11. The Tremont sub-indices produce more dramatic differences. Eighteen of the same 20 scenarios are significantly different at the 5% level.

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INCENTIVE STOCKS AND OPTIONS WITH TRADING RESTRICTIONS: NOT AS RESTRICTED AS WE THOUGHT

Melanie Cao and Jason Wei

ABSTRACT

Stock ownership and incentive options are used by companies to retain and motivate employees and managers. These grants usually come with vesting features which require grantees to hold the assets for certain periods. This vesting requirement makes the grantee's total wealth highly undiversified. As a result, as shown by previous researchers, grantees tend to value these incentive securities below market. In this case, grantees will have a strong desire to hedge away the firm-specific risk. Facing the restrictions of direct hedges such as shorting the firm's stock, employees may implement a partial hedge by taking positions in an asset highly correlated with the firm's stock, such as an industry index. In this chapter, we investigate the effects of such a partial hedge. Using the continuous-time, consumption-portfolio framework as a backdrop, we demonstrate that the hedging index can enhance the employee's optimal portfolio holding and increase his intertemporal utility. Consequently, his private valuations of these grants are higher than that without the partial hedging. However, because the partial hedge makes the employee's total wealth less sensitive to the firm's stock price, it will also undermine the incentive

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effects. Therefore, the presumed incentive effects of these restricted assets should not be taken for granted.

1. INTRODUCTION

According to the National Center for Employee Ownership (NCEO), approximately 20 million employees and executives in the U.S. received stock and stock options. The employee ownership was pioneered by high-tech start-up companies as a practical way of retaining and motivating talented employees. Usually, these companies require their employees to continue working for a minimum number of years before the stocks and options are vested. During the vesting period, employees are not allowed to sell their stock or option holdings. Retention is achieved through vesting in that the employee must forgo the holdings if he leaves the company before the stocks and options are vested; long-term motivation is achieved by linking the employee's personal wealth (i.e., the restricted holdings) to the company's stock value through ownership. Granting stocks without trading restrictions is equivalent to granting cash bonuses, since the employee can sell the stocks immediately after receiving them. In this case, although the employee values his stock grant exactly as the market does, the long-term incentive is absent and the employee can leave the company without incurring any loss. Granting stocks with an infinite vesting period does not make any sense either since they are worthless to the employee. Clearly, trading restrictions imposed for a modest period (usually 5–10 years) are necessary for both retention and long-term motivation purposes. Such restrictions will make part of the employee's wealth illiquid and impose an exposure to the company-specific or non-systematic risk. Consequently, a risk-averse employee will value such a restricted asset below its market value, resulting in a discount. Presumably, the exposure to the company-specific risk is the source of incentives, since improving the firm's overall productivity and profitability by working hard may enhance the stock price, hence the employee's personal wealth.

This incentive mechanism is not without costs though. The holding restriction makes the incentive assets illiquid and imposes an exposure to the firm's specific risk, which leads to a sub-optimal portfolio for the employee. Therefore, a risk-averse employee will always look for ways to hedge away the firm-specific risk. However, employees are not allowed to use direct hedges such as shorting the firm's stock or writing tradable options on the firm's stock. As stated explicitly by the Council of Institutional Investor, grantees are required not to enter into hedging positions (such as swaps, option positions or shorting the company's stock) that offset the alignment and risk characteristic of equity-based awards.[1]

The institutional restrictions on direct hedging will cause employees to use a partial hedge. That is, the employee will have a desire to take positions in securities highly correlated with the firm's stock, such as an industry index. To the best of our knowledge, there are only three studies which examine the impacts of holding restrictions on the private valuation of restricted assets, but none of them examines the consequence of employees' using a hedging index to undo the trading restrictions. Meulbroek (2001) shows how costly bearing non-systematic risk can be from a diversification perspective; Kahl, Liu and Longstaff (2003) (KLL hereafter) demonstrate how much the employee discounts the restricted stock in a portfolio-selection context; in a similar framework, Ingersoll (2006) focuses on the private valuation of incentive options with different features.

This chapter sets out to fill the aforementioned gap in the literature. To this end, we link the literatures on executive compensation and asset valuation with portfolio constraints, and go one step further by introducing a hedging index in the portfolio choice set.[2] As mentioned earlier, this hedging index can be considered as an industry index with a high correlation to the restricted stock. The portfolio strategy is optimized in a consumption-portfolio selection framework with respect to the employee's intertemporal utility. Overall, we find that the hedging index can increase the employee's private valuation of the restricted assets, and in the case of options, increase the delta or incentive sensitivity. Of course, the total incentive is compromised since the exposure to the company-specific risk is reduced. The specific findings are as follows.

With respect to the restricted stock, the hedging index can help improve the employee's optimal consumption-investment policy and align the employee's private valuation of the restricted stock with the market's. The difference between the employee's private value and the stock's market value represents the illiquidity discount. As expected, the discount increases with the employee's degree of risk aversion, the length of the vesting period and the volatility of the restricted stock. However, the discount is reduced when the residual correlation between the restricted stock and the hedging index is high. For example, under typical market conditions, an employee with a risk aversion parameter $\gamma = 2$ and 50% of his wealth in the firm's stock to be vested in 10 years would discount the stock by as much as 60% when the hedging index is not used. With the hedging index, the discount is reduced to 46%, a 14% improvement of the private valuation. At the same time, the non-systematic risk exposure is reduced by approximately 33%. When the residual correlation is perfect, the discount and the exposure to company-specific risk can be completely eliminated. Therefore, the trading restrictions are not as restrictive as we thought, and the intended effectiveness of incentive stocks cannot be taken for granted.

With respect to restricted options, the hedging index can also enhance the employee's private valuation and reduce the discount, regardless of whether the option is European or has vesting and early exercise features. The degree of value enhancement for options is roughly the same as that for the restricted stocks. Moreover, hedging also improves the incentive sensitivity measured by the option's delta. Therefore, for options, there appears to be a trade-off between the loss of total incentives (measured by the exposure to company-specific risk) and the gain in incentive sensitivity. That the two effects are in different measures hinders a direct trade-off analysis.

Although the continuous-time consumption-portfolio framework is similar to that of KLL (2003) and Ingersoll (2006), the present chapter differs from these two studies in three aspects. First and foremost, we focus on how hedging affects the private valuation and incentive effects. Our emphasis helps shed light on the effectiveness of the trading restrictions. Second, we augment their portfolio choice set by including a hedging index. Specifically, our portfolio choice set includes the market portfolio, a hedging index, the restricted stock and the risk-free asset. Our analysis shows that the inclusion of the hedging index can help improve the employee's intertemporal utility, increase the private valuation of the incentive stocks and options. Third, we analyze both the incentive stocks and options, while Ingersoll focuses only on options and KLL only on stocks.

It is worth noting that we study only a particular aspect of incentive stocks and options. Specifically, we study the consequence of hedging on the part of grantees assuming that the compensation package has already been decided. In this sense, issues such as how to design an optimal compensation package are outside the domain of this chapter. By the same token, our study also abstracts from many contemporary issues that can potentially impact the valuation and incentive effects of executive compensation. Backdating and forward-dating are such examples.[3]

The rest of the chapter is organized as follows. Section 2 formulates and characterizes the employee's lifetime consumption-investment decision with an expanded portfolio choice set. We illustrate numerically the extent to which the hedging index enhances the employee's utility and reduces the exposure to the company-specific risk. Section 3 analyzes and demonstrates how the hedging index can reduce the discount of restricted assets. In-depth analyses are carried out for both the restricted stocks and options. Section 4 provides concluding remarks. Proofs and tables are relegated to the appendix.

2. OPTIMAL CONSUMPTION-PORTFOLIO STRATEGY WITH A HEDGING INDEX

2.1. The Setup

Consider a risk-averse employee with a finite lifetime horizon $\bar{T}$ whose preference is described by a smooth, time-additive expected utility function: $\bar{U}(c) = E\left[\int_0^{\bar{T}} U(c_t, t)\mathrm{d}t\right]$.[4] Following the literature, we adopt the constant-relative-risk-aversion (CRRA) utility function for the employee

$$U(c_t, t) = \mathrm{e}^{-\phi t}\frac{c_t^{1-\gamma}}{1-\gamma} \qquad (1)$$

where $\phi > 0$ is the rate of time preference and $\gamma \geq 1$ the coefficient of relative risk aversion. The employee works for a company which grants restricted stocks and options over time for retention and motivation purposes. Similar to Ingersoll (2006), we approximate the ongoing nature of the incentive scheme by assuming that the employee is required to hold a fixed fraction of his total wealth (defined later) in the company's stock during the vesting period which ends at time $T \leq \bar{T}$.[5] By the end of the vesting period, the employee is free to sell these shares. The assumption of fixed fraction is equivalent to assuming proportional growths in the stock's value and the total wealth.

In the absence of trading restrictions and assuming the continuous-time Capital Asset Pricing Model (CAPM), the employee's optimal portfolio strategy is to hold the market portfolio and the risk-free asset only. With trading restrictions on the stock, his optimal portfolio strategy is no longer clear-cut. The imposed stock holding will force the employee to bear some non-systematic risk, which cannot be fully offset by the market portfolio. The employee will try to find a hedging asset to reduce this non-systematic risk. The portfolio choice set in KLL (2003) and Ingersoll (2006) includes only the market portfolio, the risk-free asset and the restricted stock. However, with trading restrictions, including other assets in the feasible set will help reduce the non-systematic risk, and as a result, improve the employee's intertemporal utility. The benefit derived from the additional assets cannot be achieved by simply re-scaling the market portfolio's weights (Section 2.3 and Table 1 delineate this point). It should be noted that, with

Table 1. Optimal Portfolio Holdings and Utility Level with or without Index.

	M	Index$_{12}$		Index$_{23}$		Index$_{23.m}$	
	Weights	Weights	$\rho_{1I.m}$	Weights	$\rho_{1I.m}$	Weights	$\rho_{1I.m}$
Asset 1	0.362	0.514	0.392		0.542		−1.000
Asset 2	0.345	0.486		0.522		0.540	
Asset 3	0.293			0.478		0.460	

Portfolio Feasible Set	Optimal Portfolio Holdings				Ultimate Portfolio Holdings				Utility Level
	x_b	x_m	x_1	x_I	w_b^*	w_1^*	w_2^*	w_3^*	
Panel A: Without trading restriction									
B, M and Assets 1–3	0.348	0.652			**0.348**	**0.236**	**0.225**	**0.191**	**100.00**
Panel B: Trading restrictions: $x_1 = 0.1$ ($< w_1^* = 0.236$)									
Case 1: B, M and Asset 1	0.295	0.605	0.100		0.295	0.319	0.209	0.178	99.36
Case 2: B, M, Asset 1 and Index$_{12}$	0.311	0.652	0.100	−0.062	0.311	0.304	0.194	0.191	99.46
Case 3: B, M, Asset 1 and Index$_{23}$	0.265	0.731	0.100	−0.095	0.265	0.364	0.202	0.169	99.55
Case 4: B, M, Asset 1 and Index$_{23.m}$	0.348	0.375	0.100	0.176	**0.348**	**0.236**	**0.225**	**0.191**	**100.00**
Panel C: Trading restrictions: $x_1 = 0.5$ ($> w_1^* = 0.236$)									
Case 1: B, M and Asset 1	0.079	0.421	0.500		0.079	0.652	0.145	0.123	85.16
Case 2: B, M, Asset 1 and Index$_{12}$	0.159	0.652	0.500	−0.311	0.159	0.576	0.073	0.191	87.30
Case 3: B, M, Asset 1 and Index$_{23}$	−0.071	1.047	0.500	−0.476	−0.071	0.879	0.112	0.080	89.29
Case 4: B, M, Asset 1 and Index$_{23.m}$	0.348	−0.731	0.500	0.882	**0.348**	**0.236**	**0.225**	**0.191**	**100.00**

Note: Parameter values: $\phi = 0.03$, $\gamma = 4.0$, $r = 0.06$, $\mu_1 = 0.10$, $\mu_2 = 0.15$, $\mu_3 = 0.20$, $\sigma_1 = 0.15$, $\sigma_1 = 0.25$, $\sigma_3 = 0.35$, $\rho_{12} = 0.2$, $\rho_{13} = 0.3$, $\rho_{23} = 0.4$. To calculate the utility level, the agent's life horizon is set to be $\bar{T} = 40$ and the vesting period is set $T = 10$. The utility level under no trading restriction is normalized to 100. B, Risk-free bond; M, market portfolio (tangent portfolio of Assets 1, 2 and 3); Index$_{12}$, tangent portfolio of Assets 1 and 2; Index$_{23}$, tangent portfolio of Assets 2 and 3; Index$_{23.m}$, portfolio consisting of Assets 2 and 3 with weights proportional to those in the market portfolio.

hedging, the employee's wealth will be less sensitive to the firm's stock price, hence compromising the incentive effects.

To capture the above setting, the employee's portfolio choice set is assumed to include a bond B earning the risk-free rate r, the market portfolio M, the restricted asset or the company's stock S and the hedging index I. The price dynamics are

$$\frac{dM}{M} = (\mu_m - q_m)dt + \sigma_m dz_m,$$
$$\frac{dS}{S} = (\mu_s - q_s) + \sigma_s dz_s,$$
$$\frac{dI}{I} = (\mu_I - q_I)dt + \sigma_I dz_I$$

where the correlation coefficient matrix of \$ $\mathbf{z} = (z_m\ z_s\ z_I)'$ is

$$\Sigma = \begin{pmatrix} 1 & \rho_{ms} & \rho_{mI} \\ \rho_{ms} & 1 & \rho_{Is} \\ \rho_{mI} & \rho_{Is} & 1 \end{pmatrix}$$

and the cum-dividend expected returns, dividend yields and the volatilities are

$$\mu = \begin{pmatrix} \mu_m \\ \mu_s = r + \beta_s(\mu_m - r) \\ \mu_I = r + \beta_I(\mu_m - r) \end{pmatrix}, \quad q = \begin{pmatrix} q_m \\ q_s \\ q_I \end{pmatrix} \quad \text{and} \quad \sigma = \begin{pmatrix} \sigma_m \\ \sigma_s \\ \sigma_I \end{pmatrix}$$

with $\beta_s = \rho_{ms}(\sigma_s/\sigma_m)$ and $\beta_I = \rho_{mI}(\sigma_I/\sigma_m)$ and the non-systematic variances for the stock and the index as $v_s^2 = (1 - \rho_{ms}^2)\sigma_s^2$ and $v_I^2 = (1 - \rho_{mI}^2)\sigma_I^2$.

With the above elements, the employee chooses an optimal consumption and portfolio policy to maximize his expected lifetime utility subject to the usual budget constraint. The first order conditions yield the following stochastic Euler equation:

$$X_t = E\left[\int_t^{\bar{T}} \frac{U_{c_\tau}}{U_{c_t}} d\tau\right] \qquad (2)$$

where X_t is the current price of an asset with a dividend yield of D, and U_{c_t} stands for the partial derivative of the utility function with respect to the consumption c_t. Intuitively, given his optimal consumption policy, the employee's private valuation of any security is the expected present value of the future yields or payoffs, discounted at his own marginal rate of substitution. It is important to emphasize that the employee takes as given the market prices of all securities (which are equilibrium outcomes under no trading restrictions). When he is free to trade any security, his private valuation equals the market's. When he is restricted in trading some of the securities, however, his private valuation may deviate from the market's. This does not imply any arbitrage since the employee is a price taker in the market place. In other words, we assume that the number of investors with trading restrictions is so small compared with the total population of investors that they cannot influence the competitive financial market.

2.2. *Optimal Consumption-Portfolio Policies: Theoretical Results*

To determine the employee's valuation of the restricted stock and options and to determine the amount of residual non-systematic risk, we first need to solve the consumption–investment problem to obtain the marginal rate of substitution. To this end, we define the employee's total wealth as $\theta_{B_t}B_t + \theta_{M_t}M_t + \theta_{S_t}S_t + \theta_{I_t}I_t$, where θ_{B_t}, θ_{M_t}, θ_{S_t} and θ_{I_t} are portfolio holdings. Denote the percentages of his total wealth invested in the risky assets as $\boldsymbol{x}_t = (x_{mt}, x_s, x_{It})'$, then we have

$$x_{mt} = \frac{\theta_{M_t}M_t}{W_t}, \quad x_{st} = \frac{\theta_{S_t}S_t}{W_t} \quad \text{and} \quad x_{It} = \frac{\theta_{I_t}I_t}{W_t}$$

Similar to Merton (1971), the budget constraint is

$$\frac{\mathrm{d}W_t}{W_t} = \left(r + x_t'(\mu - r) - \frac{c_t}{W_t}\right)\mathrm{d}t + (x_t \cdot \sigma)'\mathrm{d}z_t$$

Given the trading restrictions, the employee's optimization must be solved for two distinct periods as shown below.[6]

```
|------------------------------|------------------------------------|
t      vesting period          T       post-vesting period          T̄
```

Since there is no trading restriction during the post-vesting period, the optimal solution is similar to the standard Merton solution (1971). That is

$$x_{mt}^* = \frac{s_m}{\gamma\sigma_m}, \; x_{st}^* = 0, \; x_{It}^* = 0, \quad \forall \; T < t < \bar{T}$$

$$c_t^* = f(a, t, \bar{T})W_t \quad \text{and} \quad J(W, t, f) = \mathrm{e}^{-\phi t} f(a, t, \bar{T})^{-\gamma}\frac{W_t^{1-\gamma}}{1-\gamma}$$

$$\text{with} \quad s_m = \frac{\mu_m - r}{\sigma_m}, \quad a = \frac{\phi}{\gamma} - \frac{1-\gamma}{\gamma}\left(r + \frac{s_m^2}{2\gamma}\right), \quad f(a, t, \bar{T}) = \frac{a}{1 - \mathrm{e}^{-a(\bar{T}-t)}}$$

The optimal policy indicates that the holding of the market portfolio is positively related to the market's Sharpe ratio s_m, negatively related to the employee's risk-aversion parameter γ and the volatility of the market portfolio σ_m.

Now we turn to the vesting period. Let x_s be the fixed percentage of the employee's total wealth in the restricted stock. For any $x_s > 0$, the optimality condition implies that this constraint is binding. The employee's optimal portfolio should hold exactly x_s during the vesting period ($0 < t < T$). Unlike KLL (2003) and Ingersoll (2006), we augment the portfolio choice set by including the index due to the employee's desire to reduce the negative impact of the trading restrictions. Specifically, the employee takes x_s as given and optimizes the portfolio holdings of the market and the index. In the appendix "Proof of Proposition 1" provides the solution to this optimization problem. To facilitate the presentation of the optimal solution, we denote the residual or partial correlation between the restricted stock and the index, after controlling for the market impact, as $\rho_{Is.m} = (\rho_{Is} - \rho_{ms}\rho_{mI}) / \sqrt{(1-\rho_{ms}^2)(1-\rho_{mI}^2)}$.

Proposition 1. *When there is no trading limit on the index, the employee's optimal consumption–investment strategy during the vesting period consists of*

$$x_{mt}^* = \frac{s_m}{\gamma\sigma_m} - x_s\beta_s\frac{\rho_{ms} - \rho_{mI}\rho_{Is}}{\rho_{ms}(1 - \rho_{mI}^2)}, \quad x_{st}^* = x_s, \quad x_{It}^* = -x_s\rho_{Is.m}\frac{v_s}{v_I} \equiv x_I^*$$

$$c_t^* = F(A, t, T)W_t \quad \text{and} \quad J(W, t, F) = \mathrm{e}^{-\phi t}F(A, t, T)^{-\gamma}\frac{W_t^{1-\gamma}}{1-\gamma}, \quad \forall \; 0 < t < T$$

$$\text{where} \quad A = a - \frac{1}{2}(\gamma - 1)\Omega^*, \; \Omega^* = x_s^2v_s^2 - x_I^{*2}v_I^2 = x_s^2v_s^2(1 - \rho_{Is.m}^2),$$

$$F(A, t, T) = \frac{A}{1 + (Af(a, T, \bar{T})^{-1} - 1)\mathrm{e}^{-A(T-t)}}$$

Proposition 1 indicates that the optimal position on the index depends on four factors: the size of the restricted holding x_s, the partial or residual correlation between the stock and the index $\rho_{Is.m}$, the residual variance of the stock v_s, and the residual variance of the index v_I. The results are generally intuitive. For example, a bigger holding of the restricted stock or a higher residual variance of the stock would call for a bigger holding of the index; a higher residual variance of the index would reduce the holding of the index since the hedging purpose is to reduce the residual variance in the first place. As for the partial correlation, when it is positive, the employee will short the index; if it is negative, a long position on the index is called for. The optimal position could be large if the residual correlation is high and/or the ratio of the non-systematic risk is high. In reality, although the employee could take a long position on the index without any limit, he may face certain shorting restrictions imposed by his broker. To complete the framework, we now consider the optimal consumption–investment policy when the index position is limited. To this end, suppose the employee cannot short more than $|x_I|$ ($x_I < 0$) of his total wealth. When this constraint is not binding, i.e., when $x_I < x_I^*$, the employee's optimal consumption-portfolio strategy is still the one presented in Proposition 1. Otherwise, the following proposition characterizes the optimal holding (we omit the proof for brevity).

Proposition 2. *When $\rho_{Is.m} > 0$ and the shorting restriction on the index is $-x_s \cdot \rho_{Is.m}\, v_s/v_I < x_I < 0$, the employee's optimal consumption–investment strategy during the vesting period becomes*

$$x_{mt}^* = \frac{s_m}{\gamma\sigma_m} - x_s\beta_s - x_I\beta_I, \quad x_{st}^* = x_s, \quad x_{It}^* = x_I$$

$$c_t^* = \overline{F}(\overline{A},t,T)W_t \quad \text{and} \quad J(W,t,\overline{F}) = e^{-\phi t}\overline{F}(\overline{A},t,T)^{-\gamma}\frac{W_t^{1-\gamma}}{1-\gamma}, \quad \forall\, 0<t<T$$

where $\overline{A} = a - \frac{1}{2}(\gamma-1)\overline{\Omega},\ \overline{\Omega} = x_s^2 v_s^2 + x_I^2 v_I^2 + 2\rho_{Is.m} x_s x_I v_s v_I,$

$$\overline{F}(\overline{A},t,T) = \frac{\overline{A}}{1 + \left(\overline{A}f(a,T,\overline{T})^{-1} - 1\right)e^{-\overline{A}(T-t)}}$$

It should be stressed that the inclusion of the hedging index unambiguously increases the employee's intertemporal utility and reduce the exposure to the company-specific risk. Contrary to casual perceptions, adding the index into the optimization does not simply boil down to a re-scaling of the market portfolio's weight. If the index's role is trivial, then the optimized weight on the index should be either zero or non-zero but inconsequential to the utility level. In the follow subsection, we use a simple, three-risky-asset setup to show that the introduction of an additional asset does improve the overall consumption-portfolio decision and reduce the exposure to non-systematic risk.

2.3. Non-Trivial Role of the Index

Let us consider a simple economy where there are only three risky assets and one risk-free bond in the financial market. Suppose that the risk-free rate is $r = 6\%$ and the return characteristics of the three risky assets are given in Table 1. The market portfolio is the tangent portfolio formed with the three risky assets. The weights are 36.2% in Asset 1, 34.5% in Asset 2 and 29.3% in Asset 3. As shown in Panel A of Table 1, under no trading restrictions, an investor with risk aversion $\gamma = 4$ will invest 34.8% in the risk-free bond and 65.2% in the market portfolio. In other words, the investor ultimately holds 34.8% of his wealth in the risk-free bond, 23.6% in Asset 1, 22.5% in Asset 2 and 19.1% in Asset 3. The intertemporal expected utility is calculated via the value function in Proposition 1 and re-scaled to 100.

Without loss of generality, let us now impose a trading restriction on Asset 1. To see how the introduction of additional assets (other than the market portfolio) can affect the consumption-portfolio decisions, we examine three indices. Index_{12} is the tangent portfolio formed with Assets 1 and 2. The index's weights and residual correlation with Asset 1 (controlling for the market) are given in the top portion of Table 1. Index_{23} is defined in a similar fashion. $\text{Index}_{23.m}$ consists of Assets 2 and 3 whose weights are proportional to those in the market portfolio. Naturally, the residual correlation between Asset 1 and $\text{Index}_{23.m}$ is -1.0.[7]

Panel B of Table 1 presents results for the situation where the imposed holding on Asset 1 is 10%, which is lower than the optimal weight of 23.6% in the market portfolio. Without the hedging index (Case 1), the ultimate weight on Asset 1 is 31.9%, higher than $23.6% (the optimal weight of Asset 1 in the market portfolio), and the utility level is now lower. The holding restriction makes the investor worse-off. The overall weight on Asset 1 is higher than 10%, since the investor is also holding the market portfolio which contains Asset 1. Note that this case corresponds to the simplified portfolio choice set as in KLL (2003) and Ingersoll (2006) where the employee can only optimize his strategy over the risk-free bond and the market portfolio.

When we include Index_{12} into the choice set (Case 2), a short position is taken on this index, and the ultimate weights on all assets are different from those of Case 1. More importantly, the utility level improves. The inclusion of Index_{23} (Case 3) has similar effects, except that the utility improvement is larger since the residual correlation (0.542) is bigger than that in Case 2 (0.392). Moving on to Case 4 where we add $\text{Index}_{23.m}$ into the portfolio choice set, the ultimate portfolio weights are all restored to their optimal level, and so is the utility level.

The contribution of the additional asset is bigger when the restricted position is larger. To appreciate this point, we repeat the calculations in Panel A by setting the restriction on Asset 1 to 50%, which is higher than the optimal weight of 23.6% in the market portfolio. Panel C shows that the restriction reduces the utility by approximately 15% when the investor can only trade the market portfolio and the risk-free bond. But with the right index ($\text{Index}_{23,m}$) the restriction can be totally undone, and the loss of utility can be completely recovered.

This simple exercise clearly demonstrates that, when there are trading restrictions on certain assets, allowing only the market portfolio and the risk-free bond in the portfolio choice set will lead to sub-optimal decisions. Introducing other assets into the choice set will improve the consumption-portfolio decisions and enhance the overall utility level. This is why employees will seek for ways to undo the restrictions.

2.4. Impacts of Trading Restrictions on Optimal Portfolio Choices and Non-Systematic Risk

To show how the consumption and investment decisions are affected by the trading restrictions, we examine the optimal consumption dynamics. During the post-vesting period, the consumption evolves according

$$\frac{dc}{c} = \frac{1}{\gamma}\left[r - \phi + \frac{1}{2}\left(1 + \frac{1}{\gamma}\right)s_m^2\right]dt + \frac{s_m}{\gamma}dz_m \equiv \mu_c dt + \sigma_c dz_m$$

During the vesting period, the consumption process follows

$$\frac{dc}{c} = \left[\mu_c + \frac{1}{2}(\gamma - 1)\Omega\right]dt + \left(\frac{s_m}{\gamma} - \beta_s x_s \sigma_m - \beta_I x_{It}^* \sigma_m\right)dz_m + x_s \sigma_s dz_s + x_{It}^* \sigma_I dz_I$$

$$\text{where} \begin{cases} \Omega = \Omega^* \text{ and } x_{It}^* = x_I^* \text{ with a non-binding index shorting limit} \\ \Omega = \overline{\Omega} \text{ and } x_{It}^* = x_I \text{ with a binding index shorting limit} \end{cases} \quad (3)$$

During the post-vesting period, the employee's consumption uncertainty is completely induced by the market portfolio. During the vesting period, regardless of the index shorting limit, his consumption growth rate and volatility are affected by the uncertainty in the market portfolio, as well as by those in the stock and the index. With a well defined residual correlation between the index and the stock (i.e., $|\rho_{Is.m}| < 1$), we have $\Omega > 0$. Given that $\gamma \geq 1$, the consumption growth rate during the vesting period is higher than that during the post-vesting period. The reason is that the vesting requirement makes part of the employee's personal wealth illiquid and the employee tends to consume at a lower level. As time approaches the end of the vesting, the employee tends to increase his consumption, resulting in a higher consumption growth rate. The consumption variance during the vesting period can be easily shown as $s_m^2/\gamma + \Omega$ and is higher than that of the post-vesting period. The excess consumption variance Ω reaches to its maximum when the index is excluded and reduces to Ω^* when the employee can optimally invest in the index. Given the negative relation between the employee's intertemporal utility and the consumption volatility, it is clear that the employee obtains the highest utility when the index shorting restriction is absent or non-binding, and the lowest utility when the index is not included in the portfolio choice set. As shown later, this result translates to how much the employee discounts the market value of the restricted securities.

Since the optimal consumption rate and the total wealth are linearly related, the process for the total wealth will have the same diffusion terms as the consumption process. In other words, the variance of the wealth process shares exactly the same features as the consumption's. Specifically, during the post-vesting period, the employee's portfolio has only market risk in it and he does not bear any non-systematic risk. During the vesting period, the employee's portfolio has the most amount of non-systematic risk in it when hedging is not in place, and optimal hedging can reduce the non-systematic risk to the lowest possible level, which could be zero if the partial correlation is 1.0. When the residual variance from the stock is reduced, the incentive effect is also compromised.

To shed further light on the issue, we now present some numerical results. In order to guide ourselves in choosing parameter values, we empirically estimate correlations and the market's risk-return profile for the high-tech sector. Specifically, we download daily prices from Yahoo! Finance for three indices (S&P 500, Nasdaq, Nasdaq-100) and seven stocks (Apple, Cisco, IBM, Intel, Microsoft, Oracle and Sun) for the period of January 1, 1995 to December 31, 2004. We take S&P 500 as a proxy for the market portfolio, and Nasdaq or Nasdaq-100 as the hedging index. Table 2 reports the estimates. Several observations are in order. First, the hedging indices are highly correlated with the market, but individual stocks are correlated more with the hedging indices than with the market, suggesting the potential usefulness of the hedge indices in reducing non-systematic risk. Second, for a particular stock, its correlation with the hedging index tends to be much higher than those with other stocks, suggesting that the best candidate as a hedging vehicle is the industry index, not related stocks. Third, the narrow

Table 2. Market Profile for the High-Tech Sector, 1995–2004.

Panel A: Pair-wise correlations

	S&P 500	Nasdaq	Nasdaq-100	Apple	Cisco	IBM	Intel	Microsoft	Oracle	Sun
S&P 500	1.00									
Nasdaq	0.84	1.00								
Nasdaq-100	0.82	0.98	1.00							
Apple	**0.39**	**0.48**	**0.49**	1.00						
Cisco	**0.64**	**0.79**	**0.81**	0.39	1.00					
IBM	**0.58**	**0.57**	**0.58**	0.32	0.49	1.00				
Intel	**0.62**	**0.73**	**0.76**	0.42	0.61	0.48	1.00			
Microsoft	**0.63**	**0.71**	**0.73**	0.34	0.56	0.45	0.60	1.00		
Oracle	**0.53**	**0.65**	**0.67**	0.32	0.54	0.41	0.48	0.46	1.00	
Sun	**0.55**	**0.67**	**0.69**	0.38	0.62	0.45	0.52	0.45	0.48	1.00
Average	0.56	0.66	0.68							

Panel B: Partial correlations, returns and standard deviations

	Partial Correlation with		Annualized Return	Annualized STD
	Nasdaq	Nasdaq-100		
S&P 500			0.096	0.181
Nasdaq			0.105	0.289
Nasdaq-100			0.138	0.359
Apple	0.300	0.322	**0.120**	**0.588**
Cisco	0.598	0.648	**0.228**	**0.517**
IBM	0.190	0.231	**0.174**	**0.343**
Intel	0.492	0.565	**0.178**	**0.480**
Microsoft	0.422	0.477	**0.205**	**0.375**
Oracle	0.439	0.487	**0.183**	**0.581**
Sun	0.462	0.495	**0.157**	**0.613**
Average	0.415	0.461	0.178	0.500

Note: This table reports summary statistics for the market (S&P 500), the hedging indices (Nasdaq and Nasdaq-100), and seven stocks in the high-tech sector. Results are based on daily prices from January 1, 1995 to December 31, 2004. Panel A reports pair-wise correlations. The values under "Average" is the average of stocks correlations with the corresponding indices, i.e., the correlations in bold-type. Panel B reports, for each stock, the partial correlations, the annualized return and the annualized standard deviations. The return and standard deviation averages are for the stocks only, i.e., the numbers in bold-type. Annualization is based on 250 trading days per year.

index, Nasdaq-100, provides higher partial correlations than does the Nasdaq itself. We therefore choose Nasdaq-100 as the hedging index.[8]

For all the numerical investigations to follow, we will set the parameter values close to the empirical estimates. For the market portfolio (i.e., S&P 500) and the hedging index (i.e., Nasdaq-100), we set $\mu_m = 0.1$, $\sigma_m = 0.2$, $\sigma_I = 0.35$ and $\rho_{mI} = 0.8$. For the stock, we choose the average for each parameter: $\sigma_s = 0.5$, $\rho_{Is} = 0.7$ and $\rho_{ms} = 0.5$. The resulting partial correlation $\rho_{Is.m}$ is 0.577. For some analyses, we also examine alternative values for ρ_{ms} (0.2 and 0.8) and ρ_{mI} (0.4).

Table 3 reports optimal portfolio weights and the residual variance under different parameter combinations. To see a broader range of patterns, we preset the range for the partial correlation by using the correlation between the stock and the index as a fitter. For most cases, the absolute weight on the index is smaller than the given weight on the stock, i.e., $|x_I^*/x_s| < 1.0$. This is so since the market portfolio can also provide partial hedging. As is clear from Proposition 2, when the hedging index is absent (i.e., $x_I = 0$), the market weight is reduced by $\beta_s x_s$ from the weight under no trading restrictions. This reduction represents the hedging need due to trading restrictions. It is also interesting to observe that when the stock and the index are highly correlated, the market portfolio weight increases as the weight on the restricted stock increases. Intuitively, the higher correlation and higher weight on the stock create a stronger demand for hedging, and hence the bigger short position in the index has to be offset by holding the market portfolio.

Table 3 also reveals that, the bigger the restricted holding, the bigger the residual variance. This means that introducing the trading restrictions will increase the residual variance or non-systematic risk, and hedging via the index cannot fully eliminate the excess variance. Moreover, it is seen that the excess variance is always higher when there is a binding restriction on the index. When the correlations take the typical values from Table 2 and the restricted holding is 50% of the total wealth, the optimal hedging position is approximately 120% of the restricted holding, and the weight on the market portfolio is approximately 70% (shown in the last row of the table). The excess variance is 0.0313, approximately 30% of the total variance (0.1313). Without the hedging index, the excess variance can be calculated as 0.0469. Therefore, the exposure to non-systematic risk is reduced by 33% ((0.0313 − 0.0469)/0.0469 × 100% = 33%). One may argue that the incentive effect is reduced by 33%.

Finally, other things being equal, the size of the excess variance or the amount of non-systematic risk is inversely related to the residual correlation, $\rho_{Is.m}$. When this residual correlation is 1.0, the excess variance vanishes, and the impact of the trading restriction is completely undone. In fact, the same result ensues when the residual correlation is negative one. A perfect correlation between the stock and the index (i.e., $|\rho_{Is}| = 1.0$) can be obtained

Table 3. Optimal Portfolio Weights and Residual Variance.

					Without Trading Restriction					
		$x^*_{mt} = 0.50$						$\sigma^2_w = 0.10$		
					With Trading Restriction					
					Index Shorting: Non-Binding Restriction				Binding Restriction	
					x_s=0.1	x_s=0.5	x_s=0.1	x_s=0.5	$x_I = 0.5x_I^*$	$x_s = 0.5$
ρ_{ms}	ρ_{mI}	ρ_{Is}	$\rho_{Is.m}$	x_I^*/x_s	x^*_{mt}		Ω		x^*_{mt}	Ω
0.2	0.4	0.08	0.00	0.00	0.45	0.25	0.0024	0.0600	0.25	0.0600
0.2	0.4	0.30	0.25	−0.38	0.48	0.38	0.0023	0.0563	0.32	0.0572
0.2	0.4	0.53	0.50	−0.76	0.50	0.52	0.0018	0.0450	0.38	0.0488
0.2	0.4	0.75	0.75	−1.15	0.53	0.65	0.0011	0.0263	0.45	0.0347
0.2	0.4	0.98	1.00	−1.53	0.56	0.78	0.0000	0.0000	0.52	0.0150
0.2	0.8	0.16	0.00	0.00	0.45	0.25	0.0024	0.0600	0.25	0.0600
0.2	0.8	0.31	0.25	−0.58	0.53	0.66	0.0023	0.0563	0.45	0.0572
0.2	0.8	0.45	0.50	−1.17	0.61	1.07	0.0018	0.0450	0.66	0.0488
0.2	0.8	0.60	0.75	−1.75	0.69	1.47	0.0011	0.0263	0.86	0.0347
0.2	0.8	0.75	1.00	−2.33	0.78	1.88	0.0000	0.0000	1.07	0.0150
0.5	0.4	0.20	0.00	0.00	0.38	−0.13	0.0019	0.0469	−0.13	0.0469
0.5	0.4	0.40	0.25	−0.34	0.40	−0.01	0.0018	0.0439	−0.07	0.0447
0.5	0.4	0.60	0.50	−0.67	0.42	0.11	0.0014	0.0352	−0.01	0.0381
0.5	0.4	0.80	0.75	−1.01	0.45	0.23	0.0008	0.0205	0.05	0.0271
0.5	0.4	0.99	1.00	−1.35	0.47	0.35	0.0000	0.0000	0.11	0.0117
0.5	0.8	0.40	0.00	0.00	0.38	−0.13	0.0019	0.0469	−0.13	0.0469
0.5	0.8	0.53	0.25	−0.52	0.45	0.24	0.0018	0.0439	0.06	0.0447
0.5	0.8	0.66	0.50	−1.03	0.52	0.60	0.0014	0.0352	0.24	0.0381
0.5	0.8	0.79	0.75	−1.55	0.59	0.96	0.0008	0.0205	0.42	0.0271
0.5	0.8	0.92	1.00	−2.06	0.66	1.32	0.0000	0.0000	0.60	0.0117
0.8	0.4	0.32	0.00	0.00	0.30	−0.50	0.0009	0.0225	−0.50	0.0225
0.8	0.4	0.46	0.25	−0.23	0.32	−0.42	0.0008	0.0211	−0.46	0.0214
0.8	0.4	0.59	0.50	−0.47	0.33	−0.34	0.0007	0.0169	−0.42	0.0183
0.8	0.4	0.73	0.75	−0.70	0.35	−0.25	0.0004	0.0098	−0.38	0.0130
0.8	0.4	0.87	1.00	−0.94	0.37	−0.17	0.0000	0.0000	−0.34	0.0056
0.8	0.8	0.64	0.00	0.00	0.30	−0.50	0.0009	0.0225	−0.50	0.0225
0.8	0.8	0.73	0.25	−0.36	0.35	−0.25	0.0008	0.0211	−0.38	0.0214
0.8	0.8	0.82	0.50	−0.71	0.40	0.00	0.0007	0.0169	−0.25	0.0183
0.8	0.8	0.91	0.75	−1.07	0.45	0.25	0.0004	0.0098	−0.13	0.0130
0.8	0.8	1.00	1.00	−1.43	0.50	0.50	0.0000	0.0000	0.00	0.0056
0.5	0.8	0.70	0.577	−1.19	0.54	0.71	0.0013	0.0313	0.29	0.0352

Note: For each correlation combination, the table reports the residual correlation, $\rho_{Is.m}$; the optimal weight on the hedging index as a fraction of the given weight on the restricted stock, x_I^*/x_s; the optimal weight on the market portfolio under different weights on the restricted stock, x^*_{mt}; and the residual variance under different weights on the restricted stock, Ω. The last row of the table is based on average correlation parameters reported in Table 2. The last two columns contain the market portfolio weight and the residual variance when the restriction on the index position is binding. The restriction is set at 50% of the optimal level. Other parameter values: $\mu_m = 0.1$, $\gamma = 0.06$, $\sigma_s = 0.5$, $\sigma_I = 0.35$, $\sigma_m = 0.2$ and $\gamma = 2.0$.

even if the correlation between the stock and index is not perfect. For example, we can set $\rho_{ms}=0.2$, $\rho_{mI}=0.8$, and $\rho_{Is}=0.75$ to have $\rho_{Is.m}=1$, as shown in the table.

3. UNDOING THE HOLDING RESTRICTIONS USING THE HEDGING INDEX

3.1. The State Price Deflator

To illustrate the benefit of the hedging index to the employee, we need to determine how he values the restricted security with or without the hedging index. The employee's private valuation of an incentive security hinges upon his state price deflator U_{c_T}/U_{c_t}, i.e., the marginal rate of substitution. Given the CRRA utility function, we have $U_{c_t} = e^{-\phi t}c_t^{-\gamma}$. Applying Ito's lemma, we obtain the following processes for the marginal utility:

$$\frac{dU_c}{U_c} = -(r - \gamma\Omega)dt - (s_m - \gamma\beta_s x_s \sigma_m - \gamma\beta_I x^*_{It}\sigma_m)dz_m \\ -\gamma x_s \sigma_s dz_s - \gamma x^*_{It}\sigma_I dz_I \quad \forall\ t<T \tag{4}$$

$$\frac{dU_c}{U_c} = -rdt - s_m dz_m \quad \forall\ t \geq T \tag{5}$$

where Ω and x^*_{It} are defined in Eq. (3).

The processes in (4) and (5) apply to the vesting and post-vesting periods, respectively. Beyond the vesting period, the employee's state price deflator is determined by the risk-free rate and the Sharpe ratio of the market portfolio and is independent of the risk preference parameters ϕ and γ. In this case, the employee's valuation of any incentive security is equal to the market valuation, and the discount is zero. In other words, the optimal course of action for the employee is to sell the incentive security as soon as it is vested. Within the vesting period, the employee's valuation will deviate from the market's since the discount rate is $r - \gamma\Omega$ and the variance of the marginal utility is $s_m^2 + \gamma^2\Omega$. In the following section, we will determine precisely how much the employee discounts the restricted stock and how the hedging index can alleviate this discount.

3.2. Private Valuation of the Restricted Stock

As mentioned, when the employee faces trading restrictions, his private valuation of an incentive security no longer equals the market's. Instead, it is determined via the Euler Eq. (2) under the state price deflator (4). Taking the continuous dividend yield into consideration, the employee's subjective valuation of the restricted stock is computed as $\hat{S}_t = E_t((U_{c_T})/(U_{c_t}) e^{q_s(T-t)} S_T)$. Straightforward, albeit tedious algebra leads to

$$\hat{S}_t = S_t e^{-l_s(T-t)}$$

where the illiquidity discount l_s is defined as

$$l_s = \begin{cases} l_s^* \equiv \gamma(x_s - x_s^2)\, v_s^2\, (1 - \rho_{Is.m}^2) & \text{if the index shorting restriction is not binding} \\ l_s^* + \gamma v_I^2 (x_I - x_I^*)(x_I^* - x_I - x_I^*/x_s) & \text{if the index shorting restriction is binding} \end{cases}$$

When the trading restriction on the stock is removed, i.e., when $x_s = 0$, there is no need to take a position on the index. The illiquidity discount l_s^* becomes zero and the employee's private valuation is equal to the market's. Also, the private valuation reverts to the market value at the end of the vesting period where $t = T$. However, when $x_s \neq 0$ or $t < T$, the private value is always lower than the market value due to the employee's bearing of the company-specific risk. The extent to which the employee discounts the restricted stock depends on the length of the vesting period $T-t$ and the illiquidity discount rate, l_s^*. The illiquidity discount increases with the degree of risk aversion and the volatility of the company's stock. However, the discount decreases with the correlation between the company's stock and the index. The discount reduces to its minimum when the index position is optimized. With an optimal index weight, a perfect residual correlation between the stock and the index (i.e., $|\rho_{Is.m}| = 1$) can make the illiquidity discount completely vanish. In this case, the inherent non-systematic risk is completely avoided, and the employee's private valuation of the stock coincides with the market's, reducing the discount to zero. Admittedly, the incentive effects are also nullified in this case, making the restricted stock equivalent to a delayed cash bonus. An obvious policy implication is that, firms must be sure that there does not exist an effective hedging vehicle for the employee when granting restricted stocks.

To gain further insights, we report in Table 4 the private stock value as a fraction of its market value for various parameter combinations. One minus the fraction measures the size of discount in percentage. Under each stock restriction, we examine three scenarios: no hedging (corresponding to KLL, 2003; Ingersoll, 2006), hedging with a binding restriction on the index and optimal hedging.[9] The difference in the fractions between the no hedging ($x_I = 0$) column and optimal-hedging ($x_I = x_I^*$) column measures the reduction of discount due to hedging. Other than the obvious (e.g., a higher risk

Table 4. Private Value of the Restricted Stock as a Fraction of its Market Value.

γ	ρ_{ms}	ρ_{Is}	$T-t$	$\rho_{Is.m}$	$x_s = 0.1$			$x_s = 0.3$			$x_s = 0.5$		
					$x_I = 0.0$	$x_I = 0.5\,x_I^*$	$x_I = x_I^*$	$x_I = 0.0$	$x_I = 0.5\,x_I^*$	$x_I = x_I^*$	$x_I = 0.0$	$x_I = 0.5\,x_I^*$	$x_I = x_I^*$
2	0.20	0.34	1	0.30	0.958	0.959	0.961	0.904	0.907	0.912	0.887	0.889	0.897
2	0.20	0.34	5	0.30	0.806	0.813	0.822	0.604	0.615	0.632	0.549	0.556	0.579
2	0.20	0.34	10	0.30	0.649	0.661	0.675	0.365	0.378	0.400	0.301	0.309	0.336
2	0.80	0.75	1	0.30	0.984	0.985	0.985	0.963	0.964	0.966	0.956	0.957	0.960
2	0.80	0.75	5	0.30	0.922	0.925	0.929	0.828	0.833	0.842	0.799	0.803	0.815
2	0.80	0.75	10	0.30	0.850	0.856	0.863	0.685	0.694	0.709	0.638	0.644	0.664
2	0.20	0.54	1	0.65	0.958	0.966	0.975	0.904	0.919	0.943	0.887	0.898	0.933
2	0.20	0.54	5	0.65	0.806	0.841	0.883	0.604	0.657	0.747	0.549	0.585	0.707
2	0.20	0.54	10	0.65	0.649	0.708	0.779	0.365	0.431	0.559	0.301	0.342	0.500
2	0.80	0.87	1	0.65	0.984	0.987	0.991	0.963	0.969	0.978	0.956	0.961	0.974
2	0.80	0.87	5	0.65	0.922	0.937	0.954	0.828	0.854	0.897	0.799	0.818	0.878
2	0.80	0.87	10	0.65	0.850	0.878	0.911	0.685	0.730	0.804	0.638	0.669	0.771
2	0.50	0.92	1	1.00	0.967	0.982	1.000	0.924	0.953	1.000	0.911	0.932	1.000
2	0.50	0.92	5	1.00	0.845	0.915	1.000	0.675	0.787	1.000	0.626	0.704	1.000
2	0.50	0.92	10	1.00	0.714	0.837	1.000	0.455	0.620	1.000	0.392	0.495	1.000

Table 4. (*Continued*)

γ	ρ_{ms}	ρ_{Is}	$T-t$	$\rho_{Is.m}$	$x_s = 0.1$			$x_s = 0.3$			$x_s = 0.5$		
					$x_I = 0.0$	$x_I = 0.5\,x_I^*$	$x_I = x_I^*$	$x_I = 0.0$	$x_I = 0.5\,x_I^*$	$x_I = x_I^*$	$x_I = 0.0$	$x_I = 0.5\,x_I^*$	$x_I = x_I^*$
4	0.50	0.92	1	1.00	0.935	0.965	1.000	0.854	0.909	1.000	0.829	0.869	1.000
4	0.50	0.92	5	1.00	0.714	0.837	1.000	0.455	0.620	1.000	0.392	0.495	1.000
4	0.50	0.92	10	1.00	0.509	0.700	1.000	0.207	0.384	1.000	0.153	0.245	1.000
2	0.50	0.70	1	0.577	0.967	0.972	0.978	0.924	0.934	0.949	0.911	0.918	0.939
2	0.50	0.70	5	0.577	0.845	0.867	0.894	0.675	0.710	0.769	0.626	0.651	0.732
2	0.50	0.70	10	0.577	0.714	0.752	0.799	0.455	0.504	0.592	0.392	0.423	0.535
4	0.50	0.70	1	0.577	0.935	0.945	0.956	0.854	0.872	0.900	0.829	0.842	0.882
4	0.50	0.70	5	0.577	0.714	0.752	0.799	0.455	0.504	0.592	0.392	0.423	0.535
4	0.50	0.70	10	0.577	0.509	0.566	0.638	0.207	0.254	0.350	0.153	0.179	0.287

Note: This table reports the implied value of the restricted stock as a fraction of the market value for different combinations of the risk aversion, γ, the correlation between the stock and the market, ρ_{ms}, the correlation between the stock and the hedging index, ρ_{Is}, and the vesting period, $T-t$. For each parameter combination, we examine two levels of the stock's weight relative to the total wealth. Under each stock weight, we examine three index scenarios: no hedging ($x_I = 0$), hedging with a binding constraint on the index (50% of the optimal weight), and hedging with the optimal weight ($x_I = 0.5\,x_I^*$). The last six rows of the table are based on the average correlation parameters in Table 2. Other parameter inputs: $\sigma_s = 0.5$, $\sigma_I = 0.35$, and $\rho_{mI} = 0.8$.

aversion and a longer vesting period lead to a deeper discount), the table bears out the previous discussions: when the employee optimally chooses the position on the index, he can always reduce the illiquidity discount. Regardless of whether the stock is more correlated with the index or with the market, as long as the residual correlation $\rho_{Is.m}$ is not zero, hedging will always enhance the private valuation of the stock and undermine the incentive effect due to the reduced exposure to the company's non-systematic risk. The higher the residual correlation, the better the index can enhance the private valuation, and the weaker the remaining incentive effects. When the partial correlation is 1.0, the entire discount can be removed. In this case, the incentive effect is also completely nullified. It should be noted that many correlation combinations can lead to a perfect or a very high residual correlation $\rho_{Is.m}$. To see this, we can set $\rho_{ms}=0.2$, $\rho_{mI}=0.7$ and $\rho_{Is}=0.84$ to have $\rho_{Is.m}=1$.

Under the typical market conditions (the last two panels of the table), for an employee whose risk-aversion is 2 and who holds 50% of his wealth in the restricted stock, he discounts the holding by approximately 60% (1–0.392) when the vesting period is 10 years. With a hedging index, the discount is approximately 46% (1–0.535), representing a reduction of 14%. On the other hand, as shown in Table 3 and discussed in Section 2.4, the non-systematic risk is reduced by approximately 33% in this case.

3.3. *Private Value and Incentive Effects of European Options on Restricted Stocks*

Similar to the analyses for restricted stocks, we first obtain the marginal valuation of options and then determine how much the employee discounts the options due to trading restrictions. The valuation depends on whether the stock is still being restricted at the time of the option's exercise. When the option's maturity T_o is before the end of the vesting period T, the stock will have a private value at the option's expiry date, and an adjustment for the remaining vesting period must be made, which amounts to a discount to the stock price. When $T_o > T$, the stock attains its market value at the option's maturity and no adjustment is necessary. In this case, we must use both state price deflators. The valuation under the two cases can be summarized as follows:

$$\hat{C}_I = E_t\left[\frac{U_{c_{T_o}}}{U_{c_t}}\max(S_{T_o}e^{-l_s(T-T_o)} - K, 0)\right], \quad \forall\, T_o < T$$

$$\hat{C}_I = E_t\left[\frac{U_{c_T}}{U_{c_t}}E_T\left(\frac{U_{c_{T_o}}}{U_{c_T}}\max(S_{T_o} - K, 0)\right)\right], \quad \forall\, T_o > T$$

where K is the exercise price of the option. To facilitate presentation, let C_{BS} be the Black–Scholes call value expressed as

$$C_{BS}(S_t, T-t, K, r, q, \sigma) = S_t e^{-q(T-t)} N[d_1(r,q)] - K e^{-r(T-t)} N[d_2(r,q)]$$

where

$$d_1(r,q) = \frac{\ln(S_t/K) + (r - q + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \quad \text{and}$$

$$d_2(r,q) = d_1(r,q) - \sigma\sqrt{T-t}$$

Then the employee's valuation of the European option on the restricted stock is (see appendix, "Proof of Valuation Formula for European Options" for a proof)

$$\hat{C}_t = C_{BS}\left(S_t e^{-l_s(T-t)}, T_o - t, K, r - \gamma\Omega, q_s, \sigma_s\right), \qquad \forall\, T_o < T$$

$$\hat{C}_t = C_{BS}\left(S_t e^{-l_s(T-t)}, T_o - t, K, r - \gamma\Omega\frac{T-t}{T_o-t}, q_s, \sigma_s\right), \quad \forall\, T_o > T \tag{6}$$

It is apparent from the above that restricted European calls are always worth less than their Black–Scholes counterparts. In other words, there is a deadweight loss when granting European options. We showed earlier that the illiquidity discount l_s and the excess variance Ω can both be reduced when the index is in place. Thus, the hedging index will narrow the gap between the private valuation and the market's. When the residual correlation between the stock and the index is perfect: $|\rho_{Is.m}| = 1$, the employee's private valuation of the option is equal to the Black–Scholes value.

When studying restricted options, we must realize that there are two dimensions to incentive effects. On the one hand, similar to restricted stocks, incentives are preserved through bearing the non-systematic risk. Therefore, it is also true that when the discount on option values is reduced, incentive effects are also compromised. On the other hand, since options are nonlinear transformation of stocks, incentive effect can also be measured in terms of price sensitivities, viz., how much the private value of the option will increase for a given increase in the stock price. For this sensitivity effect, we follow the literature and examine the option's delta, i.e., the option's sensitivity to the stock price. In our case, the delta is $S_t e^{-l_s(T-t)} e^{-q(T_o-t)} N[d_1(r-\gamma\Omega, q_s)]$ or $S_t e^{-l_s(T-t)} e^{-q(T_o-t)} N[d_1(r-\gamma\Omega(T-t)/(T_o-t), q_s)]$, clearly lower than its Black–Scholes counterpart. Thus, introducing the hedging index will not only increase the private valuation of the option, but also improve its delta. The two dimensions of the incentive effects for options present an interesting trade-off when the hedging index is held: the total amount of incentive is reduced due to less exposure to the non-systematic risk, but the incentive sensitivity is enhanced due to better valuations. Since the two dimensions are measured differently, it is not clear whether an optimal trade-off exists.

To quantify the above observations, we report the option values as a fraction of their Black–Scholes counterparts in Panel A and the option delta's in Panel B of Table 5. For all calculations, we use the typical parameter values as shown in Table 2. For simplicity, we assume that the option's maturity coincides with the end of the stock's vesting period. Again, for each given weight on the restricted stock, x_s, we examine three scenarios $x_I = 0$, $x_I = 0.5x_I^*$ and $x_I = x_I^*$.

Panel A of Table 5 reveals that the discount on options is generally much bigger than that on the restricted stock (by comparing with Table 4). The deeper discount is primarily due to the non-linear nature of the option's payoff. Table 5 also reveals that the discount is bigger when (1) the weight on the restricted stock is higher, (2) the option is further out-of-the-money and (3) the option's time to maturity is longer. More importantly, the hedging index can alleviate the discount substantially. For instance, when $\gamma = 2$, $x_s = 0.5$ and $T - t = 10$ years, the discount on the at-the-money option is approximately 84% (1−0.155) when the employee does not hold the hedging index; but this discount is reduced to 68% (1−0.315) when the hedging index is optimally held, representing an improvement of 16%.

Similar observations can be made regarding delta's in Panel B of Table 5. Although the delta is lower compared with its Black–Scholes counterpart, holding the hedging index can increase the delta or incentive sensitivity. The results in Table 5 indicate that the hedging index can play an important role in reducing the discount and improving the incentive sensitivity for options. However, it should be realized that the total amount of incentive is compromised since the hedging index reduces the exposure to the stock's non-systematic risk.

3.4. Options with Early Exercise and Vesting Features

In reality, vesting requirements are usually imposed not only on the stock, but also on options. Furthermore, the incentive options are normally granted as American options which can be exercised at anytime after the option is vested. The vesting period is typically 5 years within which no exercise is allowed.

To make further investigations, we calculate American option values and delta's using a 5,000-step binomial tree. The starting price of the tree is

Table 5. Value Discount and Incentive Effect (Delta) of European Stock Options.

x_s	$\gamma = 2.0$			$\gamma = 4.0$		
	$x_I = 0.0$	$x_I = 0.5\,x_I^*$	$x_I = x_I^*$	$x_I = 0.0$	$x_I = 0.5\,x_I^*$	$x_I = x_I^*$
Panel A: Option value (as a fraction of Black–Scholes)						
	$K = \$115$, $T-t = 10$ years, Black–Scholes = \$51.208					
0.1	0.623	0.673	0.732	0.377	0.445	0.529
0.3	0.277	0.340	0.438	0.057	0.095	0.170
0.5	0.146	0.197	0.304	0.008	0.021	0.063
	$K = \$100$, $T-t = 10$ years, Black–Scholes = \$53.514					
0.1	0.629	0.679	0.736	0.385	0.452	0.536
0.3	0.287	0.349	0.447	0.061	0.100	0.178
0.5	0.155	0.207	0.315	0.009	0.023	0.068
	$K = \$85$, $T-t = 10$ years, Black–Scholes = \$56.091					
0.1	0.636	0.685	0.741	0.394	0.461	0.544
0.3	0.297	0.360	0.458	0.067	0.107	0.187
0.5	0.165	0.218	0.328	0.011	0.026	0.075
	$K = \$115$, $T-t = 5$ years, Black–Scholes = \$39.922					
0.1	0.742	0.779	0.821	0.542	0.601	0.669
0.3	0.435	0.498	0.584	0.162	0.224	0.319
0.5	0.277	0.340	0.446	0.048	0.085	0.164
	$K = \$100$, $T-t = 5$ years, Black–Scholes = \$43.574					
0.1	0.750	0.786	0.827	0.554	0.612	0.679
0.3	0.451	0.513	0.597	0.175	0.238	0.335
0.5	0.295	0.358	0.464	0.055	0.094	0.178
	$K = \$85$, $T-t = 5$ years, Black–Scholes = \$47.783					
0.1	0.759	0.794	0.833	0.568	0.624	0.690
0.3	0.469	0.529	0.612	0.190	0.254	0.353
0.5	0.316	0.378	0.484	0.064	0.107	0.196
Panel B: Option delta						
	$K = \$115$, $T-t = 10$ years, Black–Scholes = 0.680					
0.1	0.446	0.478	0.515	0.285	0.330	0.386
0.3	0.222	0.265	0.332	0.054	0.085	0.144
0.5	0.131	0.169	0.248	0.010	0.023	0.062

Table 5. (*Continued*)

x_s	$\gamma = 2.0$			$\gamma = 4.0$		
	$x_I = 0.0$	$x_I = 0.5\,x_I^*$	$x_I = x_I^*$	$x_I = 0.0$	$x_I = 0.5\,x_I^*$	$x_I = x_I^*$
	$K = \$100$, $T-t = 10$ years, Black–Scholes = 0.697					
0.1	0.462	0.494	0.531	0.298	0.344	0.400
0.3	0.235	0.278	0.346	0.060	0.092	0.154
0.5	0.142	0.181	0.263	0.012	0.026	0.069
	$K = \$85$, $T-t = 10$ years, Black–Scholes = 0.716					
0.1	0.478	0.510	0.548	0.312	0.359	0.416
0.3	0.249	0.293	0.363	0.066	0.101	0.166
0.5	0.155	0.195	0.280	0.014	0.030	0.078
	$K = \$115$, $T-t = 5$ years, Black–Scholes = 0.661					
0.1	0.514	0.535	0.559	0.393	0.429	0.471
0.3	0.332	0.371	0.424	0.143	0.187	0.255
0.5	0.233	0.274	0.345	0.051	0.083	0.149
	$K = \$100$, $T-t = 5$ years, Black–Scholes = 0.696					
0.1	0.547	0.569	0.594	0.424	0.461	0.503
0.3	0.362	0.402	0.457	0.162	0.210	0.282
0.5	0.260	0.304	0.378	0.062	0.098	0.170
	$K = \$85$, $T-t = 5$ years, Black–Scholes = 0.734					
0.1	0.583	0.605	0.630	0.457	0.495	0.538
0.3	0.396	0.436	0.493	0.186	0.236	0.313
0.5	0.293	0.337	0.415	0.076	0.116	0.197

Note: The table reports the ratio of the European stock option's value over its Black–Scholes counterpart (Panel A) and delta of the European stock option (Panel B). For each moneyness and maturity combination, we examine the ratio or value discount and delta for two levels of risk aversion ($\gamma = 2, 4$), three levels of the stock's weight relative to the total wealth ($x_s = 0.1, 0.3, 0.5$), and three index scenarios: no hedging ($x_I = 0$), hedging with a binding constraint equal to 50% of the optimal weight ($x_I = 0.5\,x_I^*$), and hedging with the optimal weight ($x_I = x_I^*$). Other parameter inputs: $\sigma_s = 0.5$, $\sigma_I = 0.35$, $\rho_{ms} = 0.5$, $\rho_{mI} = 0.8$, $\rho_{Is} = 0.7$, $\rho_{Is.m} = 0.577$, $r = 0.06$, $q_s = 0.02$. For simplicity, we assume that the option's maturity coincides with the stock's vesting period.

$S_t e^{-l_s(T-t)}$ where as before, T is the end of vesting period and t is the current time. For simplicity, we assume that the stock's vesting period coincides with the option's vesting period. In this case, the effective discount rate is $r - \gamma\Omega$ during the vesting period and r during the post-vesting period. The different interest rates lead to two distinct branching probabilities for the two periods. The jump size of the binomial tree remains the same in both periods.

Panel A of Table 6 reports the American option value with vesting features as a fraction of its market value calculated in the Black–Scholes' environment, i.e., the plain vanilla American call option. The table shows several vesting scenarios under various parameter combinations. For comparisons, we also report European values in the last panel. For consistency, in the last panel, we also calculate the percentages using the plain vanilla America option value (i.e., $56.10) as the base. There are several interesting observations. First, American options are always worth more than their European counterparts no matter how long the vesting period is. The difference is larger when the risk aversion and the weight on the restricted stock are high.[10] When the vesting period is close to 10 years, the American option is practically a European option. This becomes apparent as the vesting period increases, going from the first panel to the last panel. In addition, the impact of the restricted stock's weight and the hedging schemes is similar to that for European options as shown in Table 5, namely, partial hedging or optimal hedging can reduce the option discount substantially.

Second, other things being equal, a longer vesting period clearly reduces the option's value, which makes intuitive sense. Interestingly, the incremental impact of vesting is more pronounced when the vesting period is short. For instance, for the parameter combination of $x_s = 0.5$, $\gamma = 2$ and $x_I = x_I^*$, the private value of the restricted option as a percentage of the market value decreases from 90% to 57% (a drop of 33% points) when the vesting period increases from 1 to 5 years; however, the percentage decreases from 57% to 35% (a drop of 22% points) when the vesting period increases from 5 to 9 years.

Incidentally, it is well known that the early exercise premium should be zero for American call options if the underlying stock does not pay dividends. In Panel A of Table 6, the unrestricted American call is worth more than its European counterpart ($56.10 versus $53.51) purely due to the dividend yield. However, the substantial difference in option values between different vesting scenarios is obviously not due to dividend yield alone. For instance, for the parameter combination of $x_s = 0.5$, $\gamma = 2$ and $x_I = x_I^*$, the American option value under a vesting period of 9 years is approximately one-third of that under a vesting period of 1 year. When the vesting period is reduced from 9 to 1 year, the free exercise period is increased from 1 to 9 years. The significant improvement in value associated with the longer free exercise period is not purely due to dividend yield. Instead, the premium mainly comes from the opportunity to avoid the trading restrictions.

Table 6. Value Discount and Incentive Effects of Stock Options with Vesting and Early Exercise Features.

x_s	$\gamma = 2.0$			$\gamma = 4.0$		
	$x_I = 0.0$	$x_I = 0.5\,x_I^*$	$x_I = x_I^*$	$x_I = 0.0$	$x_I = 0.5\,x_I^*$	$x_I = x_I^*$
Panel A: Option value (as a fraction of Black–Scholes)						
American option, vesting = 1 year (Black–Scholes = $56.10)						
0.1	0.955	0.962	0.970	0.911	0.925	0.940
0.3	0.890	0.905	0.925	0.790	0.817	0.855
0.5	0.853	0.870	0.900	0.724	0.753	0.808
American option, vesting = 5 years (Black–Scholes = $56.10)						
0.1	0.789	0.819	0.853	0.621	0.671	0.729
0.3	0.544	0.596	0.670	0.280	0.342	0.439
0.5	0.424	0.476	0.574	0.150	0.201	0.307
American option, vesting = 9 years (Black–Scholes = $56.10)						
0.1	0.636	0.681	0.733	0.410	0.473	0.551
0.3	0.316	0.377	0.469	0.082	0.126	0.208
0.5	0.186	0.238	0.345	0.017	0.036	0.092
European option, vesting = 10 years (Black–Scholes = $53.51)						
0.1	0.600	0.647	0.702	0.367	0.431	0.511
0.3	0.273	0.333	0.427	0.058	0.096	0.169
0.5	0.147	0.197	0.300	0.009	0.022	0.065
Panel B: Option delta						
American option, vesting = 1 year (Black–Scholes = 0.749)						
0.1	0.717	0.722	0.726	0.687	0.695	0.704
0.3	0.675	0.683	0.695	0.606	0.622	0.649
0.5	0.650	0.664	0.682	0.565	0.586	0.624
American option, vesting = 5 years (Black–Scholes = 0.749)						
0.1	0.601	0.619	0.643	0.485	0.518	0.557
0.3	0.437	0.471	0.524	0.243	0.288	0.362
0.5	0.358	0.393	0.466	0.147	0.184	0.273
American option, vesting = 9 years (Black–Scholes = 0.749)						
0.1	0.489	0.519	0.553	0.330	0.375	0.427
0.3	0.269	0.312	0.378	0.082	0.118	0.184
0.5	0.176	0.213	0.297	0.021	0.041	0.093

Table 6. (*Continued*)

x_s	$\gamma=2.0$			$\gamma=4.0$		
	$x_I=0.0$	$x_I=0.5\,x_I^*$	$x_I=x_I^*$	$x_I=0.0$	$x_I=0.5\,x_I^*$	$x_I=x_I^*$
European option, vesting = 10 years (Black–Scholes = 0.697)						
0.1	0.462	0.494	0.531	0.298	0.344	0.400
0.3	0.235	0.278	0.346	0.060	0.092	0.154
0.5	0.142	0.181	0.263	0.012	0.026	0.069

Note: In this table, Panel A reports the value of stock options with vesting and early exercise features as a fraction of the plain vanilla American option value ($56.10). For comparison, we also report European values in the last panel, as a fraction of the same base, $56.10. Panel B reports delta of stock options with vesting and early exercise features. Similarly, we report delta's for European options in the last panel. For each vesting period, we examine the fractions and delta's for two levels of risk aversions ($\gamma = 2, 4$), three levels of the stock's wealth ($x_s = 0.1, 0.3, 0.5$), and three index scenarios: no hedging ($x_I = 0$), hedging with a binding constraint equal to 50% of the optimal weight ($x_I = 0.5\,x_I^*$), and hedging with the optimal weight ($x_I = x_I^*$). Other parameter inputs: $\sigma_s = 0.5$, $\sigma_I = 0.35$, $\rho_{ms} = 0.5$, $\rho_{mI} = 0.8$, $\rho_{Is} = 0.7$, $\rho_{Is.m} = 0.577$, $\gamma = 0.06$, $q_s = 0.02$, $S_t = 100$, $K = \$100$, $T - t = 10$ years. For American options, we assume that the stock's vesting period coincides with the options'. The American option values are calculated using a binomial tree with 5000 steps.

To examine the incentive sensitivities, we calculate the option's delta for each entry in Panel A and report the results in Panel B of Table 6. Similar to restricted European options, restricted American options have delta's lower than their Black–Scholes counterparts. The difference is exacerbated when the vesting period increases. When the vesting period is equal to the option's maturity, the American option becomes a European option, and the incentive sensitivity reaches the lowest level. Similar to the value effect in Panel A, the incremental effects of vesting is larger when the vesting period is shorter. More important is the hedging effect of the index. Introducing the index can increase delta or incentive sensitivity, especially when the stock's weight is large and when the employee is more risk-averse. The upshot is that, like other types of options, options with early exercise and vesting features will also enjoy an improvement in private valuation and incentive sensitivity when employees use a hedging index.

Comparing Table 6 with Table 5, we see that the extent to which the private value and incentive sensitivity improves with the introduction of the hedging index is similar for European and American options. For instance, with $x_s = 0.5$ and $\gamma = 2$, the private value of an at-the-money European option improves by approximately 16% (= 31.5% – 15.5%), while its delta or incentive sensitivity improves by approximately 0.121 (= 0.263 – 0.142). The corresponding quantities for an American option with a 5-year vesting period are 15% (= 57.4% – 42.4%) and 0.108 (= 0.466 – 0.358).

Before concluding the study, we would like to offer a few remarks about the general complications of incentive compensations. Intuitively, the reason that the restricted stocks and options can create incentive effects is because their appreciation in value can directly benefit the employees. The employees are willing to work harder in the hope to enhance the stock's value and hence their own holdings. The vesting requirement forces the employee to bear non-systematic or firm-specific risk, which drives a wedge between the private and market valuations of the incentive securities. The firm hopes for stronger efforts from employees during the vesting period as they work hard to enhance the stock's performance. On the other hand, bearing non-systematic risk is sub-optimal for the employees, and they will have every incentive to undo the restriction or avoid the non-systematic risk. Index hedging is thus a natural course of action for employees. As we have seen, the hedging index will definitely benefit the employee in the form of higher private valuation and higher utility, but it will also compromise the absolute incentive effect, although for options, the incentive sensitivity can also improve.

Undoubtedly, the employee's wealth must be tied to the firm's fortune in order for incentives to exist. Granting stocks or options without trading restrictions will not create long-term incentives because the employee can convert the securities into cash and leave the company at his wish. Vesting requirements will achieve the purpose of retaining the employees, and hopefully, motivating the employees at the same time. But retaining talents and enhancing incentives are two different aspects of an incentive package. Delayed cash bonus or legal contracting can both achieve retention purposes, but they do not necessarily guarantee a higher incentive. An incentive scheme is effective only when both aspects are properly considered. Presumably, granting company stocks or options with vesting features is aimed at achieving both. However, as we have shown, the employee can use a hedging index to undo the vesting effect. With a hedging index, the trading restrictions will not be as restrictive as they appear.

4. CONCLUSION

Incentive stocks and options are granted with vesting requirements and the grantees are prohibited from using direct hedges to offset the firm-specific risk. This chapter studies how private valuation and incentive effects are affected when employees use a partial hedge (e.g., by shorting the industry index) to undo the trading restrictions. The analysis is carried out in a continuous-time consumption-portfolio framework. We show that a hedging index can enhance the employee's private valuation of the restricted assets and, in the case of options, improve the incentive sensitivity. However, all the improvements are achieved via reducing the exposure to the company's non-systematic risk. In other words, the absolute incentive is compromised. The main results can be summarized as follows.

First, the hedging index can increase the employee's utility and reduce the value discount of the restricted stock. The optimal amount of the index depends positively on the stock's volatility, the residual correlation between the stock and the index, and negatively on the index's volatility, the correlation between the stock and the market. The higher the residual correlation between the stock and the index, the more effective is the index in reducing the discount. When the residual correlation is perfect, the discount can be completely removed. Therefore, the trading restrictions on the incentive stocks may not be as restrictive as we thought.

Second, the hedging index can also reduce the discount of stock options, and does so to a similar extent as for restricted stocks. Moreover, the hedging index also improves the delta's or incentive sensitivities of stock options. The value enhancement and the incentive sensitivity improvement are observed for both restricted European options and options with early exercise and vesting features.

Previous studies (e.g., Meulbroek, 2001; Kahl et al., 2003; Ingersoll, 2006) only show how and why employees would value restricted assets below their market values. The conventional wisdom is that, employees' private valuation of restricted assets is below the market's since they are exposed to the company's non-systematic risk, and that the bearing of this non-systematic risk is the source of incentives. In this study, we show that the intended effectiveness of granting restricted assets cannot be taken for granted. Employees do have the incentive and means to undo the restrictions in order to enhance their own utility. With typical parameter values, we show a reduction of approximately 33% in the exposure to non-systematic risk. With the right combination of correlations (between the stock, the market portfolio and the hedging index), the exposure to non-systematic risk can be completely removed. Interestingly, some seemingly benign correlations will do. One such an example is following: the stock's correlation with the market and the index is 0.2 and 0.84 respectively, while that between the market and the index is 0.7.

NOTES

1. For further details, please refer to page 12 of the Council's discussion paper titled "Executive Compensation Disclosure: How It Works Now, How It Can Be Improved." The paper can be found at the following web site: http://www.cii.org/site/files/pdfs/CII/%20pay/%20primer/%20edited.pdf

2. For examples of the incentive literature, see Lambert, Larcker, and Verrecchia (1991), Rubinstein (1995), Aboody (1996), Carpenter (1998, 2000), Hall and Murphy (2000, 2002), Meulbroek (2001), and Lambert and Larcker (2003). For examples of the asset valuation with portfolio constraints, see DeTemple and Sundaresan (1999) and references therein. Recent studies on constrained portfolio selections and asset valuation include Henderson and Hobson (2002), Browne, Milevsky, and Salisbury (2003), KLL (2003), and Ingersoll (2006).

3. To preserve accounting earnings and to reduce tax liabilities, firms usually grant at-the-money options by setting the exercise price equal to the prevailing stock price. If options could be granted in-the-money yet recorded at-the-money, the receiving executive will benefit at the expense of the IRS and the shareholders. Backdating refers to the practice where the effective granting date is chosen which saw the lowest stock price in the recent past; in the case of forward-dating or spring-loading, the effective granting date is purposely postponed in anticipation of a stock price decline based on either inside information or the trend of recent stock price movements. In either case, the end-result, albeit illegal, is to record options at-the-money which in effect are in-the-money. The first, major media exposure of backdating was by Forelle and Bandle (2006) in the Wall Street Journal on March 18, 2006. The Wall Street Journal maintains a webpage (http://www.online.wsj.com/public/resources/documents/info-optionsscore06-full.html) tracking the companies that are under investigation. As of December 29, 2006, the list contains more than 120 companies. For recent academic studies on this issue, please see Bebchuk, Grinstein, and Peyer (2006), Narayanan and Seyhum (2006), and Narayanan, Schipani, and Seyhum (2007).

4. Since the employee no longer faces trading constraints after the vesting period, the role of bequest is unimportant. We therefore assume a zero bequest function for simplicity.

5. KLL (2003) assume that the employee receives only one grant of the company's stock. Therefore, the number of the restricted shares is fixed during the vesting period. Such a restriction is realistic for IPO lock-up, but less so for an annual incentive scheme used by most of the companies.

6. For simplicity, we assume that the retirement time coincides with the finite horizon of the utility maximization.

7. There are many other ways in which we can introduce an additional asset into the choice set. For instance, we could introduce a tangent portfolio consisting of Assets 1 and 3. This index will be similar to $Index_{12}$ in nature. Alternatively, we could include Assets 2 and 3 as individual assets, which will be equivalent to including $Index_{23,m}$ alone.

8. Please note that exchange-traded funds are available for both the S&P 500 index and the Nasdaq-100 index. They are "Spider" and "QQQ," both traded on the American Stock Exchange. Therefore, the optimal portfolio strategy presented in this chapter can be easily implemented in reality.

9. To maintain a consistent comparison across all scenarios, in this and subsequent tables, we set the index restriction as 50% of the optimal level. We set the restriction as a fraction of the optimal level so that it is always binding to the same extent across all scenarios.

10. Although the table does not show the absolute option values, the aforementioned observations are apparent from the fractions. The fractions for American options are all higher than those for European options in the last panel.

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APPENDIX

Proof of Proposition 1

In order to determine the optimal consumption, c_t^* and portfolio holdings, x_{mt}^* and $x_{Bt}^* = 1 - x_{mt}^* - x_s - x_{It}$, we apply the optimal control rule

$$J(W, M, S, I, t) = \max_{x,c} E\left[\int_t^T U(c(\tau), \tau)\mathrm{d}\tau\right]$$

$$\Psi(x, c; W, M, S, I, t) = U(c(t), t) + \Im[J]$$

where $\Im[J]$ is the differential generator of J associated with its control function

$$\begin{aligned}\Im[J] = {} & \frac{\partial J}{\partial t} + \frac{\partial J}{\partial W} W\mu_W + \frac{1}{2}\frac{\partial^2 J}{\partial W^2} W^2\sigma_W^2 + \frac{\partial J}{\partial M} M\mu_M \\ & + \frac{1}{2}\frac{\partial^2 J}{\partial M^2} M^2\sigma_M^2 + \frac{\partial^2 J}{\partial W \partial M} WM \operatorname{cov}(\mathrm{d}W/W, \mathrm{d}M/M) \\ & + \frac{\partial J}{\partial S} S\mu_S + \frac{1}{2}\frac{\partial^2 J}{\partial S^2} S^2\sigma_S^2 + \frac{\partial^2 J}{\partial W \partial S} WS \operatorname{cov}(\mathrm{d}W/W, \mathrm{d}S/S)\end{aligned}$$

$$+\frac{\partial^2 J}{\partial S \partial M} SM\rho_{ms}\sigma_s\sigma_m + \frac{\partial J}{\partial I} I\mu_I + \frac{1}{2}\frac{\partial^2 J}{\partial I^2} I^2\sigma_I^2 + \frac{\partial^2 J}{\partial W \partial I} WI \operatorname{cov}(\mathrm{d}W/W, \mathrm{d}I/I) + \frac{\partial^2 J}{\partial I \partial M} IM\rho_{mI}\sigma_I\sigma_m + \frac{\partial^2 J}{\partial I \partial S} IS\rho_{Is}\sigma_I\sigma_s$$

Now, the bequest function for the employee at time T is $J(W, M, S, I, T) = J(W, T)$. The optimal consumption, c_t^* and the optimal weights, x_{mt}^*, x_{It}^* are solved by maximizing $\Psi(x, c; W, M, S, I, t)$. The first order conditions are

$$\frac{\partial \Psi}{\partial c} = \frac{\partial U}{\partial c} - \frac{\partial J}{\partial W} = 0 \quad \text{(A.1)}$$

$$\frac{\partial \Psi}{\partial x_m} = \frac{\partial J}{\partial W} W \frac{\partial \mu_W}{\partial x_m} + \frac{1}{2}\frac{\partial^2 J}{\partial W^2} W^2 \frac{\partial \sigma_W^2}{\partial x_m} + \frac{\partial^2 J}{\partial W \partial M} WM \frac{\partial \operatorname{cov}(\mathrm{d}W/W, \mathrm{d}M/M)}{\partial x_m} + \frac{\partial^2 J}{\partial W \partial S} WS \frac{\operatorname{cov}(\mathrm{d}W/W, \mathrm{d}S/S)}{\partial x_m} + \frac{\partial^2 J}{\partial W \partial I} WI \frac{\partial \operatorname{cov}(\mathrm{d}W/W, \mathrm{d}I/I)}{\partial x_m} = 0 \quad \text{(A.2)}$$

$$\frac{\partial \Psi}{\partial x_I} = \frac{\partial J}{\partial W} W \frac{\partial \mu_W}{\partial x_I} + \frac{1}{2}\frac{\partial^2 J}{\partial W^2} W^2 \frac{\partial \sigma_W^2}{\partial x_I} + \frac{\partial^2 J}{\partial W \partial M} WM \frac{\partial \operatorname{cov}(\mathrm{d}W/W, \mathrm{d}M/M)}{\partial x_I} + \frac{\partial^2 J}{\partial W \partial S} WS \frac{\operatorname{cov}(\mathrm{d}W/W, \mathrm{d}S/S)}{\partial x_I} + \frac{\partial^2 J}{\partial W \partial I} WI \frac{\partial \operatorname{cov}(\mathrm{d}W/W, \mathrm{d}I/I)}{\partial x_I} = 0 \quad \text{(A.3)}$$

together with

$$\Psi(x^*, c^*; W, M, S, I, t) = U(c^*(t), t) + \Im[J] = 0 \quad \text{(A.4)}$$

$$J(W, M, S, I, T) = J(W, T) \quad \text{(A.5)}$$

Tedious algebra confirms the results in Proposition 1.

Proof of Valuation Formula for European Options

Let T_o be the maturity of the option. We first prove the result for the case $T_o < T$. In this case, at the time of exercise, the stock is still being restricted. The employee's value of the non-tradable stock option is therefore given by

$$\hat{C}_t = E_t\left[\frac{U_{c_{T_o}}}{U_{c_t}} \max\left(S_{T_o} e^{-l_s(T-T_o)} - K, 0\right)\right], \quad \forall\ T_o < T$$

To compute, we need the joint conditional distribution for the state price deflator and the company's stock. We have

$$\frac{\mathrm{d}U_c}{U_c} = -(r - \gamma\Omega)\mathrm{d}t - (s_m - \gamma\beta_s x_s\sigma_m - \gamma\beta_I x_{It}^*\sigma_m)\mathrm{d}z_m - \gamma x_s\sigma_s \mathrm{d}z_s - \gamma x_{It}^*\sigma_I \mathrm{d}z_I$$

and

$$\frac{\mathrm{d}S}{S} = (\mu_s - q_s)\mathrm{d}t + \sigma_s \mathrm{d}z_s$$

Let $\tau = T_o - t$ and define

$$x = \frac{\ln(U_{c_{T_o}}/U_{c_t}) - \Phi_u}{\sigma_u\sqrt{\tau}} \quad \text{and} \quad y = \frac{\ln(S_{T_o}/S_t) - \Phi_s}{\sigma_s\sqrt{\tau}}$$

with

$$\Phi_u = -\left(r - \gamma\Omega + \frac{s_m^2 + \gamma^2\Omega}{2}\right)\tau \quad \sigma_u^2 = s_m^2 + \gamma^2\Omega$$

$$\Phi_s = \left(\mu_s - q_s - \frac{\sigma_s^2}{2}\right)\tau \quad \rho = -\frac{\mu_s - r + \gamma\Omega/x_s}{\sigma_u\sigma_s}$$

Then the joint conditional density for x and y is

$$f(x, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{x^2 + 2\rho xy + y^2}{2(1-\rho^2)}\right)$$

It is easy to show that

$$\frac{U_{c_{T_o}}}{U_{c_t}} S_{T_o} f(x, y) = S_t \exp(-l_s\tau) \exp\left(\Phi_s + \Phi_u + \frac{\tau}{2}(\sigma_s^2 + 2\rho\sigma_s\sigma_u + \sigma_u^2)\right) \times \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(y - \rho\sigma_u\sqrt{\tau} - \sigma_s\sqrt{\tau})^2}{2}\right) \times \frac{1}{\sqrt{2\pi(1-\rho^2)}} \exp\left(-\frac{(x - \rho y - (1-\rho^2)\sigma_u\sqrt{\tau})^2}{2(1-\rho^2)}\right)$$

Therefore,

$$\int_{-\infty}^{\infty}\int_{\frac{\ln K-\Phi_s}{\sigma_s\sqrt{\tau}}}^{\infty}\frac{U_{c_{T_o}}}{U_{c_t}}S_{T_o}f(x,y)\mathrm{d}x\mathrm{d}y$$
$$= S_t\exp(-l_s\tau)\exp\left(\Phi_s+\Phi_u+\frac{\tau}{2}(\sigma_s^2+2\rho\sigma_s\sigma_u+\sigma_u^2)\right)$$
$$\times\int_{-d_1(r-\gamma\Omega,q_s)}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{w^2}{2}\right)\mathrm{d}w$$
$$= S_t\exp(-l_s\tau)\exp(-q_s\tau)N[d_1(r-\gamma\Omega,q_s)]$$

where $d_1(\cdot,\cdot)$ is defined in the text and stock price takes the value $S_t\exp(-l_s\tau)$. Similarly, we have

$$\frac{U_{c_{T_o}}}{U_{c_t}}f(x,y)=\exp\left(\Phi_u+\frac{\tau}{2}\sigma_u^2\right)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(y-\rho\sigma_u\sqrt{\tau})^2}{2}\right)$$
$$\times\frac{1}{\sqrt{2\pi(1-\rho^2)}}\exp\left(-\frac{(x-\rho y-(1-\rho^2)\sigma_u\sqrt{\tau})^2}{2(1-\rho^2)}\right)$$

Therefore,

$$\int_{-\infty}^{\infty}\int_{\frac{\ln K-\Phi_s}{\sigma_s\sqrt{\tau}}}^{\infty}\frac{U_{c_{T_o}}}{U_{c_t}}Kf(x,y)\mathrm{d}x\mathrm{d}y=K\exp\left(\Phi_u+\frac{\tau}{2}\sigma_u^2\right)$$
$$\times\int_{-d_2(r-\gamma\Omega,q_s)}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{w^2}{2}\right)\mathrm{d}w$$
$$=K\exp(-(r-\gamma\Omega)\tau)N[d_2(r-\gamma\Omega,q_s)]$$

Now, for the case $T_o<T$, the stock obtains its market value at the time of exercise, but the discounting has to be done in two stages. Specifically, the employee's value of the non-tradable stock option is given by

$$\hat{C}_t=E_t\left[\frac{U_{c_T}}{U_{c_t}}E_T\left(\frac{U_{c_{T_o}}}{U_{c_T}}\max(S_{T_o}e^{-l_s(T-T_o)}-K,0)\right)\right]$$

Following similar procedures as above and using the fact that

$$\int_{-\infty}^{\infty}N(A+Bz)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)\mathrm{d}z=N\left(\frac{A}{\sqrt{1+B^2}}\right)$$

we obtain the formulas as shown in the text.

BOARD SIZE AND FIRM PERFORMANCE IN THE PROPERTY-LIABILITY INSURANCE INDUSTRY

Carl Pacini, William Hillison and David Marlett

ABSTRACT

Extant research on non-financial service firms indicates that board size is a key determinant of firm performance. Property-liability (P&L) insurers, however, face a different set of agency costs and a more intense regulatory environment than most non-financial firms. Both of these factors were reinforced by the implementation of the Financial Services Modernization Act in 2000. We document a significant inverse relation between publicly traded P&L insurer performance and board size in the post-Financial Services Modernization Act period. Publicly traded P&L insurer performance, measured by market-to-book ratio, return on revenues, and the operating ratio, was enhanced for firms with smaller board sizes in 2000 and 2001. Ironically, we find that publicly traded P&L insurers on average increased board size in 2000 and 2001. In a post-Financial Services Modernization Act environment, board size appears to be related to publicly traded P&L insurer performance, but more research is necessary to develop a complete understanding of its role in P&L insurer corporate governance.

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