Consumer informedness: A key driver of differentiation

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Abstract

Minimum Differentiation is the equilibrium in spatial models with fixed prices, while firms move apart to reduce the intensity of competition when firms set prices. Nevertheless, firms collocate in many industries where marketing-active firms compete on price. This puzzle is called the Hotelling paradox. We offer a resolution of this puzzle by noting that imperfect information about the availability of all products can soften competition, allowing firms to produce similar products without engaging in intense price competition. Specifically, we construct a model that predicts minimum differentiation when consumers have low awareness about products and maximum differentiation when they are well informed.

1 | INTRODUCTION

1.1 | Background

The principle of Minimum Differentiation (Hotelling, 1929) arises as the equilibrium outcome in spatial models when price is fixed. The idea is simple: each firm would like to locate at the place that gives them the most efficient access to a broad base of consumers. However, this principle of minimum differentiation does not hold up when firms compete on price. The reason is that when firms collocate there is no differentiation so price competition eliminates profits (this is known as Bertrand competition between homogenous goods). It follows, then, that firms should move away from each other when prices are not fixed, reducing the intensity of price competition. However, it is not possible to find equilibrium locations for competing firms that reflect this intuition in a standard Hotelling market with linear travel costs (D’Aspremont et al., 1979). In contrast, the principle of Maximum Differentiation has its roots in the idea that firms differentiate to gain pricing power over proximate consumers and to reduce their incentive to decrease prices to capture market share from the competitor. This idea explains the findings of D’Aspremont et al., who show that competing firms maximally differentiate in a Hotelling linear market when consumers have quadratic travel costs and compare the offers of competing firms. This thinking explains the observation that competing supermarkets rarely locate adjacent to each other. For example, in Thornhill, Ontario (a suburb of Greater Toronto), there is at least 1.3 km between competing supermarkets (Longos, Sobeys, and Food Basics). Because shoppers are well informed about what each supermarket offers (people shop on a regular basis), supermarkets gain by differentiating and creating geographic areas where they have an advantage.

Despite examples like supermarkets which are consistent with the predictions of D’Aspremont et al., there are many sectors where firms compete in prices yet locate close to each other. A case in point is the auto body repair market. Despite providing almost identical service, it is common to find competing body shops clustered together. For example, in Thornhill, Ontario, there are at least five auto body shops within 100 m of each other on Harlech Court, a small dead-end street. It is important to note that drivers typically have few car accidents requiring bodywork; hence, they might be poorly informed about the offerings of different body shops. We see a similar pattern in online
environments where competing firms often provide almost identical services. In the online grocery retailer market, a cursory examination of competitors such as Cornershop®, Instacart®, and Grocery Gateway® reveals uncanny similarity in the “steps to register,” “functionality,” and “participating grocers.”

In many markets with low levels of differentiation, it seems that only a small fraction of potential consumers are aware of each choice. A March 2019 article in Consumer Insights/Omnichannel reports that less than 3% of grocery shoppers shop online. Moreover, an online grocery retailer that gets a shopper to try its offering gains a huge advantage: 75% of online grocery shoppers still shop with the first online grocery retailer they tried. It appears that in March 2019, a significant majority of grocery shoppers were uninformed about any specific firm and its offer. Similarly, most consumers rarely get into car accidents, so they are likely to be aware of only a small subset of auto body shops. We examine how the level of consumer informedness affects the incentives for firms to differentiate.

In the literature, a number of explanations are proposed to explain positioning decisions of firms. These include search costs and a second unobserved level of heterogeneity between products (De Palma et al., 1985; Fischer & Harrington, 1996). However, our objective is to focus solely on informedness (the degree to which consumers are aware of competing offers) as a basis to understand the positioning decisions of firms. Given the vast number of products in the market, consumers are inevitably poorly informed about many of the products that are available. Firms spent over $250 billion on advertising in the United States in 2019 (Guttmann, 2020), a little more than 1% of GDP, in an attempt to be better known. Thus, understanding how informedness affects differentiation is of high importance.

With a two-firm model, we show how the level of informedness affects the positioning decisions of competing firms. The model shows that when the level of informedness is low (as is the case with auto body shops and online grocers), firms tend to cluster at the mid-point of the market to maximize demand from the consumers who are only aware of one firm. In contrast, when the level of informedness is high (as is the case with traditional supermarkets), firms move apart to reduce the intensity of competition for consumers who are aware of both firms. The intuition behind this result is that lower awareness of products acts as a differentiator, creating segments of customers who are not aware of all of the options in the market. This decreases the rate of substitution between products which allows firms to locate at more central locations without the threat of strong price competition.

Next, we review the literature that is relevant to our study.

1.2 Literature review

A central question in economics is to what extent will firms choose to differentiate. We focus on markets characterized by horizontal differentiation where products are of similar quality (Eaton & Lipsey, 1996). In a horizontally differentiated market, firms want to locate to maximize demand. In a linear city, this is achieved by locating in the middle of the market as suggested by Hotelling (1929). The idea is that a central location allows the firm to maximize its market share.

Another factor that may cause firms to co-locate in a central location is search costs. Here, consumer search can lead to more shopping at clusters and this provides an incentive for agglomeration, especially if products are differentiated along another dimension (Fischer & Harrington, 1996). In addition, Parakhonyak and Titova (2018) show that clustering obtains when consumers incur a fixed cost to visit a cluster. When “non-shoppers” incur incremental costs to make product and price comparisons and “shoppers” do not, consumer heterogeneity also provides a basis for clustering (Bernhardt et al., 2021; Non, 2010). But in many industries, consumer search is insignificant and consumers only patronize firms that they know about. To be specific, our focus is markets where “location” is the only source of product heterogeneity and consumers do not search for products. Consumers only consider products about which they are informed.

The desire to locate centrally is counterbalanced by the need that competitors have to differentiate to soften price competition (Tirole, 1988). Although Hotelling’s analysis considers price competition, the proposed prices do not form a Nash Equilibrium. In fact, Economides (1984) shows that firms in a linear market have a tendency to move away from each other. This obtains in a model with low reservation prices such that “jumping firms” establish local monopolies. Similarly, Pazgal et al. (2016) show that an equilibrium with moderate differentiation arises in a Hotelling model with linear travel costs if the reservation prices are not too high. When reservation prices are high, D’Aspremont et al. (1979) show that under linear travel costs a price equilibrium may not exist when firms locate close to each other. To solve this problem, the authors propose a model where firms choose a position along a linear market and consumers have quadratic travel costs; they find that firms will choose to maximally differentiate.
Other authors including Eaton and Lipsey (1975), Novshek (1980), and Ansari et al. (1998) show that models which generate equilibria with maximal or minimal levels of differentiation are sensitive to the specific assumptions associated with the model (e.g., the number of attributes or the number of firms). For example, a second level of heterogeneity is the basis for finding that firms will co-locate along a known attribute in De Palma et al. (1985). These ideas are extended in Rhee et al. (1992) by showing that similar results obtain when the differences between products come from unobservable attributes that the competing firms do not observe. This study is also related to the results of Neven and Thisse (1990) and Irmen and Thisse (1998), which show that firms differentiate in one dimension of the product space, but not in other dimensions. In contrast, the products of our model have only one attribute (along with price) which determines the value a consumer obtains from consuming that product. However, there is a second level of heterogeneity in our model - one that is created by marketing activity which creates differences in WTP based on the information set that consumers have.

In a nutshell, our approach is to study the question of positioning as a function of how marketing informs or activates consumers. Academics have made significant contributions demonstrating how advertising provides information to consumers about product attributes that are valued by some consumers and not by others. This study shows that informative advertising allows consumers to find products that better meet their needs (Bester & Petrakis, 1995; Butters, 1977; Grossman & Shapiro, 1984; Robert & Stahl, 1993). The structure we adopt is identical to the approach of Grossman and Shapiro (1984) in that advertising creates informedness, such that consumers who are exposed to advertising from a firm are informed about both the existence of the product and all of its attributes.

Our paper is also related to the literature about how advertising affects competition. Grossman and Shapiro (1984) show that increased advertising can intensify price competition. Further, they find that profits can decrease as advertising costs decline due to the introduction of more intense advertising. However, the paper does not consider how products should be positioned in a spatial market. Iyer et al. (2005) study targeted advertising and find that targeting is an effective tool to reduce price competition. Again, this study does not consider product positioning. Lauga et al. (2018, working paper) study the effect of targeted versus blanket advertising on product quality in a competitive vertically-differentiated market. The authors show that the ability to target advertising provides an incentive for firms to differentiate in terms of quality. In contrast, our analysis focuses on the relationship of advertising intensity to the horizontal positioning of products that are of equal quality.

We now present a model to analyze the impact of informedness on the positioning decisions of firms.

2 | THE MODEL

We model a consumer market using a standard Hotelling line of length 1 in which two competing firms make positioning and then pricing decisions before consumers decide to consume a single product or not at all. We assume that consumers only consider products that they are informed about. This informedness could be the result of marketing activities of firms, or it can reflect other institutional aspects of the market, such as how often consumers purchase products in the category or whether the product category is new (e.g., new technologies). Consumers are spread uniformly (with density 1) along a line segment from 0 to 1. A consumer located at location \(x_c\) obtains the following utility from a product located at location \(x_j\):

\[ U(x_c, x_j) = v - t(x_c - x_j)^2 - p_j. \]  

(1)

In this expression, \(v\) is the surplus created by a product that perfectly meets the tastes of a consumer (i.e., \(x_c = x_j\)) and \(p_j\) is the price charged by Firm \(j\). Note that a consumer also has the option not to purchase any product. In that case, the consumer earns a utility of 0. Without loss of generality, we normalize the transportation cost, \(t\), to one.

The first decision the firms make is where to locate. We constrain the firms to locate in one of three locations: at each end of the linear market (i.e., at 0 or 1) or in the center of the market (i.e., at 1/2). The choice of three possible firm locations is designed to parsimoniously reflect the choices that firms have to collocate or maximally differentiate as a function of the level of informedness in the market. The use of discrete points versus allowing location to be a continuous choice allows us to identify pure-strategy pricing equilibria for all location combinations except when there is perfect colocation (and when there is perfect colocation the mixed strategy equilibrium can be calculated straightforwardly). Our objective is to use this structure to demonstrate the basic mechanism that drives the positioning decision.
We restrict our attention to a range for $v \in \left[\frac{5}{4}, \frac{5}{3}\right]$. We set an upper bound for $v$ because high levels of $v$ lead to mixed pricing strategy equilibria for intermediate levels of informedness in which firms offer asymmetric products.\(^5\) When $v$ is greater than $\frac{5}{4}$, firms are motivated to set a high price and only serve their captive market (consumers who are only aware of that firm’s product, but not the rival product); however, when both firms set this high price then the firms have an incentive to reduce their price to capture the competitive segment (consumers who are aware of both products in the market). The pricing equilibrium is tractable when both firms are located in symmetric locations, as is the case when both firms locate in the middle of the market or when the firms locate at 0 and 1, respectively. However, when the firms are not located in symmetric locations (i.e., one firm locates centrally and the other locates at an extreme end of the Hotelling market), the equilibrium is only solvable through simulation. The lower bound on $v$ is adopted to ensure that all consumers in the competitive segment obtain positive utility. This is a standard assumption made in spatial models to ensure complete coverage (Grossman & Shapiro, 1984; Soberman, 2004).

The game has two stages. In the first stage of the game, two ex ante symmetric firms make simultaneous decisions to position themselves at 0, 1/2, or 1. Once positioned, both firms have the ability to produce a product at a marginal cost of $c$. Without loss of generality, we normalize this marginal cost to zero. In the second stage, each firm simultaneously sets its price. Finally, after the firms make these choices, each consumer is informed about each firm’s location and price with probability $\phi$.\(^6\) Similar to Grossman and Shapiro (1984), we assume that consumers are made aware of the product at random. The awareness could be the outcome of several types of marketing activities, including varied forms of advertising, the intensity of distribution and/or direct selling effort. Alternatively, the limited awareness of products may be the result of limited exposure to an industry such as the auto body shop example (in the introduction) and tour options in travel destinations or the vastness of the available options in a category (such as the choice of restaurants in metropolitan areas).

In other words, all consumers are equally likely to be exposed to the advertising effort taken by firms. Thus, the probability that a particular consumer is informed about both products is $\phi^2$. Consumers purchase at most one product and only consider products that they are informed about as in Butters (1977) and Grossman and Shapiro (1984).

Ultimately, this leads us to four groups of consumers in every location: Firm $i$’s captive demand segment of size $\phi(1-\phi)$, firm $j$’s captive demand, also of size $\phi(1-\phi)$, a competitive set of switchers who buy the product that delivers the highest utility, of size $\phi^2$, and finally a fraction $(1-\phi)^2$ of the consumers, who are informed about neither offering and are therefore inactive.

### 2.1 Timing of the game

The extensive form of the game can be summarized by the following steps:

1. The firms simultaneously choose a location (0, 1/2, or 1)
2. The firms simultaneously set prices for their products.
3. Consumers are informed about each offering—including its location and price—with probability $\phi$
4. Consumers who have been reached by the advertising of at least one firm make a decision about which, if any, firm to purchase from.

The subgame perfect equilibrium we seek is one in which neither firm has an incentive to change its decision taking the competitor’s strategy as given. This implies that if the equilibrium of the game is asymmetric, the model is agnostic about which of the two firms will be in the more favorable of the two equilibrium positions. In the next section, we present the price equilibria as a function of the locations of the firms. We focus on the positioning outcomes that are pure-strategy location outcomes.\(^7\)

### 2.2 Both firms located in center

If both firms locate in the center, the equilibrium in pricing is in mixed strategies. However, the solution is simple because all informed customers buy a product independent of their location. Such a model is similar to the informed and uninformed consumer model of Varian (1980). The difference is that the willingness to pay varies as a function of the consumer’s location but with a sufficiently high level of $v$, this difference does not affect determination of the
equilibrium. We provide the solution, and confirm that all customers informed of at least one firm’s product make a purchase within our range of \( v \). We write the objective function for Firm \( i \) as a function of the equilibrium cumulative distribution function (the \( cdf \)) of prices, which is denoted as \( F(p_i) \) for each firm.

\[
\pi_{ci} = p_i \left[ \phi(1 - \phi) + \phi^2 (1 - F(p_i)) \right].
\]

Narasimhan (1988) shows that the profits are defined by the guaranteed profit each firm earns independent of the action of its competitor, as summarized in Lemma 1.

**Lemma 1.** When the firms collocate in the center, the profit of each firm is \( \Pi = \left( v - \frac{1}{4} \right) \phi(1 - \phi) \) and the firms price according to the \( cdf \), \( F(p) = \frac{p - (v - \frac{1}{4})(1 - \phi)}{\phi} \). The lowest observed price is \( P = (v - \frac{1}{4})(1 - \phi) \).

### 2.3 Both firms located at external positions

Suppose that one firm locates at 0 and another firm locates at 1. There are three different pricing regimes that are possible within the range of \( v \) we consider, all of which have pure-strategy pricing equilibria. First, firms may set prices such that all informed consumers obtain positive utility from each firm. Second, firms can set prices such that an informed customer at the opposite end of the Hotelling line is indifferent between buying the firm’s product and not buying. Finally, the firms may set prices such that some consumers at the opposite end of the Hotelling line get negative utility from the product. We examine each of the alternatives in turn.

Suppose that all informed consumers obtain positive utility from each firm’s products. Then profits are

\[
\pi_{ci} = p_i \left[ \phi(1 - \phi) + \phi^2 \frac{1}{2} (1 - p_i + p_{-i}) \right].
\]

In such a case, the equilibrium price is \( p_i = \frac{2 - \phi}{\phi} \).

Suppose, instead, that some consumers obtain negative utility, so that the informed market is not covered. Then profits will be (assuming, as we later confirm, that the consumers at the center obtain positive utility):

\[
\pi_{ci} = p_i \left[ \phi(1 - \phi) \sqrt{v - p_i} + \phi^2 \frac{1}{2} (1 - p_i + p_{-i}) \right].
\]

Here, a pure strategy equilibrium exists based on the first-order conditions being satisfied, but it is only solvable through an implicit condition.

The final potential equilibrium is that both firms charge exactly \( v - 1 \) such that the market is perfectly covered.

In all cases, we confirm that for these equilibria, neither firm has an incentive to switch to a high price such that only captive customers (who are only informed about one product) buy. A comparison of these three possible outcomes leads to Lemma 2:

**Lemma 2.** When the firms are located at the opposite ends of the Hotelling market, profits are determined as follows:

1. If \( \phi > \frac{4}{5} \) and \( \frac{2}{\phi} \leq v \leq \frac{5}{2} \) then profits are given by Equation (2), prices are \( \frac{2 - \phi}{\phi} \), and profits are \( \frac{(2 - \phi)^2}{2} \).
2. If \( \frac{1}{2} \leq \phi \leq \frac{4}{5} \) and \( 3 - \phi \leq v \leq \frac{5}{2} \) or \( \phi > \frac{4}{5} \) and \( 3 - \phi \leq v \leq \frac{2}{\phi} \) then both firms set prices at \( v - 1 \). Profits are \( \frac{(2 - \phi)^2}{2} \).
3. If \( 0 \leq \phi \leq \frac{1}{2} \) and \( \frac{5}{2} \leq v \leq \frac{5}{2} \) or \( \frac{1}{2} \leq \phi \leq 1 \) and \( \frac{5}{4} \leq v \leq 3 - \phi \) then profits are given by Equation 3. Prices are the solution to two implicit functions given by the first-order conditions for each firm. Profits can also be written as the solution to two implicit functions.
2.4 | One firm at center and one firm at edge

We now consider the case where one firm is located at the center of the market and the other firm is located at one edge of the market. There are five potential pure-strategy pricing outcomes that need to be considered.

The first case is that both firms set their prices according to first-order conditions such that all consumers who are aware of a product choose to buy. In such a case, the firm at the end of the market earns profits of

\[ \pi_{C1} = p_C \left[ \phi (1 - \phi) + \phi^2 \left( \frac{1}{4} - p_E + p_C \right) \right]. \]  \hfill (4)

Here, \( C \) denotes the center firm and \( E \) denotes the firm located at the edge of the market. The firm located at the center of the market earns:

\[ \pi_{C1} = p_C \left[ \phi (1 - \phi) + \phi^2 \left( \frac{3}{4} - p_C + p_E \right) \right]. \]  \hfill (5)

Case 2 involves the center firm pricing according to the first-order conditions that are obtained from Equation (5), but the extreme firm prices at a kink-point where its uncontested market is perfectly covered, that is, \( p_E = v - 1 \). Case 3 involves both firms pricing according to their first-order conditions, but here some customers who are aware of the product at the edge of the market decide not to buy, even if it is their only option. In such a case, the center firm’s profits are again obtained from Equation (5), while the extreme firm’s profits are

\[ \pi_{E1} = p_E \left[ \phi (1 - \phi) \sqrt{v - p_E} + \phi^2 \left( \frac{1}{4} - p_E + p_C \right) \right]. \]  \hfill (6)

Case 4 involves the extreme firm pricing according to its first-order condition, where its profits are given by Equation (6), while the center firm prices at the kink-point where its price is \( v - \frac{1}{4} \), so that its captive market is perfectly covered. The best response price, and profits for both firms, can then be solved as a function of implicit functions. Finally, we consider a 5th case, which involves both firms pricing at kink-points such that their captive markets are fully covered, that is, \( p_E = v - 1 \) and \( p_C = v - \frac{1}{4} \). Although there are parameter values where such an equilibrium exists, it does not exist for the range of \( v \) we consider, \([5/4, 5/2]\). Thus, we focus on the first four possible outcomes for our parameter region.

The equilibrium outcome is summarized in Lemma 3:

**Lemma 3.** When one firm is located at the edge of the market while the other firm is located at the center of the market, profits are determined as follows:

1. If \( \phi > \frac{12}{25} \) and \( \frac{12 + 5\phi}{128} \leq v \leq \frac{5}{2} \), then profits are given by Equations (4) and (5). Prices are determined by taking first-order conditions of these equations.

2. If \( \frac{6}{19} \leq \phi \leq \frac{12}{25} \) and \( \frac{16 + 4\phi}{4 + 8\phi} \leq v \leq \frac{5}{2} \) or \( \frac{12}{25} \leq \phi \leq 1 \) and \( \frac{16 + 4\phi}{4 + 8\phi} \leq v \leq \frac{12 + 5\phi}{128} \), then the firm at the edge of the market sets its price as \( v - 1 \), while the center firms’ prices as determined by taking the first-order condition of Equation (5).

3. If \( L \leq \phi \leq 1 \) and \( \max [M, \frac{5}{4}] \leq v \leq \min \left[ \frac{16 + \phi}{4 + 8\phi}, \frac{5}{2} \right] \), where \( L \) solves the implicit equation \( 144 - 1432L + 5657L^2 - 10794L^3 + 8425L^4 = 0 \), and \( M \) solves the implicit equation \( 144 - 472\phi - 183\phi^2 + 6\phi^3 + 5\phi^4 + (-384\phi + 1696\phi^2 + 320\phi^3 - 32\phi^4)M + (256\phi^2 - 1856\phi^3 - 80\phi^5)M^2 + 576\phi^4M^3 = 0 \), then prices are the solution to the first-order conditions of the profits given in Equations (5) and (6).

4. If \( 0 \leq \phi \leq L \) and \( \frac{5}{4} \leq v \leq \frac{5}{2} \) or \( L \leq \phi \leq N \) and \( \frac{5}{4} \leq v \leq M \), where \( N \) solves the equation \( 144 - 808N + 1529N^2 - 956N^3 = 0 \), then the central firm prices at \( v - \frac{1}{4} \), while the firm at the edge sets prices according to the first-order conditions of the profits given in Equation (6).
3 | OPTIMAL POSITIONING FOR THE FIRMS

The optimal positioning for the firms is determined by solving the normal form for Stage 1 as shown in Table 1.

Note that our analysis does not distinguish between (0, 1) and (1, 0). Nor does it distinguish between \((0, \frac{1}{2}), (1, \frac{1}{2}), (\frac{1}{2}, 0)\), or \((\frac{1}{2}, 1)\) because our goal is to understand the level of differentiation in equilibrium, rather than the identity of the firms at each location. Moreover, we do not present the values of \(\Pi_{00}\) or \(\Pi_{11}\) because these outcomes are strictly dominated. However, the analysis provides a clear direction regarding whether the equilibrium location outcome consists of maximal differentiation, partial differentiation or collocation in the center.

We first note that markets with low informedness are characterized by minimum differentiation, while markets with high informedness exhibit maximum differentiation. This is summarized in Proposition 1.

**Proposition 1.** When \(\frac{5}{4} < v < \frac{5}{2}\) then

1. If \(\phi < \frac{2}{5}\), the unique location equilibrium involves minimum differentiation, with firms locating at the center of the market.
2. If \(\phi > \frac{9}{10}\), all pure-strategy equilibria involve maximum differentiation, with firms locating at opposite ends of the market.

For the second part of Proposition 1, the maximum differentiation outcome is qualitatively unique among pure-strategy location outcomes, with the only flexibility being that either firm may be located at 0, while the other firm will be located at 1. This proposition demonstrates our basic finding that consumer informedness affects the level of equilibrium product differentiation in the market. Thus, product differentiation is not only affected by price competition, but by the level of informedness in the market as well. When there is low consumer informedness, both firms maximize their profits by locating centrally and exploiting their captive segment. As the level of informedness increases, the fraction of consumers that see advertising from both firms increases and collocation means that profits from these consumers are competed away. This provides an incentive for the firms to move away from the middle to reduce the competitive intensity between the firms. When consumer informedness is high enough, we obtain maximum differentiation for the same reasons as in D’Aspremont et al. (1979).

A natural question to ask is what happens in between these two extreme scenarios. Here, the answer is more complex. Intuitively, there is a transition zone. When \(v\) is high, this zone involves two separate pure-strategy equilibria, supporting both maximum differentiation or minimum differentiation. On the other hand, when \(v\) is lower this transition zone involves partial differentiation where one firm locates at an edge of the market and the other firm locates in the center. Due to the complexity of the boundaries generated by our analysis, we present the equilibrium positioning locations in Figure 1.

Figure 1 shows that minimum differentiation is endemic at low levels of informedness (\(\phi\)), whereas maximum differentiation is endemic at high levels of \(\phi\). In contrast, the transition zone changes as a function of \(v\). One thing we observe in Figure 1 is that there is not always a monotonic relationship between informedness and product differentiation, holding the level of \(v\) fixed. That is, there are some cases where a small increase in informedness leads to less differentiation. This lack of monotonicity comes from shifts in which of the pricing equilibria in Lemmas 2 and 3 applies for the corresponding values of

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At the end of the appendix, we provide graphs of the profits earned by the firms under each of the potential locations for $v = \frac{5}{4}$ and $v = \frac{5}{2}$ to help the reader to better understand this transition zone.

4 | CONCLUSION

This paper examines how awareness shifts the level of product differentiation chosen by firms. In the traditional product-differentiation literature, we observe that firms differentiate to soften price competition, even though such a move sacrifices positioning at a central location that maximizes access to consumers. We demonstrate that if awareness is low then the balance of this trade-off is shifted and firms minimally differentiate. This occurs because both firms want to locate at the optimal central location: the imperfect informedness of consumers is a differentiator that softens price competition. However, as the level of informedness increases, the fraction of the market that compares the offers of each firm increases and this leads to readjustment in optimal positioning. Ultimately, at high levels of informedness the incentive to maximally differentiate, first identified by D’Aspremont et al. (1979), re-emerges.

Although our formal analysis demonstrates the results when the firms are restricted to locate at discrete points, it is possible to show that the basic result holds for nearly continuous locations by looking at the extremes of perfect informedness and nearly zero informedness. The outcome under perfect informedness and continuous location choices is the same as that of D’Aspremont et al. (1979), which is maximum differentiation. Similarly, one can construct a set of equilibria such that firms locate at the center of the market when the location choices available to the firms are equal to the whole Hotelling market in the limit. 10

These results explain why we generally find significant differentiation in markets like supermarkets where shoppers are repeat customers and less differentiation in markets like auto body shops and online retailing where potential customers are poorly uninformed about the offers in the market. These findings also provide an explanation for the striking similarity of competitive offers in some markets even when there are differences in taste across potential customers. For example, the day tours that are offered to visitors in popular tourist destinations like Rome and Paris are uncannily similar despite substantial heterogeneity in the tastes of potential customers. Of course, visitors to tourist destinations are not well informed about the offers in a destination they visit for the first time.

Finally, our results give insight into how differentiation is likely to evolve during the product life cycle. In the introductory stage of the Product Life Cycle (Levitt, 1965), consumers are relatively uninformed about product offerings. This may explain why competing products are so similar. As the market matures, consumer informedness likely grows, and with that we should expect to see more product differentiation. Consider for example, the windsurfer category which appeared in the 1970s in North America. In the early years of the windsurfer category (the 1970s), all of
the products in the market were similar in appearance and characteristics (the major competitors were Windsurfing international, Ten Cate sports, Mistral and, Bic). This was the introductory stage of the category. Over time, the category has matured (Windsurfing is now an Olympic sport), and today, there is tremendous heterogeneity in product choice (there are beginner boards, light weather boards, heavy weather boards, wave boards, sinker boards and trick boards).^{11}

## ENDNOTES

1. These observations are based on a comparison of these online grocery web sites for Toronto, Ontario in mid 2020.

2. The date of the article at https://www.thinkwithgoogle.com/consumer-insights/online-grocery-shopping-behavior/ is important because online grocery retailing has become significantly more important with the 2020 COVID-19 pandemic. These companies formed their business strategies in the pre-COVID 19 environment.

3. In Grossman and Shapiro (1984), firms exogenously assume equally spaced locations around the circular market.

4. With continuous location choices and differentiated firms, the pricing incentives are significantly more complex. Because profits for at least one firm in a standard Varian model are equal to guaranteed profits, the mixed-strategy pricing game can be solved. However, in our model with differentiation, this is not the case. When firms are not colocated, profits are not equal to the profits a firm earns by only serving its captive customers. Further, the formula for integrating the profits for any potential price in a mixed-strategy equilibrium is complex because it includes (a) obtaining a fraction of the contested segment if the competitor’s price is close to the firm’s price or (b) or one firm obtaining the whole contested segment if the price difference is large enough. This leads to substantial difficulty in the identification of pdf’s consistent with a level of guaranteed profit.

5. In the range of \( \nu > \frac{1}{2} \), minimum and maximum differentiation obtain at low and high levels of informedness respectively but our desire is to characterize the equilibrium for the complete range of informedness, parameterized by \( \phi \in [0, 1] \).

6. The equilibrium of this game is identical to a game in which marketing activity activates consumers before the pricing decision and consumers costlessly learn the prices of all products about which they are informed (Mayzlin & Shin, 2011; Meurer & Stahl, 1994).

7. We note that there are mixed-strategy location equilibria whenever there are either asymmetric pure-strategy location outcomes or multiple pure-strategy equilibria.

8. This can be seen by noting that the argument from Narasimhan (1988) shows that each firm’s expected profits in these co-location cases will be equal to their guaranteed profits (how much they would earn if they only appealed to their captive customers). If a firm deviated to the opposite end of the Hotelling line, their guaranteed profit would remain unchanged, but total profits would be larger.

9. There are, of course, mixed strategy equilibria that would lead to other combinations of locations.

10. The following forms an equilibrium for low informedness: Both firms locate at the center of the market and price at \( \nu - \frac{1}{4} \). Firms are allowed to deviate to any location except for the locations within \( \min \left\{ \frac{1}{2} \left( \frac{1 + 3 \phi}{1 - \phi} - 1 \right), \frac{2 \phi - \phi}{\pi (1 - \phi)} \right\} \) of the center location. Note that the limit of this exempt zone goes to 0 as \( \phi \to 0 \). If a firm deviates, the center firm will continue to price at \( \nu - \frac{1}{4} \), while the deviating firm that moves to location \( x \) will either price at \( \min \{ \nu - x^2; \nu - (1 - x)^2 \} \) if they are located close to the center, or with the first-order condition of

\[
\begin{align*}
\phi_x & \left( \frac{1}{2} \phi \left( \nu - \frac{1}{4} - \frac{2 \nu - \phi}{\sqrt{2} \phi} \right) \right) + \phi \left( x^2 - (\nu - \frac{1}{4})^2 \right) \\
& = \phi_x \left( \frac{1}{2} \phi \left( \nu - \frac{1}{4} - \frac{2 \nu - \phi}{\sqrt{2} \phi} \right) \right) + \phi \left( x^2 - (\nu - \frac{1}{4})^2 \right)
\end{align*}
\]

One can confirm that no firm will want to deviate from the center.

11. Another example is the cartridge razor category that appeared in the late 1960s and early 1970s (Gillette and Schick). Initially, these products were inexpensive utilitarian shaving devices with next to no differentiation. As the category matured, product differentiation expanded to include varieties with different number of blades designed for ultra smooth or sensitive skin, disposable and feminine versions.

## REFERENCES


APPENDIX A

Proof of Lemma 1. This result is a straightforward application of the results of Narasimhan (1988). Note that profits at any price in the mixing range for the proposed equilibrium are

\[ p_i[\phi (1 - \phi) + \phi^2 (1 - F(p_i))] = p_i \left[ \phi (1 - \phi) - \phi (1 - \phi) + \phi \left( \frac{v - \frac{1}{4} (1 - \phi)}{\phi^2} \right) \right] = (v - \frac{1}{2}) \phi (1 - \phi). \]

This makes each firm indifferent to any of the prices included in support for the mixed pricing strategy. For the putative equilibrium to survive, a firm cannot have an incentive to price higher or lower than this range, conditional on the strategy of the rival firm. The lower bound of pricing is \( p = (v - \frac{1}{4}) (1 - \phi). \) At this price the firm sells its product to all consumers who are aware of the product. Reducing price reduces margins but does not expand sales, leading to lower profits.

We now check to see if a firm has an incentive to defect to price that is greater than \( v - \frac{1}{4}. \) At such a price, the sales of a firm are restricted to its captive market, of size \( 2 \phi (1 - \phi) \sqrt{v - p_i}. \) Taking the first derivative of profits with respect to price yields \( \frac{\partial [p_i \phi (1 - \phi) \sqrt{v - p_i}]}{\partial p_i} = 2 \phi (1 - \phi) \left[ \frac{\sqrt{v - p_i} - \frac{p_i}{2 \sqrt{v - p_i}}}{\phi^2} \right]. \) The second term is negative whenever \( p_i > \frac{2}{3} v. \) But \( (v - \frac{1}{4}) > \frac{2}{3} v \) whenever \( v > \frac{3}{4}, \) which is satisfied in the regions we analyze.

Proof of Lemma 2. To prove the first part of the lemma, we derive the conditions by taking the first-order conditions of the profits given in Equation (2), and impose the range of \( v, \) as well as confirming that \( v - p_i = v - \frac{2 - \phi}{\phi} > \frac{1}{4} \) (this ensures that the market is covered). This condition defines the region for which the first order conditions are valid.

For prove the second part of the lemma, we confirm that the derivative of the profits given in Equation (2) is negative, while the derivative of the profits given in Equation (3) is positive. We also confirm that neither firm will raise its price to \( \frac{2}{3} v, \) the level that maximizes its profits among its captive segment.

For prove the third part of the lemma, we derive these conditions by taking simple first-order conditions of the profits shown in Equation (3). We then confirm that \( v - p_i < 1, \) so that the captive markets is not covered, but that \( v - p_i < \frac{1}{4} \) which ensures the contested market is covered. We also confirm that profits are greater than \( \frac{2v}{3} \phi (1 - \phi) \sqrt{v - \frac{2v}{3}}, \) where, as noted above, \( \frac{2}{3} v \) is the price that maximizes \( p_i \phi (1 - \phi) \sqrt{v - p_i}. \)

Proof of Lemma 3. For the first part of the lemma, we take the first order conditions of Equations (4) and (5). In this case, the firm in the center charges \( \frac{12 - 5 \phi}{12 \phi} \) and earns \( \frac{(12 - 5 \phi)^2}{144 \phi}, \) while the firm on the edge charges \( \frac{12 - 7 \phi}{144 \phi} \) and earns \( \frac{(12 - 7 \phi)^2}{144 \phi} \). The boundaries of the region are defined by the constraints that the price at the center is always below \( v - \frac{1}{4} \) (so all of its captive customers buy) and the price for the firm located at 0 or 1 is less than \( v - 1 \) (this can be binding). Moreover, we confirm that the contested segment yields positive sales to each firm (i.e., that the marginal customer is located between 0 and 1). In addition, this outcome is only an equilibrium if neither firm has an incentive to increase prices and only cater to their captive segments. To verify this, we confirm that neither firm will charge \( \frac{2}{3} v \) or \( v - \frac{1}{4} \) for the center firm, or \( \frac{2}{3} v \) or \( v - 1 \) for the edge firm, while serving only their captive customers. In the proof of Lemma 1 we show that \( \frac{2}{3} v < v - \frac{1}{4} \) for our parameter values, so it is sufficient to check the deviation of the central firm to the price of \( v - \frac{1}{4}. \) On the other hand, \( \frac{2}{3} v > v - 1, \) so we only have to check the deviation to \( \frac{2}{3} v. \)
For case 2, we solve for price as noted in the lemma, and the center firm earns profits \( \Pi_{CE} = \frac{(4 + \phi(4v - 5))^2}{64} \), while the firm at the edge earns \( \frac{\phi(v - 1)(3 + 4v - 12)}{8} \). We check that the derivative of Equation (4) with respect to the price of the firm’s own price is positive, while the derivative of Equation (6) is negative. We also check that within this region, the price for the center firm is always less than \( v - \frac{1}{4} \) and that the contested market yields positive sales to each firm. Finally, we confirm that neither firm wants to deviate to a price where they serve only their captive customers.

For case 3, we take the first order conditions of Equations (5) and (6). We confirm that the price for the firm at the edge of the market is greater than \( v - 1 \) (this implies that not all captive customers buy from the firm), while the price for the firm at the ends of the market is less than \( v - \frac{1}{4} \), as in cases 1 and 2. We also confirm that the contested market yields positive sales to each firm, and that neither firm has an incentive to deviate to a price where it serves only captive customers.

For case 4, we take the first order condition of Equation (6). We also confirm that the price for the product at the edge is greater than \( v - 1 \). For the center firm, we confirm that the derivative of Equation (5) with respect to the center firm’s price is positive, while \( \frac{\partial}{\partial p_C} \left( 2\phi(1 - \phi, v - p_C + \phi \left( v - p_C + \frac{1}{4} v - p_E \right) \right) < 0 \). We also confirm that the marginal customer in the contested segment (where customers are informed about both firms) lies in the range where the customer would get a positive utility from the firm at the edge (i.e., that \( 0 < \frac{1}{4} - p_E + p_C < \sqrt{v - p_E} \)). Finally, we confirm that the firm on the edge would not be better off increasing its price to a level where it only caters to its captive market. □

**Proof of Proposition 1 and the results from Section 3**
The solution to the normal form in Section 3 is simple in principle, but computationally difficult due to implicit functions which define the optimal prices for firms. Accordingly, we solve for the equilibria as presented in Figure 1 using computational methods in Mathematica®. First we note that \( \left( \frac{1}{2}, \frac{1}{2} \right) \) is an equilibrium when neither firm can increase profit by changing its location to the edge of the market and earning the profits of a edge firm in a partially differentiated outcome \( \left( \left( 0, \frac{1}{2} \right) \right) \) or \( \left( \left( \frac{1}{2}, 1 \right) \right) \).

Similarly, because \( \Pi_{00} = \Pi_{11} < \Pi_{01} \), maximum differentiation is an equilibrium if \( \Pi_{01} > \Pi_{MM} \).

Finally, a partially differentiated outcome is a possible equilibrium if the firm at the edge of the market does not earn more by moving toward the center, and if the firm at the center does not earn more by moving to the opposite edge of the market. That is, if \( \Pi_{MM} > \Pi_{01} \) and \( \Pi_{E} > \Pi_{MM} \).

We input these equations and generate Figure 1. The results of Proposition 1 follow immediately from these calculations. Further details on the process used to generate Figure 1 are provided below.

**Graph of profits to illustrate the process for generating Figure 1**
To demonstrate how Figure 1 is generated, we have generated two graphs, one for a low level of \( v \left( v = \frac{5}{4} \right) \) where the sequence of equilibria (as \( \phi \) increases) is minimum differentiation, minimum or maximum differentiation and then maximum differentiation. We then provide a graph for a high level of \( v \left( v = \frac{5}{2} \right) \) where the sequence of equilibria (as \( \phi \) increases) is minimum differentiation, partial differentiation and then maximum differentiation.

First, we present the graph for \( v = \frac{5}{4} \).

The X-axis is \( \phi \) and the Y-axis is profits. We present four curves: The blue curve represents profits for the firms when both firms locate at \( \frac{1}{2} \). The red curve represents profits for the firms when both firms locate at the extreme ends of the line. Finally, when there is partial differentiation, the firm located at the center of the market (orange) earns a different profit than the firm that is located at the edge of the market (green). We observe that when \( \phi \) is small, the profits under minimum differentiation are greater than the profits for an extreme firm under partial differentiation; thus minimum differentiation is the equilibrium. Also, profits under maximum differentiation are less than the profits for a central firm under partial differentiation, so locating at the center of the market is the only pure-strategy in locations outcome. At \( \phi \) just over 0.55, we observe the green
line moving above the blue curve, indicating that if both firms were located at the center of the market, one of the firms will deviate to one of the extreme positions. However, profits for the firm in the center under partial differentiation (the orange curve) is still higher than profits if each firm located at the opposite ends of the market (the red curve); hence, the equilibrium is partial differentiation. Eventually, at just under 0.85, the red curve rises above the orange curve, implying that firms will not partially differentiate at high $\phi$, but rather both firms would locate at the extreme ends of the market.

Second, we present the graph when $v = \frac{5}{4}$.

When $v = \frac{5}{4}$, we obtain similar outcome at very low $\phi$ and very high $\phi$, but at intermediate $\phi$ (approximately 0.5, where the red curve crosses the blue curve, to just below 0.9 where the green curve crosses the blue curve), we observe that the blue curve is above the green curve, indicating that if both firms are located at the center of the market no firm would move to an extreme location, while the red curve is above the orange curve, indicating that if both firms are located at the edge of the market, neither firm would deviate to the market center. Thus, both maximum differentiation and minimum differentiation are equilibria, although the equilibrium with maximum differentiation produces higher profits for the firms.