 | DITM | 1.15 | 1.46 | 1.83 | 1.35 | 2.13 | 3.00 | 0.72 | 1.73 | 2.83 |
| 0.5 | ITM | 1.41 | 2.36 | 3.40 | 0.39 | 1.70 | 3.09 | -0.50 | 0.92 | 2.41 |
| 0.5 | ATM | -0.96 | 0.90 | 2.86 | -1.47 | 0.35 | 2.24 | -1.97 | -0.20 | 1.62 |
| 0.5 | OTM | -7.77 | -4.78 | -1.76 | -4.42 | -2.06 | 0.35 | -3.89 | -1.78 | 0.37 |
| 0.5 | DOTM | -25.15 | -20.71 | -16.30 | -10.24 | -7.12 | -3.98 | -7.18 | -4.59 | -1.98 |
| 1 | DITM | 1.49 | 2.24 | 3.13 | 0.83 | 2.25 | 3.81 | -0.24 | 1.42 | 3.19 |
| 1 | ITM | 1.01 | 2.66 | 4.49 | -0.40 | 1.64 | 3.82 | -1.51 | 0.61 | 2.83 |
| 1 | ATM | -1.21 | 1.47 | 4.34 | -1.99 | 0.59 | 3.32 | -2.67 | -0.19 | 2.39 |
| 1 | OTM | -5.61 | -1.76 | 2.26 | -3.99 | -0.86 | 2.41 | -4.04 | -1.22 | 1.70 |
| 1 | DOTM | -15.85 | -10.29 | -4.57 | -7.53 | -3.62 | 0.40 | -6.16 | -2.85 | 0.53 |
| 2 | DITM | 1.22 | 2.57 | 4.12 | 0.04 | 2.08 | 4.31 | -0.99 | 1.20 | 3.54 |
| 2 | ITM | 0.39 | 2.73 | 5.32 | -0.97 | 1.67 | 4.51 | -1.88 | 0.73 | 3.49 |
| 2 | ATM | -1.21 | 2.09 | 5.67 | -1.96 | 1.16 | 4.50 | -2.68 | 0.25 | 3.32 |
| 2 | OTM | -3.78 | 0.56 | 5.22 | -3.25 | 0.37 | 4.21 | -3.57 | -0.34 | 3.05 |
| 2 | DOTM | -9.10 | -3.23 | 3.01 | -5.34 | -1.06 | 3.46 | -4.89 | -1.25 | 2.55 |
| 5 | DITM | 0.52 | 2.62 | 4.98 | -0.80 | 1.77 | 4.55 | -1.75 | 0.76 | 3.41 |
| 5 | ITM | -0.18 | 2.78 | 6.05 | -1.46 | 1.59 | 4.87 | -2.29 | 0.54 | 3.51 |
| 5 | ATM | -1.09 | 2.60 | 6.64 | -2.03 | 1.39 | 5.05 | -2.68 | 0.38 | 3.60 |
| 5 | OTM | -2.27 | 2.19 | 7.04 | -2.64 | 1.12 | 5.15 | -3.05 | 0.23 | 3.68 |
| 5 | DOTM | -4.30 | 1.25 | 7.26 | -3.54 | 0.70 | 5.22 | -3.56 | 0.02 | 3.75 |
| 10 | DITM | -0.04 | 2.51 | 5.33 | -1.20 | 1.50 | 4.39 | -2.44 | -0.06 | 2.40 |
| 10 | ITM | -0.51 | 2.73 | 6.30 | -1.65 | 1.40 | 4.67 | -2.82 | -0.23 | 2.46 |
| 10 | ATM | -0.96 | 2.83 | 6.98 | -2.01 | 1.32 | 4.86 | -3.11 | -0.35 | 2.50 |
| 10 | OTM | -1.52 | 2.82 | 7.56 | -2.34 | 1.23 | 5.04 | -3.40 | -0.48 | 2.53 |
| 10 | DOTM | -2.46 | 2.65 | 8.23 | -2.80 | 1.11 | 5.28 | -3.78 | -0.66 | 2.55 |

Notes: (1) The initial volatility is set equal to the stationary volatility for all cases. (2) The columns under different ρ values contain the percentage difference between the GARCH option price and the Black–Scholes option price, the latter of which is omitted for brevity. (3) DITM: deep in-the-money, ITM: in-the-money, ATM: at-the-money, OTM: out-of-the-money, DOTM: deep out-of-the-money.

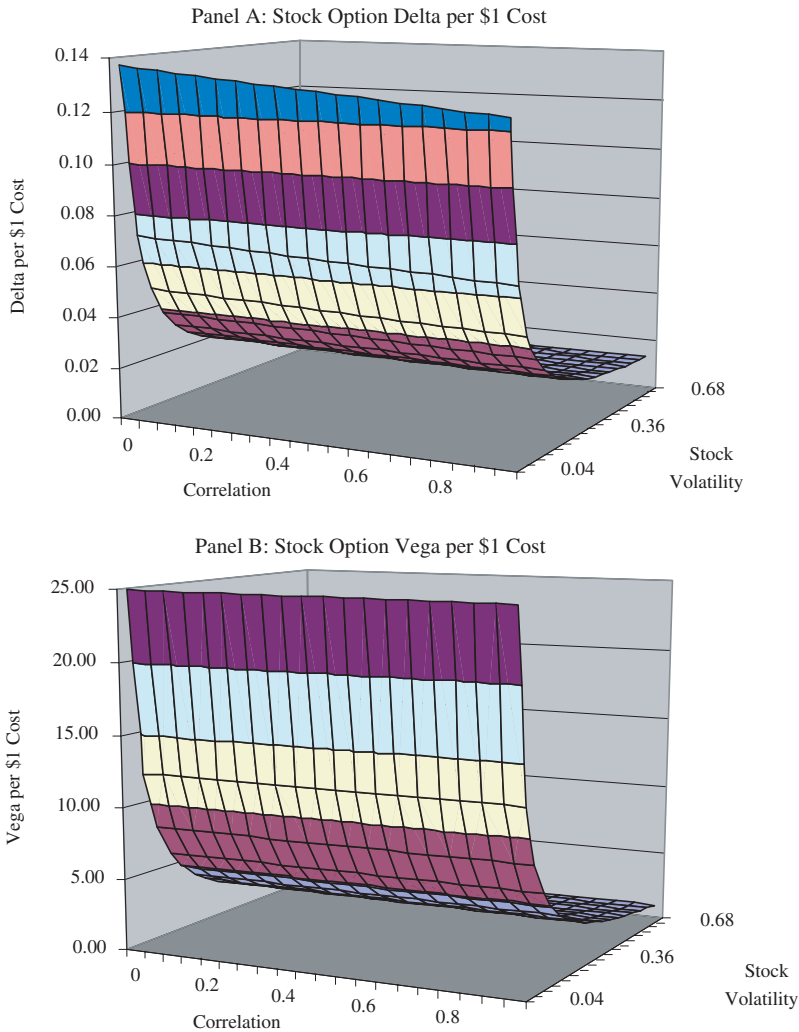


Fig. 2. Non-indexed stock option's delta and vega per \$1 cost.

Notes: (1) The plots are for at-the-money options with a time to maturity of five years. The initial volatility for the stock is set equal to the stationary volatility. The volatility asymmetry parameter θ is set at 0.6. All remaining parameters take their base values. (2) Delta's and vega's are approximated by perturbations. For delta's, the stock price is perturbed by 0.1% (or \$0.1 for a stock price of \$100); for vega's the volatility is perturbed by 0.005 (or 0.5%).

3.3. Risk incentive analysis of indexed stock options

Johnson and Tian (2000) argued that the exercise price of a stock option should be indexed, since part of the stock price movements is affected by the overall market which is beyond the CEO's control. CEOs should be rewarded only for idiosyncratic stock price appreciations, and should not be penalized for stock price declines caused

by the overall market downturn. Indexing not only aligns incentives with performance, but also saves compensating cost for the company.

Assuming constant returns and volatilities for the stock and the market index, Johnson and Tian (2000) proposed the following indexed exercise price (with a zero dividend yield):

$$H_t = S_0(I_t/I_0)^\beta e^{[r(1-\beta)+0.5\rho\sigma_s\sigma_1(1-\beta)]t}, \tag{3.1}$$

where S_0, I_0 are the stock price and the market index at time 0 (the option granting time), I_t is the market index at time t , σ_s and σ_1 are the stock and market index volatilities, ρ is the correlation between the stock and the market index returns, r is the risk-free rate, and $\beta = \rho\sigma_s/\sigma_1$. Clearly, at the time of granting, the exercise price is equal to the prevailing stock price. As shown by Johnson and Tian (2000), one can easily re-scale the exercise price to make it in-the-money or out-of-the-money at the time of granting. With the above setup, the option’s value can be expressed as

$$C_t^{\text{indexed}} = S_t N(d_1) - H_t N(d_2), \tag{3.2}$$

where

$$d_1 = \frac{\ln(S_t/H_t) + 0.5\sigma_a^2(T-t)}{\sigma_a\sqrt{T-t}}, \quad d_2 = d_1 - \sigma_a\sqrt{T-t}, \quad \sigma_a = \sigma_s\sqrt{1-\rho^2},$$

$T-t$ is the option’s time to maturity, and $N(\bullet)$ is the cumulative probability function for a standard normal distribution.

To study the value and risk incentive effects of indexed CEO stock options within the GARCH framework, we must make some further assumptions for the simulations. Specifically, (1) we index the exercise price according to (3.1), (2) we use the long run or stationary volatilities in place of σ_s and σ_1 , (3) the current stock and index prices are assumed to be the same as those at the time of granting, and (4) the overall exercise price is scaled to make the option in-the-money or out-of-the-money at the time of granting. The Black–Scholes value will be given in (3.2), and is used as the control variate in the simulations. Similar to non-indexed stock options, the difference between the Black–Scholes and GARCH option values stems from time varying volatilities. Within the Black–Scholes framework, the systematic risk exerts its impacts through indexing; within the GARCH framework, the systematic risk exerts its impacts through both indexing and the return dynamics.

Similar to the study of non-indexed stock options, we calculate option values under different systematic risk levels by varying the correlation coefficient, ρ . For brevity, we only report in Table 4 the numerical results for the base case where the initial volatility is set at 0.4, the long run volatility, and the volatility asymmetry parameter is set at 0.6. Several observations are in order.

First, unlike the non-indexed options whose value is totally independent of the systematic risk in the Black–Scholes environment, the Black–Scholes value for an indexed option is negatively affected by the systematic risk. As already established by Johnson and Tian (2000), a higher systematic risk means a smaller contribution of the idiosyncratic risk; most of the stock movements is attributable to the market

Table 4

Impact of systematic risk on indexed CEO stock option prices – Initial volatility equal to stationary volatility

| Maturity (years) | Money-ness | Black–Scholes price (\$), GARCH price (\$), and % difference | | | | | | | | |
|------------------|------------|--|-------|--------|------------------------|-------|-------|------------------------|-------|------|
| | | Panel A, $\rho = 0.00$ | | | Panel B, $\rho = 0.35$ | | | Panel C, $\rho = 0.70$ | | |
| | | B–S | GARCH | % | B–S | GARCH | % | B–S | GARCH | % |
| 0.5 | DITM | 21.31 | 21.60 | 1.35 | 20.81 | 21.28 | 2.28 | 19.21 | 19.63 | 2.16 |
| 0.5 | ITM | 15.01 | 15.07 | 0.39 | 14.36 | 14.67 | 2.12 | 12.11 | 12.53 | 3.39 |
| 0.5 | ATM | 11.23 | 11.07 | -1.47 | 10.52 | 10.64 | 1.12 | 8.03 | 8.37 | 4.13 |
| 0.5 | OTM | 8.32 | 7.95 | -4.42 | 7.61 | 7.56 | -0.65 | 5.16 | 5.39 | 4.44 |
| 0.5 | DOTM | 5.41 | 4.86 | -10.24 | 4.77 | 4.55 | -4.54 | 2.66 | 2.77 | 4.29 |
| 1 | DITM | 24.86 | 25.07 | 0.83 | 24.07 | 24.70 | 2.60 | 21.40 | 22.08 | 3.16 |
| 1 | ITM | 19.33 | 19.25 | -0.40 | 18.39 | 18.82 | 2.31 | 15.13 | 15.78 | 4.26 |
| 1 | ATM | 15.85 | 15.54 | -1.99 | 14.86 | 15.11 | 1.64 | 11.36 | 11.92 | 4.95 |
| 1 | OTM | 12.99 | 12.47 | -3.99 | 11.98 | 12.03 | 0.46 | 8.44 | 8.91 | 5.56 |
| 1 | DOTM | 9.87 | 9.13 | -7.53 | 8.89 | 8.72 | -1.94 | 5.53 | 5.86 | 5.93 |
| 2 | DITM | 30.23 | 30.24 | 0.04 | 29.05 | 29.82 | 2.65 | 24.99 | 25.94 | 3.80 |
| 2 | ITM | 25.41 | 25.16 | -0.97 | 24.10 | 24.69 | 2.46 | 19.47 | 20.42 | 4.87 |
| 2 | ATM | 22.27 | 21.83 | -1.96 | 20.90 | 21.35 | 2.18 | 16.01 | 16.91 | 5.66 |
| 2 | OTM | 19.56 | 18.92 | -3.25 | 18.15 | 18.47 | 1.79 | 13.15 | 13.99 | 6.41 |
| 2 | DOTM | 16.43 | 15.56 | -5.34 | 15.01 | 15.15 | 0.92 | 10.03 | 10.77 | 7.43 |
| 5 | DITM | 40.95 | 40.62 | -0.80 | 39.13 | 40.09 | 2.45 | 32.63 | 33.99 | 4.19 |
| 5 | ITM | 37.12 | 36.57 | -1.46 | 35.15 | 36.02 | 2.49 | 28.06 | 29.51 | 5.17 |
| 5 | ATM | 34.53 | 33.83 | -2.03 | 32.47 | 33.29 | 2.51 | 25.06 | 26.54 | 5.93 |
| 5 | OTM | 32.19 | 31.34 | -2.64 | 30.07 | 30.82 | 2.48 | 22.42 | 23.92 | 6.69 |
| 5 | DOTM | 29.38 | 28.34 | -3.54 | 27.19 | 27.82 | 2.32 | 19.34 | 20.85 | 7.77 |
| 10 | DITM | 52.35 | 51.72 | -1.20 | 49.97 | 51.27 | 2.60 | 41.23 | 42.95 | 4.15 |
| 10 | ITM | 49.35 | 48.54 | -1.65 | 46.81 | 48.06 | 2.67 | 37.42 | 39.19 | 4.73 |
| 10 | ATM | 47.29 | 46.34 | -2.01 | 44.65 | 45.86 | 2.72 | 34.85 | 36.64 | 5.13 |
| 10 | OTM | 45.39 | 44.33 | -2.34 | 42.65 | 43.81 | 2.72 | 32.52 | 34.34 | 5.59 |
| 10 | DOTM | 43.05 | 41.84 | -2.80 | 40.21 | 41.28 | 2.68 | 29.72 | 31.59 | 6.30 |

Notes: (1) The initial volatility is set equal to the stationary volatility for all cases. (2) The columns under different ρ values contain the Black–Scholes price in dollars, the GARCH option price in dollars, and the percentage difference between the two. (3) DITM: deep in-the-money, ITM: in-the-money, ATM: at-the-money, OTM: out-of-the-money, DOTM: deep out-of-the-money. (4) The indexing is according to the scheme in Johnson and Tian (2000).

movements, and the CEO is rewarded much less than if the exercise price is not indexed. Incidentally, comparing Panel A of Table 4 with Panel B of Table 2, we notice that the indexed and non-indexed options have the same value when the correlation is zero. This is expected since the indexed exercise price in (3.1) reduces to $S_0 e^{r(T-t)}$ at maturity, which is identical to the present-value-adjusted exercise price for the non-indexing case.

Second, in sharp contrast with the non-indexed options, the indexed option values appear to be negatively affected by the systematic risk. Increasing systematic risk is no longer in the CEO's interest. This finding has profound implications for optimal

risk incentives, as will be discussed in the next section. It should be noted, however, that the table has only presented the results for three values of ρ . Once we consider the whole spectrum of possible values, the GARCH option value for the indexed contract actually increases with the systematic risk for a small range of ρ and then starts to decline, as reflected in Fig. 4. This result is due to the interplay between the increasing force from the GARCH dynamic and the dampening effect of indexing the contract. Once the systematic risk reaches to a certain level, the dampening effect becomes the dominant force.

Third, as long as the systematic risk is not close to zero, GARCH option prices are higher than their Black–Scholes counterparts. Thus, for indexed options, using Black–Scholes model to estimate the compensation cost can lead to under-estimation. The higher the systematic risk, the bigger the under-estimation. When the return correlation is zero, the Black–Scholes model tends to over-estimate the true option value, similar to the non-indexed options.

Fourth, comparing option values across panels (both Black–Scholes and GARCH values), we notice that, systematic risk exerts a bigger impact (in percentage terms) on out-of-the-money options than on in-the-money options. Intuitively, out-of-the-money options have less cushion against exercise price fluctuations than in-the-money options.

Again, the above analysis is on a per option basis. For completeness, we also calculate the indexed option's delta and vega on a per dollar basis. Fig. 3 is an indexed counterpart of Fig. 2. It is seen that the indexed option's vega behaves in a similar fashion as the non-indexed option's vega; however, the delta's of the two options behave quite differently. For indexed options, a higher correlation or systematic risk leads to a higher delta on a per dollar basis. The same is observed within the Black–Scholes framework. Interestingly, for both types of options, the CEO's and firm's desired levels of risk are opposite to each other. Take the indexed option as an example. The CEO prefers a lower systematic risk since it leads to a higher option value, while the firm prefers a higher systematic risk since it brings about a higher value incentive on a per dollar basis. No matter which type of option is being used, as soon as the option is granted, the CEO will have an incentive to alter the stock's systematic risk. It is therefore imperative that the firm uses an option package that will induce the CEO to adhere to the systematic risk level. We show in the following section how this can be achieved.

3.4. Optimal risk incentives

We have shown that, the CEO has an incentive to increase the systematic risk for non-indexed options, and decrease the systematic risk for indexed options. Neither of the two alone can induce the CEO to opt for a level of systematic risk desirable to the firm. However, a basket of the two has the potential of inducing the CEO to choose a level of systematic risk deemed desirable for the company. In other words, an "optimal" compensation scheme can be devised by partially indexing the stock options. "Partial" can be implemented in many ways. For instance, for each option grant, the company can index only a fraction of the total number of

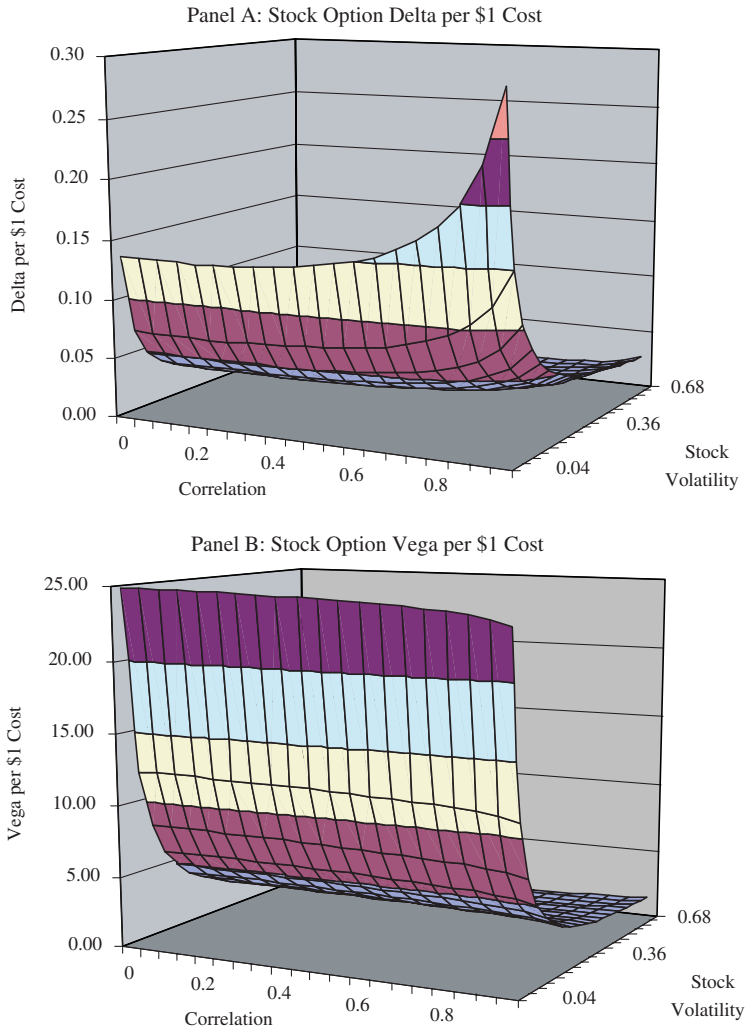


Fig. 3. Indexed stock option's delta and vega per \$1 cost.

Notes: (1) The plots are for at-the-money options with a time to maturity of five years. The initial volatility for the stock is set equal to the stationary volatility. The volatility asymmetry parameter θ is set at 0.6. All remaining parameters take their base values. (2) Delta's and vega's are approximated by perturbations. For delta's, the stock price is perturbed by 0.1% (or \$0.1 for a stock price of \$100); for vega's the volatility is perturbed by 0.005 (or 0.5%).

options. Alternatively, the company can grant indexed and non-indexed options in alternate years.

To illustrate the above idea, for the correlation range of $[0.0, 0.80]$, we calculate values for the indexed and non-indexed at-the-money options using the base parameter values and a maturity of 10 years. We then form a basket of the two options at each level of correlation or systematic risk. For simplicity but without loss of

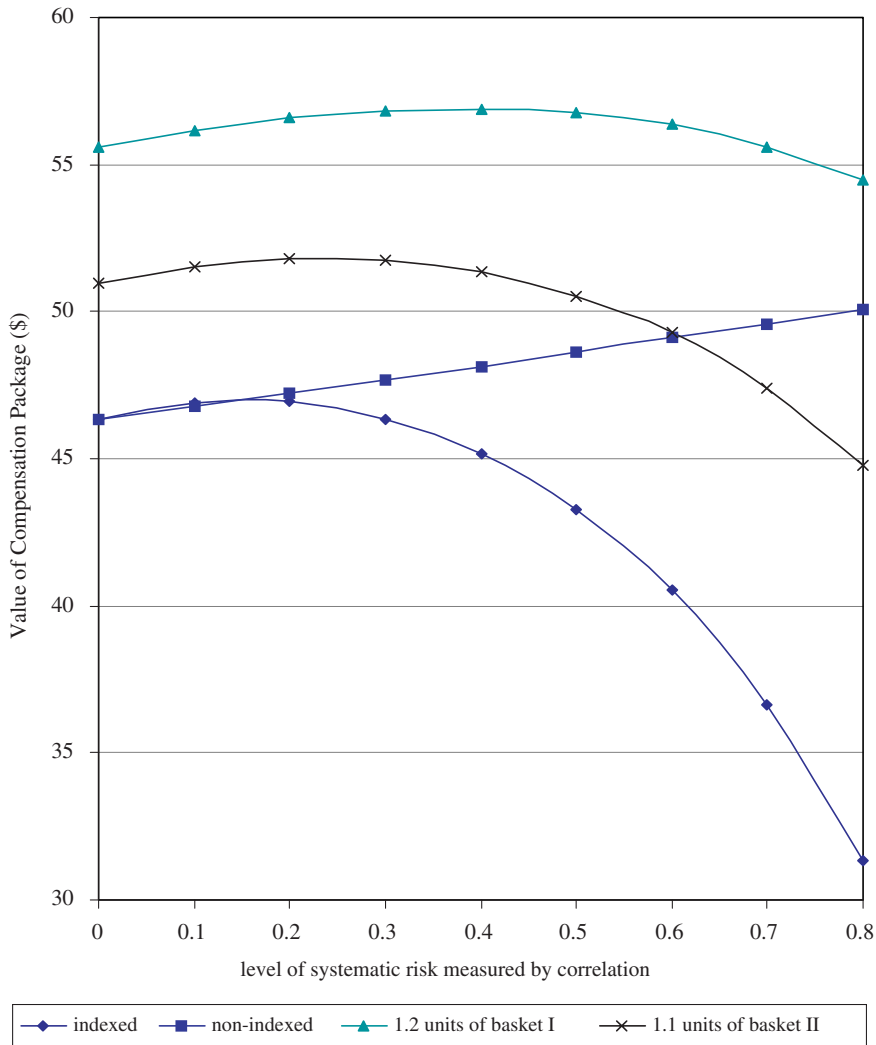


Fig. 4. Optimal contract design aimed at a fixed level of systematic risk.

Note: Basket I contains 0.25 units of indexed and 0.75 units of non-indexed options; Basket II contains 0.5 units of indexed and 0.5 units of non-indexed options. We plot the values for 1.2 units of Basket I and 1.1 units of Basket II in order to separate the curves for better visual effects.

generality, we form the basket using fractional units which add up to unity. For instance, if we have 0.25 units of the non-indexed option in the basket, then we will have 0.75 units of the indexed option. In Fig. 4, we plot the basket values for two particular combinations along with individual contracts. With a combination of (0.25, 0.75), the basket value peaks when the correlation is 0.4; with a combination of (0.5, 0.5), the basket value peaks when the correlation is 0.2. Once a company has

decided on its suitable systematic risk level, it can choose the basket to ensure that the CEO will be motivated by his/her own interest to pursue it. For instance, if the desired level of correlation is 0.4, then the CEO should be granted 0.25 units of non-indexed and 0.75 units of indexed options. The unit values of the two options are \$48.15 and \$45.19 respectively. If the total grant value is set at \$1,000,000, then the total number of baskets to be granted will be $1,000,000/[0.25 \times 48.15 + 0.75 \times 45.19] = 21,772$. In other words, the CEO should be granted with 5,443 units of the non-index option and 16,329 units of the indexed option.

Admittedly, this scheme works perfectly only at the time of grant. Over time, due to changes in the stock price and volatility, the option value's sensitivity to the systematic risk will also change. But this is the same challenge faced by any board of directors when they design stock option compensation packages. A simple practical remedy would be to re-evaluate the CEO's entire stock option portfolio at each granting date, and decide on a mixture that is risk-incentive compatible for the whole portfolio.

Before concluding the paper, we would like to discuss the applicability of our results when the CEOs face trading restrictions. As mentioned before, we do not distinguish private value from market value in this paper. When trading restrictions are introduced, CEOs' private valuation will be different from the market's. The natural concern is whether our qualitative conclusions on risk incentives would remain valid under private valuations. Careful thought experiments seem to suggest a positive answer. When CEOs face trading restrictions, they are forced to bear non-systematic risk and, as a result, will under-value their stock or option holdings. Had we modeled the trading restrictions directly, we would have introduced a wedge between the private and market values. However, the nature of systematic risk's impact will remain the same for private values. Specifically, a higher systematic risk would still lead to a higher value for non-indexed options and a lower value for indexed options. For non-indexed options, the additional non-systematic risk borne by the CEO due to trading restrictions will make him/her prefer systematic risk even more. In other words, the risk incentive effects are intensified under private valuations. For indexed options, the two effects compete against each other. On the one hand, a higher systematic risk still causes a lower option value due to indexing; on the other hand, a higher systematic risk offsets some of the additional non-systematic risk from constrained holdings, narrowing the gap between the private and market values. It is reasonable to predict that the former effect dominates the latter since it is a direct valuation effect. This prediction is even more reasonable when trading restrictions are less stringent. Therefore, in a whole, we would expect the conclusions concerning risk incentives to remain valid under private valuations.

4. Summary and conclusions

There is a large body of literature on CEO stock options that addresses such key issues as valuation, repricing, and risk incentives. It is generally recognized that, due

to CEOs' risk-aversion and trading restrictions, the value of a stock option to the CEO can be different from the cost to the company. It is accepted that the cost of a stock option can be estimated by applying the Black–Scholes formula because the company itself does not face trading restrictions. Within this framework, the study of risk incentives will necessarily be confined to total risk only. Given that the Black–Scholes value of an option is positively related to the volatility input, CEOs will have an incentive to increase the total risk in order to enhance their own wealth. This in itself is a useful insight for designing stock option packages and monitoring the CEOs.

However, standard asset pricing theory stipulates that the breakdown of total risk into systematic and non-systematic risks is quite relevant in determining the value of an asset. CEOs do have the ability to influence the composition of the total risk. Therefore, an important question is, "Will the granting of stock options create any incentive for the CEO to alter the total risk breakdown?" If the answer is yes, then how do we design stock option packages that are risk-incentive compatible?

We use the GARCH option valuation framework to address the above questions. We first establish theoretically that the option's value does depend on the amount of systematic risk. We then carry out a numerical analysis of risk incentives induced by stock options. For non-indexed stock options, CEOs always have an incentive to increase the systematic risk. This incentive becomes stronger when the total risk level is lower. In addition, when the systematic risk is high, the Black–Scholes formula tends to under-estimate the true option value because of its inability to incorporate the systematic risk into pricing. When the systematic risk is close to zero, the Black–Scholes model tends to over-estimate the true option value.

For indexed options, CEOs usually have an incentive to reduce systematic risk. The opposite risk incentives induced by the two types of options suggest an optimal compensation scheme to control the systematic risk level. It involves granting a mixture of indexed and non-indexed stock options such that the CEO is induced to pursue the systematic risk level deemed appropriate by the board of directors.

The main implication of our analysis is that, the board of directors should not simply focus on the total risk when designing the stock option and monitoring their CEOs. Systematic risk should also be a key factor of consideration. Even if the board restricts the total risk to be within a certain range, the CEO could still alter the composition of the total risk in order to enhance his/her own wealth. Moreover, granting non-indexed or indexed options alone will induce the CEO to target at a systematic risk level that is not appropriate for the company. A mixture of the two options offers a promising possibility for risk control.

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Appendix A. Examples of stock option grants for CEOs

| Granting year | Number of call options | Original exercise price | Adjusted exercise price | Maturity time | Vesting period (year) |
|--|------------------------|-------------------------|-------------------------|---------------|-----------------------|
| <i>Panel A: Stock options for CEO of Hewlett Packard</i> | | | | | |
| 1997 | 37,500 | 52.5 | 25.77 | November 2006 | 1 |
| | 37,500 | | | | 2 |
| | 37,500 | | | | 3 |
| | 37,500 | | | | 4 |
| 1988 | 125,000 | 61.75 | 30.31 | November 2007 | 1 |
| | 125,000 | | | | 2 |
| | 125,000 | | | | 3 |
| | 125,000 | | | | 4 |
| 1999 | 75,000 | 59.19 | 59.19 | November 2008 | 1 |
| | 75,000 | | | | 2 |
| | 75,000 | | | | 3 |
| | 75,000 | | | | 4 |
| <i>Panel B: Stock options for CEO of Philip Morris</i> | | | | | |
| 1998 | 400,000 | 39.72 | 33.20 | June 2008 | 1 |
| 1999 | 500,000 | 40.00 | 34.44 | June 2009 | 1 |
| | 250,000 | 40.00 | 34.44 | June 2009 | 3 |
| | 198,188 | 35.81 | 30.84 | June 2003 | 0.5 |
| | 80,497 | 35.81 | 30.84 | June 2004 | 0.5 |
| 2000 | 1,171,400 | 21.34 | 21.34 | January 2010 | 1 |
| | 267,567 | 33.09 | 33.09 | June 2004 | 0.5 |

Notes: (1) Panel A is for Lewis E. Platt, CEO of Hewlett Packard; Panel B is for Geoffrey C. Bible, CEO of Philip Morris.

(2) Source: DEF14A Form from the EDGAR data base of the Securities Exchange Commission. Note that the exercise price is equal to the stock price at the time of grant, as all stock options are granted at-the-money. The “adjusted exercise price” is the original exercise price adjusted for all the stock splits since the granting date.

Appendix B. Characterization of the risk premium parameter, λ_t

Our starting point is the theoretical setup of Duan (1995). In this setup, the fundamental asset pricing equation in the representative agent exchange equilibrium is

$$V_t = E^P \left[e^{-\kappa} \frac{U'(C_{t+1})}{U'(C_t)} V_{t+1} \middle| \phi_t \right],$$

where V_t is the asset price, $U(C_t)$ is the utility function of time- t consumption C_t , κ is the impatience parameter reflecting a discount over time, and $E^P(\cdot)$ is an expectation operator with respect to measure P . We assume that the negative of the logarithmic stochastic discount factor (i.e., $Y_{t+1} = \kappa - \ln \frac{U'(C_{t+1})}{U'(C_t)}$) and the continuously compounded asset return follow a bivariate normal distribution, conditional on the information set at time t , denoted by ϕ_t .

When the fundamental pricing equation is applied to the one-period risk-free bond, we obtain

$$\begin{aligned} e^{-r_{t+1}} &= E^P \left[e^{-\kappa} \frac{U'(C_{t+1})}{U'(C_t)} \middle| \phi_t \right], \\ &= E^P \left[e^{-Y_{t+1}} \middle| \phi_t \right], \\ &= e^{-\mu_{t+1} + \frac{1}{2} \delta_{t+1}^2}, \end{aligned}$$

where $\mu_{t+1} = E^P[Y_{t+1} | \phi_t]$, and $\delta_{t+1}^2 = \text{Var}^P[Y_{t+1} | \phi_t]$. The above equation then implies

$$\mu_{t+1} - r_{t+1} - \frac{1}{2} \delta_{t+1}^2 = 0. \tag{A.1}$$

Now we project $\ln \frac{S_{t+1}}{S_t}$ on Y_{t+1} (conditional on ϕ_t) to yield

$$\ln \frac{S_{t+1}}{S_t} = w_{1,t+1} + w_{2,t+1} Y_{t+1} + Z_{t+1}, \tag{A.2}$$

where $w_{1,t+1}$ and $w_{2,t+1}$ are ϕ_t measurable, Y_{t+1} and Z_{t+1} are independent by the bivariate normality assumption, and $E^P[Z_{t+1} | \phi_t] = 0$. Equating the first two moments of $\ln \frac{S_{t+1}}{S_t}$ from (A.2) and the EGARCH(1, 1) process in (2.1), we obtain

$$r_{t+1} + \lambda_{t+1} \sigma_{t+1} - \frac{1}{2} \sigma_{t+1}^2 = w_{1,t+1} + w_{2,t+1} \mu_{t+1}, \tag{A.3}$$

$$\sigma_{t+1}^2 = w_{2,t+1}^2 \delta_{t+1}^2 + v_{t+1}^2, \tag{A.4}$$

where $v_{t+1}^2 = E^P[Z_{t+1}^2 | \phi_t]$.

Finally, applying the fundamental pricing equation to the stock, we obtain

$$\begin{aligned} 1 &= E^P \left[e^{-\kappa} \frac{U'(C_{t+1})}{U'(C_t)} \frac{S_{t+1}}{S_t} \middle| \phi_t \right], \\ &= E^P \left[e^{-Y_{t+1} + \ln \frac{S_{t+1}}{S_t}} \middle| \phi_t \right], \\ &= E^P \left[e^{w_{1,t+1} + (w_{2,t+1} - 1) Y_{t+1} + Z_{t+1}} \middle| \phi_t \right], \\ &= E^P \left[e^{w_{1,t+1} + (w_{2,t+1} - 1) \mu_{t+1} + \frac{1}{2} (w_{2,t+1} - 1)^2 \delta_{t+1}^2 + \frac{1}{2} v_{t+1}^2} \right], \end{aligned}$$

which in turn implies that

$$w_{1,t+1} + (w_{2,t+1} - 1)\mu_{t+1} + \frac{1}{2}(w_{2,t+1} - 1)^2\delta_{t+1}^2 + \frac{1}{2}v_{t+1}^2 = 0. \quad (\text{A.5})$$

Conditions in (A.1), (A.3), (A.4) and (A.5) together give rise to the following result:

$$\lambda_{t+1} = \frac{w_{2,t+1}\delta_{t+1}^2}{\sigma_{t+1}}. \quad (\text{A.6})$$

Now, defining $q_{t+1} \equiv \text{Corr}(\ln \frac{S_{t+1}}{S_t}, Y_{t+1} | \phi_t) = \frac{w_{2,t+1}\delta_{t+1}^2}{\sigma_{t+1}}$. Eq. (A.6) can be rewritten as

$$\lambda_{t+1} = q_{t+1}\delta_{t+1}.$$

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