Designing the content of advertising in a differentiated market

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Abstract

In many markets, consumers use attribute information to assess the value they expect from purchasing a product or service. This includes many low involvement experience goods including take-out food, many packaged good categories and restaurants. In these markets, quality differences exist but many differences are horizontal in nature: the consumer is interested in finding a product that meets her unique tastes. Beyond ensuring that consumers know the brand, the category and the price; it seems advertising should provide consumers with attribute information. However, a significant proportion of advertising does not provide it. In fact, within the same category, competitors respond to messages that emphasize detailed attribute information with messages that are devoid of attribute information. These messages are uninformative about product attributes. We explore how competition in a differentiated market is affected by the ability of a firm has to choose uninformative messages. We construct a model to investigate the factors that affect a firm’s decision to use advertising with detailed attribute information or advertising that does not provide it. The model demonstrates that content decisions about advertising are affected by the differences between products, the range of heterogeneity in consumer tastes and the degree to which costs increase as a function of the quantity of information in advertising. Surprisingly, even when the cost to increase the quantity of information in advertising is low, uninformative campaigns can be more profitable than campaigns with attribute information. The analysis also demonstrates that firms can be more likely to provide attribute information when there are less consumers that are attribute-sensitive. Finally, the model shows that uninformative messages can create “artificial differentiation” in some situations.

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1. Introduction

In many low involvement markets, marketing managers make choices about how to inform potential consumers about their firm’s offer and consumers use this information to assess the value they expect from purchasing a given product or service. Markets that fit this description include many fast moving consumer goods, take-out food and restaurants. In these markets, quality differences exist but many differences are horizontal in nature: the consumer is interested in finding a product that meets her unique tastes. Here, common sense suggests that advertising should provide consumers with attribute information. In Table 1, we provide a list of categories that fit this context. For each category, the table shows advertising that might be used and an attribute that affects how well the product meets the needs of consumers.

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Despite the importance of attributes as determinants of how well products meet the needs of consumers, a significant proportion of advertising does not provide such information. Four studies of TV advertising content show that 37.5% of advertising does not contain information about product attributes (Abernethy & Butler, 1992).\(^1\) Abernethy and Franke (1996) report that more than 37% and 25% of advertising messages for many services and OTC products respectively are uninformative.

In these categories, a key challenge for the consumer is to find an offer that meets her tastes, a “match” using the terminology of Meurer and Stahl (1994), yet uninformative advertising does not assist in this regard. In fact, within the same category, competitors often respond to messages that emphasize attribute information with messages that are devoid of attribute information. These messages are uninformative about product attributes.\(^2\) We are interested in learning how firms make decisions about the quantity of information to include in their messages and how these decisions affect competition.

This discussion leads to three research questions that we address in this study. First, how is competition affected by a firm’s willingness to implement uninformative marketing in a differentiated market? Second, is there always an increase in the level of uninformative marketing as firms become more undifferentiated? Finally, are there conditions (in terms of customer heterogeneity and the cost of advertising), where uninformative advertising allows undifferentiated firms to generate positive returns? We construct a model with two competing firms that first make a decision about the level of information to provide to consumers and second, set price to maximize profits. In the model, we consider (a) a range of heterogeneity in consumer tastes and (b) a range of costs to provide information to consumers. In addition, we assume a firm incurs an incremental cost to send additional information to consumers.

Our analysis demonstrates that both the range of heterogeneity in consumer tastes and the range of costs to provide information affect firm decisions about how much information to provide. When competing products are different, strong heterogeneity in consumer needs leads to symmetric content-rich advertising. When the range of consumer needs is less heterogeneous, uninformative advertising becomes attractive as a response to content-rich advertising. In fact, when the heterogeneity in consumer needs is below a threshold, the firm that employs uninformative advertising is more profitable than the content-rich advertiser. However, the likelihood of uninformative marketing does not increase monotonically as the heterogeneity in consumer needs falls; in an intermediate range, firms may be more willing to adopt information-rich strategies. Finally, we find that uninformative advertising can be used to create artificial differentiation when products are functionally identical. There are limits with respect to when “artificial differentiation” occurs; when it is infeasible, the market is monopolized by one of the two firms. We now move to a discussion of key literature related to this topic.

### 2. Literature review

There is a long battle between researchers who claim that advertising is largely uninformative and those who claim its purpose is to provide information to consumers. Empiricists have directed significant effort to distinguish between the persuasive and informative roles of advertising (Ackberg, 2001; Gurun, Matvos, & Seru, 2013; Hastings, Hortacsu, & Syverson, 2013).

There are two key claims made by people who argue that advertising is uninformative. First, critics of “image-based” advertising use the term “uninformative” to describe image-based advertising because it does not contain information about the physical product. Image-based advertising certainly contains information but researchers such as Bain (1956, 1967) and Solow (1967) contend that it creates an illusory perception of superiority for identical products. This is referred to as spurious differentiation and reflects a negative view of advertising (Tirole, 1988).\(^3\) It is important to note that our objective is not to study “image-based” advertising. Rather, it is to analyze the effect of advertising that is truly uninformative; it solely makes consumers aware of a brand as a choice within a category.

The second claim is that uninformative advertising can provide a signal of quality in a vertically differentiated market (Nelson, 1974). Here advertising, while uninformative, helps consumers to find products that meet their requirements. The explanation is that the quantity of advertising (not its content) signals quality to consumers (Bagwell & Ramey, 1994; Feng & Xie, 2012; Klein & Leffler, 1981; Milgrom & Roberts, 1986; Nelson, 1974; Schmalensee, 1978). This explanation is known as the money-burning theory of advertising.

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1. Almost always, marketing ensures that consumers know the brand name, the category and the price before buying.
2. In the literature, attribute-free advertising is typically described as uninformative despite the fact that it transmits the brand and the product’s availability (in the category) to consumers (Bagwell, 2007; Bagwell & Ramey, 1994; Mayzin & Shin, 2011; Milgrom & Roberts, 1986; Tirole, 1988).
3. This school of thought leads many to believe that the terms image-based, persuasive and uninformative are synonymous adjectives for advertising.
The counterpoint to this literature is the partial view of advertising wherein advertising is seen as informative. Here, consumers obtain information that better (and truthfully) informs about products and their unique attributes. This view argues that advertising provides information that leads to higher benefits for consumers. The canonical example of informative advertising adding value for consumers is when advertising provides consumers with better pricing information (Telser, 1964). This allows consumers to find products that deliver more surplus.

Informative advertising can also provide information on product attributes that are valued by some consumers and not by others (horizontal differentiation). There are numerous studies which demonstrate how informative advertising allows consumers to find products that better meet their needs (Bester & Petrakis, 1995; Butters, 1977; Grossman & Shapiro, 1984; Robert & Stahl, 1993). In particular, exposure to a firm’s advertising in Grossman and Shapiro (1984) makes consumers aware of the firm’s product and its characteristics (including price) in a horizontally differentiated market. More recently, studies on search advertising examine strategies in keyword bidding to improve the match between products and consumer needs (Du, Su, Zhang, & Zheng, 2017).

A number of studies highlight a preponderance of uninformative advertising in markets where informative advertising should dominate (Abernethy & Butler, 1992). These are markets where advertising informs consumers about the brand but provides little information on product attributes. In an empirical study, Clark, Doraszelski, and Draganska (2009) show that advertising can simply activate brands as alternatives without impacting their quality perceptions. Mayzlin and Shin (2011) show that uninformative advertising can lead consumers to search for information about products and this can help them make better decisions. In Mayzlin and Shin (2011), advertising activates interest in the brand similar to Butters (1977) and products can be evaluated through inspection. Importantly, the distinguishing feature between products in Mayzlin and Shin (2011) is quality based. That is, one product is simply better than another. Note that the mechanism in Mayzlin and Shin (2011) is different from the money-burning theory of advertising and the theory of persuasive advertising, both of which are alternative explanations for why high levels of advertising are associated with high priced products.

Abstracting away from vertically differentiated markets (the context where the money-burning theory of advertising and persuasive advertising provide insight), we observe significant uninformative advertising in horizontally differentiated markets where consumers need information about attributes to assess competing offers (Abernethy & Franke, 1996). When a consumer is aware of a brand but does not have information about its attributes, her only choice is to form expectations about the product characteristics and base her purchase decision on those expectations (Meurer & Stahl, 1994). Our model is designed to explore when (and why) in a horizontally differentiated market, a competitive firm might choose uninformative advertising.

It is important to distinguish our study of managing “the message in advertising” from research which examines how the targeting of advertising (or media) affects markets. Esteban, Gil, and Hernández (2001) consider a monopolist that targets different levels of advertising to different segments within a market. They find that the monopolist directs heavier advertising weight to consumers who are willing to pay more for the product and that with targeting, the level of advertising falls. Roy (2000) considers competition for a homogeneous good where two firms target consumers with advertising. Firms then compete on prices after observing each other’s advertising. This work shows that the firms have strong motivation to avoid each other and target specific segments. In contrast, Iyer, Soberman, and Miguel Villas-Boas (2005) consider the impact of targeting in a market where products are differentiated ex ante. In this context, the authors show that firms endogenously create differentiation through advertising choices. In our analysis, firms cannot target specific segments in the market; the best they can do is transmit advertising to consumers who are likely to be interested in the category. Nevertheless, the link of our study to research which considers the targeting of advertising is important. In particular, we find that targeted media is not the only tool that marketers have to create perceived differentiation between products.

More specifically, we seek to understand how modulating the content of the advertising message affects competition. To reiterate, the model allows firms to choose advertising that (a) is uninformative and does not provide information about product attributes or (b) has rich content and provides attribute information. Our interest is to understand how firms respond to each other in equilibrium and ultimately to assess how these choices affect market outcomes.

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4 Examples of this include Ehrlich and Fisher (1982) and Becker and Murphy (1993) in which advertising increases demand for a product by reducing the cost of consuming a product or increasing the marginal value of consumption respectively. In contrast to uninformative advertising, the information contained in the messages delivers value.

5 This reinforces findings in behavioral research showing that brand activation is critical for a product to be included in a consumer’s consideration set (Mitra & Lynch, 1995; Nedungadi, 1990).

6 The tendency of uninformed consumers to search for product information underlies models that consider how the distribution of information in a population affects firm pricing (Anderson & Renault, 2000; Wolinsky, 1986).

7 Anderson and Renault (2006) examine the decision of a monopolist to include pricing information, attribute information or both in its advertising. In contrast, our objective is to examine the message content decision of competitive firms in a context where all ads activate the brand and the category. The question is whether or not the messages also provide attribute information.
3. The model

3.1. Model structure

The market is assumed unitary and each consumer buys (at most) one product from either Firm 1 or Firm 2. Each firm offers one product with distinct attributes and we normalize the marginal cost of the products to 0. For the sake of simplicity, we assume that a product has either a left attribute or a right attribute but not both.

Prior to the start of the game, the firms develop products and each firm has a 50% chance of developing a product with a left attribute and a 50% chance of developing a product with a right attribute. In other words, there is a 50% chance that the two firms have differentiated products and a 50% chance that the products are identical. If a consumer becomes aware of a product, her prior probability of that product having a left or right attribute is 50/50. However, if the consumer sees informative advertising about the product (or if she consumes the product), she learns the actual attribute of the product with 100% probability. We assume that each firm knows the attribute of the competitor; the challenge for firms is to manage the information set of consumers.

The assumption that a firm has a 50/50 chance of being left or right is important because it implies that consumers cannot infer the attribute of a firm by “knowing” the attribute of the competitor. In a simple market with only two products, this implies that 50% of the time, one of the desired attributes will not be available. However, this is an artifact of the simplified market. In general, consumers cannot infer the attributes of a firm by observing the advertising of a competitor.

To motivate our study, consider the choice of restaurants by cruise passengers who arrive for a brief visit to a port of call. To simplify the example, we assume there are two restaurants in the port, Sputini and the Spaghetti Factory, which are similar in terms of food quality. However, they have different atmospheres. Sputini’s has a romantic atmosphere and the Spaghetti Factory is family friendly. What defines horizontal differentiation is that to provide more of what some consumers want, a firm must provide less of what other consumers want. In our example, it is difficult for a restaurant to be simultaneously romantic and family friendly.

The vast majority of advertising mentions the brand and the brand’s category. Without loss of generality, we assume that all advertising contains the brand name and the category. The difference between uninformative (U) and content-rich advertising (C) is that content-rich advertising also contains information about the product attribute (see Table 2). In all cases, we assume that the information provided by advertising is truthful.

Similar to Meurer and Stahl (1994), consumers costlessly learn the price of products for which they have seen advertising. This implies that price information is available when the purchase decision is made (through price boards, printing on the package or shelf signs); information about the key attributes is obtained through advertising.

When a firm chooses content-rich advertising, information about the firm’s attribute (left or right) is added to the firm’s communication. This reflects the idea that each firm makes a conscious (and costly) decision about whether to communicate its attribute to the market. As noted earlier, a product has either a left attribute or a right attribute but not both. This represents a horizontal attribute where the designer of the product manages a trade-off. Typical examples of such attributes where the consumer learns the attribute through experience include spicy versus bland, tight-fitting versus loose and so on. Absent the possibility where firms have the ability to launch a second product, a manager cannot simultaneously provide left and right in the same offer. Moreover, we assume that the left and right attributes are equally attractive. The objective is to examine the “information” strategies of firms when their products “match” with the preferences of some consumers and not others. Without marketing, consumers are uninformed about firms and their attributes. As noted earlier, there is heterogeneity in consumer preferences across the market. We assume there are three segments of consumers, denoted by L, R and S. The L and R segments are of equal size. Because of this, the left and right attribute are equally attractive. The size of the S segment is s and this implies that both the L and R segments are of size: 1−s.

Consumers in the L segment prefer products with the left attribute. Similarly, consumers in the R segment prefer products with the right attribute and dislike products with the left attribute. Consumers in the third segment S are indifferent between the left and right attributes. These are consumers whose enjoyment is not affected by whether the product is spicy/bland or tight-fitting versus loose for example. In reality, market segments are not this clean. Nevertheless, this structure allows us to represent a market where (a) firms have an incentive to differentiate (homogenous products lead to Bertrand competition) and this implies that both the L and R segments are of size: 1/2

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8 This reflects a context where firms conduct market research independently and do not coordinate their offers. In this market, absent coordination, the expected profitability of either attribute is equal.
9 To be specific, there is a 25% chance that both products have the left attribute, a 25% chance that both products have the right attribute and a 50% chance that one product is left and the other right.
10 A situation where one product provides more of what all segments want (such that it is a dominant choice for all segments) is a vertically differentiated product.
11 False (or misleading) advertising is prohibited by the Competition Act in Canada and the guidelines of the Federal Trade Commission in the USA.
12 Alternatively, the model reflects a market where prices are obtained through search and search is costless (price is a search attribute). However, the product is an experience good so without advertising (or experience), knowledge of the attributes is limited.
13 Equally attractive does not mean that each and every consumer is willing to pay the same for left or right. It means that the potential demand for each attribute (which in part comes from different consumers) is equivalent. This assumption allows us to analyze the effect of differentiation on advertising strategies (and not the advantage one firm might have over its competitor).
price competition) and (b) the incentive to differentiate can be modulated by varying the size of the S segment (the larger the S segment is, the less important the Left and Right attributes are).

We denote by $V$, the overall benefit that a consumer obtains by consuming a product that meets her preferences. A consumer in either the L or R segment places a value of $V = r$ on the product with preferred features and $V = 0$ otherwise. The assumption that a poor match results in zero utility simplifies the analysis without loss of generality. In contrast to the L and R segments, a consumer in segment S buys the least expensive product she knows about and receives a value $r$ from a purchased product.15

As in Butters (1977) and Grossman and Shapiro (1984), exposure to marketing by a firm means that consumers are aware of the firm (as an option within the category) and the price. As noted earlier, firms cannot target their advertising to specific segments. So, if a firm decides to advertise, its advertising reaches everyone. We return to the example of competing restaurants for cruise passengers who arrive for a short visit to a port of call. The advertising channel where passengers find out about restaurants is the ship’s cruise magazine that all passengers receive at the beginning of the cruise.

As noted earlier, content-rich advertising is assumed to be more expensive than uninformative advertising. It may seem that little cost should be associated with adding an additional piece of information (Left or Right) into advertising. However, the model represents a context wherein the attribute represents detailed information about the offer (the uninformative ad does not provide this). Returning briefly to our competing restaurants for cruise passengers who arrive for a short visit to a port of call, the ship’s cruise magazine charges advertisers based on the space requested. If one of the competing restaurants wishes to communicate the restaurant atmosphere, additional advertising space needs to be purchased. The difference in the cost for uninformative advertising versus content-rich advertising has a theoretical basis which has been confirmed experimentally and in the field. This stream of work which started in the 1970s finds that more complexity in a message.

1. makes it more difficult for consumers to process.
2. forces the sender to repeat the message more or to lengthen the exposure of the message to effectively transmit the content. These points are highlighted in Kasulis and Zaltman (1977).
3. presents a greater creative challenge (to make the message fluent).

All three of these points are demonstrated experimentally with voice mail in Reinsch and Beswick (1990) and banner ads in Wang, Shi, and Perrachio (2013). The bottom line is that with more complex ads, the marketer needs to make the ad easier to process or needs to show it more times to get the same effect (Magrath, 1989). In fact, Tellis (1999) argues that effective frequency, the prevailing wisdom for effective media planning, does not depend solely on the number of consumer exposures, but on three factors: brand familiarity, message complexity and message novelty.

From a practical point of view, this means that managers need to decide how much information they wish to transmit to consumers. As shown in Table 3, the more information the manager wishes to transmit, the more expensive it is (this reality extends across the entire range of media options).

In our model, we normalize all factors except message complexity (content-rich ads are more complex). The cost of content-rich advertising is assumed to be $c_r$, the cost of uninformative advertising is assumed to be $c_u$, and $c_r > c_u$. This means that when a manager wants to send a more complex message to the market, she spends more to produce the message (or in media).16 Admittedly, some uninformative advertising is of extraordinary quality and is expensive to produce (due to high costs of production and talent). In this model, we assume that the quality of the both message-types is identical in terms of production value and talent. The difference is that content-rich messages convey more information. That is, content-rich advertising as communication entails the transfer of more “bits of information”.

The prior probability of a firm having either a Left or Right attribute is 50% and consumers know this. We represent the beliefs of consumers about the product attribute as $\mu_i$ ($i$ is the subscript for the firm), the probability that product has a Right attribute. Thus, the prior probability for a consumer who sees an uninformative ad for a product is $\mu_l = \frac{1}{2}$.

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14 Another interpretation of the model is that a base level of utility is provided by any product and the equilibrium prices in the model are simply the markups over a product that provides the base level of utility.

15 Conceivably, switchers might have a lower reservation price than loyal segments. Intuitively, a lower reservation value from the switchers makes the segment less attractive so firms should have less incentive to fight for it. The effect of this change is likely to be similar to what happens when the segment of switchers is smaller. Unless the switchers’ reservation price is less than $\frac{1}{4}$ this would have limited impact on the results.

16 Leigh and Menon (1987) show that “under strategy-limited (low-involvement) exposure conditions, increases in complexity tended to result in decreased memory performance for the communicated features. In other words, to achieve the same level of memory performance increased exposure is needed.
Table 3  
Media costs vary with the quantity of information transmitted.

<table>
<thead>
<tr>
<th>Media Channel</th>
<th>The Expense</th>
<th>The Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Television</td>
<td>15 s ads cost much less to produce than 30 s ads</td>
<td>beverlyboy.com/tv-commercial-production/</td>
</tr>
<tr>
<td>Radio</td>
<td>15 s ads contain up to 45 words and cost half of 30 s ads which contain up to 90 words</td>
<td>jstudios.ca/word-count-commercial-script-timing.html</td>
</tr>
<tr>
<td>Print</td>
<td>Black &amp; white ads in TIME cost $78 K for a 1/3 page ad and $172 K for a full page. Color is $119 K and $265 K respectively</td>
<td>bizfluent.com/facts-5625807-average-cost-advertising-magazine-.html</td>
</tr>
<tr>
<td>Classified</td>
<td>Classified ads cost $6.00 for the first 20 words and 30 cents per word thereafter</td>
<td>vernonpublishing.com/resources/newspapers/Index/RateCard.pdf</td>
</tr>
</tbody>
</table>

Suppose a consumer in segment L or R sees an uninformative ad from a firm. She knows that the firm’s product is available. Unless she can update her prior, she believes that the product has a 50% chance of meeting her preferences. Following Meurer and Stahl (1994) and Anderson and Renault (2009), when a consumer is uninformed about a product’s attribute, her expected value for the product is $\frac{V}{2}$.

In contrast, if a consumer in segment L or R sees a content-rich ad from a firm, she knows whether the firm’s product has a Left or a Right attribute. Thus, depending on the match between the product and the consumer’s preference, she expects a value of $V = r$ or $V = 0$ from the product.

A consumer buys the product which maximizes her utility, defined as $u_i = V_i - p_i$, where $p_i$ is the price of the product from firm $i$. We also assume that $u_i > 0$ for a consumer to buy. In other words, the outside option in the model provides 0 utility to consumers. The actual level of utility provided by the outside option does not affect our findings as long as the products provide an incremental benefit. When $u_i = u_j$, the consumer randomly chooses between Firms 1 and 2, and this leads to an even split of demand in that segment.

If a consumer in segment S sees an uninformative ad from a firm, she does not know the attributes of the product but because she is indifferent to the product attribute, her expected value of the product is $r$. If she is exposed to a content-rich message, her value is also $r$.

This underlies a source of tension in the model. The unwillingness of the attribute sensitive segments (L and R) to pay for a product unless it aligns with the segment’s preference undermines the ability of firms to extract surplus from the segment of consumers (S) that is not sensitive to product attributes. We now discuss the equilibrium concept used in the game.

3.2. The equilibrium concept

Following Fudenberg and Tirole (1991), the model is solved using the Perfect Bayesian Equilibrium (PBE). The PBE restricts players to choosing strategies that are best responses (P) and imposes “logical consistency” on the beliefs of uninformed players (B). The beliefs of uninformed players (i.e., consumers) are derived using Bayes’ Rule from the actions of the informed player (the firm knows its attribute) before the uninformed players make decisions. This means that signalling is possible if the actions of the informed player (the firm) transmit information to the uninformed player (the consumer). In summary,

(P) The strategies of the informed player are optimal given the beliefs of the uninformed players.

(B) The beliefs of uninformed players are based on strategies that are consistent with Bayes’ Rule.

As noted earlier, $\mu_i$ is the prior belief of consumers about Firm $i$’s attribute if Firm $i$ advertises. We let the advertising decision of each firm be represented as $a_i$ where $a_i = 0$. $U$ or $C$ (no advertising, uninformative advertising or content-rich advertising). Consumers update their prior after observing advertising from one or both firms and we represent the updated “posterior” belief as $\tilde{\mu}_i(a_1, a_2)$.\footnote{All consumers in the market have identical priors and posteriors because everyone is assumed to see exactly the same marketing from each firm.}

The quadruplet $(a_1, a_2, \tilde{\mu}_1, \tilde{\mu}_2)$ constitutes a Perfect Bayesian Equilibrium (PBE) if and only if it satisfies the following conditions related to sequential rationality (P) and Bayesian consistency in beliefs (B).

(P) $a_i^* = \arg\max a_i \pi_i(a_i, a_j, \tilde{\mu}_1(a_1, a_2), \tilde{\mu}_2(a_1, a_2))$.

(B) $(\tilde{\mu}_1, \tilde{\mu}_2) = (\tilde{\mu}_1, \tilde{\mu}_2)$ given that $(a_1, a_2) = (a_1^*, a_2^*)$.

The PBE only imposes logical consistency on the beliefs of uninformed players over actions on the equilibrium path. We further restrict the beliefs of uninformed players (consumer) over actions off the equilibrium path with the Intuitive Criterion.
(IC) of Cho and Kreps (1987). This eliminates the possibility of multiple equilibria based on consumers believing that firms choose strategies that are equilibrium dominated. In the next section, we present the analysis and equilibrium for the game.

4. Analysis

A firm can choose uninformative advertising independent of which attribute its product has and an equilibrium relies on logically consistent consumer beliefs about the firm’s attribute, were it to make such a choice. Said differently, consumer beliefs are important even if uninformative advertising is not chosen in equilibrium: the PBE relies on every player choosing an optimal strategy. In order to ensure that a choice is optimal, we need to know what the firm would earn were it to choose an alternative strategy. We start by discussing the updating process employed by consumers to form posterior beliefs.

4.1. The updating of consumer beliefs

A key requirement to identify the equilibrium when firms advertise is to specify how consumers form beliefs about a firm’s attribute. If a firm does not advertise then consumers are unaware of its existence. Hence, the beliefs of consumers about that firm’s attribute are moot. Conversely, if a firm chooses content-rich advertising then its type is perfectly known to consumers. Thus, the challenge is to explain how consumers update their beliefs when a firm employs uninformative advertising.

The prior belief \( \mu_u \) about a firm that employs uninformative advertising is \( \mu_u = \frac{1}{2} \). When only one firm advertises with an uninformative message, consumers cannot update their beliefs about the attribute of the advertised firm. The reason is that independent of whether the firms are identical or not, the firm that advertises could be Left or Right. The focus of our study is parameter conditions where both firms advertise (at least sometimes).20

When a firm’s advertising is content rich, the firm’s attribute is known. When a firm’s advertising is uninformative, consumers try to infer the firm’s attribute from firm actions that precede the buying decision. Here, there are two types of signals that may inform consumers whether the firms are identical or differentiated: price and advertising. When a firm chooses uninformative advertising and the competitor does too, the pricing strategies of differentiated and undifferentiated firms are identical. Accordingly, our focus is to identify conditions where the advertising decisions of the two firms reveals the uninformative advertiser’s attribute.

In a nutshell, the technical challenge is to specify posterior beliefs when one firm employs content-rich advertising and the other employs uninformative advertising. As noted earlier, in a PBE, (i) consumer beliefs about the player types are consistent with actions that the players choose and (ii) the players’ actions are consistent with the beliefs that are held by consumers after observing the players’ actions.

We start by assuming the existence of a region where the posterior belief held by consumers (after observing the advertising choices of both firms) is identical to the prior, i.e. \( \tilde{\mu}_u = \mu_u = \frac{1}{2} \). We then define the parameter space where differentiated (identical) firms exhibit \((U, C)\) as \( \Omega_{\text{diff}}(U, C) \) given \( \tilde{\mu}_u = \mu_u \). Similarly, we define the parameter space where identical firms would exhibit \((U, C)\) as \( \Omega_{\text{id}}(U, C) \) given \( \tilde{\mu}_u = \mu_u \). Reasoning provided in the Appendix leads to Lemma 1.

**Lemma 1.** \( \Omega_{\text{diff}}(U, C) \subseteq \Omega_{\text{id}}(U, C) \).

**Lemma 1** implies that parameter conditions where differentiated firms employ asymmetric advertising strategies \((U, C)\) also lead identical firms to exhibit \((U, C)\). But the reverse is not true. In fact, identical firms both advertise if and only if differentiated firms exhibit \((U, C)\) in equilibrium. Within \( \Omega_{\text{id}}(U, C) \), there are conditions where differentiated firms prefer advertising strategies \((C, C)\). In this region, \( (U, C) \) is not sustainable for identical firms because \((U, C)\) is equilibrium dominated for differentiated firms when \( \tilde{\mu}_u = \mu_u \). The intuitive criterion implies that consumers update their beliefs in this region such that \( \tilde{\mu}_u = 1 \) if the content-rich advertiser is Right and \( \tilde{\mu}_u = 0 \) if the content-rich advertiser is Left (the firms are identical).21

Accordingly, we first examine the sub-game equilibrium for the case when the firms are differentiated. Both the \((U, C)\) and \((C, C)\) are viable outcomes for differentiated firms. We then move to the sub-game equilibrium when the firms are identical. If there are conditions where differentiated firms exhibit \((U, C)\), we know that identical firms choose the same strategies. In these conditions, the posterior belief about the firm that employs uninformative advertising \( \tilde{\mu}_u \) is identical to the prior. This process allows the identification of a PBE for identical firms where both advertise.

This pre-amble provides a basis to solve the two-stage game. In the first stage, the firms simultaneously set prices and consumers decide which product to buy.

---

19 An outcome fails the IC if consumers hold beliefs \( off \) the equilibrium path about a firm that would result in that firm earning less than an alternative strategy.
20 These conditions are derived in the course of solving the game.
21 When consumers have beliefs that the firms are identical, price competition leads to zero profits in the \((U, C)\) pricing subgame for both firms.
4.2. The case when firms are differentiated

For differentiated firms, we first examine the outcome when there is no S segment (s = 0). This reflects a market where consumer heterogeneity is at a maximum. We then examine conditions where s > 0. To facilitate exposition of the differentiated case, we assume that Firm 1 has the Left attribute and Firm 2 has the Right attribute.

4.2.1. No S segment

When all consumers are sensitive to product attributes (s = 0), the conditions reflect maximum consumer heterogeneity because every consumer likes the product of Firm 1 or Firm 2 and not both. Here, we consider the pricing equilibria and corresponding firm profits under different advertising scenarios.

When neither firm advertises, consumers in the market are not aware of the products so nobody buys and both firms earn zero profits. When only one firm chooses uninformative advertising, say Firm 1, consumers cannot update their prior beliefs and all consumers expect a value of \( z \) (when s = 0, all consumers in the market are attribute sensitive). As long as the price is lower than \( z \), all consumers buy from Firm 1. Consequently, Firm 1 charges the monopoly price of \( z \) and earns a profit of \( \frac{z}{4} - c_0 \).

When both firms choose uninformative advertising, consumers do not know the attributes of either product. Consequently, they expect a value of \( z \) from the two choices and consumers perceive the products to be identical. As a result, the firms engage in Bertrand-like price competition in the second stage of the game. This leads to an equilibrium price of zero (\( p_1 = p_2 = 0 \)) and the firms earn negative profits because the cost of advertising is sunk: \( \pi_1 = \pi_2 = -c_0 \).

Alternatively, consider what happens when both firms employ content-rich advertising and consumers are informed about the attributes of both products. Here, consumers in the L and R segments only buy from Firms 1 and 2 respectively, because the product from the other firm provides them with zero value. In this situation, each firm spends \( c_4 \) to be a de facto monopolist in its corresponding segment. A monopoly price of \( r \) is charged and each firm earns a profit of \( \pi_1 = \pi_2 = \frac{r}{4} - c_4 \).

In the final advertising scenario, one firm uses uninformative advertising and the other firm employs content-rich advertising. For ease of exposition, Firm 1 is the uninformative advertiser and Firm 2 is the content-rich advertiser. The prior beliefs of consumers about Firm 1 are that it is 50/50 Left/Right. Following the discussion in Section 4.1, we know that identical firms will exhibit U,C whenever differentiated firms exhibit U,C. Hence, the posterior belief for Firm 1 is also 50/50 Left/Right. As a result, if consumers observe U,C, they are willing to pay \( z \) for Product 1. Furthermore, because Firm 2’s advertising provides truthful information about its product attribute, consumers in the L segment will not buy from Firm 2 while those in the R segment are willing to pay \( r \). In other words, Firm 1 has monopoly power in the L segment and can compete with Firm 2 in the R segment. To compete for the R segment, Firm 1 has to charge a price that is \( \frac{3r}{4} \) less than the price of Firm 2.

This gives rise to an equilibrium in mixed pricing strategies (Varian, 1980). Here, Firm 1 can focus on the L segment, charge the monopoly price (given that its advertising is uninformative) and earn guaranteed profit of \( \frac{1}{2} z = \frac{3r}{4} \) (net of advertising costs). Firm 1 can also choose to compete for the R segment and if successful, its demand is 1 and its profit is \( p_1 \). Because Firm 1 can earn \( \frac{3r}{4} \) by focussing on the L segment, it will not charge a price less than \( \frac{3r}{4} \). Understanding this, Firm 2 does not price less than \( \frac{3r}{4} \) (consumers in the R segment will pay \( \frac{3r}{4} \) more for Firm 2’s product than for Firm 1’s product). This implies that Firm 2’s guaranteed profit is \( \frac{3r}{4} \) i.e., the product of the minimum price needed to guarantee sales from the R segment \( \frac{3r}{4} \) and the size of the R segment. The price support for Firm 1’s mixed pricing strategy is therefore \( \frac{3z}{4} \). The price support for Firm 2’s mixed pricing strategy is \( \left( \frac{3z}{4}, r \right) \).

Here, the equilibrium entails Firm 1 choosing a price of \( \frac{3r}{4} \) with positive probability \( z \). We define \( F_1(p) \) as the cumulative distribution function of Firm 1’s price conditional on Firm 2 not setting \( p = \frac{3r}{4} \) and \( F_2(p) \) is the cumulative distribution of Firm 2’s price. We now write the profit functions for Firms 1 and 2.

\[
\pi_1 = \frac{p}{2} + \left(1 - F_2(p + \frac{r}{2})\right) \frac{p}{2} = \frac{r}{4} \tag{1}
\]

\[
\pi_2 = \frac{3}{2} p + (1-z) \left(1 - F_1(p - \frac{r}{2})\right) \frac{1}{2} p = \frac{3r}{8} \tag{2}
\]

Because Firm 1 does not charge a price higher than \( \frac{3r}{4} \), we know that \( F_1(p - \frac{3r}{4}) = 1 \) at \( p = r \). This implies that \( z = \frac{1}{4} \), i.e. Firm 1’s mass point at \( z \) occurs with probability \( \frac{1}{4} \). The cumulative distribution functions for each firm’s pricing strategy satisfy Eqs. (1) and (2).

When the firms choose different advertising strategies, the sub-game profits of the content-rich advertiser (Firm 2) are higher than the sub-game profits of the firm that uses uninformative advertising. This results from the pricing power that the content-rich advertiser has over the segment for which its product attributes are a match. Whether this is attractive depends on the cost of content-rich advertising, \( c_4 \). Throughout the paper, we focus on parameter conditions where

\[\text{Firm 2’s price support does not include the end point } r \text{ because } r \text{ is not a best response to any price charged by Firm 1.} \]
both firms have an incentive to advertise. In the absence of attribute insensitive consumers, the relevant condition is
\( c_u < c_c < \frac{r}{2} \).

Using the equilibrium profits in the sub-games, we evaluate the first decision of the firms, whether to implement uninformative or content-rich advertising. Table 4 presents the normal form for the first stage of the game (and the profits for each firm).

The equilibrium advertising strategy is found by determining the strategy combination that is a best response for both firms. This is summarized in Proposition 1 (proofs for all propositions are provided in the appendix).

**Proposition 1.** When there is no S segment, if \( c_c - c_u \leq \frac{r}{4} \) then both firms engage in content-rich advertising. If \( c_c - c_u > r/4 \) one firm chooses content-rich advertising and the other implements uninformative advertising.

**Proposition 1** underlines how the advertising cost affects the equilibrium. When the incremental cost of content-rich advertising is sufficiently high, one of the firms has an incentive to use uninformative advertising in response to content-rich advertising.

When this happens, two points are worth noting. First, the profits earned by the content-rich advertiser (Firm 2) are reduced because the content-rich advertiser faces competition in the segment that prefers its attribute (in this case the Right attribute). When both firms employ content-rich advertising, they are fully differentiated and de facto monopolists in their respective segments.

A second important point is that when \( c_c - c_u > \frac{r}{4} \), the uninformative advertiser earns a minimum of \( \frac{r}{4} \) more profit than the content-rich advertiser. When there is no S segment, the relative profitability of the firms when they employ different advertising strategies is primarily driven by the higher cost of content-rich advertising. In other words, even though content rich advertising provides more pricing power and higher sub-game profit (i.e., profit excluding advertising cost), its competitive advantage is offset by the higher cost. Interestingly, even in a highly heterogeneous market (i.e., without a switcher segment), uninformative advertising can be attractive.

We now proceed to the analysis of a market with consumers who are not sensitive to attributes.

4.2.2. With an attribute insensitive segment

We now examine the firms’ optimal strategies when there is a segment of consumers (size \( s \)) who are not sensitive to product attributes. Similar to the previous section, we determine the pricing equilibrium under different advertising scenarios.

As before, when neither firm advertises, both firms earn zero profit. Consider a situation where only one firm advertises. In one case, the firm chooses to employ uninformative advertising and in the second case, the firm implements content-rich advertising. In Table 5, the willingness to pay of each segment for the advertised product is shown (\( i, j = L \) or \( R \) and \( i \neq j \)). For the ease of exposition, we assume that the content-rich advertising indicates a fit with segment \( i \) and a mismatch with segment \( j \).

When a firm engages in uninformative advertising and the other firm does not advertise, consumers in segment S have an expected value of \( r \) from the firm that advertises and consumers in segments \( i \) and \( j \) have a willingness to pay of \( r/2 \). In this situation, the firm compares the profit associated with selling to everyone \( \frac{r}{2} - c_u \) with the profit earned by selling to the S segment alone, i.e. \( sr - c_u \). As a result, the uninformative advertiser’s optimal price depends on the size of the attribute insensitive segment. When one firm engages in content-rich advertising and the other firm does not advertise, consumers in segment S and the segment for which the firm’s attribute is a match \( i \) buy as long the price is less than or equal to \( r \). Consequently, the content-rich advertiser charges a price of \( r \) and realizes profit of \( \frac{r^2}{4} - c_c \).

When both firms choose uninformative advertising, the products are perceived as homogeneous. This leads to profits of zero in the pricing sub-game because of Bertrand competition.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Advertising game without S segment.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Advertising</td>
</tr>
<tr>
<td>No Advertising</td>
<td>0,0</td>
</tr>
<tr>
<td>Uninformative</td>
<td>( \frac{1}{2} - c_u, 0 )</td>
</tr>
<tr>
<td>Content-rich</td>
<td>( \frac{1}{2} - c_c, 0 )</td>
</tr>
</tbody>
</table>

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23 When \( c_c > \frac{r}{4} \) there are two possible equilibria, one of which is a degenerate outcome where only one firm employs uninformative advertising. Our goal is to examine markets where given \( r \) and \( s \), consumers observe firms competing with each other, i.e. we focus our attention on values where content-rich advertising is feasible. In Section 4.3, we examine conditions where the firms are identical. There, despite the feasibility of advertising, one firm may choose to not advertise.

24 In the equilibrium, consumers believe that Firm 1 is 50/50 Left/Right because identical firms exhibit the same outcome.

25 When both firms choose uninformative advertising, consumers’ posterior beliefs are the same as their priors. As a result, consumers in the L and R segments are willing to pay \( r/2 \) while attribute insensitive consumers are willing to pay \( r \). The incentive of firms to undercut each other is unaffected by the difference in reservation prices across segments. Therefore, the firms undercut each other on price until prices and profits are zero.
Table 5
Consumers’ willingness to pay.

<table>
<thead>
<tr>
<th></th>
<th>i</th>
<th>s</th>
<th>j</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uninformative Advertising</td>
<td>( \frac{s}{3} )</td>
<td>r</td>
<td>( \frac{s}{2} )</td>
</tr>
<tr>
<td>Content-rich Advertising</td>
<td>r</td>
<td>r</td>
<td>0</td>
</tr>
</tbody>
</table>

When both firms choose content-rich advertising, consumers in the L segment only buy from the firm that offers an L attribute (and as before, we assume that the L attribute is offered by Firm 1 and the R attribute is offered by Firm 2) and those in R segment only buy from the firm that offers an R attribute. The attribute insensitive consumers simply choose the least expensive product. This resembles a special case of Narasimhan (1988). The pricing equilibrium is in mixed strategies and each firm earns guaranteed profit of \( \pi_1 = \pi_2 = \frac{p_1(s)}{2} - c \). Because of symmetry, there is no mass point in the price support; profit from serving the attribute insensitive segment is competed away.

When one firm chooses uninformative advertising (Firm 1) and the other chooses content-rich advertising (Firm 2), the pricing equilibrium depends on the size of the attribute insensitive segment. Obviously, Firm 1 can capture the L segment as long as its price is less than \( r/2 \). Therefore, Firm 1’s pricing decision depends on whether it wants to compete for the S-segment or both the S and R segments. When Firm 1 chooses to compete only for the S segment, its price is \( p_1 = p_2 - \epsilon \). If Firm 1 competes for the S and R segments \( p_1 < p_2 - \frac{s}{2} \) is necessary and this implies that Firm 1 captures the entire market and earns profit of \( \pi_1 = p_2 - \frac{s}{2} \). When \( p_2 \leq \frac{s}{2} \), Firm 1 only competes for the L and S segments because its price needs to be \( \frac{s}{2} \) less than \( p_2 \) to compete for the R segment which implies a negative price. When \( p_2 > \frac{s}{2} \), Firm 1 can compete for the R segment but the profit needs to be higher than the profit that would result by capturing only segments L and S.

In the asymmetric advertising scenario, Proposition 2 summarizes the pricing equilibrium and equilibrium profits.

**Proposition 2.** When one firm chooses uninformative advertising and the other chooses content-rich advertising, the firms play mixed strategies in prices and their profits are:

\[
\begin{align*}
\pi^* (\text{uninformative advertiser}) &= \begin{cases} 
\frac{1}{4} s & \text{if } 0 < s \leq 1/3, \\
\frac{1}{8} (1-s) & \text{if } 1/3 < s < 1 
\end{cases} \\
\pi^* (\text{content rich advertiser}) &= \begin{cases} 
\frac{3}{8} (1-s) & \text{if } 0 < s \leq \sqrt{2} - 1, \\
\frac{1}{8} (1-s) & \text{if } \sqrt{2} - 1 < s < 1 
\end{cases}
\end{align*}
\]

Under asymmetric advertising, the sub-game pricing behavior of the firms depends on the size of the switcher segment. To be specific, there are three cases, each entailing a different mixed pricing equilibrium. Fig. 1 illustrates the support of the mixed pricing equilibrium.

When \( s \) is small \((s \leq 1/3)\), the pricing equilibrium is similar to the case with no attribute insensitive consumers. In fact, as \( s \to 0 \), the price support approaches the price support derived in Section 4.2.1. Firm 1 has a mass point at \( r/2 \) and it captures both the L and S segments (for expositional purposes of the differentiated case, Firm 1’s product has the L attribute and Firm 2 the R attribute). Here, Firm 2’s equilibrium profit is equal to its lowest profit times the size of R segment.

When \( s \) is medium \((1/3 < s \leq \sqrt{2} - 1)\), Firm 1 still prices below \( r/2 \), but Firm 2 (the content-rich advertiser) gradually reduces its price to compete with Firm 1 for the switcher segment. Notice that the maximum price in Firm 2’s price support (the content-rich advertiser) is substantially less than the reservation price \( r \) (the maximum price that the R segment is willing to pay for Firm 2’s product). This stands in contrast to the pricing behavior of the content-rich advertiser when the S segment is small \((s < 1/2)\). When the S segment is small, the content-rich advertiser randomizes its price over a range that approaches \( r \) at its upper limit. Furthermore, the lower section of Firm 2’s price support overlaps with Firm 1’s upper part. Firm 1 has two mass points: one at \( s_{1,2}/4 \) and the other at \( r/2 \). Firm 2 has a mass point at \( 2s_{1,2}/4 \).

When the S segment is large \((s > \sqrt{2} - 1)\), the firms share a continuous price support in the region \( [s_{1,2}/4, \sqrt{2}/4] \) and they both have mass points: Firm 1 at the upper end of the range \( [s_{1,2}/4, \sqrt{2}/4] \) and Firm 2 at a price of \( s_{1,2}/4\sqrt{2} \) which is higher than the maximum price in the range \( [s_{1,2}/4, 2s_{1,2}/4] \).

In this pricing equilibrium, there are two forces at work. On the one hand, the content-rich advertiser (Firm 2) can charge a high price to the R segment because these consumers obtain \( s \) more utility from Firm 2’s product than from Firm 1’s product. The implication of this for Firm 2 is that if it wishes to compete for the attribute insensitive segment by choosing a price less than Firm 1, it walks away from at least \( s \) of margin that can be earned on product sold to the R segment. As a result, Firm 2 (the content-rich advertiser) has limited willingness to undercut Firm 1 to capture business from the attribute insensitive segment.

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26 Throughout the analysis of the differentiated case, we assume that consumers’ posterior about Firm 1 is identical to the prior since identical firms will exhibit the same outcome if it is optimal for differentiated firms.
On the other hand, the uninformative advertiser, Firm 1, is simultaneously in a weak position to compete for the R segment and in a strong position to compete for the attribute insensitive segment. As a result, Firm 1 competes aggressively for the attribute insensitive segment and this limits the degree to which Firm 2 can set a high price for the R segment. This explains why Firm 2 has a mass point in the support for its mixed pricing strategy at the limit price where it captures the R segment 100% of the time (recall that Firm 2 has the R-attribute). Were Firm 2 to price higher than $r_1 + s(r_2/4s)$ and Firm 1 to randomize its price between $r_1/4s$ and $r_2$, at times Firm 1 would capture the R segment.

Notice that as $s \to 1$, the mass point of the content-rich advertiser decreases towards the upper boundary of the continuous price support $r_1 + s(r_2/4s)$. Not surprisingly, the probability masses for both firms approach zero as $s \to 1$, i.e., $1 - (1-s)^2 \to 0$ and $21/2s - 1 \to 0$. This happens because as $s \to 1$, the two firms become increasingly undifferentiated: most consumers are attribute insensitive and are willing to pay $r$ for the products of either firm.

As shown in Fig. 2, when $s$ is small, the content rich advertiser earns higher subgame profit than the uninformative advertiser. In the interval $s \in \left(\sqrt{5} - 2, \sqrt{2} - 1\right)$, this changes and the uninformative advertiser earns higher subgame profit than the content-rich advertiser. Once $s > \sqrt{2} - 1$, both firms earn identical profit in the pricing subgame.

The explanation for this is as follows. Firm 1 (the uninformative advertiser) always captures the L segment for prices less than $r_2$: Firm 2’s product is unacceptable to the L segment because, from Firm 2’s advertising, they know that it has the R attribute. To ensure that the L segment buys, Firm 1 anchors its prices in a range less than $r_2$.

When $s$ is small, differentiation is high. Here, both firms have significant loyal segments and the content-rich advertiser prices to protect demand from the R segment. This allows the uninformative advertiser (Firm 1) to focus on capturing the S and the L segments. In fact, Firm 1 keeps its price less than $r_2$ so that the L segment buys. This appears to be a weakness yet as $s$ increases it becomes a strength. Firm 1 has its price anchored in a range where it always competes for a fraction $1 + s/4s$ of the market (the L segment and the S segment). This explains why the uninformative advertiser earns more profit than the content-rich advertiser when the S segment is in an intermediate range.

When $s$ is sufficiently large ($s \geq 1/3$), the switcher segment is attractive enough such that both firms compete for it. As a result, their pricing behaviors are more homogenous. While both firms have an incentive to focus on their loyal segment, as $s \to 1$, that incentive shrinks to a mass point. The similarity in the pricing strategies of the firms leads to both firms earning equal profits in the subgame.

We now solve the game of advertising choice, as represented by the normal form shown in Table 6. Our focus is advertising outcome that are in pure strategies.27

Proposition 3 summarizes the equilibrium in the first stage.

27 Equilibria that involve asymmetric outcomes also mean that mixed advertising strategies exist because firms can mix over uninformative and content-rich advertising.
Proposition 3. Let $f(s)$ be a function of $s$, when $c_c - c_u < f(s)$, both firms engage in content rich advertising; when $c_c - c_u \geq f(s)$, firms engage in asymmetric advertising, where $f(s)$ is described below:

$$f(s) = \begin{cases} \frac{(1-3s)r}{4} & \text{if } s \leq 1/3, \\ \frac{(4s - 3s^2 - 1)r}{8s} & \text{if } 1/3 < s < 1. \end{cases}$$

Fig. 3 summarizes the equilibrium advertising strategies in the feasible parameter space. In the figure, the Y axis is the cost difference between content-rich and uninformative advertising $c_c - c_u$. The function $f(s)$ is what defines the boundary between (Content-rich, Content-rich) and (Content-rich, Uninformative) as an outcome.

Notice that when differentiation is high ($s < 1/3$), the first derivative of the threshold in Proposition 3 with respect to $s$ is negative: $\frac{\partial f(s)}{\partial s} = -\frac{3r}{8s}$. This means that as $s$ increases, the threshold decreases implying that the likelihood of an asymmetric outcome increases. As a result, the market is likely to move from a (Content-rich, Content-rich) outcome to a (Uninformative, Content-rich) outcome as $s$ increases. This is explained by the cumulative distribution of the pricing strategies employed by the firms in the asymmetric sub-game (Eq. i in the Appendix). Prices increase with $s$ in the asymmetric subgame meaning that decreases in differentiation lead to higher (and not lower) prices. This unusual dynamic is explained by recognizing that sometimes Firm 1, the uninformative advertiser, captures business from the R segment. This happens because Firm 2 is motivated to set high prices to capitalize on the high willingness to pay of the R segment for its product. However, as $s$ increases (the level of differentiation in the market decreases), Firm 1 has less incentive to discount to attract the R segment because it is smaller. This relaxes price competition and ultimately, has a positive effect on Firm 1’s profits. As noted earlier, when $\sqrt{5} - 2 < s < \sqrt{2} - 1$, the subgame profit of the uninformative advertiser is higher than that of the content-rich advertiser. As $s$ increases, this dynamic explains why the probability of a firm choosing uninformative advertising in response to a competitor that employs content-rich advertising increases.

When differentiation is low ($s$ is close to 1), the equilibrium between firms is more likely to be asymmetric as $s$ increases. However, the explanation for this is different compared to conditions of high differentiation. When differentiation is high, 

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Table 6

<table>
<thead>
<tr>
<th>Advertising game with S segment.</th>
<th>No Advertising</th>
<th>Uninformative</th>
<th>Content-rich</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Advertising</td>
<td>0, 0</td>
<td>0, $\frac{1}{4} - c_u$</td>
<td>0, $\frac{2(1-s) - r}{3} - c_c$</td>
</tr>
<tr>
<td>Uninformative</td>
<td>$\frac{1}{4} - c_u, 0$</td>
<td>$-c_u, -c_u$</td>
<td>$\pi_{uc} - c_c, \pi_{uc} - c_c$</td>
</tr>
<tr>
<td>Content-rich</td>
<td>$\frac{(1-3s)r}{4} - c_u, 0$</td>
<td>$\pi_{uc} - c_c, \pi_{uc} - c_c$</td>
<td>$\pi_{uc} - c_c, \pi_{uc} - c_c$</td>
</tr>
</tbody>
</table>

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28 Recall that $s$ is inversely related to differentiation in this model.
responding to content-rich advertising with uninformative advertising delivers benefits of differentiation and lower costs. In contrast, when differentiation is low ($s$ is close to 1), the profits of the uninformative advertiser and the content-rich advertiser in the pricing subgame are identical (see Fig. 2). Here, it solely the lower cost of uninformative advertising that motivates the response.

When $s > 1/3$, an important point emerges, related to the likelihood of observing uninformative advertising. As $s$ increases (differentiation in the market decreases), it seems that detailed product information (through content-rich advertising) should be less valuable. In other words, a symmetric (Content-rich, Content-rich) equilibrium should be less likely as $s$ increases. This is true when differentiation is high or very low ($s$ is small or close to 1). However, when $s$ is in an intermediate range, this is not the case. Proposition 4 relates to the likelihood of seeing uninformative advertising i.e. it summarizes the shape of the boundary where at least one firm engages in uninformative advertising.

**Proposition 4.** When $1/3 < s < 1/\sqrt{3}$, firms are more likely to both employ content-rich advertising the level of differentiation decreases ($s$ increases).

Proposition 4 shows that when $1/3 < s < 1/\sqrt{3}$, reductions in the level of differentiation (higher $s$) lead to an increase in the amount of content-rich advertising. Consider the boundary in Fig. 3 between (Uninformative, Content-rich) and (Content-rich, Content-rich) for $s > 1/2$. The positive slope of the boundary in $s \in \left(1/3, 1/\sqrt{3}\right)$, implies that the equilibrium changes from (Uninformative, Content-rich) to (Content-rich, Content-rich) as $s$ increases. Therefore, in this range, less differentiation (an increase in $s$) leads to more content-rich advertising.

To understand why decreases in differentiation lead to more content-rich advertising in this range, consider the effect that decreases in differentiation have when differentiation is high ($s \leq 1/2$). In this range, expansion of the S segment lowers the incentive of the uninformative advertiser to cut price to capture the competitor’s loyal segment (large S means a smaller loyal segment). This leads to higher profits for an uninformative advertiser that faces a content-rich advertiser. As a result, the (Uninformative, Content-rich) outcome is more likely when $s$ increases.

In contrast, when firms are less differentiated ($s > 1/3$), there is a significant difference in how a reduction in differentiation affect firm profits. In this range, the comparative statics on subgame profits for each firm highlight the difference. An increase in $s$ reduces profits for both firms in the symmetric (Content-rich, Content-rich) outcome ($\frac{\partial \pi_{\text{cc}}}{\partial s} < 0$) and the asymmetric (Uninformative, Content-rich) outcome ($\frac{\partial \pi_{\text{uc}}}{\partial s} < 0$ and $\frac{\partial \pi_{\text{cc}}}{\partial s} < 0$). In contrast, when $s \leq 1/2$, $\frac{\partial \pi_{\text{uc}}}{\partial s} > 0$. The difference between the ranges of differentiation (high and low) come from distinct pricing equilibria that occur in each range. Unusually, average prices increase with reductions in differentiation (increases in $s$) when differentiation is high. The opposite and more typical relationship between prices and differentiation is observed when differentiation between firms is weaker.

This observation alone does not explain Proposition 4. To understand why there is an increase in content-rich advertising, we make a detailed comparison of the profit associated with an uninformative response ($\pi_{\text{uc}}$) and a content-rich response ($\pi_{\text{cc}}$) to a content-rich advertiser. This is the comparison a firm makes to determine its best response to a competitor that employs content-rich advertising. To compare the shape of $\pi_{\text{uc}}$ and $\pi_{\text{cc}}$ as function of $s$ (holding $r$ constant), we examine

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29 In Fig. 3, the scale on the y-axis is units of $r$. 

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![Fig. 3. Advertising equilibrium when firms are differentiated.](image-url)
the second order derivatives of $\pi_{uc}$ and $\pi_{cc}$ and find that $\frac{\partial^2 \pi_{uc}}{\partial s^2} > 0$ and $\frac{\partial^2 \pi_{cc}}{\partial s^2} = 0$. This means that $s$ has a negative and convex impact on the profit associated with an uninformative response to a content-rich advertiser. In contrast, the relationship of $s$ to the profits associated with a content-rich response to a content-rich advertiser is negative and linear. Because of the convexity of $\pi_{uc}$ in $[1/3, 1/\sqrt{3}]$, $\pi_{uc}$ declines faster than $\pi_{cc}$ as a function of $s$. In $[1/\sqrt{3}, 1]$, the reverse is true.

Why does $\pi_{uc}$ decrease faster than $\pi_{cc}$ as a function of $s$ in the interval $[1/3, 1/\sqrt{3}]$? The explanation is that the $S$ segment is large enough to attract the attention of the content-rich advertiser (who prices aggressively to capture it from time to time). This makes price competition intense when the firms use different advertising strategies. Understanding this, a firm relaxes the intensity of price competition for the $S$ segment by employing content-rich advertising. It is also important that the attribute sensitive segments (L and R) remain significant in $[1/3, 1/\sqrt{3}]$. Consequently, as $s$ increases, a firm remains willing to pay the incremental cost of content-rich advertising to achieve a less damaging outcome in the pricing sub-game.

This dynamic applies as long as the attribute sensitive segments remain significant. When $s > 1/\sqrt{3}$, the market approaches a condition of no differentiation and the firms become more homogenous. Here, the attribute sensitive segments are small so the advantage of content-rich advertising (allowing a high price to be charged to the attribute sensitive segment from time to time) diminishes. This reverses the trend observed when $s < 1/\sqrt{3}$ and makes uninformative advertising a more attractive response to content-rich advertising. In particular, the rapid decline in $\pi_{uc}$ as a function of $s$ when $s < 1/\sqrt{3}$ is muted and $\pi_{cc}$ declines more rapidly in the upper part of the interval.

4.2.3. Summary for differentiated firms

When consumer needs are highly differentiated, the most likely outcome is that differentiated firms employ content-rich advertising. As the level of differentiation decreases ($s$ increases), the likelihood of uninformative advertising being a best response to content-rich advertising increases. The intuition is that reduced differentiation in consumer taste increases the strength of a generic appeal (uninformative advertising) and decreases the strength of a differentiated appeal (content-rich advertising). In fact, when $s > 0.23607$, the subgame profit of the uninformative advertiser is higher than that of the content-rich advertiser. However, the analysis of Proposition 4 shows that the trend of uninformative advertising becoming more likely as a response to content-rich advertising reverses when $s > 1/3$. The reason is that the equilibrium changes: when $s > 1/3$, the $S$ segment is big enough to attract the attention of the content-rich advertiser who prices more aggressively. This intensifies competition and reduces the attractiveness of the uninformative response. However, as $s$ continues to increase, the level of differentiation between the firms becomes less important and the relative attractiveness of uninformative advertising as a response to content-rich advertising returns. This means that the likelihood of a firm responding to a competitor’s content-rich advertising with uninformative advertising is non-monotonically related to the level of differentiation in the market. The non-monotonicity of the advertising equilibrium is the combined result of providing information to the market and alleviating price competition through advertising.

4.3. The equilibrium when firms are undifferentiated

As noted earlier, it is equally likely that the firms develop identical products and differentiated products. Here, we analyze the equilibrium when the firms have identical products (both possess the L attribute or both possess the R attribute). Before presenting the analysis, it is useful to highlight two points.

First, symmetric strategies where firms with identical products both advertise result in negative profits. Symmetric strategies cause consumers to perceive the products as identical. This leads to Bertrand competition in the sub-game and losses related to the expenditure on advertising (either $c_s$ or $c_r$). Indeed, the key difference with Section 4.2 is that the (Content-rich, Content-rich) outcome leads to negative profits because the products are identical.

Second, there are only two ways for positive profits to be earned when firms are identical. The first is that only one firm advertises. The second is if consumers perceive two advertised products to be different. Following Section 4.1, the only outcome that allows two advertised product to be perceived as different entails one firm using content-rich advertising and the other using uninformative advertising. This outcome is only possible in conditions where $\hat{\mu}_i = 1/2$ where $i$ is the firm that uses uninformative advertising (the posterior belief for Firm $i$ equals the prior belief). This is the case whenever $U, C$ is the chosen strategy of differentiated firms. In parameter conditions $(c_s, c_r, s, r)$ where differentiated firms exhibit $C, C$, the outcome for identical firms entails only one active firm. With $f(s)$ as defined in Proposition 3, this reasoning leads to Proposition 5.

**Proposition 5. When the firms are identical and**

1. $s < 1/3$, the advertising outcome is

---

30 The parameters $c_s$, $c_r$, $s$ and $r$ are common knowledge for all players in the game.
one firm employs content-rich advertising when \( c_c - c_u < \frac{f(s)}{2} \)

one firm employs uninformative advertising when \( c_c - c_u \in (\frac{f(s)}{2}, f(s)) \)

the firms exhibit asymmetric advertising when \( c_c - c_u \geq f(s) \).

2. \( s > \frac{1}{2} \), the advertising outcome is

one firm employs content-rich advertising when \( c_c - c_u \in (0, f(s)) \)

the firms exhibit asymmetric advertising when \( c_c - c_u \geq f(s) \).

Proposition 5 shows that firms use advertising content to create “artificial differentiation” when the difference in cost between content-rich and uninformative advertising exceeds a threshold. For “artificial differentiation” to occur, the symmetric content-rich advertising is not the equilibrium for differentiated firms. Another insight from Proposition 5 is that when the difference in cost between content-rich and uninformative advertising is small, only one firm operates when the firms have identical products. It is useful to note that in the zones where only one firm operates, there exist mixed advertising strategies where firms can mix over (a) content-rich advertising and not advertising or (b) uninformative advertising and not advertising. Thus, while we do not observe firms competing against each with symmetric pure advertising strategics, they indeed do in symmetric mixed advertising strategies.

The equilibrium outcomes are illustrated in Fig. 4.

The observation that uninformative advertising can be used to create perceived differentiation between identical products is important; it echoes the accusation of advertising critics who claim that advertising leads to spurious differentiation (Chamberlin, 1933; Comanor & Wilson, 1974; Galbraith, 1967). However, the mechanisms suggested by these critics entail advertising that misrepresents the benefits of a brand or that creates “valueless” positive emotional associations for brands. In contrast, this analysis shows that perceived (or spurious) differentiation comes from using advertising that is truly uninformative. The advertising provides no attribute information to consumers and it allows a firm to capitalize on how consumers form expectations about attributes.

5. Conclusion

The objective of this study is to understand how competitive firms choose between content-rich advertising and uninformative advertising. Our focus is low involvement goods where consumers need information (from advertising) or experience, to know a product’s attribute. The model includes the assumption that advertising which transmits information about product attributes (content-rich) costs more than uninformative advertising. As expected, this assumption leads to an asymmetric outcome when the cost of content-rich advertising is sufficiently high. However, the model goes further than this. It provides an explanation for a preponderance of uninformative advertising that is not cost-based.

The key model parameter we vary is the degree to which consumer preferences are differentiated. We start from a situation of maximum differentiation in the market: in a 2 segment market, one segment wants one thing and the other segment wants another. At the other extreme, we consider a situation where there is no heterogeneity in consumer needs: consumers are happy with the product of either firm. We examine cases between the extremes by using a third segment of attribute insensitive consumers. We progressively increase the fraction of attribute insensitive consumers until the entire market is comprised of attribute insensitive consumers.

The analysis is based on the firms developing products independently and as a result, the products are differentiated or identical with 50/50 probability. First, we discuss the findings when the firms have differentiated products.

Not surprisingly, in conditions where consumer tastes are maximally differentiated, content-rich advertising is optimal. It facilitates perfect matching of consumers to products. Here, despite the effectiveness of content-rich advertising, a firm may choose uninformative advertising as a response to content-rich advertising if the cost difference exceeds a threshold.

For intermediate levels of differentiation, one of the firms also chooses uninformative advertising when content-rich advertising is sufficiently expensive but two distinct pricing regimes arise: one when the attribute insensitive fraction of the market is small (the level of differentiation in consumer preferences is high) and another, when the attribute insensitive fraction of the market is large (the level of differentiation in consumer preferences is low).

When the fraction of attribute insensitive consumers is small, the firms choose different advertising strategies when the incremental cost of content-rich advertising exceeds a threshold. Here, the content-rich advertiser sets high prices to capture surplus from the segment of consumers whose preferences are aligned with its product. These prices allow the uninformative advertiser to attract this segment with low prices. Unusually, as differentiation decreases, average prices increase. As a result, prices and the uninformative advertiser’s profits increase as differentiation decreases. This dynamic means that when differentiation is high, the likelihood of an asymmetric outcome increases as differentiation decreases.

When the fraction of attribute insensitive consumers is large, the model predicts that outcome is likely to be one firm using uninformative advertising and the other using content-rich advertising. It seems reasonable to assume that the chocolate bar market and the beer market have a relatively large S segment i.e., consumers who obtain high value from more than one of the choices in the market. Accordingly, as a form of validation for our findings, we present key messages of recent outdoor and transit advertising for chocolate bars and shelf talker advertising for craft beer in Toronto in Table 7. The first category is well established and the second is relatively new (and the brands are unknown). The model applies to both sit-
uations because even in well-established low involvement categories, advertising creates awareness of brands and their attributes.31

While it is beyond the scope of our analysis to assess the actual size of the S segment in the chocolate bar or beer markets, several observations from Table 7 seem to validate the findings generated by our analysis. First, competitors within these categories seem to compete with messages that contain varying levels of information. Some executions only contain the brand name to activate awareness. Second, some brands alternate between different executions that have different amounts of information. As noted earlier, the asymmetric outcome in our model implies a mixed strategy outcome where firms alternate between uninformative and content-rich messages. Finally, when a brand wants to transmit attribute level information, the message is typically longer and has more words.

When firms have identical products, the only feasible outcomes are that one firm operates by itself or that the firms choose asymmetric advertising strategies. In fact, when the firms employ different advertising strategies, advertising creates “artificial differentiation” between identical products. However, identical firms are constrained by how consumers update their beliefs in a PBE. As a result, identical firms both advertise whenever differentiated firms would themselves exhibit a \((U, C)\) outcome. Indeed, the likelihood of observing a \((U, C)\) outcome is unaffected by whether firms are differentiated or not. The primary difference between outcomes when firms are differentiated versus identical is that markets with identical firms are more likely to be dominated by a single firm.

Finally, the likelihood of observing uninformative advertising as a response to content-rich advertising is non-monotonically related to the degree by which consumer preferences are differentiated (independent of whether the firms have different products). As discussed, this arises because, when consumer tastes are highly differentiated, the natural outcome is that differentiated firms employ content-rich advertising to maximize efficient matching. However, as the fraction of

---

31 Awareness is essential for a brand to be included in the consumer’s evoked set (Howard & Sheth, 1969).
the market that is attribute insensitive increases, the likelihood of an asymmetric outcome increases, then decreases and then increases as we approach a fully undifferentiated market.

In sum, the impact of changes in the level of customer heterogeneity on advertising strategies depends strongly on the initial level of customer heterogeneity. The model shows that decreases in differentiation can lead to increases or decreases in the occurrence of asymmetric advertising strategies. The model also provides an explanation for why firms opt for uninformative advertising in categories where consumers have heterogeneous preferences. At times, firms sacrifice the strength of preference delivered by content-rich advertising for the breadth of preference provided by uninformative advertising.

Appendix A

Proof of Lemma 1

We start by considering the case where the firms are identical and one firm employs content-rich advertising and the other employs uninformative advertising. Here, consumers learn the attribute of the firm that employs content rich advertising through the advertising. In Table A.1, the key question is “can consumers deduce the attribute of uninformative advertiser?”.

It is clear that identical firms will exhibit U and C anytime differentiated firms exhibit the same strategies (the first row of Table A.1). The explanation is that the only profitable strategy for identical firms where both advertise is U and C. However, a requirement for this outcome is that consumers do not update their beliefs.

We now move to the case where the firms are differentiated and one firm employs content-rich advertising and the other employs uninformative advertising. As before, the key question in Table A.2 is “can consumers deduce the attribute of the uninformative advertiser?”.

From Row 1 of Table A.2, we see that in conditions where identical firms choose U,C differentiated firms will choose U,C if a firm increases profit from the profits earned in the C,C outcome by choosing U. Conversely, in Row 2 of Table A.2, whenever a differentiated firm perfectly identifies itself as having the opposite attribute to its competitor by choosing U in response to C, U is dominant because identical Stage 2 profits are earned but with less expensive advertising.

Row 2 provides the starting point to solve the game. Previewing what we explain later, we do not observe the conditions of Row 2 in equilibrium. Identical firms never exhibit either a C,C or U,U outcome because both outcomes lead to negative profits (consumers perceive the products as identical).

In summary, Table A.1 implies that parameter conditions which lead differentiated firms to exhibit U,C as an outcome, also lead identical firms to exhibit U,C as an outcome. In fact, the only time identical firms both advertise is when U,C is the outcome. This obtains because the posterior beliefs of consumers are the same as the priors. Table A.2 is a guide for when differentiated firms might exhibit U,C as an outcome. It occurs if a firm increases profit from the profits earned in the C,C outcome by choosing U.

Pricing Equilibrium in the Asymmetric Advertising Sub-game

Under asymmetric advertising, firms’ pricing behaviours depend on the size of the switcher segment s. In particular, there are three cases: s ≤ 1/3, 1/3 < s < √2 − 1, and s > √2 − 1, each entailing different pricing equilibrium. We provided detailed proof for the case of s ≤ 1/3 and s > √2 − 1. For the case of 1/3 < s < √2 − 1, because the firms’ response functions are not upper-hemicontinuous, we characterize the equilibrium and provide proof that no firm will deviate from the equilibrium. The following Lemmas summarise the price equilibria.

Lemma A.1. s ≤ 1/3, Firm 1 randomises its price on [r(1+s)/4, r/2], with a mass point at r/2, and Firm 2 randomises its prices on [r(1+s)/4, r], the density functions for the mixed pricing equilibrium are described by:

\[
\begin{align*}
F_1(p) &= \begin{cases} 
\frac{4p - r - r_1}{4p - r}, & 0 \leq p < r/2, \\
\frac{3s}{4}, & p = r/2, \\
\frac{4p - 3r - r_1}{4p - r}, & r/2 \leq p \leq r.
\end{cases} \\
F_2(p) &= \begin{cases} 
\frac{4p - 3r - r_1}{4p - r}, & 0 \leq p < r/2, \\
\frac{3s}{4}, & p = r/2, \\
\frac{4p - r - r_1}{4p - r}, & r/2 \leq p \leq r.
\end{cases}
\end{align*}
\]

Proof. To identify the pricing equilibrium, we start by examining Firm 1’s price response. Depending on p_2, Firm 1 can choose to: compete for the attribute insensitive segment (p_1 = p_2 - ε), compete for R and S segment (p_1 = p_2 - r/2 - ε), or restrict its attention to the L segment (p_1 = r/2),\(^33\) The corresponding profits are:

\(^{32}\) In the first row of Table 1, an outcome where only one firm advertises is not stable because the non-advertising firm has a profitable deviation.

\(^{33}\) Throughout these proofs, we assume that μ_i = 1/2 in the asymmetric advertising outcome (i is the firm that employs uninformative advertising) This follows from the reasoning of Section 4.1.
The Updating of Consumer Beliefs when Firms are Differentiated.

When firms are perceived identical, Bertrand competition reduces profit to zero in Stage 2 of the game.

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Table A.1
The Updating of Consumer Beliefs when Firms are Identical.

<table>
<thead>
<tr>
<th>Beliefs about the firm that employs U</th>
<th>Updated beliefs</th>
<th>Profits of identical firms</th>
<th>Best response of identical firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumers observe U and C</td>
<td>If identical &amp; differentiated firms choose the same strategy, 50/50 Left/Right (−c_u, −c_v)</td>
<td>U, C profits where U firm is believed to be 50% Left and 50% Right</td>
<td>U, C firms will employ U, C if profits positive</td>
</tr>
<tr>
<td>If differentiated firms choose C,C or U,U, identical to C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>When firms are perceived identical, Bertrand competition reduces profit to zero in Stage 2 of the game.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A.2
The Updating of Consumer Beliefs when Firms are Differentiated.

<table>
<thead>
<tr>
<th>Beliefs about the firm that employs U</th>
<th>Updated beliefs</th>
<th>Profits of differentiated firms</th>
<th>Best response of differentiated firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumers observe U and C</td>
<td>If identical &amp; differentiated firms choose the same strategy, 50/50 Left/Right (−c_u, −c_v)</td>
<td>U, C profits where U firm is believed to be 50% Left and 50% Right</td>
<td>U, C if U firm increases profit from C,C</td>
</tr>
<tr>
<td>If identical firms choose C,C or U,U, the opposite of C</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We examine these three profit possibilities to determine Firm 1’s best price response. When \( p_2 > r/2 \), \( \pi_1 > \pi_u \iff p_2 > \frac{r(1-s)}{2(1-s)} \). Therefore, when \( p_2 > \frac{r(1-s)}{2(1-s)} \), we just compare \( \pi_1 \) and \( \pi_1^u \) (we have: \( \pi_1 > \pi_1^u \iff p_2 > \frac{r(1-s)}{2(1-s)} \)). When \( \frac{r(1-s)}{2(1-s)} > p_2 > r/2 \) so we compare \( \pi_1 \) and \( \pi_1^u \). In this case, \( \pi_1 > \pi_1^u \iff p_2 > \frac{r(1-s)}{2(1-s)} \).

When \( p_2 < r/2 \), the calculation is straightforward, \( \pi_1 > \pi_1^u \iff p_2 > \frac{r(1-s)}{2(1-s)} \). It is straightforward to show that:

\[
s > \sqrt{2} - 1 \iff \frac{1 + s}{4s} < \frac{3 + s}{4} < \frac{1}{2(1-s)} \tag{iii}
\]

\[
s < \sqrt{2} - 1 \iff \frac{1 + s}{4s} > \frac{3 + s}{4} > \frac{1}{2(1-s)} \tag{iv}
\]

We now have Firm 1’s price response:

\[
\begin{align*}
\text{when } s &\leq \sqrt{2} - 1
\begin{cases}
 p_1 = p_2 - r/2 - \epsilon & \text{if } p_2 > \frac{r(1-s)}{2(1-s)}, \\
p_1 = p_2 - r/2 & \text{if } \frac{r(1-s)}{2(1-s)} \geq p_2 > \frac{r(1-s)}{4}, \\
p_1 = p_2 - \epsilon & \text{if } r/2 \geq p_2 > \frac{r(1-s)}{2(1-s)}, \\
p_1 = r/2 & \text{if } p_2 \leq \frac{r(1-s)}{2(1-s)}.
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\text{when } 1/2 > s &> \sqrt{2} - 1
\begin{cases}
 p_1 = p_2 - r/2 - \epsilon & \text{if } p_2 > \frac{r(1-s)}{2(1-s)}, \\
p_1 = p_2 - \epsilon & \text{if } \frac{r(1-s)}{2(1-s)} \geq p_2 > \frac{r(1-s)}{4}, \\
p_1 = r/2 & \text{if } r/2 \geq p_2 > \frac{r(1-s)}{2(1-s)}, \\
p_1 = r/2 & \text{if } p_2 \leq \frac{r(1-s)}{2(1-s)}.
\end{cases}
\end{align*}
\]

Now we look at Firm 2. Similar calculations imply that Firm 2’s best response is:

\[
\begin{align*}
p_2 &= p_1 - \epsilon & \text{if } p_1 > \frac{r(1-s)}{4s}, \\
p_2 &= p_1 + r/2 - \epsilon & \text{if } p_1 \leq \frac{r(1-s)}{4s}.
\end{align*}
\]
From Eq. (v), we know that when \( s \leq \sqrt{2} - 1 \), Firm 1 does not set price greater than \( r/2 \). Notice that when \( s \leq 1/3, \frac{(1-s)}{4s} \geq r/2 \). Because \( 1/3 < \sqrt{2} - 1 \), we know that if \( s \leq 1/3, p_1 \leq \frac{(1-s)}{4s} \). From Eq. (vi), we conclude that when \( s \leq 1/3 \), Firm 2’s best response is \( p_2 = p_1 + r/2 - \epsilon \).

From Eq. (ii), we know that Firm 1 will not lower its price below \( \frac{(1-s)}{4s} \). As a result, Firm 2’s lowest price is \( p_2 = \frac{(1-s)}{4s} \). This leads straightforwardly to the support for the mixed pricing strategy when \( s \leq 1/3 \):

\[
\begin{align*}
\{ p_1 \in \left[ \frac{(1-s)}{4s}, r/2 \right], \\
p_2 \in \left[ \frac{r(1-s)}{4s}, r \right].
\end{align*}
\]  

(vii)

The response functions are not upper hemi-continuous (a condition that ensures the existence of an equilibrium). However, when \( s \leq 1/3 \), the game has a unique equilibrium. Firm 1 has a mass point at \( r/2 \) and captures both the L and the attribute insensitive segment. Firm 2’s equilibrium profit is its lowest price times the size of R segment. The equilibrium profits net of advertising expenditures are:

\[
\begin{align*}
\pi_{1e}^* &= \frac{1}{2} \frac{1-s}{2}, \\
\pi_{2e}^* &= \frac{r(1-s)}{4s}.
\end{align*}
\]  

(vi i)

The cumulative density function for the price distribution is obtained by solving the equal profit equation:

\[
\begin{align*}
F_1(p)(p + \frac{1}{2}) - 0 + (1 - F_1(p))(p + \frac{1}{2}) \frac{1-s}{2} &= \frac{r(1-s)}{4s}, \\
F_2(p)(p - \frac{1}{2}) + (1 - F_2(p))(p - \frac{1}{2}) &= \frac{r(1-s)}{4s}.
\end{align*}
\]  

(ix)

Lemma A.2. When \( s > \sqrt{2} - 1 \), Firm 1 randomises its price on \( \left[ \frac{(1-s)}{4s}, r/2 \right] \), with a mass point at \( r/2 \), and Firm 2 randomises its prices on \( \left[ \frac{(1-s)}{4s}, r/2 \right] \), with a mass point at \( \frac{(1-s)}{4s} \) and the density functions for the mixed pricing equilibrium are described by:

\[
\begin{align*}
F_1(p) &= \left\{ \begin{array}{ll}
\frac{(1-s)(4p - r(1-s))}{8ps^2}, & p \in \left( \frac{(1-s)}{4s}, \frac{r}{2} \right), \\
\frac{(1-s)2}{4s^2}, & p = \frac{r}{2},
\end{array} \right.

F_2(p) &= \left\{ \begin{array}{ll}
\frac{(1-s)(4p - r(1-s))}{8ps^2}, & p \in \left( \frac{r(1-s)}{4s}, \frac{r}{2} \right), \\
\frac{(1-s)2}{4s^2}, & p = \frac{r(1-s)}{4s}.
\end{array} \right.
\]  

(x)

Proof. We start with Firm 1’s (the uninformative advertiser) price response. Since it will not compete for the R segment, it has two options in pricing: \( p_1 = p_2 - \epsilon \) or \( p_1 = \frac{r}{2} \). The first option leads to two possible profits:

\[
\pi_1 = \left\{ \begin{array}{ll}
sp_2, & p_2 > \frac{r}{2}, \\
(s + \frac{1-s}{2})p_2, & p_2 \leq \frac{r}{2}.
\end{array} \right.
\]  

(xi)

The second pricing option also leads to two possible profits:

\[
\pi_{1i} = \left\{ \begin{array}{ll}
(s + \frac{1-s}{2}) \frac{r}{2}, & p_2 > r/2, \\
\frac{1-s}{2} \frac{r}{2}, & p_2 \leq r/2.
\end{array} \right.
\]  

(xii)

Firm 1 compares the choices:

\[
\pi_1 - \pi_{1i} = \left\{ \begin{array}{ll}
sp_2 - \frac{r(1-s)}{4s}, & p_2 > r/2, \\
\frac{p_2(1-s)}{2} - \frac{r(1-s)}{4s}, & p_2 \leq r/2.
\end{array} \right.
\]  

(xi i)

We solve the above equation to obtain: \(^{34}\)

\[
\begin{align*}
\pi_1 > \pi_{1i} & \text{ if } p_2 > \frac{r(1-s)}{4s}, \\
\pi_1 < \pi_{1i} & \text{ if } \frac{r}{2} \leq p_2 \leq \frac{r(1-s)}{4s}, \\
\pi_1 > \pi_{1i} & \text{ if } \frac{r}{2} \geq p_2 > \frac{r(1-s)}{2(1-s)}, \\
\pi_1 < \pi_{1i} & \text{ if } p_2 \leq \frac{r(1-s)}{2(1-s)}.
\end{align*}
\]  

(xiv)

\(^{34}\) Note that \( \frac{(1-s)}{4s} > \frac{r}{2} \) for all \( s \in (0, 1) \).
We know that Firm 2 has a guaranteed profit of \( \frac{r(1-s)}{4s} \) if it charges a price of \( r/2 \). Therefore it will not lower its price below \( \frac{r(1+s)}{4s} \). Therefore, the last case in Eq. (xiv) is not feasible. This allows us to write Firm 1’s best response function:

\[
\begin{align*}
p_1 &= p_2 - \epsilon & \text{if } p_2 > \frac{r(1+s)}{4s}, \\
p_1 &= r/2 & \text{if } \frac{r}{2} < p_2 \leq \frac{r(1+s)}{4s}, \\
p_1 &= p_2 - \epsilon & \text{if } p_2 \leq \frac{r}{2}.
\end{align*}
\] (xv)

Understanding that Firm 1 will not undercut when \( \frac{r}{2} < p_2 \leq \frac{r(1+s)}{4s} \), Firm 2 will not charge a price between \( \left[ \frac{r}{2}, \frac{r(1+s)}{4s} \right] \). In other words, there is a hole in its price support between \( \left[ \frac{r}{2}, \frac{r(1+s)}{4s} \right] \).

Next we examine Firm 2’s price response. Given any \( p_1 \), Firm 2 has two options: undercut Firm 1 (\( p_2 = p_1 - \epsilon \)) or charge a price of \( p_2 = p_1 + r/2 - \epsilon \). The latter case means that Firm 2 does not compete with Firm 1 for the attribute insensitive segment and focuses on the R segment. Comparing the profit of the two price options, we have:

\[
\pi_2(p_2 = p_1 - \epsilon) - \pi_2(p_2 = p_1 + r/2 - \epsilon) = p_1 \frac{1 + s}{2} - \left( p_1 + \frac{r}{2} \right) \frac{1 - s}{2} = p_1 s - \frac{r(1-s)}{4}.
\] (xvi)

Therefore, when \( p_1 \leq \frac{r(1-s)}{4s} \), Firm 2 will not undercut Firm 1. Instead, it will raise its price to \( p_1 + \frac{r}{2} \) and focus on the R segment. In other words, Firm 2 will not lower its price below \( \frac{r(1-s)}{4s} \). Notice that when \( s > 1/3 \iff \frac{r(1-s)}{4s} < \frac{r}{2} \) this leads to Firm 2’s options in terms of price response:

\[
\begin{align*}
p_2 &= p_1 - \epsilon & \text{if } p_1 > \frac{r(1-s)}{4s}, \\
p_2 &= p_1 + r/2 - \epsilon & \text{if } p_1 \leq \frac{r(1-s)}{4s}.
\end{align*}
\] (xvii)

From the above equation, we know that when Firm 1 prices at \( p_1 = \frac{r(1-s)}{4s} \), Firm 2 will increase its price by \( \frac{r}{2} \). In other words, Firm 2 never prices below \( \frac{r(1-s)}{4s} \). Understanding this, Firm 1 will not price less than \( \frac{r(1-s)}{4s} \). Therefore the price support for the mixed pricing strategy lies “within” \( \left[ \frac{r(1-s)}{4s}, \frac{r}{2} \right] \cup \left[ \frac{r(1-s)}{4s}, r \right] \). However, not all prices in this parameter region are included in the price support for the mixed pricing equilibrium. In fact, consider Firm 1 choosing a price of \( r \). In that situation, Firm 2’s best response is to undercut Firm 1. The price undercutting continues until the price reaches \( \frac{r(1-s)}{4s} \). At that point, Firm 1 dramatically drops its price to \( r/2 \), and Firm 2 again undercut at \( r/2 \). The price undercutting resumes until the price reaches \( \frac{r(1-s)}{4s} \). At this point, Firm 2 stops undercutting and dramatically raises its price by \( r/2 \) to \( \frac{r(1+s)}{4s} \) and then, the price undercutting resumes. The above strategic tournament implies that no firm chooses a price greater than \( \frac{r(1-s)}{4s} \) in the equilibrium. It can be easily checked that if Firm 1 randomizes its price over \( \left( \frac{r(1-s)}{4s}, r/2 \right] \) for Firm 2, any positive probability allocated to a price greater than \( \frac{r(1+s)}{4s} \) is dominated by the mass point at \( \frac{r(1-s)}{4s} \). Similarly, if Firm 2 randomizes on \( \left[ \frac{r(1-s)}{4s}, \frac{r}{2} \right] \) with a mass point at \( \frac{r(1+s)}{4s} \), Firm 1’s incentive to choose a price greater than \( \frac{r}{2} \) is dominated by a mass point at \( \frac{r}{2} \). In equilibrium, each firm has equal profit at any price in the support. Denote by \( \alpha \) the probability mass for Firm 1 at \( r/2 \), and by \( \beta \) the probability mass at \( \frac{r(1+s)}{4s} \) for Firm 2. Calculating the profit for Firms 1 and 2 at prices of \( \frac{r(1-s)}{4s} \) and at \( r/2 \) respectively, we can write the equal profit conditions for both firms as functions of \( \alpha \) and \( \beta \).

\[
\begin{align*}
\text{Firm 1:} & \quad \frac{r(1-s)}{4s} \frac{1 + s}{2} = \frac{r}{2} \left( 1 - \beta \right) \frac{1 - s}{2} + \beta \frac{1 + s}{2}, \\
\text{Firm 2:} & \quad \frac{r(1-s)}{4s} \frac{1 + s}{2} = \frac{r}{2} \left( 1 - \alpha \right) \frac{1 - s}{2} + \alpha \frac{1 + s}{2}.
\end{align*}
\] (xvi ii)

(xix)

These conditions allows us to determine the equilibrium mass point probabilities:

\[
\begin{align*}
\alpha &= \frac{(1-s)^2}{4s^2}, \\
\beta &= \frac{(1-s)^2}{4s^2}.
\end{align*}
\] (xx)

The equilibrium profits are determined by using the probability masses.

\[
\begin{align*}
\pi_1^* &= \frac{1 + (1-s)r}{4s}, \\
\pi_2^* &= \frac{1 + (1-s)r}{4s}.
\end{align*}
\] (xxi)

The cumulative density functions are obtained by solving the equal profit condition.
Lemma A.3. 1/3 < s ≤ \sqrt{2} − 1, Firm 1 randomises its price on \( \left( \frac{r(1+s)}{4} , \frac{r}{2} \right) \), with a mass point at \( \frac{r(1-s)}{4s} \) and another mass point at \( r/2 \), and Firm 2 randomises its prices on \( \left( \frac{r(1-s)}{4s} , \frac{r(1+s)}{4} \right) \cup \left( \frac{r(1+s)}{4} , \frac{r(1+s)}{2} \right) \), with a mass point at \( \frac{r(3+s)}{4} \), and the density functions for the mixed pricing equilibrium are described by:

\[
F_1(p) = \begin{cases} 
\frac{4p-r-r-s}{4p+2r}, & p \in \left( \frac{r(1+s)}{4}, \frac{r(1-s)}{4s} \right) \\
\frac{(s+1)^2}{2s(1-s)}, & p = \frac{r(1-s)}{4s} \\
\frac{4p(1+s-r)-r(1-s)(3+s)}{8ps}, & p \in \left( \frac{r(1+s)}{4}, \frac{r}{2} \right) \\
\frac{1-s^2}{4s^2}, & p = \frac{r}{2} \\
\frac{(1+s)(4ps-r+r)}{8ps^2}, & p \in \left( \frac{r(1+s)}{4}, \frac{r(1+s)}{2} \right) \\
\frac{8ps-r(1+s)-s^2}{2s(1-s)(2p-r)}, & p \in \left( \frac{r(1+s)}{4}, \frac{r(1+s)}{2} \right)
\end{cases}
\]

\[
F_2(p) = \begin{cases} 
\frac{4p-r-r-s}{4p+2r}, & p \in \left( \frac{r(1+s)}{4}, \frac{r(1-s)}{4s} \right) \\
\frac{(s+1)^2}{2s(1-s)}, & p = \frac{r(1-s)}{4s} \\
\frac{4p(1+s-r)-r(1-s)(3+s)}{8ps}, & p \in \left( \frac{r(1+s)}{4}, \frac{r}{2} \right) \\
\frac{1-s^2}{4s^2}, & p = \frac{r}{2} \\
\frac{(1+s)(4ps-r+r)}{8ps^2}, & p \in \left( \frac{r(1+s)}{4}, \frac{r(1+s)}{2} \right) \\
\frac{8ps-r(1+s)-s^2}{2s(1-s)(2p-r)}, & p \in \left( \frac{r(1+s)}{4}, \frac{r(1+s)}{2} \right)
\end{cases}
\]

Proof. The search for the pricing equilibrium for this case largely follows that of Lemma A.2. Let 
\( \phi = \text{Prob}(p_1 = \frac{1-sr}{s}) \) and \( \psi = \text{Prob}(p_1 = r/2) \), \( \omega = F_1(p_1 < \frac{1-sr}{4s}) \). We have:

\[
\pi_2(p_2) = \left( 1 + \frac{s}{4s} \right) = (1 - \omega)(1-s)p_2/2, \quad \pi_2(p_2 = \frac{3+s}{4}) = \frac{p_2(1-s)}{2}, \quad \pi_2(p_2 = r/2) = \frac{1-s}{2}.
\]

Applying equal profit condition to the above profit equations, we can have the probability masses for the two mass points of Firm 1. This gives us the equilibrium profit: \( \pi_1 = \frac{(1-s)(3+s)}{8s} \). Follow the similar process, we have the equilibrium profit of Firm 2: \( \pi_2 = \frac{(1-s)(3+s)}{8s} \). The cumulative density functions are obtained through the equal profit condition. One can check the the density functions reach 0 and 1 at the upper and lower end of the price support.

Proof of Proposition 2:

The proof is straightforward following the above Lemmas. The equilibrium profits are also provided in the proofs of the Lemmas.

Proof of Proposition 3:

The threshold \( f(s) \) in the proposition obtains by comparing the profits earned by a firm that responds to content-rich advertising with uninformative advertising to the profits earned by a firm that responds to content-rich advertising with content-rich advertising. If \( \pi_{cc} - c_u > \pi_{ec} - c_c \), the best response is uninformative advertising, resulting in an asymmetric advertising equilibrium.

Proof of Proposition 4:

When \( s > 1/3 \), take the derivative of \( f(s) \) with respect to \( s \) to obtain:

\[
\frac{df(s)}{ds} = \frac{1-3s}{s}\frac{df(s)}{ds} > 0.
\]

Obviously, when \( s < \frac{1}{3} \), \( \frac{df(s)}{ds} > 0 \) and \( f(s) \) is increasing in \( s \). Recall that when \( c_c - c_u < f(s) = \pi_{cc} - \pi_{ec} \), the equilibrium in advertising is (Content-rich, Content-rich). Therefore, holding \( c_c - c_u \) constant, as \( s \) increases, a higher \( f(s) \) means the equilibrium moves from (Uninformative, Content-rich) to (Content-rich, Content-rich), i.e., more firms are willing to do content-rich advertising.

References
