Pricing Nikkei Put Warrants

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ABSTRACT. Nikkei put warrants are put options on the Nikkei 225 index which are traded in dollars and subject to exchange rate risk. This paper develops close-form Black-Scholes type pricing formulas for three categories of European Nikkei put warrants. Directions are given for numerically valuing American Nikkei put warrants. Despite the fact that there are two underlying state variables (i.e., the Nikkei index and the exchange rate), all warrants can be priced within a single-state-variable framework. The effects on pricing of various factors such as exercise delay and limit options are also discussed.

1. INTRODUCTION

In the past several years, cross-currency options have gained increasing popularity. There are approximately 30 listed Nikkei put warrants in Europe, six listed on the American Stock Exchange, and six listed on the Toronto Stock Exchange, excluding many private deals. (See Appendixes 1 and 2 for summaries of Nikkei put warrants listed on the American Stock Exchange and the Toronto Stock Exchange.) The American Stock Exchange also lists two Nikkei call warrants (one by Salomon Inc. and another by Paine Webber Group). The variety of cross-currency products seems to

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be limited only by imagination. For example, the American Stock Exchange has listed warrants on other foreign indexes such as FT-SE 100 and CAC 40. The Chicago Mercantile Exchange has introduced Nikkei index futures and options on Nikkei index futures, all traded in U.S. dollars. Meanwhile, the Toronto Stock Exchange has listed such products as call warrants on U.S. Treasury Bonds (issued by BT Bank of Canada), put warrants on Canadian dollars vis-à-vis U.S. dollars (issued by Canadian Imperial Bank of Canada), and call warrants on the New York Composite Index (issued by Citibank Canada). These contracts have a number of different exchange rate/payoff specifications and have created a rich array of investment and hedging opportunities.

Given the globalization of financial markets, it is most probable that options on foreign indexes (or securities in general) will continue to flourish. A theoretical investigation into the pricing of these options is therefore of interest. The pricing of financial products based on the exchange rate alone has been well studied. Shortly after the Philadelphia Exchange commencing trading of currency options in 1982, several authors have explicitly discussed the pricing of currency options (Biger and Hull, 1983; Garman and Kohlhagen, 1983; and Grabbe, 1983). Recent work on currency options includes Adams and Wyatt (1987), Bodurtha and Courtadon (1987), Choi and Hauser (1990), Hull and White (1987a), Rogalski and Seward (1991), Shastri and Tandon (1986), and Shastri and Tandon (1987). However, little work exists in the literature that deals with cross-currency instruments.[1] This study develops theoretical pricing models for Nikkei put warrants that are currently in existence in North American markets. It should be pointed out that the models developed in this paper are perfectly general. To apply the models to a warrant on other foreign indexes such as FT-SE 100 and CAC 40, we need only to classify the warrant into a proper category and re-specify the parameters.

The remainder of the paper is organized as follows. Section 2 classifies and describes the Nikkei put warrants. In section 3, theoretical pricing models are developed for both European and American style Nikkei put warrants. Section 4 justifies some of the modelling assumptions. The paper is concluded in section 5.

2. AN OVERVIEW OF THE NIKKEI PUT WARRANTS

2.1 Classifying the Nikkei Put Warrants

Nikkei put warrants (NPWs or warrants hereafter) are put options that entitle a holder (outside Japan) to receive (from the issuer) the difference (if any) between the strike price and the level of the Nikkei 225 index, taking into consideration the exchange rate and the exercise multiple.[2,3] Therefore, NPWs are instruments for investors who are bearish on the Japanese market or wish to hedge existing holdings on Japanese stocks. In the U.S., Kingdom of Denmark and the Salomon Inc. launched the first NPW issue in January 1990. In Canada, the first NPW was issued by Bankers Trust Bank of Canada in February 1989.

As mentioned earlier, there are six different NPWs trading on the American Stock Exchange (AMEX) and six on the Toronto Stock Exchange (TSE), all of which are American style put options on the Nikkei 225 index with 3 years to maturity at issue.[4] AMEX warrants are traded in U.S. dollars, while TSE warrants are traded in Canadian dollars. Let K denote the strike price, S_T the index level on the exercise day, and X_T the exchange rate on the exercise day (X_T is the dollar value of one Japanese yen). An examination of warrants' payoffs (see Appendixes 1 and 2 for payoff functions) reveals that there are three general classes or categories of NPWs:

Category	*Payoff upon Exercise in Domestic Currency*
Category I NPWs:	Max $[0, X_T(K - S_T)]$
Category II NPWs:	Max $[0, X_0(K - S_T)]$
Category III NPWs:	Max $[0, X_0K - X_TS_T]$

A category I NPW pays off *multiple*max[0,* $X_T(K-S_T)$*]* upon exercise.[5] For this category, a warrant holder, upon exercising the warrant, will be entitled to the difference in yen between the exercise price and the Nikkei 225 index closing level. This amount in yen will then be converted into dollars at the prevailing exchange rate. One example of category I NPWs is Bankers Trust New York NPW, with parameter values of 0.5 for the multiple, and ¥37,206.42 for K, the exercise price. Therefore the payoff on this NPW would be:

$$\$0.5*(37206.42 - 25000)*0.00714 = \$43.58$$

if the index closes at ¥25,000 on the exercise date, and the exchange rate on that date is 0.00714 US$/Jap¥ (or 140 Jap¥/US$). See Appendixes 1 and 2 for examples of other category I NPWs.

A category II NPW has a payoff function: $multiple*X_0*max[0, K-S_T]$. The payoff is the yen difference between the strike price and the index level upon exercise, converted to dollars at a pre-specified exchange rate. The Kingdom of Denmark NPW is an example of category II NPWs. The multiple parameter, exercise price K, and the pre-specified exchange rate are 0.2, ¥37,516.77, and 0.00688 US$/Jap¥, respectively. Therefore, on the exercise date, the warrant's payoff would be:

$$\$0.2*0.00688*(37516.77 - 25000) = \$17.27$$

if the Nikkei index closes at ¥25,000 on that day. It can be seen from Appendixes 1 and 2 that most NPWs are of category II.

A category III NPW has a payoff function: $multiple*max[0, KX_0 - S_TX_T]$. Here the exercise price K is converted into dollars up front at a pre-specified exchange rate, while the index is converted at the future spot exchange rate. For example, the Bankers Trust Bank of Canada Series II NPWs' exercise price is ¥33,403. It is converted to Cdn$270.54 at the exchange rate prevailing on the issue date, which is 0.0081 Cdn$/Jap¥ (or 123.47 Jap¥/Cdn$). The multiple for this warrant is 0.10311. Thus, the payoff would be

$$Cdn\$0.10311*(270.54 - 25{,}000*0.008) = Cdn\$7.27$$

if the Nikkei 225 index closes at 25,000 on the exercise date, and if the spot Cdn$/Jap¥ exchange rate is 0.008 on that day. The

Bankers Trust Bank of Canada Series II NPW is the only category III warrant. See Appendix 2 for details.

2.2 The Exercise of Nikkei Put Warrants

Due to the time zone difference, a warrant holder will not be able to determine the cash settlement value at the time of exercise. When the North American exchanges are open (say from 9:30 a.m. to 4:30 p.m. EST–Eastern Standard Time), the Tokyo Stock Exchange remains closed. When the Tokyo Stock Exchange is open, the North American exchanges stay closed. Realizing this non-simultaneity in trading, warrant issuers explicitly specify how the warrants can be exercised. Typically, after delivering an exercise notice, a warrant holder has to wait for one or two business days before the exercise can be settled. An *Exercise Date* is the business day (EST) on which a warrant holder gives instructions to exercise the warrant. A *Valuation Date* is the business day (EST) on which the exercise is actually settled. Depending on the issuers, the Valuation Date is either the business day immediately following the Exercise Date or the second business day after the Exercise Date. To better understand how the exercise delay clause works, let's look at the following example. Suppose you decide to exercise your Bankers Trust NY Corp. NPWs. You tender your warrants and deliver your exercise notice on Thursday (EST). Then your exercise will be settled on Friday (EST), using the closing quotations of the Nikkei 225 index and the exchange rate from the last trading session in Tokyo. Similarly, if you give exercise instructions on Friday (EST), then your exercise will be settled on Monday. Since the index/exchange rate may move adversely between the Exercise Date and the Valuation Date, a warrant holder incurs an "exercise risk." To mitigate this "exercise risk," issuers usually give warrant holders a free "limit option" upon exercise.

Applicable to almost all NPWs trading on the American Stock Exchange and the Toronto Stock Exchange, a "limit option" works in the following way. When delivering an exercise notice, if the warrant holder chooses the limit option (free of charge), and if, on the settlement day (i.e., the Valuation Date), the Nikkei closes at a level which is more than 500 points above the Nikkei close on the exercise day (i.e., the Exercise date), then the warrant is deemed not to be exercised, i.e., the warrant holder can (and must) get the warrant back. If, on the other hand, the limit option is not chosen, the warrant is deemed to be exercised, no matter how much the Nikkei index has moved during the one or two business day waiting period. Essentially, the limit option is a "protector" against adverse index movements during the waiting period. It can be seen that the effects of the exercise delay and limit option offset each other, with the former adversely affecting the warrant value and the latter favourably affecting the warrant value.

3. *THE PRICING OF NIKKEI PUT WARRANTS*

There are two direct underlying factors that affect NPWs' value: the Nikkei 225 index level and the exchange rate. We assume that each factor follows a geometric Brownian motion:[6]

$$\frac{dS}{S} = \mu_s dt + \sigma_s dz \qquad (1)$$

$$\frac{dX}{X} = \mu_x dt + \sigma_x dw \qquad (2)$$

where dz and dw are standard Wiener processes with instantaneous correlation ρ.

Other notation is as follows:

S = Nikkei 225 index level in yen;
X = the US$/Jap¥ (or Cdn$/Jap¥) exchange rate;
μ_s = instantaneous drift rate of dS/S;
μ_x = instantaneous drift rate of dX/X;
σ_s = constant instantaneous standard deviation of dS/S;
σ_x = constant instantaneous standard deviation of dX/X.

We also make the following assumptions:

1. there are no taxes and transactions costs;
2. securities are traded continuously and there are no restrictions on short selling;
3. the risk free interest rates r (domestic) and r_f (Japanese) are constant;
4. the Nikkei 225 index pays a continuous dividend yield, q;
5. simultaneity exists between the warrant price and the Nikkei index level (and the exchange rate).[7]

Given the above assumptions, it can be shown that the risk-neutral valuation technique pioneered by Cox and Ross (1976) can be applied to the pricing of NPWs.[8] Specifically, the processes of S and X in a risk-neutral world can be written as

$$\frac{dS}{S} = (r_f - q - \sigma_{sx})dt + \sigma_s dz \qquad (3)$$

$$\frac{dX}{X} = (r - r_f)dt + \sigma_x dw \qquad (4)$$

With the above risk-neutral processes, the NPWs can be priced in the same way as ordinary equity options. We will first examine the pricing of European NPWs.

3.1 Pricing European Nikkei Put Warrants

Although all exchange-listed NPWs are American warrants, we first examine their European counterparts in order to better understand the pricing mechanism. To derive values of European warrants, we first find the expected payoff of the warrant, and then discount back (in a risk-neutral world) this expected payoff at the domestic risk free interest rate. For example, the value of a European category I NPW can be found by evaluating the following expression:

$$e^{-r(T-t)} \int_0^{+\infty} \int_0^K X_T(K-S_T) f(X_T, S_T, \rho) dX_T dS_T$$

where $f(X_T, S_T, \rho)$ is a bivariate lognormal density function based on the processes in (3) and (4). In what follows, we will use $P_i(S,X,t)$ (i = I, II, III) to denote the values of European NPWs of each category.

3.1.1 Category I Nikkei Put Warrants. An application of the above procedures to a European category I NPW leads to:[9]

$$P_I(S, X, t) = X[Ke^{-r_f(T-t)}N(-a_2) - Se^{-q(T-t)}N(-a_1)] \quad (5)$$

where N(•) is the cumulative standard normal density function, and

$$a_1 = \frac{\ln(\frac{S}{K}) + (r_f - q + \frac{\sigma_S^2}{2})(T-t)}{\sigma_S\sqrt{T-t}}, \qquad a_2 = a_1 - \sigma_S\sqrt{T-t}$$

Note that the NPWs value is simply the value of an ordinary Japanese put option (denominated in Japanese yen), converted to dollars at the *current* exchange rate. The correlation between S and X is irrelevant. Indeed, (5) could have been derived by the following simple arbitrage arguments. The value of the ordinary Japanese put option is

$$P = Ke^{-r_f(T-t)}N(-a_2) - Se^{-q(T-t)}N(-a_1) \quad (6)$$

where a_1 and a_2 are defined as before. One such put would cost $P \cdot X$ now for a domestic investor and give a payoff (in dollars) of max[0, $(K - S_T)X_T$] at maturity. Notice that this terminal payoff is exactly equal to that of a European category I NPW. To avoid arbitrage opportunities, the European category I NPW must be priced at XP, which is identical to (5).

The comparative statics are the same as those for conventional put/call options, except that every derivative is multiplied by X. For brevity, they will not be duplicated here.[10]

3.1.2 Category II Nikkei Put Warrants. Discounting the expected maturity payoff of a European category II NPW leads to:

$$P_{II}(S, X, t) = X_0[Ke^{-r(T-t)}N(-b_2) - Se^{-(r+q+\sigma_{SX}-r_f)(T-t)}N(-b_1)] \quad (7)$$

where

$$b_1 = \frac{\ln(\frac{S}{K}) + (r_f - q - \sigma_{sx} + \frac{\sigma_S^2}{2})(T-t)}{\sigma_s\sqrt{T-t}}, \qquad b_2 = b_1 - \sigma_s\sqrt{T-t}$$

There are several interesting features about the above results. First, the value of a category II NPW depends on both the domestic and the Japanese riskfree interest rates. Second, a European category II NPW can be thought of as a domestic option on an asset paying a continuous dividend yield of $(r + q + \sigma_{sx} - r_f)$. Finally, the payoff from a European category II NPW is independent of the exchange rate, yet the current value of the warrant does depend on the covariance between the index level and the exchange rate. This "counter-intuitive" result can be better understood in the Black-Scholes "perfect hedge" context (Black and Scholes, 1973). The Black-Scholes option pricing formula is derived by noting that, at each instant, an option can be replicated by a portfolio of the underlying asset and a discount bond. To avoid arbitrage opportunities, the option's price must equal the present value of all the costs (for example, borrowing costs) incurred during the life of the option if the replicating portfolio is continuously rebalanced to track the option's value. In the case of a category II NPW, the underlying asset (i.e., the Nikkei 225 index) is denominated in foreign currency (Japanese yen). Every time a domestic investor rebalances the portfolio which tracks the warrant's value, it is necessary to take positions in the index, which causes foreign exchange rate risk exposure. As a result, part of the replication costs is attributable to the exchange rate fluctuations. The "perfect hedge" and arbitrage free arguments then dictate that the covariance between the index and the exchange rate should be reflected in the price.

The way in which the index level S and time to maturity $(T\text{-}t)$ affect a category II NPWs value is conventional. The effects of other parameters are less clear-cut and are shown below. It can be seen that the domestic and the Japanese riskfree interest rates affect the warrant's value in a conventional way. The effect of the correlation coefficient is as expected, since σ_{sx} can be thought of as part of the dividend yield. The effects of volatilities are not always positive. For the index volatility, the sign of the derivative depends, among other things, on the correlation coefficient and the degree to which the warrant is in-the-money. For the exchange rate volatility, the sign of the derivative is solely determined by the correlation coefficient.

$$\frac{\partial P_{II}}{\partial r} = -(T-t)P_{II} < 0\,,$$

$$\frac{\partial P_{II}}{\partial r_f} = -(T-t)X_0 S e^{(r_f-r-q-\sigma_{sx})(T-t)} N(-b_1) < 0\,,$$

$$\frac{\partial P_{II}}{\partial \rho} = \sigma_s \sigma_x X_0 S(T-t) e^{(r_f-r-q-\sigma_{sx})(T-t)} N(-b_1) > 0,$$

$$\frac{\partial P_{II}}{\partial \sigma_s} = \sqrt{T-t}X_0 K e^{-r(T-t)} N'(-b_2) + \rho\sigma_x X_0 S(T-t) e^{(r_f-r-q-\sigma_{sx})(T-t)} N(-b_1) \geq (\leq)\, 0,$$

$$\frac{\partial P_{II}}{\partial \sigma_x} = \rho\sigma_s X_0 S(T-t) e^{(r_f-r-q-\sigma_{sx})(T-t)} N(-b_1) \geq (\leq)\, 0.$$

3.1.3 Category III Nikkei Put Warrants. Applying the above risk-neutral valuation technique to European category III NPWs leads to:

$$P_{III}(S,X,t) = X_0 K e^{-r(T-t)} N(-c_2) - (S\cdot X) e^{-q(T-t)} N(-c_1) \qquad (8)$$

where

$$c_1 = \frac{\ln\left(\frac{S\cdot X}{KX_0}\right) + \left(r-q+\frac{v^2}{2}\right)(T-t)}{v\sqrt{T-t}}, \qquad c_2 = c_1 - v\sqrt{T-t}$$

$$v = \sqrt{\sigma_S^2 + 2\rho\sigma_S\sigma_X + \sigma_X^2}$$

A careful examination of the above formula reveals that a European category III NPW can be likened to a domestic option with strike price KX_0, written on an asset $f = S\cdot X$ which is denominated in the domestic currency and pays a continuous dividend yield q.

The value of a category III NPW is independent of the Japanese riskfree interest rate. The effects of the domestic riskfree interest rate, the index level, the current exchange rate, and time-to-maturity are easy to see. The derivatives with respect to the volatilities and the correlation coefficient are shown below.

$$\frac{\partial P_{III}}{\partial \sigma_S} = \frac{\sigma_S + \rho\sigma_X}{v} S\cdot X\sqrt{T-t}N'(c_1)e^{-q(T-t)} \geq (\leq)\, 0$$

$$\frac{\partial P_{III}}{\partial v} = S\cdot X\sqrt{T-t}N'(c_1)e^{-q(T-t)} > 0$$

$$\frac{\partial P_{III}}{\partial \sigma_X} = \frac{\sigma_X + \rho\sigma_S}{v} S\cdot X\sqrt{T-t}N'(c_1)e^{-q(T-t)} \geq (\leq)\, 0$$

$$\frac{\partial P_{III}}{\partial \rho} = \frac{\sigma_S\sigma_X}{v} S\cdot X\sqrt{T-t}N'(c_1)e^{-q(T-t)} > 0$$

Although the value of a category III NPW is positively related to the overall volatility of SX, the effects of individual volatilities are no longer clear-cut. If the proportional changes of S and X are positively correlated, then the volatilities affect the warrant's value in a conventional way (that is, having positive effects). However, if the correlation is sufficiently negative, then it is possible that the volatilities inversely affect the warrant's value.[11] Finally, given that a category III NPW can be thought of as an ordinary option on an asset $f = SX$, it is easy to understand that the NPWs value is positively related to the correlation coefficient between the proportional changes of S and X.

3.2 Pricing American Nikkei Put Warrants

It is well known that an American call option on a non-dividend paying stock should never be exercised prematurely but an American put on the same stock may be exercised prior to maturity (Merton, 1973). When the underlying stock pays a continuous dividend yield, an American put option may still be exercised early. As shown in section 3.1, NPWs of categories I and III can be considered as ordinary put options (with careful interpretations), thus American NPWs of these two categories may be exercised prior to maturity. The early exercise of an American category II NPW is not immediately obvious. We show below that it is always possible to prematurely exercise an American category II NPW.[12]

If $AP_{II}(S,X,t)$ is the price of an American category II NPW, then by definition

$$AP_{II}(S, X, t) \geq \max(X_0(K - S), P_{II}(S, X, t))$$

where $P_{II}(S,X,t)$ is defined in (7). If we can establish that the probability for $X_0(K-S)$ to be greater than $P_{II}(S,X,t)$ is not zero, then there is always a positive probability that the American NPW will be exercised prematurely. To see this, note that an American category II NPW is worth at least $X_0(K-S)$. Therefore the relationship $X_0(K-S) > P_{II}(S,X,t)$ implies $AP_{II}(S,X,) > P_{II}(S,X,t)$, which in turn implies a positive probability of early exercise, because the

American NPW will be priced the same as its European counterpart if the probability of early exercise is zero.

Now referring to (7), when $S \rightarrow 0$, $P_{II}(S,X,t) \rightarrow X_0[Ke^{-r(T-t)} - Se^{-(r+q+\sigma_{sx}-r_f)(T-t)}]$. Therefore it is possible that $X_0(K-S) > P_{II}(S,X,t) \approx X_0[Ke^{-r(T-t)} - Se^{-(r+q+\sigma_{sx}-r_f)(T-t)}]$. This is because we can always find an asset price S so that $S < K(1-e^{-r(T-t)})/(1-e^{-(r+q+\sigma_{sx}-r_f)(T-t)})$, as long as $r + q + \sigma_{sx} - r_f > 0$ is satisfied. Therefore the probability of prematurely exercising an American category II NPW is not zero. When the American NPW is optimally exercised prematurely we must have $AP_{II}(S^*,X,t) = X_0(K-S^*)$. Therefore an S^* always exists such that whenever $S < S^*$ an American category II NPW will be exercised immediately.

The early exercise of NPWs causes complications for pricing, since analytical pricing formulas do not generally exist for American options that can be prematurely (and optimally) exercised. Numerical procedures have to be employed. When there are two underlying state variables, a two-state-variable numerical model has usually to be employed. Although a NPW generally depends on two state variables (the index level and the exchange rate), it can be shown that all American NPWs can be valued using a single state variable numerical model. To see this, note that the pricing formulas for all European NPWs are in the same spirit as the Black-Scholes (1973) model. They all collapse to a single state variable problem. For example, to apply the binomial model (Cox, Ross and Rubinstein, 1979) to each category of NPWs, the single state-variable pricing framework (in a risk-neutral world) can be characterised as follows:[13]

	Current Value of the State Variable	Strike Price	Drift Rate	Variance	Dividend Yield	Discount Rate
Category I	SX	KX	r_f	σ_s^2	q	r_f
Category II	SX_0	KX_0	$r_f-\sigma_{sx}$	σ_s^2	q	r
Category III	SX	KX_0	r	$\sigma_s^2+2\rho\sigma_s\sigma_x+\sigma_x^2$	q	r

Each American NPW can then be priced using a binomial tree based on a proper "underlying state variable."[14] For example, an American category III NPW can be priced as if it were an American put option on an asset with a current price SX, a risk-neutral drift rate r, an instantaneous variance $\sigma_s^2 + 2\rho\sigma_s\sigma_x + \sigma_x^2$, and a continuous dividend yield q. The fictitious American put option has a strike price KX_0, and is evaluated in a risk-neutral world with a riskfree rate r. It should be noted that the S and X in the above framework simply stand for the current value of the index and the exchange rate. For instance, for an American category I NPW, the single state variable *does not* follow a process with a drift rate r and a variance $\sigma_s^2 + 2\rho\sigma_s\sigma_x + \sigma_x^2$, the process followed by SX if S and X stand for stochastic variables. By the same token, the strike price, KX, is *not* meant to be a random variable by specification. What we really mean is that, at any given moment, for pricing purposes, the strike price of the option takes on the value KX.

It is not difficult to understand why a single-state-variable numerical framework can handle the pricing of American NPWs. For American category I NPWs, we can think of the pricing procedure as using the binomial tree to price the ordinary American option in Japan and then converting the value into domestic currency at the current exchange rate X. An American category II NPW involves only one state variable whose drift rate and volatility in the (domestic) risk-neutral world are $r_f - \sigma_{sx}$ and σ_s respectively, so a single-state-variable numerical pricing framework based on these parameters applies. Finally, for a category III NPW, the two state variables S and X combine into one, SX, which is a "tradable" asset domestically.

Due to the time zone difference, we can not obtain "true" quotations for the index when we price the warrants. (We do not have this problem with the exchange rate since the yen currency is traded in the North American markets.) The best we can do is to use the closing quotation of the Nikkei index from the previous trading session on the Tokyo Stock Exchange. We will illustrate a detailed example to show how a NPW can actually be priced. Suppose we want to price a category III American NPW with the following

variable/ parameter inputs (the Nikkei 225 index value is supposed to be the closing quotation from the previous trading session on the Tokyo Stock Exchange):

S = ¥30000	K = ¥30000	r = 0.10
X = 0.008 US$/Jap¥	X_0 = 0.008333 US$/Jap¥	$T-t$ = 2
σ_s = 0.30	ρ = -0.10	
σ_x = 0.10	q = 0.01	

Suppose also we use a binomial lattice with 100 steps (n = 100). Then using the notation in Cox, Ross, and Rubinstein (1979), the binomial tree starts at SX = $30000*0.008 = $240, with a strike price KX_0 = $30000*0.008333 = $250. The volatility is

$$\sigma = \sqrt{\sigma_S^2 + 2\rho\sigma_S\sigma_X + \sigma_X^2} = 0.3066.$$

At each node on the lattice the amplitude and the probability of upward / downward movement are:

$$u = e^{\sigma\sqrt{(T-t)/n}} = 1.0443 \qquad d = e^{\sigma\sqrt{(T-t)/n}} = 0.9576$$

$$p = \frac{e^{(r-q)(T-t)/n} - d}{u-d} = 0.5098 \qquad 1-p = \frac{u - e^{(r-q)(T-t)/n}}{u-d} = 0.4902$$

Working backwards through the lattice, we get the value of this American NPW: $31.06. By (8), the value of its European counterpart is $24.37.

4. JUSTIFICATIONS FOR SOME OF THE PRICING ASSUMPTIONS

4.1 *Simultaneity of Observable Variables*

As discussed earlier, the warrant prices and the Nikkei index level are not quoted simultaneously. The natural strategy is to use the closing quotation from the previous trading session in Tokyo (which is at least 8 hours old). The question is, Would this approximation cause a significant pricing bias? To answer this question properly we must first make clear what approximation we are actually making. The true value of a NPW is the *expected* value of the NPW at time t given the index quotation at time $t-h$, where h is between 8 and 15 hours.[15] Therefore the approximation involves calculating this true NPW value at time t as if the quotation for time $t-h$ were the true quotation for time t.

The problem of finding the *expected* value of a European option at time t given the variable/parameter values at time $t-h$ has been solved by Rubinstein (1984). Assuming that all investors agree that the underlying stock price S follows a lognormal distribution with an instantaneous drift rate μ and variance σ, Rubinstein (1984) showed that the expected value of a put option on this underlying stock for time t is

$$Expected\ P(t) = Ke^{rh}e^{-r(T-(t-h))}N(-a_2') - Se^{\mu h}N(-a_1') \qquad (9)$$

where K is the strike price, r the riskfree interest rate, T the maturity date, and

$$a_2' = \frac{\ln\left(\frac{Se^{\mu h}}{Ke^{rh}}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-(t-h))}{\sigma\sqrt{T-(t-h)}}, \qquad a_2' = a_1' - \sigma\sqrt{T-(t-h)}$$

It can be seen that the expected put option value is like that of an ordinary put at time $t-h$, except that the inputs for the stock price and the strike price are $Se^{\mu h}$ and Ke^{rh} respectively.

A closed-form formula does not exist when the above problem is solved for an American put (or call) option. But we can approximate the expected value using a Taylor series expansion.[16] Using the above notation and letting S_t^e denote the expected value of S_t, the expected value of an American put at time t can be written as

$$\int_0^{+\infty} P(S_t)f(S_t)dS_t$$

where $f(S_t)$ is the probability density function for S_t in a normal world (as opposed to a risk-neutral world). Applying Taylor series expansion to $P(S_t)$ at S_t^e and omitting terms with higher orders lead to

$$\int_0^{+\infty} P(S_t)f(S_t)dS_t = \int_0^{+\infty} [P(S_t^e)+P'(S_t^e)(S_t-S_t^e) + \frac{1}{2}P''(S_t^e)(S_t-S_t^e)^2]f(S_t)dS_t \qquad (10)$$

where the single prime and the double prime denote first order and second order derivatives respectively. Note that the second term on the right hand side of the equation vanishes after evaluating the integral, and that the last term is simply one-half of the product of the variance of S_t and the Gamma of the put.[17] Let Γ denote the Gamma of the put, then the above equation can be simplified to

$$\int_0^{+\infty} P(S_t)f(S_t)dS_t = P(S_t^e) + \frac{1}{2}\Gamma Var(S_t) \qquad (11)$$

where by definition

$$S_t^e = S_{t-h}e^{\mu h} \qquad \Gamma = \frac{\partial^2 P(S_t^e)}{\partial (S_t^e)^2} \qquad Var(S_t) = S_{t-h}^2 e^{2\mu h}(e^{\sigma^2 h}-1)$$

In words, the expected value of an American put at time t (given the stock price at time $t-h$) is the value of an American put evaluated at time t with a stock price $S_{t-h}e^{\mu h}$, adjusted by one-half of the product of the variance of S_t and the Gamma of the put.

Now we are ready to investigate the pricing biases. Our original theme is to approximate the expected put value at time t by the value of an American put at time t with a stock price S_{t-h}, which we will call the "approximated value." Similarly, we will call the expected value obtained from (11) the "real value." Using a trinomial tree with 100 steps and setting the forward looking period to one day, we obtain the pricing biases for various input value combinations.[18] The results are shown in Table 1. It can be seen that the percentage biases are minimal in all cases. The largest relative bias (0.37%) occurs with an in-the-money option when the volatility is low. In addition, it is apparent that the approximation procedure over-prices (under-prices) the option when the volatility is low (high). Applying the above findings to NPWs leads us to conclude that the use of the closing quotation for the Nikkei 225 index is justified.

4.2 *Exercise Delay and the Limit Option*

Assumption 5 implies that the NPWs can be exercised based on the concurrent (but non-existent) index level. The exercise delay and the limit option features invalidate this assumption. To evaluate the impacts of these two complications we will assume here that the non-simultaneity problem is absent, i.e., concurrent quotations exist for both the index and the exchange rate. We will first illustrate how the exercise delay and the limit option can be incorporated into the binomial tree for an ordinary put option, and then generalize the methods to the NPWs.

We work backwards along a binomial tree when we value an American put option. At each node we compare two values of the option: the value if we exercise it immediately and the value if we hold the option. Referring to Figure 1, the exercise value at node i is $\max(0, K-S)$, where K is the strike price. The value of the option, if we continue to hold it, is given by Put $= p$*(Put_up) + $(1-p)$*(Put_down), where p and $(1-p)$ are the probabilities for S to move up to S_up and down to S_down, respectively; and Put_up and Put_down are the put option's values at node $(i+1)$, corresponding to the upward and downward movements of the stock price. At node i, the option's value is then given by max{max(0, $K-S$), Put}. Working backwards along the tree and carrying out the above evaluation for every node, we get the current value of the American put option.

Table 1

Pricing Biases due to Approximation for the Expected Value of an American Put

$\mu = 0.15$ $r = 0.10$ $h = 1$ day

S: the current stock price K: the exercise price S/K: the degree of being in-the-money

		VOLATILITY								
		0.10			0.20			0.30		
		S/K			S/K			S/K		
		0.9	1.0	1.1	0.9	1.0	1.1	0.9	1.0	1.1
TIME	0.5	0.37	0.21	0.00	0.06	-0.05	-0.31	-0.03	-0.15	-0.30
TO	1.0	0.37	0.25	0.30	0.05	0.00	-0.10	-0.02	-0.05	-0.11
MATURITY	1.5	0.37	0.18	0.00	0.05	0.02	0.00	-0.01	-0.03	-0.05
	2.0	0.37	0.29	0.00	0.05	0.02	0.00	-0.01	-0.02	-0.04

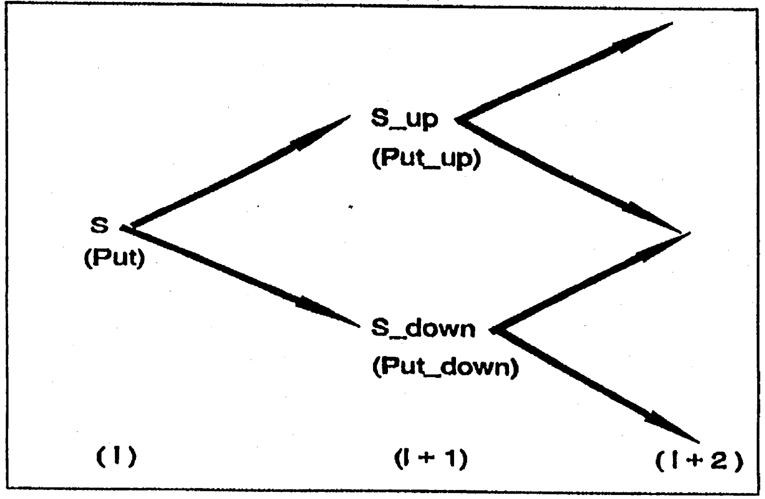
Note:

a) The pricing biases are calculated as (approximated value - real value)/(real value)*100%. The "approximated value" is the value of an American put at time t with the stock price at time (t - h) as input. The "real value" is an American put option's value calculated according to Eq(11). It is the true option's value when the concurrent stock price is not observable.

b) All American put option values are calculated using a trinomial tree with 100 steps.

Figure 1.

A Two-Step Binomial Tree Illustrating Stock Price (S) and Put Option Price (Put) Movements



Now suppose there is a one-day exercise delay. First we make the time interval between two adjacent nodes equal one day. The exercise value at node i (denoted by "delay") would be delay $= p^*\max(0, K-S\_up) + (1-p)^*\max(0, K-S\_down)$, and the option's value at node i is given by max{delay, Put}.[19] Repeating this evaluation for every node along the tree (backwards), we get the option's current value.

What if we introduce the limit option feature? Suppose the option's contract specifies, in connection with the one-day exercise delay, (a) that the option holder can (and must) get the option back if the stock price moves up by more than $x, (b) that the option is deemed to be exercised if the upward movement of the stock price during the one-day waiting period is less than $x, and (c) that if the option holder does not choose this limit option when delivering the exercise notice then the option is deemed to be exercised as if it were a normal exercise-delay option. In this case, at each node, we

must compare three values: (1) the value if we do not exercise the option, which is equal to "Put," (2) the value if we exercise the option but do not choose the limit option, which is equal to "delay," (3) and the value if we exercise the option and choose the limit option. Let "limit" denote the value of the option for situation 3, then

$$\text{limit} = \begin{cases} p^*\text{Put\_up} + (1-p)^*\max(0, K-S\_down) & \text{if } S\_up - S > x, \\ p^*\max(0, K-S\_up) + (1-p)^*\max(0, K-S\_down) = \text{delay} & \text{if } S\_up - S < x. \end{cases}$$

Therefore the value of the option at node i is max{Put, delay, limit}. Again, working backwards along the tree leads to the current value of an American option with one-day exercise delay and a limit option.

The above ad hoc modification of the binomial lattice can be directly applied to category II NPWs. For categories I and III NPWs, a two-state-variable lattice must be used, since when we look ahead for one day, we have two random variables: the index and the exchange rate. Notwithstanding, the above logic flow still applies. We will show below that the impacts of the exercise delay and the limit option on the NPWs' value are minimal for all three categories of NPWs.

A two-state-variable lattice framework (Boyle, 1988) is used for category I warrants. Three values are calculated for each set of hypothetical parameter inputs: the value of a plain warrant (denoted by V), the value of a warrant with one-day exercise delay (denoted by VD), and the value of a warrant with both one-day exercise delay and a limit option (denoted by VDL).[20] We compare the value of a plain warrant with those of the other two and examine the pricing differences. Since the time interval on the lattice must be at least as small as one day, we choose a time-to-maturity of 0.3 years which translates to 110 steps when the interval is one day.[21] The results are summarized in Table 2. Two observations can be made. First, the exercise delay and the limit option affect the warrant value in opposite directions, with the effect of exercise delay being greater than that of the limit option. Second, the exercise delay and the limit

Table 2

Pricing Biases due to Ignoring the Exercise Delay and the Limit Option Features

--------Category I NPWs

$r_f = 0.06$ $\sigma_x = 0.20$ $\rho = -0.05$ $q = 0.0043$
$t = 0.3$ years $X = 0.008333$ US$/Jap¥ $K = 30000$
multiple = 0.1 exercise delay = one day limit size = 500

V: Value of a plain NPW; VD: Value of a NPW with exercise delay; VDL: Value of a NPW with exercise delay and limit option.

		S/K = 0.8333 (S = 25000)			S/K = 1 (S = 30000)			S/K = 1.167 (S = 35000)		
		V	VD	VDL	V	VD	VDL	V	VD	VDL
	0.15	$4.167	4.163	4.163	$0.665	0.664	0.664	$0.015	0.015	0.015
			0.10%	0.10%		0.15%	0.15%		0.00%	0.00%
INDEX VOLATILITY	0.35	$4.236	4.233	4.233	$1.459	1.457	1.457	$0.346	0.346	0.346
			0.07%	0.07%		0.14%	0.14%		0.00%	0.00%
	0.45	$4.689	4.686	4.687	$2.263	2.261	2.262	$0.964	0.963	0.963
			0.06%	0.04%		0.09%	0.04%		0.10%	0.10%

Note:

a) This table calculates and compares (with different parameter inputs) three sets of category I NPWs values: the NPW value without exercise delay and limit option features (V), the NPW value with the exercise delay feature only (VD), and the NPW value with both the exercise delay and the limit option features (VDL).

b) Pricing biases are caused due to ignoring the exercise delay feature and the limit option feature. Thus the percentage numbers in columns with the heading VD are calculated as (V-VD)/VD*100%, and the percentage numbers in columns with the heading VDL are calculated as (V-VDL)/VDL*100%.

c) For all calculations, a two-state-variable trinomial tree with 110 steps is used. The length of each step interval is one day.

option complications do not significantly affect the warrant value. The largest relative difference occurs with at-the-money warrant when the index volatility is low. But even this largest difference (0.15%) is negligible.

A single-state-variable binomial model is used for category II NPWs. The step size is one day and the time-to-maturity is 2.5 years which translates to 913 steps. The calculations and comparisons are in the same manner as for category I NPWs. The results are summarized in Table 3. It can be seen that the results are very similar to those in Table 2. The largest pricing difference (0.14%) also occurs with an at-the-money warrant when the index volatility is low.

Similar to the case of category I NPWs, a two-state-variable lattice with 110 steps is used for category III NPWs. The step size is also one day. The results are shown in Table 4. Compared with categories I and II NPWs, the pricing differences here are relatively more manifest. But the largest percentage deviation is only 0.31% and is negligible.

The above results on a whole indicate that, for pricing purposes, the exercise delay and the limit option complications can be ignored. Although our results hinge upon the assumption of concurrent quotations for the index and the exchange rate, given the evidence shown in the early part of this section, we do not expect the results to change substantially if this assumption is dropped.

To sum up, relaxing some of the assumptions does not lead to a significant change in pricing results obtained from the basic models we proposed. This is especially true for the approximation assumptions, namely, the assumption of simultaneous quotations of the index/exchange rate and the warrant prices, and the assumption of immediate exercise settlement based on concurrent quotations.

5. CONCLUSIONS

This paper has developed closed-form pricing formulas for three categories of European Nikkei put warrants. Directions are given for numerically valuing American Nikkei put warrants. It has been

Table 3
Pricing Biases due to Ignoring the Exercise Delay and the Limit Option Features
———Category II NPWs

$r_f = 0.06$ $r = 0.12$ $\sigma_z = 0.20$ $\rho = -0.05$ $q = 0.0043$
$t = 2.5$ years $X_0 = 0.008333$ US\$/Jap¥ $K = 30000$
multiple = 0.12 exercise delay = one day limit size = 500

V: Value of a plain NPW; VD: Value of a NPW with exercise delay; VDL: Value of a NPW with exercise delay and limit option

	S/K = 0.8333 (S = 25000)			S/K = 1 (S = 30000)			S/K = 1.167 (S = 35000)		
	V	VD	VDL	V	VD	VDL	V	VD	VDL
INDEX VOLATILITY 0.15	\$5.000	4.994	4.994	\$1.472	1.470	1.470	\$0.414	0.413	0.413
		0.12%	0.12%		0.14%	0.14%		0.24%	0.24%
0.25	\$5.460	5.455	5.455	\$2.942	2.940	2.940	\$1.605	1.604	1.604
		0.09%	0.09%		0.07%	0.07%		0.06%	0.06%
0.35	\$6.609	6.605	6.605	\$4.453	4.450	4.450	\$3.055	3.052	3.053
		0.06%	0.06%		0.07%	0.07%		0.10%	0.07%
0.45	\$7.892	7.888	7.888	\$5.960	5.956	5.957	\$4.580	4.577	4.577
		0.05%	0.05%		0.07%	0.05%		0.07%	0.07%

Note:

a) This table calculates and compares (with different parameter inputs) three sets of category II NPWs' values: the NPW value without exercise delay and limit option features (V), the NPW value with the exercise delay feature only (VD), and the NPW value with both the exercise delay and the limit option features (VDL).

b) Pricing biases are caused due to ignoring the exercise delay feature and the limit option feature. Thus the percentage numbers in columns with the heading VD are calculated as (V-VD)/VD*100%, and the percentage numbers in columns with the heading VDL are calculated as (V-VDL)/VDL*100%.

c) For all calculations, a single-state variable binomial tree with 913 steps is used. The length of each step interval is one day.

Table 4
Pricing Biases due to Ignoring the Exercise Delay and the Limit Option Features
———Category III NPWs

$r_f = 0.06$ $r = 0.12$ $\sigma_z = 0.20$ $\rho = -0.05$ $q = 0.0043$
$t = 0.3$ years $X = 0.008333$ US\$/Jap¥ $X_0 = 0.008333$ US\$/Jap¥ $K = 30000$
multiple = 0.1 exercise delay = one day limit size = 500

V: Value of a plain NPW; VD: Value of a NPW with exercise delay; VDL: Value of a NPW with exercise delay and limit option

	S/K = 0.8333 (S = 25000)			S/K = 1 (S = 30000)			S/K = 1.167 (S = 35000)		
	V	VD	VDL	V	VD	VDL	V	VD	VDL
INDEX VOLATILITY 0.15	\$4.167	4.159	4.159	\$0.969	0.966	0.966	\$0.123	0.123	0.123
		0.05%	0.05%		0.31%	0.31%		0.00%	0.00%
0.35	\$4.107	4.159	4.159	\$1.505	1.502	1.502	\$0.427	0.426	0.426
		0.05%	0.05%		0.20%	0.20%		0.23%	0.23%
0.45	\$4.507	4.501	4.501	\$2.181	2.177	2.180	\$0.952	0.950	0.951
		0.13%	0.04%		0.28%	0.05%		0.21%	0.11%

Note:

a) This table calculates and compares (with different parameter inputs) three sets of category III NPWs' values: the NPW value without exercise delay and limit option features (V), the NPW value with the exercise delay feature only (VD), and the NPW value with both the exercise delay and the limit option features (VDL).

b) Pricing biases are caused due to ignoring the exercise delay feature and the limit option feature. Thus the percentage numbers in columns with the heading VD are calculated as (V-VD)/VD*100%, and the percentage numbers in columns with the heading VDL are calculated as (V-VDL)/VDL*100%.

c) For all calculations, a two-state-variable trinomial tree with 110 steps is used. The length of each step interval is one day.

demonstrated that all NPWs (both European and American) can be priced within a one-state-variable framework, despite the fact that there are two underlying variables. It also shows that the pricing models are robust with respect to many of the pricing assumptions. The models are directly applicable to other foreign index warrants.

Throughout the paper, constant riskfree interest rates have been assumed. It is shown elsewhere (Wei, 1992b) that close-form pricing formulas still exist for European Nikkei put warrants when stochastic riskfree interest rates are incorporated. However, it is far more difficult to incorporate stochastic riskfree interest rates into the pricing of American Nikkei put warrants. Another important assumption made in the paper is that the volatilities of the index and the exchange rate are constant over time. Some authors have built stochastic volatilities into the Black-Scholes option pricing model. The general consensus is that the randomness of the volatility (of the underlying stock) has very minimal effects on the pricing of options. The Black-Scholes model is satisfactorily robust. Moreover, the procedures of incorporating a stochastic volatility are quite involved and cumbersome. In the case of NPWs there are two volatilities. Therefore it will be even more difficult to model. Ultimately it is an empirical question as to whether the above unrelaxed modelling assumptions are realistic.

ENDNOTES

1. A paper closely related to the pricing of cross-currency options is by Rumsey (1991).

2. The exercise multiple will be defined later.

3. Since the warrants are settled in cash upon exercise, no actual stocks are delivered. This is one of the differences between index options and stock options.

4. "American style" means the warrants can be exercised anytime before maturity.

5. The "multiple" is applied due to specifications in the prospectuses. Issuers include this multiple either to rescale the warrant payoff or to compensate themselves for "risk." For example, the multiple 0.1168 for Bankers Trust Bank of Canada Series I NPWs is calculated as (106*3.55)/(32174*0.1), where 106 is the Jap¥/Cdn$ exchange rate on the issue date; 3.55 is the warrant issue price;

32174 is the warrant strike price in yen; and 0.1 (10%) is the "risk premium" demanded by the issuer. The multiple remains a constant throughout the life of the warrant, so it does not affect the nature of the warrant. Different warrants tend to have different multiples, due to different exercise price, premium, etc.

6. Nikkei 225 index is an arithmetic average. If individual stock prices are lognormal, then the index will not be so. Nevertheless, it is a standard practice to use lognormal distribution to approximate an arithmetic index's distribution.

7. This assumption is somewhat redundant since assumption 2 implies assumption 6. But we still explicitly state it here, because it has direct implications for pricing.

8. See Wei (1992a) for detailed arguments. Also see Derman, Karasinski and Wecker (1990) for a derivation of the risk-neutral processes. For a general treatment on the change of probability measures, see Harrison and Pliska (1981).

9. For a bivariate lognormal density function, see Abramowitz (1972).

10. Many textbooks on options contain basic comparative statics. See, for example, Hull (1989).

11. Specifically, for a negative ρ, when $|\rho\sigma_x| > \sigma_s$ the value of a category III NPW is negatively related to the index volatility. Similarly, for a negative ρ, when $|\rho\sigma_s| > \sigma_x$ the value of a category III NPW is negatively related to the exchange rate volatility. Given the fact that exchange rate volatility is normally smaller than the index volatility, it is rare that the relationship $|\rho\sigma_x| > \sigma_s$ is satisfied.

12. The following approach is similar to that in Grabbe (1983).

13. See Wei (1992a) for a formal proof.

14. To enhance calculation accuracy, the control variate technique (Hull and White, 1988) can be used in conjunction with the binomial model.

15. This is based on the assumption that the Tokyo Stock Exchange is open between 9:00 am and 3:00 pm local time, while the American Stock Exchange is open between 9:30am and 4:30pm EST, and that the time zone difference is 14 hours.

16. I am grateful to Prof. Alan White for suggesting this approach to me.

17. An option's Gamma is the second derivative of the option's value with respect to the underlying stock price.

18. The gamma is calculated using the same lattice. Specifically, we let the tree start one step early and use the stock prices and option values on the nodes of the second step to calculate the gamma.

19. At maturity, at each node *i*, the option's value is given by *p**max(0, *K*–*S*_up) + (1–*p*)*max(0, *K*–*S*_down). This is also true when we consider the limit option feature.

20. The size of the limit is chosen as 500 points, as applied to the actual NPWs.

21. The choice of the time-to-maturity (and hence the number of steps) is restricted by the computing time. For a two-state variable five-jump lattice, the computing time increases exponentially with the number of steps.

22. Typical papers include Hull and White (1987b), Johnson and Shanno (1987), Scott (1987) and Wiggins (1987).

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Appendix 1
Summary Information Of Nikkei Put Warrants Listed on The AMEX

Warrants	Issue Date	Issue Price	Issue Size—#wts	Expiration Date	Payoff Upon Exercise [b]	Classification
Kingdom of Denmark Series I (DXA.WS)[a]	Jan. 3, 1990	$4.05	6,500,000	Jan. 3, 1993	$\$0.2*\text{Max}[0, X_0*(37516.77-S_T)]$ $X_0 = 1/145.325$	Category II
Kingdom of Denmark Series II (DXA.WS)[a]	Jan. 12, 1990	$5.375	3,850,000	Jan. 3, 1993	$\$0.2*\text{Max}[0, X_0*(37516.77-S_T)]$ $X_0 = 1/145.325$	Category II
Salomon Inc. Series I (SXA.WS)	Jan. 18, 1990	$3.54	13,800,000	Jan. 19, 1993	$\$0.2*\text{Max}[0, X_0*(36821.14-S_T)]$ $X_0 = 1/145.52$	Category II
Salomon Inc. Series II (SXO.WS)	Jan. 15, 1990	$3.76	10,000,000	Feb. 16, 1993	$\$0.2*\text{Max}[0, X_0*(37471.99-S_T)]$ $X_0 = 1/144.55$	Category II
BT Trust New York Corp. (BTB.WS)	Feb. 1, 1990	$9.17	6,000,000	Jan. 16, 1993	$\$0.5*\text{Max}[0, (37206.42-S_T)X_T]$	Category III
Paine Webber Group (PXB.WS)	Apr. 10, 1990	$3.20	7,000,000	Apr. 8, 1993	$\$0.2*\text{Max}[0, X_0*(29249.06-S_T)]$ $X_0 = 1/159.80$	Category II
A/S Eksportfinans Corp. (EXW.WS)	Apr. 26, 1990	$3.75	3,625,000	Apr. 22, 1993	$\$0.2*\text{Max}[0, X_0*(29424.51-S_T)]$ $X_0 = 1/151.24$	Category II

Note:

a. These two series are actually traded on the AMEX as a single issue, due to the same specifications of terms.

b. The decimal numbers in front of the maximum operator are called "multiples", and are calculated based on specifications in the prospectuses.
S_T : closing level of Nikkei 225 in yen on the exercise date.
X_T : US$/Jap¥ exchange rate on the exercise date.

Appendix 2
Summary Information Of Nikkei Put Warrants Listed on The TSE

Warrants	Issue Date	Issue Price	Issue Size—#wts	Expiration Date	Payoff Upon Exercise [b, c]	Classification
AB Svensk Exp. Corp. I (SEK.WT)[a]	Dec. 1, 1989	$3.24	2,366,181	Nov. 16, 1992	$\$0.11680*\text{Max}[0, (35963.74-S_T)X_T]$	Category I
AB Svensk Exp. Corp. II (SEK.WT)[a]	Feb. 7, 1990	$2.65	1,726,651	Nov. 16, 1992	$\$0.11680*\text{Max}[0, (35963.74-S_T)X_T]$	Category I
BT Bank of Canada Series I (NKP.WT)	Feb. 17, 1989	$3.55	9,100,000	Feb. 17, 1992	$\$0.11680*\text{Max}[0, (32174.00-S_T)X_T]$	Category I
BT Bank of Canada Series II (NKP.WT.A)	Jun. 15, 1989	$2.65	12,375,000	Jun. 15, 1992	$\$0.10311*\text{Max}[0, 33403*X_0-S_TX_T]$ $X_0 = 1/123.47$	Category III
BT Bank of Canada Series III (NKP.WT.B)	Feb. 16, 1990	$2.50	4,800,000	Mar. 16, 1993	$\$0.00092*\text{Max}[0, 37460.32-S_T]$	Category II
BT Bank of Canada Series IV (NKP.WT.C)	Mar. 22, 1990	$2.50	6,000,000	Apr. 12, 1993	$\$0.00116*\text{Max}[0, 29843.34-S_T]$ [c]	Category II
Trilon Financial Corp. (TFC.WT.N)	Feb. 22, 1990	$2.75	3,734,900	Feb. 22, 1993	$\$0.00105*\text{Max}[0, 37460.32-S_T]$ [c]	Category II

Note:

a. These two series are actually traded on the TSE as a single issue, due to the same specifications of terms.

b. The decimal numbers in front of the maximum operator are called "multiples", and are calculated based on specifications in the prospectuses.
S_T : closing level of Nikkei 225 in yen on the exercise date.
X_T : Cdn$/Jap¥ exchange rate on the exercise date.

c. The pre-specified exchange rate X_0 is reflected in the "multiple".