

A Game-Theoretic Analysis of Skyscrapers

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Abstract

Skyscrapers are urbanism in the extreme, but they have received surprisingly little direct attention in urban economics. The standard urban model emphasizes differentials in access across locations, which determine land price differentials and building heights. This explanation leaves out an important force that appears to have historically influenced skyscraper construction: an inherent value placed on being the tallest. In this paper, we present a game-theoretic model of skyscraper development that captures this additional force. The model predicts dissipative competition over the prize of being tallest, a prediction consistent with the historical record. The paper discusses the implications of this result for the nature and efficiency of urban development and for the operation of urban real estate markets.

I. Introduction

Skyscrapers are urbanism in the extreme, but they have received surprisingly little direct attention in urban economics. Typically, these very tall buildings are treated as being one of many phenomena explained by the standard urban model (Alonso (1964), Mills (1967), and Muth (1969)). As Mills (1967, pp 197-199) puts it,

It is not unusual for land values to vary by a factor of from ten to one hundred within a distance of ten or twenty miles in a large metropolitan area. And the tremendous variation in capital-land ratios – from skyscrapers and high-rise apartments downtown to single story factories and single family homes on two-acre lots in the suburbs – is the market’s response to those dramatic variations in relative factor prices.

Thus, skyscrapers are seen as manifestations of the fundamental tradeoffs of land economics, with differentials in access across locations determining land price differentials, which in turn determine building heights differentials.

However, tall buildings have never been about economy alone, at least in the narrow sense discussed above. Ever since the development of the first skyscrapers, the actions and statements of builders suggest that building height has importance in and of itself, beyond the pro forma attribution of value to saleable or leaseable space. One dimension of building height that appears to have been important to builders is relative height. Five-and-dime entrepreneur F.W. Woolworth revised the plans for his eponymous New York City building to ensure that it was larger than the Metropolitan Life Building, and so the largest in the world at the time, save for the unoccupied Eiffel Tower. The Manhattan Company Building (40 Wall Street) was planned to be the largest in the world. It was, if only for the brief period prior to the vertex being added to the Chrysler Building. This secret addition to his building’s height allowed Walter Chrysler to achieve his ambition of developing the world’s tallest building. Unlike the Manhattan Company Building, the Chrysler Building was taller than the Eiffel Tower, and so was tallest on every list. The Chrysler Building’s place at the top of the tallest-buildings list was also short-lived. The Empire State Building initially had five floors added to its planned height to top the Chrysler Building, but this gave it the lead by only four feet. Given the lengths to which Chrysler had gone to top the Manhattan

Company Building, such a slim margin was not enough. Consequently, a dirigible mooring mast was added to the Empire State Building, putting it at the top of the list of tallest-buildings for forty years. In all these instances, it seems clear that builders assigned value to being tallest that was independent of the narrow value of a skyscraper as a piece of real estate.

Placing value on being tallest was not a phenomenon confined to the boom years of early 20th Century New York City. Later in New York's history, Governor Nelson Rockefeller pushed the development of the World Trade Center with the hope that developing the world's tallest building would spur the renewal of Lower Manhattan. The Sears Tower, however, took the top position soon after the World Trade Center was completed. Such contests continue today. The Petronas Towers in Kuala Lumpur were the tallest in the world from 1998 until they were exceeded by Taipei 101 in 2004. In the summer of 2007, the height of completed floors of the Burj Dubai, currently under construction, surpassed the height of Taipei 101. The bottom line of all of this is that skyscraper construction sometimes takes place in the context of a contest between rival builders.

This paper carries out a game-theoretic analysis of such a skyscraper contest. A builder is assumed to have a payoff function that depends in part on profits, as derived from a standard model of urban spatial structure. Builder payoff also depends on whether the builder has developed the tallest structure in his or her market. The paper considers both simultaneous- and sequential-move skyscraper games.

The main conclusion that emerges from the analysis is that the contest results in dissipation, with the value of the tallest-building prize at least partially lost in the poor economics of skyscrapers. In the simultaneous-move version of the game, all but the highest-value builder gain no expected value whatsoever from competing in the skyscraper contest. Although the highest-value builder does enjoy positive expected surplus from the contest, there is partial expected dissipation of the fruits of victory for this builder as well. This sort of race is at least broadly consistent with the historical evidence sketched above. In a sequential version of the game, the dissipation takes the form of costly pre-emption, where the leader builds a tall-enough building to deter competitors. This pre-emption also seems to be consistent with observation, in particular with the Empire State Building's long stay at the top of the tallest-building list.

These results are relevant to several significant issues in urban economics. First, they bear on agglomeration in that skyscrapers allow the concentration of great numbers of workers and businesses in very close proximity. It is well-known that in the presence of positive externalities associated with agglomeration, there exists a tendency for density to be inefficiently low in the market equilibrium. The results of our analysis of skyscraper contests suggest that an opposing tendency may exist to build at excessive densities. Second, the results also bear on the related issues of the health of central cities and so-called “urban sprawl.” Opponents of sprawl argue that the tendency to decentralize spatially is inefficiently strong in equilibrium since suburbanites and exurbanites do not bear the full costs of commuting or decentralized public good and service provision. The skyscraper contest results suggest an opposing tendency, one that tends towards a more centralized urban spatial structure. Third, the results bear on the tendency towards overbuilding in real estate markets. Overbuilding has been identified as an important aspect of real estate cycles in nearly every city. It has been variously attributed to irrationality on the parts of builders and lenders, to incentive problems in banking, and to particular features of tax codes. This paper’s results suggest another possible foundation, one arising from strategic interactions between builders.

In carrying out a game-theoretic analysis of skyscrapers, the paper fills an important gap in the urban economics literature. As noted above, the typical way to explain skyscrapers has been to consider them in the context of the standard model. There has been almost no work that has isolated skyscrapers as being important in their own right. One exception is Grimaud (1989), who considers the relationship of building heights to agglomeration economies in a model of spatial interactions. Another is Sullivan (1991), who shows that the low cost of vertical transportation by an elevator can encourage the construction of tall buildings even on cheap land. Helsley and Strange (2007) also employ a model of spatial interactions. They show that skyscrapers can be seen as a second-best internalization of agglomeration externalities. There is no paper in the urban economics literature that considers the implications of rivalry among builders of tall buildings.

The paper also builds also on work in game theory. The contest that we consider is a kind of all-pay auction. Builders expend resources (which is like bidding for an object) and the highest bidder wins (like winning the auction). Unlike the most common auction forms, however, all bidders pay what they have bid, even the losers. The all-pay auction under complete information is considered by Moulin (1986) and Baye et al (1996). It has been

applied previously to political lobbying (Baye et al (1993)) and to the arms race (O'Neill (1986)). To the best of our knowledge, skyscraper contests have never been mentioned previously as instances of all pay-auctions.

The remainder of the paper is organized as follows. Section II presents a history of skyscraper development, establishing the importance to builders of being tallest. Section III specifies and solves a version of the standard model of spatial structure that is suitable to considering skyscrapers. Section IV sets out and solves a sequential model of a skyscraper development. Section V analyzes skyscrapers through a simultaneous-move game. Section VI concludes.

II. A brief history of skyscrapers

A. Tall buildings in pre-industrial times: masonry and monuments

This section will selectively review the history of tall buildings, focusing on the identification of contests between rival builders. For much of recorded history, the world's tallest building has been the Great Pyramid of Giza. It is difficult to think of a better illustration of the importance that builders can assign to a building's size. It is also difficult to think of a better illustration of the limits of masonry construction. Given the building technology of the ancient period, the higher levels of a pyramid become narrower and narrower, while the lower levels must be very large in order to support what will rise above them.

The Great Pyramid remained the world's tallest building until the middle-ages, when it was supplanted by a series of cathedrals. Again, it is easy to understand the inherent importance of size. It is worth noting that this importance is not limited to being biggest in the world. Importance was also placed on a simple church steeple being the tallest building in a town or city, and so being closest to God.

At the conclusion of the era of masonry construction, the tallest building in the world was the Washington Monument (1884 -1889). It remains the tallest free-standing stone structure in the world. As world's tallest, it was supplanted by the Eiffel Tower (1889), whose height was made possible by structural steel.

B. Early skyscrapers

None of the structures mentioned thus far were built for human habitation. It is easy to understand why. As discussed above, as a masonry building grows taller, the loads borne by lower floors require wider and wider walls. This means less usable space. In addition, the need to walk up to higher floors means lower rents, further reducing the marginal revenue associated with building height. Structural steel allowed building height to grow without increasingly wide exterior walls. The elevator improved street access to high floors, allowing them to command greater rents. The Equitable Life Building (New York, 1870) was the first office building to employ Elisha Otis' invention of a steam powered elevator. William LeBaron Jenney's Home Insurance Building (Chicago, 1885) was the first to be completely supported by a steel skeleton. At this point, the limits to the size of an occupied building were no longer largely technological, but were instead imposed by the ambition of builders.

This ambition would soon prove to be substantial. Table 1 presents a history of the tallest occupied buildings from the beginning with the New York World Building (also known as the Pulitzer Building, for the newspaper's editor), built in 1890 at the dawn of the age of skyscrapers. Taking advantage of its elevators, Joseph Pulitzer's office was in the building's dome, allowing him a commanding view of the city spread out below him. The New York World Building was succeeded by the Manhattan Life Insurance Building in 1894 and by the Milwaukee City Hall in 1895. It is notable that with a population just over 200,000, Milwaukee was the country's 16th largest city according to the 1890 Census. Given the close relationship between population and land rents, it is difficult to explain the choice of height by economic forces alone. The Park Row Building became the tallest occupied building in the world in 1899. All of these buildings were smaller than 400 feet in height, making all of them smaller than the Washington Monument at 555 feet and much smaller than the Eiffel Tower at nearly 1000 feet.¹

The next holder of the title to the world's tallest buildings was sewing machine entrepreneur Isaac Singer. The Singer Building opened in 1908 at more than six hundred feet in height, having doubled in height from its initial plans in order to ensure its position as

¹ A tall unoccupied building seems to have less claim to greatness than a building that is both tall and in some sense useful. Many will know that the Empire State Building was once the world's tallest, and that the current tallest building is in Asia. Fewer will be able to identify the world's tallest structure and the world's tallest unsupported structure as of this writing. They are respectively the KVLV mast in North Dakota and Toronto's CN Tower. It has been common to refer to the world's tallest building with the qualification about occupation being understood. Unless otherwise noted, we will follow this convention.

tallest. It was the first building bigger than the Great Pyramid and the Washington Monument, although it was much smaller than the Eiffel Tower. In 1909, the Singer Building was exceeded by the Metropolitan Life Building. In 1913, the Woolworth Building was completed, displacing the Metropolitan Life Building at the top of the list of the world's tallest buildings. In private conversations with builder Louis Horowitz, Frank W. Woolworth made it clear that an accounting that related the costs of the building to its leasing revenues failed to capture the great value that accrued to being tallest:

Woolworth told Horowitz that he had something up his sleeve: The intangible profits in publicity and brand-name recognition that his firm would receive for erecting the world's tallest building far outweighed any real losses he might suffer from the venture. Woolworth realized that all successful entrants in the biggest or highest of something, from the time when pharaoh vied with pharaoh and matched tomb against tomb, were essentially in the same race and reaped the same benefits. The day after the world's tallest building opened in 1913, Woolworth knew, practically every newspaper would cover the story. The building would be pointed out to every tourist visiting the city, it would be written up in every guidebook to the city, and entered in every almanac and encyclopedia. Whatever the medium, the corporate name would be forever attached to the building. (Tauranac, 1995, p. 48)

Put simply, Woolworth assigned value to being tallest that was independent of the narrow value of the skyscraper as a piece of real estate. As will be seen below, this situation is one that has repeated itself several times, with builders assigning value to being biggest and so topping each other with structures of undeniable symbolic significance but doubtful economy.

C. The great skyscraper race

The Woolworth Building retained its position at the top of the tallest buildings list through the 1920s. The first challenge to the Woolworth Building was mounted by the Chrysler Building. It was initially planned at a height of 809 feet and 62 stories. This height would have made the Chrysler Building the world's occupied structure, although it would still have been smaller than the unoccupied Eiffel Tower. Not long after the Chrysler

Building's construction had begun, the Manhattan Company Building (40 Wall Street) was announced. It was planned to be even taller at 840 feet and 68 stories. The contest between these two buildings had both geographic and personal dimensions. The geographic aspect of the rivalry was between Midtown Manhattan, whose growth had accelerated explosively after the construction of the Grand Central Terminal in 1913, and Downtown Manhattan. The personal aspect of the rivalry was between William Van Allen, the chief architect of the Chrysler Building, and H. Craig Severance, Van Allen's former partner and now bitter competitor.

To this mix must be added Walter Chrysler's obvious desire to establish his own position by building the world's tallest building. He was clear in instructing his architect: "Make this building higher than the Eiffel Tower." He elaborates in the language of his industry:

Van, you've just got to get up and do something. It looks as if we're not going to be the highest after all. Think up something. Your valves need grinding. There's a knock in you somewhere. Speed up your carburetor. Go to it! (Bascomb, 2003, p. 112)

It should be no surprise that Chrysler was willing to add floors in order to top the Manhattan Company Building. Van Allen's plans were altered, being changed to call for a 925 foot tower with 72 stories. The response of the builders of the Manhattan Company Building was to add four stories to the Manhattan Company Building, ultimately bringing it to a height of 927 feet, barely larger than the Chrysler Building. Upon its completion, therefore, the Manhattan Company Building was the world's tallest. Since the Chrysler Building's steel skeleton had already been completed, the contest seemed to be over.

This victory was to be short lived. Severance did not know that the plans for the Chrysler Building had been changed. Previously, a dome had been planned for the building's pinnacle. This was replaced with a tapered vertex that would take the Chrysler Building to 1048 feet. At this height, it would not only be much taller than the Manhattan Company Building, it would also be taller than the Eiffel Tower, and so it would be the world's tallest in every sense. Predictably, those associated with the Manhattan Company Building would challenge the Chrysler Building's legitimacy as tallest, since the former had higher occupied

floors. To the great dismay of Severance and others, however, these challenges did not seem to affect public perceptions of which building was really tallest.

The great skyscraper race of the 1920s and 1930s was not over even at this point, however. Prior to the construction of either the Chrysler Building or the Manhattan Company Building, a group of investors assembled by Pierre du Pont and John Raskob bought the Waldorf Astoria Hotel with the intention of replacing it with a skyscraper, the Empire State Building. Despite the deepening of the Great Depression, Raskob pressed on with his building, reassuring investors about the economy and employing former New York governor and presidential candidate Al Smith to market the building. The size of the building was increased during the planning stages, ultimately stopping at 1050 feet. This was only slightly larger than the Chrysler Building, an uncomfortably slim edge given the demonstrated abilities and inclinations of Chrysler and Van Allen to find creative ways to add to their building's height. In order to ensure victory, a dirigible mooring mast was added to the Empire State Building at a cost of more than \$750,000. This feature of the building was quietly forgotten after its completion, and it is difficult to see it as being serious. It did, however, bring the Empire State Building's height to an unassailable 1250 square feet.

In the end, all of these buildings were completed shortly after the onset of the Great Depression. All were built on already tenuous economic foundations, and none proved to be a good investment. The Manhattan Company Building's lead investor George L. Ohrstrom would later apologize to fellow investors. The Empire State Building became known as the Empty State Building, with more than forty vacant floors during the depression.

D. Other skyscraper contests

The Empire State Building remained the world's tallest building until the completion of the North Tower of the World Trade Center in 1972. The World Trade Center project (as conceived by New York governor Nelson Rockefeller, his banker brother David, and others) had as its goal the revival of Lower Manhattan. Only later was it decided to construct the world's tallest building. The owners of the Empire State Building responded by undertaking analysis of adding another 11 stories and retaking top position. This clearly illustrates the importance of being tallest. The possibility of adding height was actually announced publicly. This idea was ultimately dropped, either because of its questionable feasibility or because of the announcement of the Sears Tower in Chicago, which was even higher than the

World Trade Center. The importance of being tallest was illustrated again at this point, with the antennae on top of the World Trade Center being lengthened in an attempt to be perceived as being taller than the Sears Tower.

The 1990s have brought renewed vigor to the contest to have the world's tallest building. In 1998, the Petronas Towers in Kuala Lumpur replaced the Sears Tower as the world's tallest. They held the crown for six years, being replaced by Taiwan's Taipei 101 in 2004 at 1670 feet in height. There are currently buildings under construction that are expected to surpass Taipei 101. Most noteworthy among these is the Burj Dubai in Dubai. It has been speculated that this "superscraper" will reach a height of 2313 feet. This estimate is quite imprecise because Emaar Properties, the builder, will not release its final planned height, in part out of concern with what rival builders might do with this information. One can claim with some precision, though, that the skyscraper contest is ongoing as of this writing.

We have focused thus far on the race to build the world's tallest building. There are many other situations where a contest might exist. For instance, many buildings are referred to as being tallest in some other context. When Los Angeles' Library Tower (U.S. Bank Tower) was announced to have been a potential terrorist target, it was referred to as the tallest building West of the Mississippi. It is, of course, also the tallest in Los Angeles and in California. Previously, the Smith Tower in Seattle (1913) was referred to as the tallest building outside of New York and Chicago. These are only a few examples. The tallest building in any country, state, or city presumably has prestige attached to it. So does the tallest building in a class, such as tallest education building (at Moscow University) or the tallest residential building (Q1 Tower in Queensland Australia). The bottom line is that prestige is assigned to being tall.

All of this suggests several general patterns characterizing skyscraper contests. First, builders assign value to being tallest, and there are situations where there are contests among builders. Second, the contests are sometimes resolved when one participant builds at a preemptive height that thus deters successive builders from further building-height competition. Third, building at great height has frequently proven to be uneconomical. Later in the paper, we will specify and solve game-theoretic models that are consistent with these patterns. Before doing this, however, we will set out the place of skyscrapers in standard urban economic analysis.

III. Skyscrapers in the standard urban model

A. Overview

This section will examine the place of skyscrapers in the standard urban model associated with Alonso (1964), Mills (1967), and Muth (1969). This approach solves simultaneously for land rent, the rent on structural space the spatial structure of a city. The aspect of spatial structure of concern here is building height or, equivalently, the ratio of capital to land in the production of space. The key determinant of this is factor prices. Since the price of capital is not thought to vary much within a city, the price of land is decisive.

B. Model

The model begins by considering rents, which are derived from the profits earned by land users. We will refer to these land users as tenants. In our model, tenants are identical business service producers, broadly conceived. Each is assumed to occupy one unit of building space. We normalize this to equal one floor of a building. We consider a discrete location space where the location of a tenant, or, more precisely, the location of the building in which the tenant rents space, is denoted by $i = 1, 2, \dots, N$. For simplicity, we do not consider the labor demands of tenants.

A tenant's output is given by the increasing and concave production function $q(K)$. K represents the quality of the city's business environment. As such, it includes the full range of agglomeration economies. Marshall (1890) identifies input-sharing, labor market pooling, and knowledge spillovers as aspects of a city that have the potential to augment productivity. Jacobs (1969) argues for an even broader range of interactions that create "new work."² We treat K as being fixed, but it is obviously possible to endogenize K by supposing it to be determined by the individual contributions made by tenants. See Helsley and Strange (2007) for one approach to endogenizing K .

Tenants incur a location-specific cost of obtaining K and thus interacting with the city's other businesses. We suppose that the interaction cost for a tenant at location i is given

² See Duranton and Puga (2004) for a survey of models of the microfoundations of agglomeration economies and Rosenthal and Strange (2004) for evidence.

by t_i . Locations are ranked by accessibility, so $t_1 < t_2 < t_3$ and so on. The price of tenant output is p . Under these assumptions, the profit of a tenant at location i is

$$pq(K) - t_i - r_i, \quad (3.1)$$

where r_i denotes the rent per unit space (per floor) at location i .

The market for space is competitive. Bidding for land must therefore ensure that the profits of tenants are the same at every location, with ρ^0 denoting the common level. The bid-rent for floor-space at location i is therefore equal to

$$r_i = pq(K) - t_i - \rho^0. \quad (3.2)$$

There is one unit of land at each location. Each location is owned by a builder, also indexed by i . The profit in the next-best use of a builder's land is for simplicity assumed to equal zero. If a builder chooses to put up a building, height (equivalent to density) will be chosen to maximize profits. The profit of builder i is

$$\pi_i(h_i) = r_i h_i - c(h_i), \quad (3.3)$$

where $c(-)$ is the cost of construction. We suppose $c(-)$ to be increasing and convex, giving a concave profit function. It is worth pointing out that revenues may decline with height at an increasing rate. As a building grows taller, the elevator and stair systems take up a greater fraction of the building's footprint, reducing the amount of saleable space. This tends to reinforce the concavity of the profit function.

The profit-maximizing building height for location i satisfies

$$r_i - c'(h_i) = 0, \quad (3.4)$$

with the second-order condition satisfied by the convexity of $c(-)$. This implicitly defines profit-maximizing building height in the standard model, h_i^* .

The key comparative statics are:

Proposition 1: The height of the building constructed on site i , h_i , (a) increases in the value of the local business environment, K , and (b) decreases in interaction cost, t_i .

Proof: Both claims can be obtained by substituting (3.2) into (3.4) and applying the implicit function theorem. QED.

Since output increases in K , building height increases in K at all locations. Differences in heights across locations will be driven by differences in accessibility, broadly conceived. A corollary to the obvious result (b) is that locations with similar access characteristics two (e.g., lots that are nearby) should be built to similar heights in equilibrium.

Composing h_i^* into the expression for builder profit defines the firm's maximum profit $\pi_i^* = \pi(h_i^*)$. Since output increases in K , builder profit increases in K at all locations. Since the locations are ranked by accessibility, builder profit decreases in i . The extent of development is determined as follows. For all developed sites, we must have $\pi_i^* \geq 0$. Since π_i^* is decreasing in i , the last occupied location I will have the property that $\pi_I^* \geq 0$ and $\pi_{I+1}^* < 0$. The set of occupied locations will be larger the larger is K .

Throughout this paper, we treat the price of output, p , as given. It is worth discussing an alternative specification. Suppose that business services are consumed locally, with tenants facing a downward sloping demand function $p(Q)$, where Q is the aggregate output of all firms, given by

$$Q = q(K) \sum_{i=1}^I h_i. \quad (3.5)$$

In this case the height choices of builders at different locations would be interdependent, and familiar issues related to small numbers competition and strategic interactions between builders would arise. For example, in a Stackelberg building height game, where builders choose heights to maximize profit but one builder chooses first, the leader will increase building height relative to the Cournot-Nash equilibrium and capture a larger share of the market for space. The model's basic results would persist if p were endogenous.³

³ See Helsley and Strange [1994] for a model of strategic interaction between rival developers. In this model, the focus is on very large developers and competition between cities. The key strategic interaction is

C. Is the standard model consistent with patterns of skyscraper construction?

To summarize the preceding analysis, the standard model makes several predictions regarding skyscraper construction. First, if larger cities offer greater agglomeration economies to firms locating there (larger K), then larger cities should have taller buildings. Second, nearby lots (similar t_i) should be developed to similar heights. Third, skyscrapers should be economical. These predictions are not entirely consistent with observation.

The first prediction of the standard model is that since bigger cities offer greater agglomeration economies, they should also have taller buildings. For the U.S., there does appear to be a positive relationship between building height and city size. Looking at the tallest buildings in the twenty largest U.S. cities, the correlation between metropolitan area population and height of the tallest building is 0.65. The correlation between metropolitan area population density and height of the tallest building is 0.50. These are broadly consistent with the standard model. However, the correlation is not perfect. Chicago has the tallest building in North America, but is smaller than New York City. Similarly, Kuala Lumpur, Taipei, and Dubai are all smaller than other cities in their regions (i.e., Singapore, Hong Kong, and Riyadh). The latter does not seem to be consistent with the standard model. It is possible, however, to argue that an extended version of the standard model could better explain the patterns. Skyscrapers are durable, and so building height should relate not just to the current degree of agglomeration but to anticipated growth in agglomeration. While this explanation fits the construction of tall buildings in early 20th Century New York and in Shanghai currently, it does not seem to explain the construction of very tall buildings in, for instance, Dubai. Regardless, expected future growth does not explain the historical tendency to build the tallest building, it instead only offers a possible explanation for building large buildings.

The standard model's second prediction is that nearby lots should be developed at similar heights. This prediction does not hold. The Sears Tower is 96 meters larger than any other building in Chicago. The Library Tower is 49 meters larger than the next largest building in Los Angeles. The Empire State Building is 62 meters larger than its nearest rival in New York. Table 2 presents the difference between the largest and next largest buildings

between rivals' choices of the quantity of construction rather than their choices of the relative heights of particular buildings.

in the twenty largest cities in the U.S. The average difference is 27 meters, or approximately 11% of the average height of the tallest building in one of these cities. Despite the possibility of building a similar building nearby, the tallest buildings in America's large cities are much taller than their rivals.

A corollary to the second prediction is that when it is efficient to build a very large structure at one point in time, it should be efficient to build a very large structure slightly later. This corollary also fails to be consistent with observation. In the relatively brief era of skyscrapers, there have been three long periods when the world's tallest building retained its position in the hierarchy for many years. The pre-eminence of the Woolworth Building lasted 17 years, the Empire State Building was at the top of the list for 41 years, and the Sears Tower was world's tallest for 24 years. The situation is similar when one considers the tallest buildings in individual cities. Of the twenty largest cities, the median date of the construction of the tallest building is 1980.

It is sensible to ask whether a richer version of the standard model would be able to predict these patterns. For instance, vintage models of urban spatial growth (Brueckner [1980]) allow for discontinuities in building heights, reflecting variations in development dates and economic conditions over time. However, within a particular time period, nearby lots continue to be developed at similar heights, as in the static model. Thus, dynamics and durable capital alone do not explain radical discontinuities in building heights for contemporaneous development. Land assembly is also worth considering. A skyscraper nearly always requires a very large lot. For literally all of the buildings we have discussed in this paper, prior to the construction of the skyscraper, the lots had been in other urban use for some time. The construction of a skyscraper requires that the new development's revenues cover both the out-of-pocket costs of construction and the opportunity cost of lost rents from current use. It is possible, then, that nearby lots are developed differently because of idiosyncrasies in pre-skyscraper land use. We cannot rule out that skyscraper development has followed a pattern consistent with this sort of extension of the standard model. However, we believe that the differences discussed above between heights of buildings built near each other in time and space are too large to be completely explained in this way.

The third prediction of the standard model is that skyscraper construction is economical. It is difficult to see the *ex post* performance of New York's great skyscrapers as evidence for their economy. In retrospect, it is also difficult to see the *ex ante* economic case for these buildings as persuasive. Pioneering urban economist W. Colin Clark carried out an

analysis of the economics of New York City skyscrapers.⁴ His main conclusion was that at a land price of \$200 per square foot, characteristic of midtown Manhattan in 1929 (**review**), a 63-story building was optimal. To justify the construction of a 75-story building, the price of land would have had to double to \$400 per square foot. To put these heights into context, the Manhattan Company Building had 70 floors, the Chrysler Building 77, and the Empire State Building 101.⁵ In any case, it is even more difficult to see design features such as the Chrysler Building's vertex or the Empire State Building's mooring mast as being economically motivated.

In sum, observed patterns of skyscraper construction suggest that the standard model does not provide a complete explanation of the construction of very tall buildings. The remainder of the paper will consider a game-theoretic model of skyscraper construction that augments the standard model and better fits the historical record.

IV. A strategic analysis of skyscrapers

A. Primitives

This section will specify and solve a simple game of skyscraper development. In order to focus on the strategic issues, we begin by considering a situation where there are only two builders. We assume them to be risk-neutral. The two builders are denoted $i = 1, 2$. Each owns a site and chooses building height. As above, we suppose that the indexes are chosen so that builder 1's location offers better access. Thus, under the standard model we would have $h_1^* > h_2^*$. Unlike the standard model, we now assume that there is an exogenous value $v > 0$ associated with constructing the tallest building in the market.

Formally, we suppose that the payoff to builder i is:

$$\delta v + \pi_i(h_i), \tag{4.1}$$

⁴ As reported in the *New York Times*, September 22, 1929.

⁵ The number of floors for the Manhattan Company Building (currently the Trump Building) is reported as 70 (Emporis) and 72 (Trump Building website). This presumably reflects changes to the internal configuration of the building.

where δ is an indicator variable equal to 1 if the builder's skyscraper is strictly tallest and equal to 0 otherwise. In this setup, there is a contest among builders. This is consistent with Section III, which presented extensive evidence that builders attach value to having the tallest structure in a given market. The market in question in this section's model could be an industry, with value accruing to having a taller building than one's rivals. This seems to have been one relevant aspect of the rivalry between Walter Chrysler and his eponymous building and the Empire State Building, spearheaded by John Raskob and Pierre duPont of General Motors. The market in question could instead be geographic, with value accruing to having the tallest building in a city, region, nation, or in the entire world.

There are many reasons a premium might be placed on winning the skyscraper contest. First, the premium may be a matter of taste alone. An oversized building is a good match for an oversized ego. Second, a building's stature may serve as advertising. It may make consumers aware of a product, or it may change the image that a product has. These are the sorts of "intangible benefits" that motivated F.W. Woolworth, as noted in Section II. Third, there may be signaling. It is clear that Walter Chrysler saw his building's competition with the Bank of Manhattan Building as being partly competition between Chrysler and General Motors. One can conceive of Chrysler's dogged pursuit of victory in the skyscraper race as an attempt to signal his company's fitness. This could have favorable effects on product market competition. Similarly, when the public sector is heavily involved in skyscraper construction, it may be motivated by a desire to signal the fitness of the city. For instance, the recent construction boom in Dubai signals that Dubai's institutions are favorable to business.

In the absence of a contest, builder i would choose height h_i^* as discussed above. A builder would also choose h_i^* if defeat in the skyscraper race were certain. In these cases, the builder's payoff would be $\pi_i(h_i^*)$. If the builder were to win the contest with certainty, a larger height would give the same payoff. This height is defined by:

$$v + \pi_i(h_i^P) = \pi_i(h_i^*). \quad (4.2)$$

For any $h_i > h_i^P$, $v + \pi_i(h_i^P) < \pi_i(h_i^*)$. The superscript "P" refers to pre-emption in the sense that if a rival builder j chose height $h_j \geq h_i^P$, builder i would concede the contest because it

would never be in the builder's interest to choose a height that would win. See Figure 1 for an illustration of the determination of h_i^P .⁶

In general, because $h_1^* > h_2^*$, (4.2) implies $h_1^P > h_2^P$. It is possible that the builders are so different that the contest is uninteresting. If $h_1^* > h_2^P$, then builder 1 would pre-empt builder 2 even if $v = 0$. This case is parallel to the case of blockaded entry in industrial organization. Because it is uninteresting, we suppose that $h_1^* < h_2^P$ in what follows. Returning to the standard model, this assumption is equivalent to assuming that access is similar for the properties owned by rival builders.

B. A simple sequential game

We will consider several ways that a skyscraper contest might be specified. It is natural to begin with a particularly simple sequential game. Specifically, we suppose for now that builders choose h_i sequentially, with builder 1 choosing first. There is no possibility of waiting.

The next result characterizes the solution of this simple game:

Proposition 2. If two builders choose heights sequentially, with builder 1 choosing first, then the subgame perfect equilibrium outcome is for builder 1 to pre-empt ($h_1 = h_2^P$) and for builder 2 to concede ($h_2 = h_2^*$).

Proof: Builder 2's equilibrium strategy is to choose h_2^* if builder 1 chooses $h_1 \geq h_2^P$ and to choose $\max\{h_2^*, h_1 + \varepsilon\}$ for some small ε if builder 1 chooses $h_1 < h_2^P$. It is clearly optimal for builder 2 to concede if builder 1 builds beyond builder 2's pre-emptive height. Likewise, it is optimal for builder 2 to build at the profit-maximizing height h_2^* if builder 1 picks a lower height. Finally, if builder 1's height is between these levels, builder 2 will just top builder 1's height in order to win the contest. This strategy is a best response to builder 1 choosing $h_1 = h_2^P$ by the definition of h_2^* . Builder 1 cannot improve on choosing $h_1 = h_2^P$ when playing against this strategy. If builder 1 were to choose $h_1 < h_2^P$, then builder 2 would

⁶ In the closed specification, where p is endogenous, the maximizing choice of each builder depends on the choice of the rival. Denoting this best response by $h_i^*(h_j)$, the condition that defines the pre-emption height becomes $v + \pi_i(h_i^P) = \pi_i(h_i^*(h_i^P))$. The analysis and basic results of this section are essentially unchanged in this case.

win, and builder 1 would receive at most $\pi_1(h_1^*)$. Builder 1 would prefer to play h_2^P , win the contest, and earn $v + \pi_1(h_2^P)$. Because builder 1 has been assumed to occupy the better site, $h_2^P < h_1^P$, and so $v + \pi_1(h_2^P) > \pi_1(h_1^P) = \pi_1(h_1^*)$. Choosing $h_1 > h_2^P$ would generate a lower payoff for builder 1, since $\pi_1(h_1)$ is decreasing for $h_1 > h_1^*$. QED.

The key characteristic of the equilibrium is costly pre-emption. The builder of the tallest building wins the race by putting up a building that the rival chooses not to top. In equilibrium, builder 1 will retain surplus relative to a case where there is no prize for the tallest building. This surplus will equal $v + \pi_1(h_2^P) - \pi_1(h_1^*)$. Builder 2 receives a payoff exactly equal to the payoff that would be received if $v = 0$. The increase in builder 1's payoff is

$$v + \pi_1(h_2^P) - \pi_1(h_1^*) < v, \quad (4.3)$$

by the definition of h_1^* . The pre-emption is therefore dissipative. In order to win the contest, the winning builder overbuilds relative to the profit-maximizing level. In the case where the two builders are identical, the result is even stronger. When $h_1^P = h_2^P$, both the winner and loser earn $\pi_i(h_i^*)$ as in the $v = 0$ situation. In this case, dissipation is complete. It is worth noting that our model is, in a sense, biased against dissipation since it does not allow for the price of space to be negatively impacted by overbuilding.

We believe that the simple race model captures forces that the standard model does not. It thus helps to explain some otherwise puzzling patterns of skyscraper construction. The existence of a tallest building premium allows for significant differences in the heights of nearby buildings. It also allows for long periods of pre-emption, where one building is sufficiently tall that construction of taller buildings is discouraged. The model is also consistent with tall buildings being built in medium-sized cities if the premium for being tallest is large enough. Finally, and probably most importantly, the race model is consistent with the common observation that skyscrapers have tenuous economics. The nature of a skyscraper contest is that victory is always Pyrrhic, at least in a narrow economic sense.

These results extend readily to a situation where there are many builders moving in inverse order of access. By pre-empting builder 2, builder 1 automatically pre-empts all the other builders. The results do not extend to changes in move order. Suppose that there are

only two builders and that the high-type builder moves second. In this case, the subgame perfect equilibrium outcome is for builder 2 to begin the game by choosing $h_2 = h_2^*$ and for builder 1 to respond by building a slightly taller building. In this case builder 1 (the second mover), receives the entire surplus. This shows that the game has a second-mover advantage. It also shows that the dissipation result requires pre-emption.

This suggests that the crucial question is whether a builder typically anticipates rivalry after construction. The historical record suggests that a builder should anticipate such competition. The Waldorf Astoria Hotel (site of the Empire State Building) had been purchased for redevelopment prior to the construction of the Chrysler Building and the Manhattan Company Building. Equitable Life announced a very large building while the Empire State Building was being constructed. There were many additional plans for skyscrapers that either were built as smaller buildings or not built at all. More recently, when Taipei 101 was completed, there were already plans for other tall buildings, possibly taller. It seems clear that no builder can safely assume that his or her building is the last one to be built, thus allowing for cheap pre-emption. Builders face strong potential competition, and this presumably impacts any pre-emptive strategies that they pursue. Our sense, therefore, is that the simple game where the highest-type builder moves first is more reasonable than one where the highest-type builder moves last because it captures potential competition. Even so, given these complications, it seems advisable to consider a model where results do not depend on move order. The next section does this by considering a simultaneous-move game.

V. A model of a skyscraper race

A. Basics

One aspect of the previous section's solutions that seems at odds with evidence is that the skyscraper contest features one builder pre-empting and the other (or others) conceding. In the case of the NY race of the 1920s and 1930s and the current Asian situation, the evidence suggests a race where many builders actively compete for the prize. In this section, we will consider a simultaneous-move game which will have this feature.

Suppose that the game is as above with the modification that builders choose heights simultaneously. It can be readily seen that the equilibrium with pre-emption described above

$(h_1 = h_2^P$ and $h_2 = h_2^*)$ is not a pure strategy Nash equilibrium of the simultaneous game. While builder 2's choice to concede is a best-response to builder 1's choice of the preemptive height, builder 1's choice is not a best response. Instead builder 1 would be better off choosing h_1^* . The next result generalizes this.

Proposition 3: The simultaneous move game has no equilibrium in pure strategies as long as builder 2 is not blockaded, that is, so long as $h_2^P > h_1^*$.

Proof: Suppose not, so there exists a pure strategy equilibrium (h_1, h_2) . It must always be the case that $h_1 \in [h_1^*, h_1^P]$ and $h_2 \in [h_2^*, h_2^P]$, by the definitions of h_1^* and h_1^P . Suppose that in the candidate equilibrium $h_1 > h_2$. In this candidate equilibrium, builder 2 loses the contest and so sets $h_2 = h_2^*$. In this case, builder 1's best response is to set $h_1 = h_1^*$. Thus, the candidate equilibrium is (h_1^*, h_2^*) . However, this cannot be an equilibrium. Because $h_1^* < h_2^P$, builder 2 could improve on the candidate equilibrium by topping builder 1. Thus, there can be no pure strategy equilibrium with $h_1 > h_2$. The case for $h_2 > h_1$ is parallel. If $h_1 = h_2$, then neither builder wins. If $h_1 = h_2 > h_1^*$, then both builders could raise payoffs by building at lower height. If $h_1 = h_2 < h_1^*$, then builder 1 could raise its payoff by topping builder 2. Thus, there can be no pure strategy equilibrium with $h_1 = h_2$. QED.

Intuitively, there can be no pure strategy equilibrium because either the winner would like to win more cheaply or the loser would be unwilling to incur the costs of building a tall building without the prospect of winning.⁷

B. Mixed strategy equilibrium

In order to find the equilibrium with simultaneous choices, we therefore consider mixed strategies. This seems to be appropriate given the nature of the skyscraper contest. It is clear from Section II's discussion of revised and re-revised plans that a builder was never completely certain how high rival buildings would go. In addition, there were many credible announcements of new contenders for the world's tallest building. Particularly notable among these was the Metropolitan Life Company, which began construction of a building

⁷ See Moulin (1986) and Baye et al (1996) for parallel results for all-pay auctions.

that was slated to rise to 100 stories, and thus could potentially have denied the Empire State Building the title of world's tallest that its builders so fervently sought. Further evidence of the uncertainty that confronted a builder in this environment is that bookmakers placed odds on the height that the Empire State Building would achieve. This uncertainty continued long after the great skyscraper race was concluded. The owners of the Empire State Building considered adding 11 stories in order to top the World Trade Center. The heights of buildings in planning and in construction are sometimes closely guarded secrets, as with the Burj Dubai. In sum, the participants in contests do not always know for sure what the other participants are doing. Supposing that players in the game play mixed strategies is a sensible way to characterize this situation.

The basic characteristic of a mixed strategy equilibrium is that each player must be indifferent among all pure strategies that it plays with positive probability. Let the cumulative distribution function $\Phi_i(h)$ denote the probability that builder i chooses a building height less than or equal to h . For builder 1 the indifference requirement means that

$$\Phi_2(h)v + \pi_1(h) = k_1 \tag{5.1}$$

for all $h \in H_1$. k_1 is a positive constant equal to builder 1's payoff for playing any pure strategy in its support, which is denoted by H_1 . Intuitively, the left side of (5.1) is the probability that builder 1 wins the contest when choosing h_1 times the value of the prize plus the economic value of the building. The complication of this and all mixed strategies is that this probability depends on the randomization chosen by the other player, captured in (5.1) by $\Phi_2(h)$. For builder 2, the indifference requirement is

$$\Phi_1(h)v + \pi_2(h) = k_2 \tag{5.2}$$

for all $h \in H_2$. k_2 is a positive constant equal to builder 2's payoff for playing any pure strategy in its support, H_2 .

The following result characterizes the mixed strategy equilibrium of the skyscraper game.

Proposition 4: The mixed strategy equilibrium of the two-player skyscraper contest is: (a) builder 1 randomizes over heights according to the distribution function $\Phi_1(h) = 1 + [\pi_1(h_2^P) - \pi_2(h)] / v$ for $v \in [h_1^*, h_2^P]$, and (b) builder 2 randomizes over heights according to the distribution function $\Phi_2(h) = [\pi_1(h_1^*) - \pi_1(h)] / v$ for $v \in [h_1^*, h_2^P]$, with a point-mass equal to $[\pi_2(h_2^*) - \pi_2(h_1^*)] / v$ at $h = h_2^*$.

Proof: See the Appendix.

The mixed strategy equilibrium has several key features. The first is that, as in the sequential games considered above, skyscraper construction is dissipative. In equilibrium, builder 2 places positive probability weight on h_2^* . This implies that the expected payoff for builder 2 is equal to $\pi_2(h_2^*)$. Builder 2 is thus no better off in expected payoff than in the absence of the prize. The expected payoff of builder 1 equals the probability that builder 2 plays h_2^* times the value of the prize, plus $\pi_1(h_1^*)$. Thus, the expected payoff of builder 1 rises by less than the amount of the prize. Together, these results establish the robustness of our earlier result that in a race skyscraper construction is likely to be uneconomical.

The second key feature of the mixed strategy equilibrium is that, unlike the sequential games, there is active participation by both builders. This is unlike the sequential games where builder 2 effectively conceded the prize in response to pre-emption by builder 1. This simultaneous game thus resembles the skyscraper races discussed in Section II. As with the sequential game, the simultaneous game can explain phenomena that are unaccounted for in the standard urban model. In the mixed strategy equilibrium, nearby buildings need not be similar in height, there is no guarantee of a tight relationship between city size and the height of the tallest building, and tall buildings may be uneconomical.

C. Overbuilding

One especially notable feature of the mixed strategy equilibrium is the existence of overbuilding. Since neither builder places any probability weight on heights less than h^* , expected building height must exceed the profit maximizing building height in the mixed strategy equilibrium. This “overbuilding” is easy to characterize when the builders and lots

are identical. Let $\pi(h)$ denote the common profit function of a builder under these conditions. All builders' mixed strategies now have the same support, $H = [h^*, h^P]$.

Proceeding as above, in the case where there are only two builders, playing h^* guarantees payoff $\pi(h^*)$ for both. Thus, (5.1) and (5.2) yield

$$\Phi_j(h_i)v + \pi(h_i) = \pi(h^*) \quad (5.3)$$

for all $h_i \in \{h^*, h^P\}$ and $j = 1, 2$. This defines the common distribution function

$$\Phi_j(h_i) = [\pi(h^*) - \pi(h_i)]/v, \quad h_i \in [h^*, h^P]. \quad (5.4)$$

Differentiation yields

$$\phi(h_i) = -\pi'(h_i)/v, \quad (5.5)$$

$$\phi'(h_i) = -\pi''(h_i)/v > 0, \quad (5.6)$$

where $\phi'(h_i) > 0$ by the concavity of $\pi(h)$. Expected building height is

$$E[h] = \int_{h^*}^{h^P} -(\pi'(h)/v)h \, dh. \quad (5.7)$$

Integrating by parts, using the definition of h^P and rearranging terms yields:

$$E[h] = h^* + \frac{1}{v} \left[\int_{h^*}^{h^P} \pi(h) \, dh - \pi(h^P)(h^P - h^*) \right] > h^*, \quad (5.8)$$

where the term in brackets is positive since $\pi(h)$ is decreasing on $[h^*, h^P]$. Thus, there is overbuilding in the sense that expected building height exceeds the profit maximizing building height for each active builder.

If there are I identical builders, then the basic equilibrium condition (5.3) becomes

$$\Phi_j(h_i)^{(I-1)}v + \pi(h_i) = \pi(h^*), \quad (5.9)$$

which generates the equilibrium function:

$$\Phi_j(h_i) = [[\pi(h^*) - \pi(h_i)]/v]^{1/(I-1)}, \quad (5.10)$$

$\Phi_j(h_i)$ is increasing in I . This means that, for any h_i , the equilibrium strategy places more probability weight below h_i as I increases. Intuitively, as I increases, the probability that any height wins the contest decreases, and so each builder reduces his equilibrium "bid" or height. Expected building height in this case can be written:

$$E[h] = h^p - \int_{h^*}^{h^p} \left[\frac{\pi(h^*) - \pi(h_i)}{v} \right]^{1/(I-1)} dh_i, \quad (5.11)$$

where the integrand is just the distribution function $\Phi_j(h_i)$. Since $\Phi_j(h_i)$ is increasing in I , it must also be the case that $E[h]$ is decreasing in I . In fact, in the limit as I approaches infinity, $\Phi_j(h_i)$ approaches 1 for all h_i , and so the expected building height approaches h^* . In this sense, competition discourages overbuilding in a skyscraper contest. This analysis is summarized in the following proposition.

Proposition 5: The mixed-strategy equilibrium features overbuilding: expected building height exceeds the profit maximizing building height for each active builder. When builders and lots are identical, the degree of overbuilding decreases as the number of active builders rises.

Proof: see above.

Our analysis suggests that contests to build tall buildings result in the construction of buildings that are, in a probabilistic sense, too large. Most of the discussion in the paper has been concerned with the most notorious of these contests, the building of the world's tallest structure. It is important to recognize that the phenomenon of building tall to win a prize is more general than that in several ways. First, as noted above, there are many markets in which it is possible to be biggest. A web search by geography or use or both will reveal that

building owners find it worth advertising their buildings' positions at the top of lists of tallest in various categories. Second, there may be value in being among the tallest in some market without necessarily being the tallest.

VI. Conclusion

This paper has considered the game-theoretics of a contest between rival builders of skyscrapers. A review of the history of skyscraper construction is consistent with the existence of such contests, as is the current state of skyscraper construction. We conclude by considering the implications of our results for the nature and efficiency of urban development and the operation of urban real estate markets.

Skyscraper contests impact urban development in several ways. First, by constructing a very tall building, a builder can pre-empt later rivals. Thus, one can expect long periods where the tallest building's primacy is unchallenged, as with the Woolworth Building (world's tallest 1913-1930), the Empire State Building (1931-1972), and the Sears Tower (1974-1998). Second, winning the skyscraper contest involves dissipation, in some situations complete dissipation. This can help to explain the dubious economy of very tall buildings. The nature of the skyscraper game ensures that the tallest buildings will be uneconomical in this narrow sense. Third, the distortions associated with the skyscraper contest work in the opposite direction from other important market imperfections. It is well known that positive Marshallain externalities associated with the spatial concentration of production can lead to an equilibrium that is excessively decentralized. Similarly, according to opponents of "sprawl," congestion and pollution externalities lead to excessive suburbanization. A tendency to overbuild in order to win a skyscraper contest pushes the city toward a more centralized spatial structure.

Skyscraper contests also have implications for the operation of urban real estate markets. First, a skyscraper contest can help explain overbuilding. The results in Sections IV and V show that every participant in a skyscraper contest chooses a building height that weakly exceeds the height dictated by profit maximization. Aggregating all of the choices results in overbuilding. Second, this overbuilding can also contribute to real estate cycles. Although skyscrapers are typically not given special attention in the cycles literature, it is not hard to see how the construction of tall buildings can contribute to increases in vacancies and declines in rents, leading to subsequent slowdowns or even shutdowns in construction. The

magnitude of the effect of a skyscraper race on a local real estate market can be significant. Together, the Empire State Building, Manhattan Company Building, and Chrysler Building added more than 4,000,000 square feet of commercial space to the New York market. This amounted to roughly 20% of the stock. It seems likely that this construction frenzy prolonged the slump in commercial real estate in New York that began with the Great Depression. More recently, Taipei 101 added an amount to Taipei's office market equal to roughly one year's construction. If there were a downturn in the Taipei market, one would expect the size of Taipei 101 to impact the rate of the real estate sector's recovery.

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Appendix

Proposition 4: The mixed strategy equilibrium of the two-player skyscraper contest is: (a) builder 1 randomizes over heights according to the distribution function $\Phi_1(h) = 1 + [\pi_1(h_2^P) - \pi_2(h)] / v$ for $v \in [h_1^*, h_2^P]$, and (b) builder 2 randomizes over heights according to the distribution function $\Phi_2(h) = [\pi_1(h_1^*) - \pi_1(h)] / v$ for $v \in [h_1^*, h_2^P]$, with a point-mass equal to $[\pi_2(h_2^*) - \pi_2(h_1^*)] / v$ at $h = h_2^*$.

Proof: The first step is to characterize the supports for the probability distributions. It is easy to see that builder i would not place positive probability on any height below h_i^* .

Furthermore, builder 2 will place no probability on any height in the interval (h_2^*, h_1^*) because in this region builder 2 loses for sure and earns a lower payoff than at h_2^* . In addition, neither builder would place positive probability on any height greater than h_2^P . Builder 2 would sacrifice payoff relative to h_2^* , while builder 1 would win for sure at h_2^P , and would earn lower profits at greater heights. In sum, builder 1's support must be contained in $[h_1^*, h_2^P]$, while builder 2's support is contained in $\{h_2^*\} \cup [h_1^*, h_2^P]$.

Consider the upper bounds of the two builders' supports, denoted H_1^+ and H_2^+ respectively. Suppose that one is bigger than the other. Without loss of generality, let $H_1^+ > H_2^+$. In this case, the builder 1 could raise its profits by reallocating some probability down from the interval $(H_2^+, H_1^+]$, since profits would be greater and the probability of winning would remain unchanged. The argument would be identical for the case $H_2^+ > H_1^+$. Thus, the upper supports must be equal.

Let H_1^- and H_2^- respectively denote the lower bounds of the two builders' supports on the interval $[h_1^*, h_2^P]$. Suppose that one is bigger than the other. Without loss of generality, let $H_1^- > H_2^-$. In this case, the builder 2 could raise its profits by reallocating some probability from the interval $[H_2^-, H_1^-]$ to h_2^* , since costs would be lower and the probability of winning would remain unchanged. The argument for the case $H_2^- > H_1^-$ would be the same with the only modification being that builder 1 would reallocate the probability to h_1^* . Thus, the lower supports on the interval $[h_1^*, h_2^P]$ must be equal as well.

It is straightforward to show that there can be no height greater than h_2^* played with strictly positive probability (atoms). Suppose first that there were such a height on the interior of the interval $[h_1^*, h_2^P]$. Denote the height by $\eta \in (h_1^*, h_2^P)$. If builder i chose height

η with positive probability, then builder j would have a payoff that increased discontinuously at $h_j = \eta$. Thus, there would exist an ε -neighborhood below η over which builder j would assign no probability. In this case, builder i would not want to place an atom at η , since a lower height would increase profit. Thus, there can be no height in the open interval (h_1^*, h_2^P) played with positive probability in a mixed strategy equilibrium.

There can also be no atoms at the common upper bound of the supports, h_2^P . If builder 1 had an atom at h_2^P , then there would exist an ε -neighborhood below h_2^P where builder 2 would do better by reallocating probability to h_2^* . This gap would imply that builder 1 would not want to play h_2^P with positive probability. If builder 2 had an atom at h_2^P , there would be an ε -neighborhood below h_2^P where builder 1 would do better by reallocating probability to slightly above h_2^P because of the discontinuity in the probability of winning. Finally, neither builder can place strictly positive probability on h_1^* . For builder 2, doing so would be dominated by h_2^* . For builder 1, an atom at h_1^* would encourage builder 2 to place an atom at a slightly greater height, which has been ruled out above.

To complete the solution, we must solve for the constants k_1 and k_2 from (5.1) and (5.2). First, note that there must be a probability mass at h_2^* for builder 2. Suppose not. Without a probability mass at h_2^* , $k_1 = \pi_1(h_1^*)$. However, builder 1 could instead play a pure strategy of h_2^P , which would earn a strictly greater payoff. It is thus necessary that there be a probability mass at h_2^* . Since builder 2 loses with certainty at height h_2^* , this implies that $k_2 = \pi_2(h_2^*)$. By the definition of h_2^P , this implies that the upper support for the builder 2 must equal h_2^P . If this were not true, then there would exist a height where builder 2 earned a payoff strictly greater than $\pi_2(h_2^*)$. The upper support for builder 1 must also equal h_2^P . Evaluating (5.1) at h_2^P then implies that $k_1 = \pi_1(h_2^P) + v$.

Substituting for k_1 in (5.1) and rearranging gives

$$\Phi_2(h_1) = [\pi_1(h_1^*) - \pi_1(h_1)]/v \quad (\text{A.1})$$

for $v > 0$ and for all $h \in H_1$. Substituting for k_2 in (5.2) and rearranging gives

$$\Phi_1(h_2) = 1 + [\pi_1(h_2^P) - \pi_2(h_2)]/v, \quad (\text{A.2})$$

for $v > 0$ and for all $h \in H_2$. Evaluating (A.2) at $h_2 = h_2^*$ gives the value of the probability atom at h_2^* ,

$$\alpha(h_2^*) = [\pi_2(h_2^*) - \pi_2(h_1^*)] / v. \quad (\text{A.3})$$

This completes the characterization of the mixed strategy equilibrium.

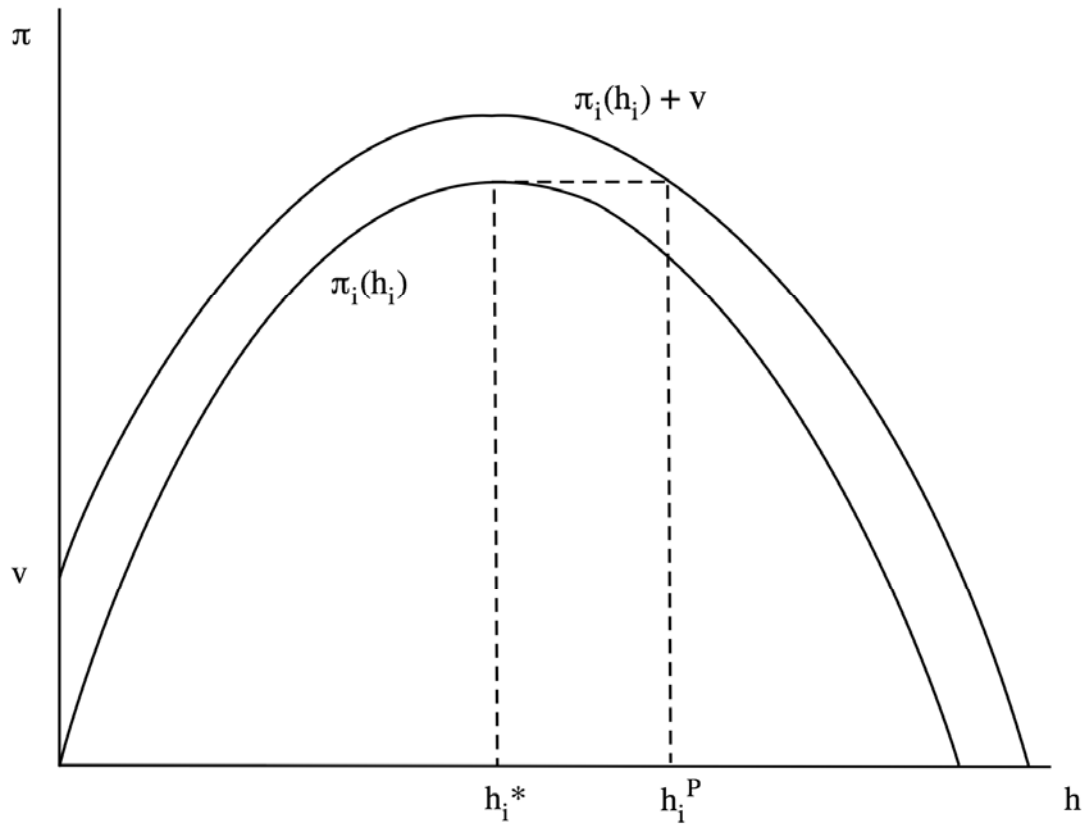
Figure 1. Pre-emptive building height

Table 1. The world's tallest buildings since 1873

Built	Building	City	Country	Floors	Roof	Pinnacle
1890	New York World Building	New York City	U.S.	20	309 ft	349 ft
1894	Manhattan Life Insurance Building	New York City	U.S.	18	348 ft	
1895	Milwaukee City Hall	Milwaukee	U.S.	9	350 ft	
1899	Park Row Building	New York City	U.S.	30	391 ft	
1908	Singer Building	New York City	U.S.	47	612 ft	
1909	Met Life Tower	New York City	U.S.	50	700 ft	
1913	Woolworth Building	New York City	U.S.	57	792 ft	
1930	40 Wall Street	New York City	U.S.	71		927 ft
1930	Chrysler Building	New York City	U.S.	77	925 ft	1046 ft
1931	Empire State Building	New York City	U.S.	102	1250 ft	1472 ft
1972	World Trade Center (North tower)	New York City	U.S.	110	1368 ft	1729 ft
1974	Sears Tower	Chicago	U.S.	108	1451 ft	1729 ft
1998	Petronas Towers	Kuala Lumpur	Malaysia	88		1483 ft
2004	Taipei 101	Taipei	Republic of China (Taiwan)	101	1474 ft	1671 ft

Source: data are © Emporis.

Table 2. Tallest Buildings by City Size (20 Largest Cities 1990)

Rank	City	Population (thousands)	Land Area (sqr. miles)	Population Density (thousands /sqr. mile)	Height of Tallest (meters)	Height of Next Tallest (meters)	Difference	As Percentage of Tallest Height
1	New York, NY	7323	309	23.69903	381	318.9	62.1	0.162992
2	Los Angeles, CA	3485	469	7.430704	310.3	261.5	48.8	0.157267
3	Chicago, IL	2784	227	12.26432	442.3	346.3	96	0.217047
4	Houston, TX	1631	540	3.02037	305.4	302.4	3	0.009823
5	Philadelphia, PA	1586	135	11.74815	288	242	46	0.159722
6	San Diego, CA	1111	324	3.429012	152.4	152.1	0.3	0.001969
7	Detroit, MI	1028	139	7.395683	221.5	192.6	28.9	0.130474
8	Dallas, TX	1007	342	2.944444	280.7	270.1	10.6	0.037763
9	Phoenix, AZ	983	420	2.340476	148.1	124.1	24	0.162053
10	San Antonio, TX	936	333	2.810811	135.3	123.1	12.2	0.09017
11	San Jose, CA	782	171	4.573099	86.9	85.3	1.6	0.018412
12	Baltimore, MD	736	81	9.08642	161	155.1	5.9	0.036646
13	Indianapolis, IN	731	362	2.019337	253	154	99	0.391304
14	San Francisco, CA	724	47	15.40426	260	237.4	22.6	0.086923
15	Jacksonville, FL	635	759	0.836627	188	163.1	24.9	0.132447
16	Columbus, OH	633	191	3.314136	191.7	169.2	22.5	0.117371
17	Milwaukee, WI	628	96	6.541667	183.2	167.3	15.9	0.08679
18	Memphis, TN	610	256	2.382813	143.3	131.1	12.2	0.085136
19	Washington, DC	607	61	9.95082	57	57	0	0
20	Boston, MA	574	48	11.95833	240.7	228.4	12.3	0.051101
						Average	27.44	0.1097

Source: 1990 Census for population and land area; height data are from www.skyscraperpage.com