The Dynamics of the Implied Volatility Surface*

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August 7, 2018

Abstract

The term structure of the U.S. market implied volatility is upward-sloping in good and normal times but downward-sloping in bad times. In addition, the implied volatility features a smirk in bad and normal times; the implied volatility of out-of-the-money put options is larger than that of at-the-money and in-the-money puts. As economic conditions improve, the implied volatility resembles more and more to a smile. We show that an asset-pricing model with rare disasters followed by recoveries is able to explain the aforementioned empirical regularities, whereas ignoring recoveries yields inconsistent predictions.

Keywords: Implied Volatility Surface Dynamics, Rare Disasters, Recoveries

*We are particularly grateful to the Canadian Derivatives Institute (CDI) for their generous financial support.

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1 Introduction

Extensive research has been devoted to the pricing of options (Black and Scholes, 1973; Merton, 1973) and to the modeling of the implied volatility surface (Gatheral and Taleb, 2011) because the size of the option market and its influence on the global financial market has grown exponentially over the years. One of the most famous model is that of Heston (1993), which assumes that stock returns feature stochastic volatility. In contrast to Black and Scholes' model, Heston's model yields an implied volatility that varies with the moneyness of the option, which is consistent with empirical evidence (Derman, Miller, and Park, 2016).

In this paper, we show empirically that the implied volatility increases with the maturity of the option in good and normal times. As economic conditions deteriorate, the term structure of implied volatility becomes downward-sloping. Furthermore, the dependence of the implied volatility on moneyness varies with economic conditions. The implied volatility features a smirk in bad and normal times, but resembles more and more to a smile as economic conditions improve. We build an equilibrium model in which rare economic disasters are followed by recoveries, and show that the aforementioned empirical findings are consistent with the predictions of the model.

Our empirical study uses an extensive database of daily S&P 500 (SPX) European options. The sample starts in January 1996 and ends in April 2016. The slope of the term structure is measured daily as the difference between the IV of two (synthetic) options with equal moneyness and varying maturities. To study the conditional movements of the term structure slope, business and financial cycles (which allow us to separate "good times" and "bad times") are identified using a set of 10 economic and financial indicators.[1] We also compute aggregate measures of economic and financial cycles, namely the Aggregate Economic Indicator and the Aggregate Financial Indicator, corresponding to the normalized sum of all individual indicators in each category. Further, we explore the dynamics of the implied volatility smile/smirk with regards to economic and financial conditions to provide a comprehensive view on the conditional patterns of the IVS.

Empirical results show a cyclical term structure slope, where slopes are lower in bad times (and typically negative) than in good times. The average slope between at-the-money SPX options with 120 and 60 days to maturity is 0.57% in good times and -0.79% in bad times, based on NBER recession dates. We obtain similar results with the other economic and

[1]Economic indicators include the NBER Recession Indicator, the OECD Recession Indicator, Hamilton's GDP-based Recession Indicator, the Aruoba-Diebold-Scotti Business Conditions Index, and the FRB Diffusion Index for Current General Activity. Financial indicators include year-on-year returns of the S&P 500 Index, the TED spread, the BofA Merrill Lynch US High Yield Option-Adjusted Spread, the CBOE Volatility Index (VIX), and the Excess-Bond Premium (EBP).

financial indicators. The differences between the average IV slopes are always statistically significant. When regressing daily term structure slopes against our set of economic and financial indicators, we find that the main drivers of the slope are the VIX and the EBP. The slope is also more sensitive to fluctuations of financial conditions than of economic conditions. In addition, the influence of economic and financial conditions on the slope is more pronounced during bad times. For example, our Aggregate Financial Indicator explains 45% of the time-variation in the IV slope for at-the-money SPX options. When conducting the regression for good and bad times separately, the explanatory power becomes 6.9% and 36.9%, respectively, which shows a stronger relationship between the IV slope and market conditions in periods of stress. We find similar results when considering aggregate economic conditions.

Regarding the shape of the volatility surface, we find that the smile is also more pronounced in good times than in bad times. The average skew, capturing the steepness of the smirk between 1-month options with moneyness levels of 1.05 and 1.00 (defined as the strike price divided by the price of the underlying asset), declines from -2.35% in good times to -3.27% in bad times, based on NBER recession dates. On the other hand, the skew between options with moneyness levels of 1.00 and 0.95 increases from -4.73% in good times to -4.13% in bad times. The differences between the average skews are statistically significant. Overall, the opposite variations for the 1.05-1.00 and 1.00-0.95 specifications confirm a more pronounced smile in good economic and financial times.

To explain the observed relationship between the implied volatility surface and economic conditions, we consider a pure-exchange Lucas (1978) economy populated by a representative agent with Epstein and Zin (1989) preferences. In equilibrium, the agent consumes the aggregate output of the economy, which is subject to rare disasters occurring with stochastic intensity (Wachter, 2013). The occurrence of each disaster is followed by a recovery period, which is defined as a period of abnormally high economic growth. Empirical evidence of economic disasters followed by recoveries are provided by Gourio (2008), Nakamura, Steinsson, Barro, and Ursua (2013), and Cavallo, Galiani, Noy, and Pantano (2013).

In a model with recoveries, the implied volatility increases monotonically with the option's maturity in good and normal times. As economic conditions deteriorate, the implied volatility becomes a hump-shaped function of the maturity for in-the-money and at-the-money puts, and a monotonically decreasing function of the maturity for out-of-the money puts. In contrast, the implied volatility is monotonically increasing in the maturity irrespective of economic conditions in a model without recoveries. The reason is that recoveries dampen the risk in the long run, and this effect is particularly important when the intensity of disaster is high, i.e., in bad times.

The existence of post-disaster recoveries implies an implied volatility smirk in bad and normal times, and an implied volatility smile in good times. The economic mechanism is as follows. On the one hand, the potential occurrence of a disaster—a period of abnormally low economic growth—makes the implied volatility of out-of-the-money put options higher than that of at-the-money and in-the-money puts (Benzoni, Collin-Dufresne, and Goldstein, 2011). On the other hand, a recovery—a period of abnormally high economic growth—has the exact opposite impact. It makes the implied volatility if in-the-money put options higher than that of at-the-money and out-of-the-money puts. The former effect dominates the latter when the intensity of disasters is not too low, leading to an implied volatility smirk in bad and normal times. In contrast, both effects have impacts of similar magnitudes when the disaster intensity is low, therefore generating an implied volatility smile in good times. At odds with our empirical findings, a model abstracting from recoveries yields an implied volatility smirk irrespective of economic conditions.

Finally, we show that recoveries also help explain the observed relationships between economic conditions and the term structures of interest rates, dividend strip return volatility, and dividend strip risk premia. Consistent with the empirical findings of Kurmann and Otrok (2013), the model with recoveries implies that the term structure of interest rates is upward-sloping in bad and normal times but downward-sloping in good times. In contrast, a model ignoring recoveries yields a decreasing term structure of interest rates unless economic conditions are particularly bad. Furthermore, the slope of the term structures of dividend strip return volatility and risk premia is procyclical in the presence of recoveries, which is in line with empirical evidence (van Binsbergen, Hueskes, Koijen, and Vrugt, 2013; van Binsbergen and Koijen, 2016). In particular, the term structure of dividend strip return volatility is downward-sloping in bad and normal times, and becomes upward-sloping in good times. In contrast, the term structures of dividend strip return volatility and risk premia are monotone increasing in the model without recoveries, which is inconsistent with the data.

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section 3 describes the data. Sections 4 and 5 provide an empirical investigation of the relationship between the U.S. equity market's implied volatility surface and economic conditions. Section 6 presents and solves the model. Section 7 illustrates the predictions of the model and Section 8 concludes. Derivations, robustness checks, and additional results are provided in the Appendix.

2 Literature Review

This section surveys the literature on the determinants of IVS movements, which relates to variations in the term structure and in the smile patterns. We also review applications of IV term structure variability in the option literature.

2.1 Term Structure Patterns

The variability of the IV term structure is well documented in the finance literature, in particular for currencies. Xu and Taylor (1994) analyze term structure movements of at-the-money foreign exchange options on various currency pairs (USD/GBP, USD/DEM, USD/JPY and USD/CHF options traded on the Philadelphia Stock Exchange), between 1985 and 1989. They show that IV variability is greater for short-term options. They also discern different average levels of volatility, possibly signaling the presence of time-varying regimes. Campa and Chang (1995) extend the research of Xu and Taylor (1994) by studying IV term structure dynamics for the same set of currencies, between 1989 and 1992. They note that: i) the term structure varies significantly over time, with similarities across all currencies; ii) the higher variability of short-term implied volatilities is the leading driver of term structure movements; and iii) term structure slopes react strongly to market shocks, such as the 1992 crisis.[2]

Following Campa and Chang (1995), Mixon (2007) examines the IV term structure for multiple international stock indices, between 1994 and 2001. The slope of the term structure is measured using the IV differential between at-the-money call options with 1 and 12 months to maturity. Although his research finds that the term structure is generally upward sloping (except for the Japanese Nikkei Index), Mixon (2007) notes that the slope becomes negative in times of market turbulence (including late 1997, 1998 and 2001).

Overall, studies on the determinants of term structure variations remain relatively scarce in the option literature. The papers mentioned above support a high degree of variability in IV term structures and a negative correlation between the term structure slope and the underlying asset's volatility. In addition, Aijo (2008) finds that term structure variations for options on a given asset can have a meaningful impact on term structure dynamics for other securities. In this case, he demonstrates that movements in the term structure of DAX options influence IVS dynamics for options on the Swiss Market Index and the EuroStoxx 50.[3] In this paper, we will seek to understand IVS variations for SPX options as well as for

[2]This crisis refers to the "Black Wednesday" that occurred in the U.K. in 1992, when the government was compelled to withdraw the pound sterling from the European Exchange Rate Mechanism (ERM).

[3]The DAX is a blue-chip stock market index consisting of the 30 largest firms on the Frankfurt Stock Exchange. The Euro Stoxx 50 is an index of Eurozone equities designed by STOXX (Deutsche Borse Group).

various international stock indices.

2.2 Smile Patterns

Although this paper mainly examines conditional variations of the IV term structure, reviewing previous studies on the determinants of smile patterns provides valuable insights on the potential drivers of IVS variations and the techniques to uncover them. Pena, Rubio, and Serna (1999) study smile dynamics on the IBEX-35 Index (the benchmark of the Bolsa de Madrid, Spain's largest stock exchange) between 1994 and 1996. Daily smile shapes are estimated using a volatility averaging process in pre-established moneyness intervals. A deterministic function of moneyness is then calibrated to the data to obtain slope and curvature coefficients. Granger tests indicate that transaction costs (i.e., bid-ask spreads), market uncertainty (the annualized standard deviation of the IBEX), market momentum (a moving average measure of the IBEX) and time to expiration are the key determinants of smile variability.

Hafner, Wallmeiery, and G. (2000) arrive at comparable conclusions in their analysis of the DAX Index between 1995 and 1999. Implied volatilities are estimated for a maturity of 45 days, using weighted least squares spline regressions. The steepness of the smile, or skew, is the differential between implied volatilities at distinct moneyness levels. Hafner et al. (2000) find that market frictions (measured as a function of trading volume), changing asset volatility and jumps in the underlying security are important determinants of smile patterns.

Bates (2000) addresses the impact of economic instability on IV patterns. He portrays the evolution of the smile for S&P 500 options, following the 1987 financial crisis. Cubic splines are used to fit daily data, with IV differentials to measure the skew-similar to the approach of Hafner et al. (2000). Empirical observations show a smirk-shaped IV profile between 1988 and 1993. Higher IV spreads are observed between OTM puts and ATM options in times of market turbulence, notably during the crisis of October 1989 and the Kuwait crisis of 1990.

Finally, Bollen and Whaley (2004) investigate the impact of supply and demand pressures on the level and shape of the IV smile. The study examines the IV behavior of S&P 500 Index options, between June 1988 and December 2000, as well as options on 20 individual stocks, between January 1995 and December 2000. To characterize the IV smile, Bollen and Whaley (2004) compute the average implied volatility for five different moneyness categories. The skew is the difference between the average implied volatilities in two different categories. Their results suggest that option supply and demand levels have a significant influence on the level and skew of the IV smile. Changes in IV are most strongly affected by net buying

pressures on out-of-the-money S&P 500 put options.

2.3 Model with Disasters and Recoveries

Our theoretical model is closely related to the asset pricing literature considering rare economic disasters. Gabaix (2012) and Wachter (2013) show that a large number of asset pricing puzzles can be resolved when accounting for either a time-varying severity of disasters or a time-varying probability of occurrence. Tsai and Wachter (2016) show that a model with rare booms and disaster can explain the characteristics of the value premium. Seo and Wachter (2017) explain the high level of implied volatility relative to realized volatility together with the implied volatility smirk by considering a stochastic intensity rare disaster model. Hasler and Marfè (2016) show that considering post-disaster recoveries helps explain the empirically observed downward-sloping term structures of dividend strip return volatility and risk premia. Motivated by the empirical findings of Gourio (2008) and Nakamura et al. (2013) which provide evidence that disasters are followed by recoveries, we contribute to the literature by showing that post-disaster recoveries have to be taken into account in order to understand the observed relationship between the implied volatility surface and economic conditions.

3 Data Set

This section describes the option data set employed throughout this study, along with the various filters applied to the initial database. Various estimation techniques are presented for the computation of implied volatility values at specific maturity and moneyness levels. We also outline the economic and financial indicators used in the conditional analysis of the term structure.

3.1 Database Description

We retrieve end-of-day option data from the OptionMetrics Ivy database.[4] Each observation includes the contract bid and ask prices, strike price, expiration date, exercise style, trading volume, open interest and implied volatility, as well as the closing price of the underlying security. The focus of this paper is on the SPX options market, i.e., the most liquid product offered on the Chicago Board Options Exchange, between January 1996 and April 2016. We

[4]OptionMetrics Ivy DB US includes historical data for all US listed equities and market indices, as well as all US listed index and equity options, from 1996 to 2016.

also conduct an analysis of IV behavior for options on the Russell 2000 and other non-US securities for robustness (see Appendix A)

OptionMetrics adopts industry standards to compute implied volatilities. A standard inversion of the Black-Scholes formula is used for European options. Implied volatilities for American options are obtained through OptionMetrics' proprietary algorithm based on the Cox-Ross-Rubinstein binomial tree model (Cox, Ross, Rubinstein, et al., 1979). For all options, the OptionMetrics algorithm computes interest rates from ICE IBA LIBOR rates and settlement prices of CME Eurodollar futures,[5] with linear interpolation between market data points. Finally, the algorithm uses a linear regression model based on the put-call parity to compute the continuous dividend yield on stock indices. For more information, refer to the OptionMetrics Reference Manual.

Various filters are applied onto the OptionMetrics database to generate the final data set, following standard practices in the option literature (see, e.g., Bakshi, Cao, and Chen (1997); Dumas, Fleming, and Whaley (1998); Skiadopoulos, Hodges, and Clewlow (2000)). These measures, described below, are implemented to minimize market microstructure concerns and to remove the impact of outliers. The descriptive statistics for the final sample of S&P 500 options are presented in Table 1.

i. Options for which time to expiry is inferior to a week are removed from the sample. When options are close to maturity, the corresponding implied volatility metrics become very sensitive to pricing errors and often behave erratically.

ii. Options with maturities over 365 days are omitted, as they are often illiquid.

iii. Options with moneyness levels below 0.8 or above 1.2 are excluded, due to liquidity concerns and IV sensitiveness to pricing errors. We define moneyness as the ratio of the strike price to the value of the underlying asset.

iv. Options with IV values below 4% or above 90% are removed from the sample. Such outliers are often associated with pricing errors.

v. Options priced below $0.10 (as determined by the bid-ask midpoint) are excluded to reduce distortions from variations in discrete market prices.

vi. Options with zero volume or open-interest are removed from the sample.

vii. We only consider out-of-the-money options - i.e., call options with moneyness equal or above 1, and put options with moneyness equal or below 1. In-the-money options are often more expensive and less liquid.

[5]Overnight London Interbank Offered Rate (LIBOR) rates provided by ICE Benchmark Administration Limited (IBA). Eurodollar Futures quotes provided by the CME Group (Chicago Mercantile Exchange & Chicago Board of Trade).

	# Obs.	Median	Average	Std. Dev.	Min.	Max.
Implied Volatility	1,247,116	19.2%	20.7%	8.8%	4.9%	89.9%
Moneyness Level	1,247,116	97.4%	97.2%	8.5%	80%	120%
Maturity (Days)	1,247,116	44.0	67.2	70.4	6.0	365.0
Daily Volume, Total	5,113	207,777	265,900	228,970	4,450	2,003,114
Daily Volume, Per Option Average	5,113	960	1,226	819	63	7,625
Daily Open Interest, Total	5,113	2,826,014	2,873,700	2,045,600	196,000	7,535,554
Daily Open Interest, Per Option Average	5,113	11,619	14,087	7,807	2,820	38,377
Options Traded per Day	5,113	126	244	270	29	1,727
Available Maturities per Day	5,113	6.0	8.7	4.9	4.0	27.0
Av. # of Mon. Levels per Maturity, per Day	5,113	20.0	23.1	10.2	7.2	67.9

Table 1: Descriptive Statistics of the Sample of S&P 500 (SPX) Options
This table reports the descriptive statistics for the implied volatility, the moneyness level (defined as the ratio of the strike price to the value of the underlying asset), the maturity of the options (in days), the daily trading volume (number of traded contracts) for all available options, the daily average trading volume per option, the daily open interest (number of contracts outstanding) for all options, the daily average open interest per option, the number of options traded per day, the number of available maturities per day, and the average number of moneyness levels per available maturity, per day. The data set covers the period between January 4, 1996 and April 29, 2016.

3.2 Implied Volatility Estimation

Option contracts are traded for a finite number of unevenly distributed maturities and moneyness levels. We consider various estimation techniques in this paper to compute daily IV values at any given maturity or moneyness level. A multivariate interpolation approach, inspired by Carr and Wu (2008), will serve as the basis for this study. Appendix A presents variations of that method, as well as an alternative estimation technique based on cubic spline interpolation, for robustness.

The multivariate interpolation approach that we propose is as follows. For a target time to maturity τ, we first choose the two closest surrounding maturities among all available options. At each of the two maturities, we linearly interpolate between traded options to estimate volatility metrics at the desired moneyness level k. Lastly, we interpolate between the two estimated values to obtain $IV(\tau, k)$. This approach is hereby termed the Proximal Trilinear Interpolation Technique (PTIT). In all steps of the PTIT, we allow for extrapolation of IV values when market data is insufficient to interpolate - within pre-determined limits of maturity, moneyness, and implied volatility. Appendix B describes this technique in greater detail and proposes several variations.

We consider the PTIT for two main reasons. First, only four surrounding options are

sufficient to estimate one IV at specific moneyness and maturity coordinates. As such, we can precisely calculate an IV without using all the points on the surface - which, in turn, reduces calculation time. Second, a slight variation of the basic PTIT allows us to incorporate a liquidity factor in the selection of the "closest neighbors." We can thus estimate IV values using the most traded options available near the target point, without having to increase liquidity filters on the original database.

3.3 Economic and Financial Conditions

We now present the various economic and financial indicators that we consider to capture the business cycle and, thus, to distinguish between good and bad times.

3.3.1 Economic Indicators

We measure economic activity using the following indicators: 1) The NBER Recession Indicator (NBER), provided by the National Bureau of Economic Research, reflects the US business cycle based on real GDP, real income, employment, industrial production, as well as wholesale-retail sales; 2) The OECD Recession Indicator (OECD) combines economic and market gauges to determine specific start- and end-dates for economic downturns, notably consumer sentiment, industrial confidence, NYSE share prices, and interest rate spreads; 3) Hamilton's GDP-Based Recession Indicator Index (HGR) uses a two-state regime switching model to determine periods of recessions, based on GDP growth data (see Hamilton (2011)) The Aruoba-Diebold-Scotti Business Conditions Index (ADS) tracks business conditions through weekly initial jobless claims, monthly payroll employment, industrial production, personal income less transfer payments, manufacturing and trade sales, as well as quarterly real GDP; 5) the FRB Diffusion Index for Current General Activity (FRB) - based on the Manufacturing Business Outlook Survey of the Federal Reserve Bank of Philadelphia - reports the change in general business activity for manufacturing firms. Table 2 reports the descriptive statistics of these economic indicators, with visual representations in Figure 1. All data are retrieved from FRED (website of the Federal Reserve Bank of St. Louis), except for the ADS Index, issued by the Federal Reserve Bank of Philadelphia.

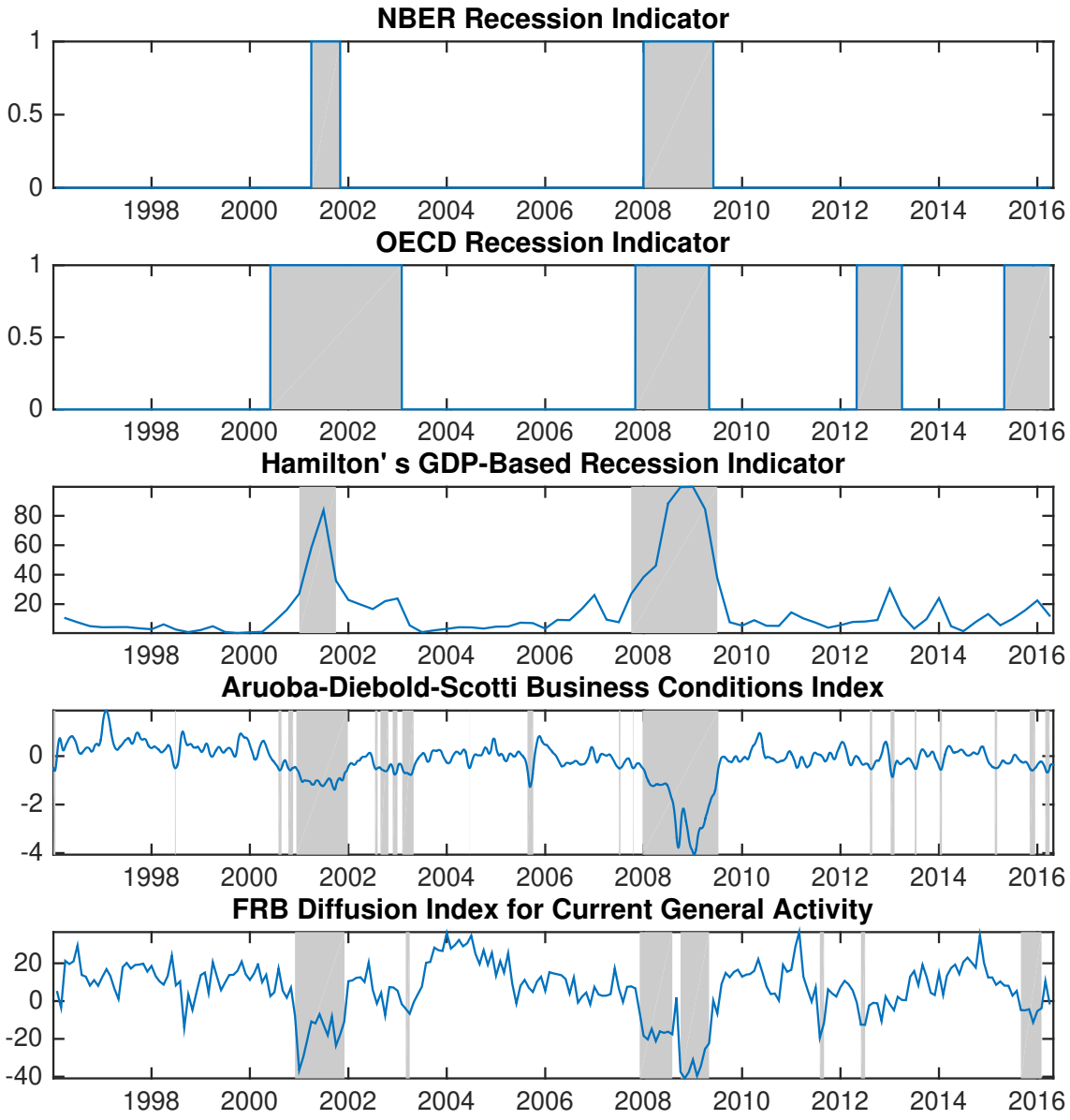

Figure 1: Time Series of the Economic Indicators
This figure displays the NBER Recession Indicator (monthly), the OECD Recession Indicator (monthly), Hamilton's GDP-based Recession Indicator Index (quarterly), the Aruoba-Diebold - Scotti Business Conditions Index (daily), and the FRB Diffusion Index for Current General Activity (monthly). Grey-zones denote periods of high economic stress, as defined by each indicator. Section 3.3 defines the periods of good and bad times for each indicator. The sample covers the period between January 4, 1996 and April 29, 2016.

	# of Obs.	Mean	Std. Dev.	Minimum	Maximum
Panel 1: Full Period					
NBER	243	0.11	0.31	0.00	1.00
OECD	243	0.31	0.46	0.00	1.00
HGR	81	16.33	22.30	0.33	99.70
ADS	5113	-0.19	0.76	-4.07	1.88
FRB	243	6.36	14.43	-40.90	36.50
Panel 2: Good Times					
NBER	217	0.00	0.00	0.00	0.00
OECD	167	0.00	0.00	0.00	0.00
HGR	65	7.57	5.47	0.33	22.88
ADS	4090	0.08	0.38	-0.49	1.88
FRB	194	12.01	8.57	-2.30	36.50
Panel 3: Bad Times					
NBER	26	1.00	0.00	1.00	1.00
OECD	76	1.00	0.00	1.00	1.00
HGR	16	51.93	29.00	23.83	99.70
ADS	1023	-1.27	0.91	-4.07	-0.49
FRB	49	-15.99	10.71	-40.90	-3.50

Table 2: Descriptive Statistics of the Economic Indicators
This table reports the descriptive statistics for the NBER Recession Indicator (NBER), the OECD Recession Indicator (OECD), Hamilton's GDP-based Recession Indicator Index (HGR), the Aruoba-Diebold-Scotti Business Conditions Index (ADS), and the FRB Diffusion Index for Current General Activity (FRB). Panel 1 includes statistics for the entire period. Panels 2 and 3 cover good and bad times, respectively. Section 3.3 defines the periods of good and bad times for each indicator. The sample covers the period between January 4, 1996 and April 29, 2016.

We apply decision-rules for each indicator to distinguish between good and bad times. The NBER and OECD indicators are straightforward to define: values of 0 and 1 designate times of low and high economic stress, respectively. For the other indicators, dates corresponding to the bottom quintile of the time-series capture bad times. The exception is the HGR indicator, which is countercyclical. As such, the top quintile reflects bad times in this case.

3.3.2 Financials Indicators

Five financial indicators are considered in this study: 1) Year-on-year returns of the S&P 500 Index (YOY) determine the level of stock market performance; 2) The TED spread (TED) - the rate differential between the 3-Month U.S.-based LIBOR and the 3-Month Treasury Bill - is an indication of monetary liquidity and overall economic stability, and thus measures the perceived level of risk in the financial system; 3) The BofA Merrill Lynch US High Yield Option-Adjusted Spread (BAML) indicates the state of the corporate economy through credit quality. It is measured as the spread between a computed Option-Adjusted Spread (OAS)

index of junk-bonds (rated BB or below) and a spot Treasury curve; 4) The VIX, provided by the Chicago Board Options Exchange, reflects the market's 30-day volatility expectations and measures market uncertainty. 5) the EBP, introduced by Gilchrist and Zakrajsek (2012), serves as a measure of investor sentiment. Descriptive statistics are reported in Table 3, with visual representations in Figure 2. All data is retrieved from FRED (website of the Federal Reserve Bank of St. Louis), except for S&P 500 values and the Excess-Bond Premium indicator, issued by OptionMetrics and the U.S. Federal Reserve, respectively.

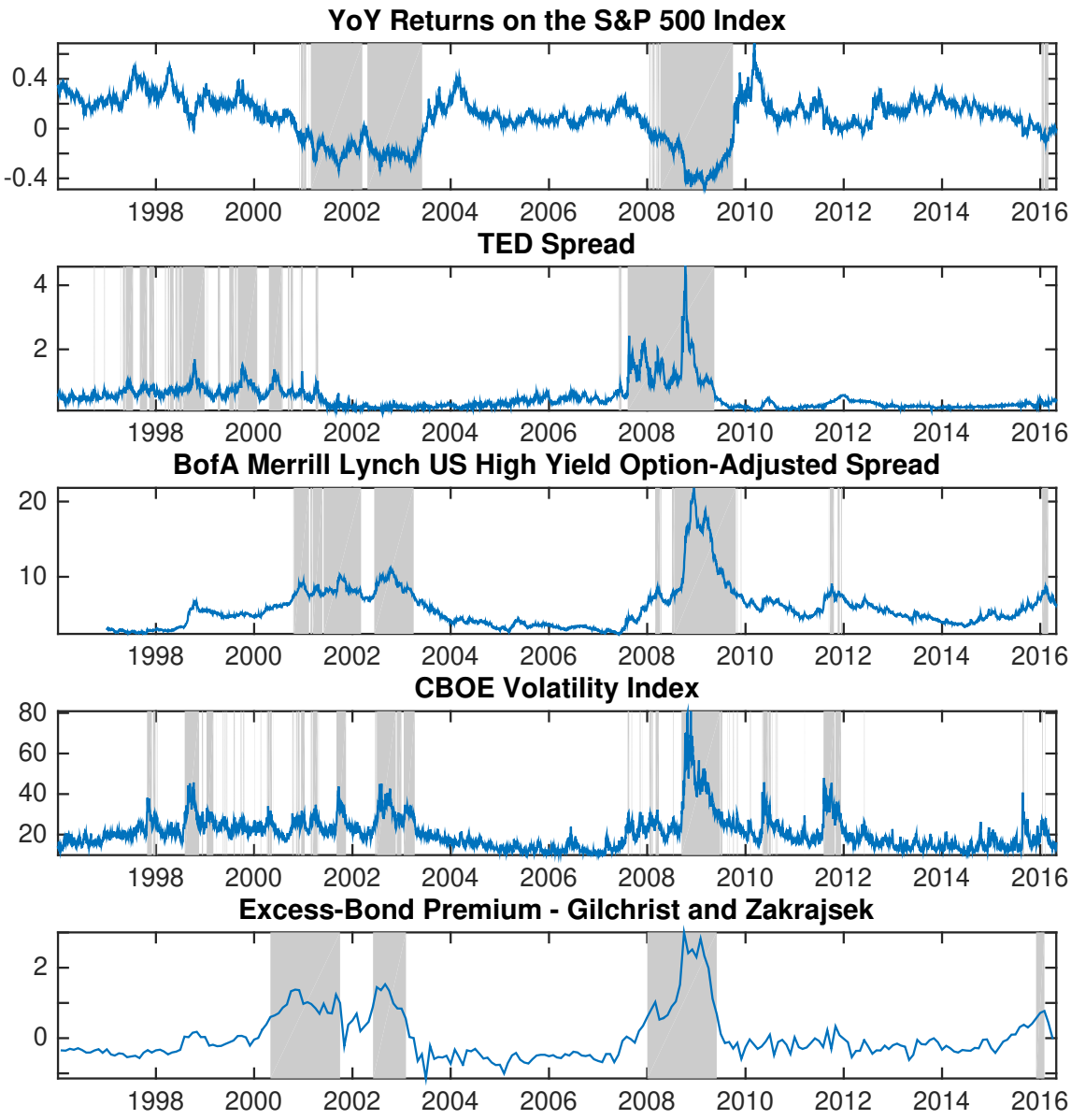

Figure 2: Time Series of the Financial Indicators
This figure displays year-on-year returns of the S&P 500 Index (daily), the TED spread (daily), the BofA Merrill Lynch US High Yield Option-Adjusted Spread (daily), the CBOE Volatility Index (daily), and the Excess-Bond Premium (monthly). Grey-zones denote periods of financial instability, as defined by each indicator. Section 3.3 defines the periods of good and bad times for each indicator. The sample covers the period between January 4, 1996 and April 29, 2016.

	# of Obs.	Mean	Std. Dev.	Minimum	Maximum
Panel 1: Full Period					
YOY	5113	0.08	0.18	-0.49	0.69
TED	4987	0.49	0.42	0.09	4.58
BAML	5043	5.82	2.84	2.41	21.82
VIX	5111	20.95	8.28	9.89	80.86
EBP	243	0.02	0.68	-1.14	3.00
Panel 2: Good Times					
YOY	4090	0.16	0.11	-0.05	0.69
TED	3958	0.34	0.15	0.09	0.68
BAML	4031	4.77	1.39	2.41	7.57
VIX	4088	17.81	4.12	9.89	25.65
EBP	194	-0.26	0.29	-1.14	0.50
Panel 3: Bad Times					
YOY	1023	-0.21	0.10	-0.49	-0.05
TED	1029	1.10	0.53	0.69	4.58
BAML	1012	10.00	3.25	7.58	21.82
VIX	1023	33.49	8.85	25.66	80.86
EBP	49	1.15	0.62	0.53	3.00

Table 3: Descriptive Statistics of the Financial Indicators
This table reports the descriptive statistics for the year- on-year returns of the S&P 500 Index (YOY), the TED spread (TED), the BofA Merrill Lynch US High Yield Option- Adjusted Spread (BAML), the CBOE Volatility Index (VIX), and the Excess-Bond Premium (EBP). Panel 1 shows statistics for the entire considered period. Panels 2 and 3 cover good and bad times, respectively. Section 3.3 defines the periods of good and bad times for each indicator. The sample covers the period between January 4, 1996 and April 29, 2016.

We apply quintile-based decision rules to each financial indicator to determine good and bad times. Dates corresponding to the top quintile of the time-series are consistent with periods of financial stress. The YOY indicator is an exception, with the bottom quintile serving as an indication of financial stress.

4 Term Structure Analysis

The following section examines the dynamics of the IV term structure for S&P 500 options. In particular, we study how the slope of the IV term structure varies with economic and financial conditions.

4.1 Term Structure Measurement

We measure the daily slope of the term structure by means of an IV differential method, using the implied volatility span between two options with equal moneyness levels and different

maturities.

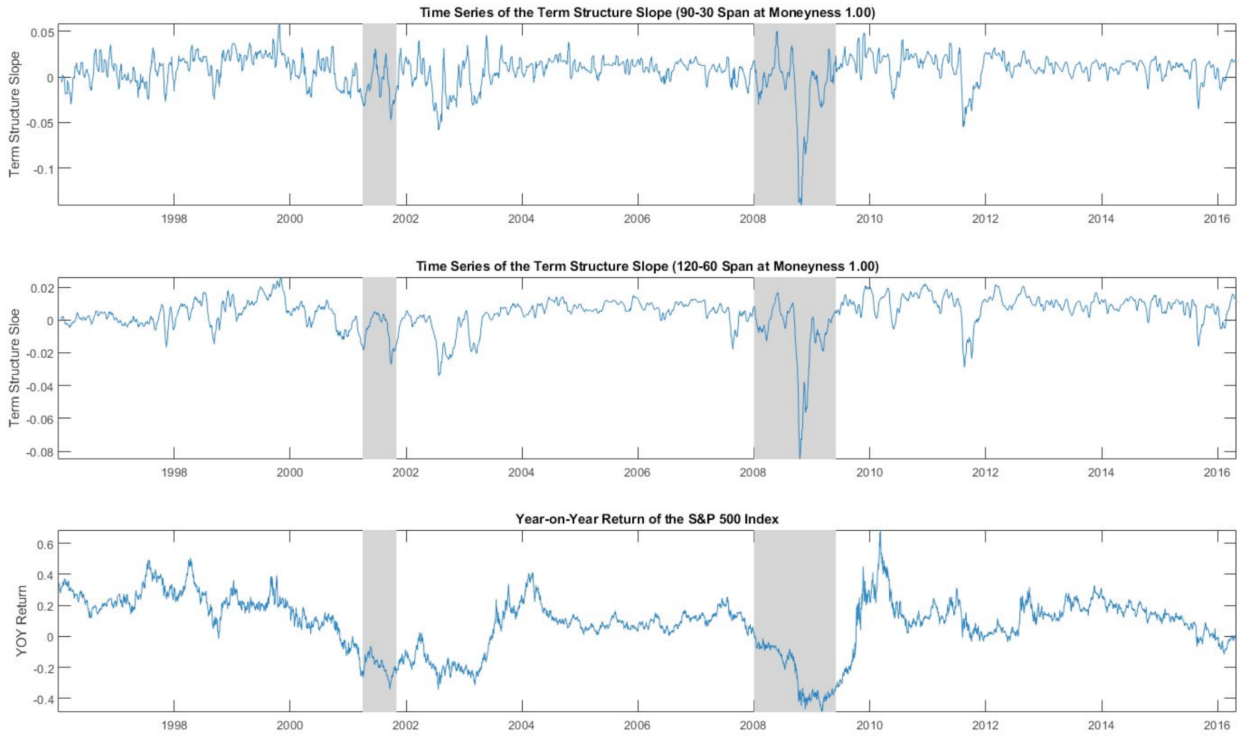

Figure 3: Dynamics of the Slope of the Term Structure - ATM options
This figure shows the evolution of the term structure slope for S&P 500 options, which is the implied volatility (IV) differential between two options of different maturities but identical moneyness. The top panel considers options with 90 and 30 days to maturity, while the middle panel considers options with 120 and 60 days to maturity. All options are at-the-money (ATM). IV values are computed using the Proximal Trilinear Interpolation Technique. The time series of the top two panels are smoothed using a 10-day moving average for visual clarity. The bottom panel shows the year-on-year return of the S&P 500 Index for comparative purposes. Shaded areas represent periods of NBER recessions. The sample covers the period between January 4, 1996 and April 29, 2016.

The use of IV differentials is standard in the option literature - see Mixon (2007) and Vasquez (2017) with regards to term structure slopes, or Bates (2000) and Hafner et al. (2000) for representations of smile shapes. The slope is defined as follows:

$$TS_t = IV_t(k, T_2) - IV_t(k, T_1)$$

where TS_t is an indication of the term structure slope on day t, k is the moneyness level, and T is the time to maturity for each option (with $T_2 > T_1$). Both IV values are obtained through the Proximal Trilinear Interpolation Technique. We consider two sets of maturities - namely options with 90 and 30 days to maturity, as well as options with 120 and 60 days

to maturity. Descriptive statistics of the term structure time series are reported in Table 4, with visual representations in Figure 3.

	Panel 1: 90-30 Term Structure			Panel 2: 120-60 Term Structure		
	0.95 Mon.	1.00 Mon.	1.05 Mon.	0.95 Mon.	1.00 Mon.	1.05 Mon.
Average Slope	-0.0130	0.0065	0.0080	-0.0036	0.0042	0.0087
Standard Deviation	0.0254	0.0250	0.0231	0.0117	0.0117	0.0114
P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 4: Slope of the Term Structure by Moneyness
This table reports descriptive statistics for the slope of the term structure for S&P 500 options. Statistics include the average term structure slope, the standard deviation of the term structure, and the p-value of the data set for a one-sample t-test. Panel 1 considers the slope between implied volatility (IV) observations for options with 90 and 30 days to maturity. Panel 2 considers options with 120 and 60 days to maturity. Each panel reports statistics for three moneyness levels (defined as the strike price of the option to the price of the underlying asset): 0.95, 1.00, and 1.05. All IV values are computed using the Proximal Trilinear Interpolation Technique. The sample covers the period between January 4, 1996 and April 29, 2016.

Observations for S&P 500 Index options indicate a positive average slope at moneyness levels of 1.00 and 1.05, and a negative average slope at a moneyness level of 0.95. For the 90-30 term structure specification, the average slopes at moneyness levels of 0.95, 1.00 and 1.05 are -1.30%, 0.65% and 0.80%, respectively. For the 120-60 specification, the average slopes at moneyness levels of 0.95, 1.00 and 1.05 are -0.36%, 0.42% and 0.87%, respectively. Based on one-sample t-tests, all slopes are statistically significant. Furthermore, slopes appear to be highly time varying (see Figure 3).

4.2 Conditional Term Structure Analysis

We now analyze the dynamics of the slope of the term structure. We first compare average term structure slopes in good and bad times and then carry a regression analysis. In this section, slopes are computed daily using the IV differential between two options with 120 and 60 days to maturity. Section A.2 analyzes other maturities for robustness.

	(1) Unconditional	(2) Good Times	(3) Bad Times	(4) Difference (2-3)	(5) P-Value
Panel 1: Moneyness of 0.95					
NBER	-0.0036	-0.0024	-0.0136	0.0112	0.0000
OECD	-0.0036	-0.0013	-0.0087	0.0074	0.0000
HGR	-0.0036	-0.0021	-0.0096	0.0075	0.0000
ADS	-0.0036	-0.0017	-0.0112	0.0095	0.0000
FRB	-0.0036	-0.0017	-0.0111	0.0094	0.0000
YOY	-0.0036	-0.0012	-0.0130	0.0117	0.0000
TED	-0.0036	-0.0026	-0.0074	0.0048	0.0000
BAML	-0.0036	-0.0007	-0.0142	0.0135	0.0000
VIX	-0.0036	-0.0002	-0.0169	0.0166	0.0000
EBP	-0.0036	-0.0013	-0.0128	0.0115	0.0000
Panel 2: Moneyness of 1.00					
NBER	0.0042	0.0057	-0.0079	0.0135	0.0000
OECD	0.0042	0.0069	-0.0018	0.0087	0.0000
HGR	0.0042	0.0059	-0.0028	0.0087	0.0000
ADS	0.0042	0.0065	-0.0048	0.0113	0.0000
FRB	0.0042	0.0064	-0.0044	0.0108	0.0000
YOY	0.0042	0.0070	-0.0068	0.0137	0.0000
TED	0.0042	0.0054	-0.0002	0.0055	0.0000
BAML	0.0042	0.0074	-0.0078	0.0152	0.0000
VIX	0.0042	0.0076	-0.0093	0.0169	0.0000
EBP	0.0042	0.0070	-0.0068	0.0138	0.0000
Panel 3: Moneyness of 1.05					
NBER	0.0087	0.0100	-0.0027	0.0128	0.0000
OECD	0.0087	0.0111	0.0032	0.0079	0.0000
HGR	0.0087	0.0102	0.0021	0.0082	0.0000
ADS	0.0087	0.0109	-0.0002	0.0110	0.0000
FRB	0.0087	0.0105	0.0012	0.0093	0.0000
YOY	0.0087	0.0114	-0.0022	0.0136	0.0000
TED	0.0087	0.0094	0.0058	0.0036	0.0000
BAML	0.0087	0.0118	-0.0030	0.0148	0.0000
VIX	0.0087	0.0115	-0.0027	0.0143	0.0000
EBP	0.0087	0.0113	-0.0019	0.0132	0.0000

Table 5: Conditional Descriptive Statistics of the S&P 500 IV Term Structure – Maturities of 120 and 60 Days

This table reports descriptive statistics of the term structure slope for various economic and financial conditions. The term structure slope is defined as the difference between the estimated implied volatilities of options with 120 and 60 days to maturity. Panel 1 to 3 show the results for moneyness levels of 0.95, 1.00 and 1.05, respectively. Columns (1) to (3) show average IV term structure slopes unconditionally, in good times, and in bad times, respectively. Column (4) presents the difference between (2) and (3), while Column (5) displays p-values for the two-sample t-test. Economic indicators include the NBER Recession Indicator (NBER), the OECD Recession Indicator (OECD), Hamilton's GDP-based Recession Indicator Index (HGR), the Aruoba - Diebold - Scotti Business Conditions Index (ADS), and the FRB Diffusion Index for Current General Activity (FRB). Financial indicators include year-on-year returns of the SP 500 Index (YOY), the TED spread (TED), the BofA Merrill Lynch US High Yield Option-Adjusted Spread (BAML), the CBOE Volatility Index (VIX), and the Excess-Bond Premium (EBP). Section 3.3 defines these various indicators and the indicator-specific periods of good and bad times. The sample covers the period between January 4, 1996 and April 29, 2016.

Table 5 shows that the slope is higher in good times than in bad times. For ATM options (Panel 2), the average slope is positive in good times (between 0.57% and 0.69% with economic indicators, and between 0.54% and 0.76% with financial indicators) but negative in bad times (between -0.79% and -0.18% with economic indicators, and between -0.93% and -0.02% with financial indicators). Panels 1 and 3 provide similar results for moneyness of 0.95 and 1.05. Results are thus consistent for in, at, or out of the money options.

The difference between the average slopes in good and bad times is statistically significant for all economic and financial indicators. Options with longer maturities thus display lower IV values than their shorter-term counterpart during periods of stress, suggesting market expectations that volatility will revert to long-term levels.

4.3 Regression Analysis

We now analyze the relationship between the slope of the term structure and the financial and economic indicators. We consider at-the-money S&P 500 options with 120 and 60 days to maturity.[6]

4.3.1 Methodology

All economic and financial indicators described in Tables 2 and 3 will serve as independent variables in the regression analysis below, either individually or jointly. Monthly indicators are transformed by keeping the indicator value for a given month to all days in that month. All independent variables are standardized - i.e., centered at a mean of 0 and variance of 1 - to allow for adequate comparisons between regression coefficients. The slope of the term structure is multiplied by 100. Altogether, we estimate the following model on a daily basis:

$$IV_t(T_2, k) - IV_t(T_1, k) = \beta_{i,0} + \beta_{i,1} \times IND_{i,t} + \varepsilon_{i,t}$$

where IV_t is the implied volatility that we estimate on day t using the PTIT approach, as a function of moneyness level k and maturity T (with $T_2 > T_1$); $\beta_{i,0}$ is the constant coefficient associated with the specification with indicator i, which corresponds to the average IV slope from the standardization of the independent variables; $\beta_{i,1}$ is the slope coefficient for indicator i; $IND_{i,t}$ is the value of indicator i on day t; and $\varepsilon_{i,t}$ is the error term for indicator i. Regressions are conducted for each indicator separately and for combinations of the various economic and financial indicators. We compute all t-statistics using Newey-West standard errors with 10 lags.

[6]Regressions with alternative maturities (e.g., 90-30) and other moneyness generate very similar results. The tables are unreported for space consideration but remain available upon request.

Additionally, we compute aggregate measures of business cycle variations that we use as independent variables for the regression analysis - namely the Aggregate Economic Indicator (AEI) and the Aggregate Financial Indicator (AFI). The AEI is computed as the sum of the economic indicators listed in Table 2, each standardized and adjusted so that bad times correspond to the lower part of the time series (i.e., the NBER, OECD and HGR indicators are reversed). The AEI is also centered at a mean of 0 and variance of 1. Similarly, the AFI is calculated as the standardized sum of the centered and directionally-adjusted financial indicators. Figure 4 illustrates the aggregate indicators and show that both measures capture distinct periods of economic and financial stress.

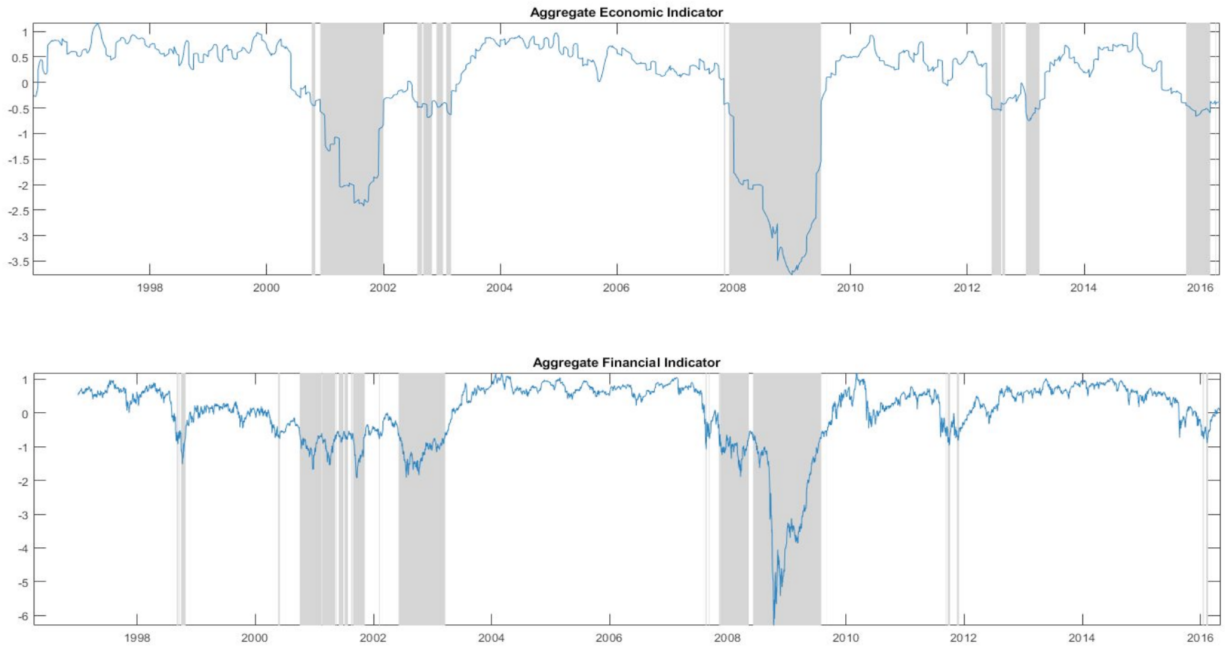

Figure 4: Dynamics of the Aggregate Economic and Financial Indicators
The top panel displays the evolution of the Aggregate Economic Indicator between January 4, 1996 and April 29, 2016, measured as the sum of the economic indicators listed in Table 2, each standardized and directionally adjusted. The AEI is centered at a mean of 0 and scaled at a variance of 1. The bottom panel shows the Aggregate Financial Indicator between January 2, 1997 and April 29, 2016, also measured as the standardized sum of the centered and directionally-adjusted financial indicators. Shaded areas represent periods of stress according to each indicator, i.e., the observations corresponding to the bottom quintile of the time series.

4.3.2 Main Results

Table 6 presents the regression results based on the individual economic indicators. Constant parameters, representing the average term structure slope, amount to 0.42% - in line with

our previous findings. The slope coefficients show the positive linear relationship between variations in economic conditions and the term structure slope. We obtain negative coefficients for the NBER, OECD and HGR indicators, which are high in bad times. Inversely, the cyclical ADS and FRB indicators display positive coefficients. All regression parameters are statistically significant, which indicates that the slope of the term structure is always higher in good times than in bad times. This finding is consistent across the various measures of economic conditions.

	(1) NBER	(2) OECD	(3) HGR	(4) ADS	(5) FRB	(6) Full
NBER	-0.4173***					0.0923
	(5.1800)					(1.0524)
OECD		-0.4037***				-0.1055
		(6.9467)				(1.6227)
HGR			-0.5011***			-0.4072***
			(5.6303)			(2.8481)
ADS				0.4331***		-0.0669
				(5.2716)		(0.8258)
FRB					0.4901***	0.2480***
					(6.5718)	(3.5328)
Constant	0.4215***	0.4215***	0.4215***	0.4215***	0.4215***	0.4215***
	(9.1916)	(9.1423)	(9.5732)	(9.2629)	(9.5461)	(9.8525)
Observations	5113	5113	5113	5113	5113	5113
Adj. R-Squared	0.1271	0.1189	0.1834	0.1369	0.1754	0.2190

Table 6: Relation between the Slope of the Term Structure and Individual Economic Indicators
This table reports the results of the regression analysis of the term structure slope for S&P 500 options against economic indicators. The dependent variable is the slope of the term structure, which is computed as the IV differential between at-the-money options with 120 and 60 days to maturity. All IV values are computed using the Proximal Trilinear Interpolation Technique. The independent variables are: the NBER Recession Indicator (NBER), the OECD Recession Indicator (OECD), Hamilton's GDP-based Recession Indicator Index (HGR), the Aruoba-Diebold-Scotti Business Conditions Index (ADS), and the FRB Diffusion Index for Current General Activity (FRB). We compute t-statistics (reported below the coefficients) using Newey-West standard errors with 10 lags. See section 3.3 for a definition of the economic indicators. The sample covers the period between January 4, 1996 and April 29, 2016.

The coefficients of determination are similar across indicators, from 11.9% (OECD) to 18.3% (HGR). The multivariate regression of term structure slopes against all economic indicators shows that economic conditions explain about 22% of term structure slope variations. The FRB Diffusion Index for Current General Activity (FRB) and Hamilton's GDP-based Recession Indicator Index (HGR) are the main drivers of the relationship, with t-statistics of 3.53 and 2.85, respectively. Note that the FRB and HGR are also, individually, the indicators that contribute the most to the variations of the IV term structure, as indicated by

their coefficients of determination of 17.5% and 18.3%, respectively.

The study of financial indicators yields similar conclusions (see Table 7). Results for univariate regressions suggest that term structure slopes decrease during bad times, as indicated by the negative slope coefficients for the TED, BAML, VIX, and EBP indicators. The relation is positive between the slope and the YOY indicator, where bottom quintile observations of YOY define periods of financial instability. All regression parameters are statistically significant. The coefficients of determination vary substantially across financial indicators, ranging between 16.9% (TED) and 51.8% (VIX). Results for the multivariate linear regression, combining all financial indicators, indicate that market conditions explain 65% of term structure slope variations, mostly driven by the year-on-year returns of the S&P 500 Index, the BAML and the VIX. The explanatory power of the financial variables to term structure slope variations is about three times larger than the explanatory power of the economic indicators.

	(1) YOY	(2) TED	(3) BAML	(4) VIX	(5) EBP	(6) Full
YOY	0.5680***					0.5100***
	(7.8350)					(10.0543)
TED		-0.4818***				0.0284
		(4.4136)				(0.3547)
BAML			-0.5998***			0.9282***
			(7.0146)			(12.1066)
VIX				-0.8420***		-1.0708***
				(12.8250)		(17.6869)
EBP					-0.6617***	-0.3646***
					(8.1913)	(5.9602)
Constant	0.4215***	0.4215***	0.4362***	0.4215***	0.4215***	0.4362***
	(9.8537)	(9.4325)	(9.9207)	(12.4372)	(10.7310)	(14.9469)
Observations	5113	5113	4861	5113	5113	4861
Adj. R-Squared	0.2356	0.1694	0.2521	0.5180	0.3199	0.6534

Table 7: Relation between the Slope of the Term Structure and Individual Financial Indicators
This table reports the results of the regression analysis of the term structure slope for S&P 500 options against financial indicators. The dependent variable is the slope of the term structure, which is computed as the IV differential between at-the-money options with 120 and 60 days to maturity. All IV values are computed using the Proximal Trilinear Interpolation Technique. The independent variables are: the year-on-year returns of the S&P 500 Index (YOY), the TED spread (TED), the BofA Merrill Lynch US High Yield Option-Adjusted Spread (BAML), the CBOE Volatility Index (VIX), and the Excess-Bond Premium (EBP). We compute t-statistics (reported below the coefficients) using Newey-West standard errors with 10 lags. See section 3.3 for a definition of the financial indicators. The sample covers the period between January 4, 1996 and April 29, 2016 (except for columns (3) and (6), covering the period between January 2, 1997 and April 29, 2016).

Table 8 reports the results when using our aggregate indicators, which provide further

evidence of the cyclicality of the term structure slope. The slope coefficients amount to 0.52 and 0.80 with regards to the AEI and AFI, respectively, suggesting a decrease in term structure slopes following a deterioration of economic and market conditions. All regression parameters are statistically significant. Also, the IV term structure appears to be more sensitive to variations in the Aggregate Financial Indicator than to variations in the Aggregate Economic Indicator, which confirms our previous finding based on individual series. The adjusted R^2 value is 45.04% for the AFI, whereas it is 20.09% for the AEI - again supporting a stronger relationship between the slope of the term structure and financial conditions.

	Aggregate Economic Indicator			Aggregate Financial Indicator		
	Full Period	Good Times	Bad Times	Full Period	Good Times	Bad Times
Constant	0.4215***	0.6387***	-0.4468***	0.4362***	0.7656***	-0.8817***
	(9.7031)	(16.5111)	(2.9778)	(11.7812)	(23.3087)	(7.2484)
Slope Coefficient	0.5245***	0.1342***	0.6139***	0.8017***	0.1968***	1.0046***
	(6.4960)	(2.8297)	(3.1418)	(10.0355)	(5.4883)	(4.6837)
Observations	5113	4090	1023	4861	3889	972
Adj. R-Squared	0.2009	0.0240	0.1280	0.4504	0.0692	0.3688

Table 8: Relation between the Slope of the Term Structure and the Aggregate Economic and Financial Indicators.
This table reports the results of the regression analysis of the term structure slope for S&P 500 options against the aggregate economic and financial indicators. The dependent variable is the slope of the term structure, which is computed as the IV differential between at-the-money options with 120 and 60 days to maturity. All IV values are computed using the Proximal Trilinear Interpolation Technique. The independent variables are the Aggregate Economic Indicator and the Aggregate Financial Indicator. For each indicator, results are presented for three simple linear regressions: for the full period, good times and bad times. See section 3.3 for a definition of the aggregate indicators and a definition of good and bad times. We compute t-statistics (reported below the coefficients) using Newey-West standard errors with 10 lags. The sample covers the period between January 4, 1996 (January 2, 1997) and April 29, 2016 for the Aggregate Economic (Financial) Indicator.

We now extend the analysis by conducting regressions of term structure slopes against the Aggregate Economic Indicator and the Aggregate Financial Indicator, for good and bad times separately. Bad times are defined for both aggregate indicators as the periods associated with the bottom quintile of their time series, while good times correspond to the remaining observations. The independent variables are standardized to facilitate the interpretation of the results.

Table 8 suggests that the constant parameters (i.e., the average IV term structure slopes) are positive in good times and negative in bad times: 0.64 against -0.45 for the AEI, and 0.77 vs. -0.88 for the AFI. Slope coefficients show that the term structure is more sensitive to variations in the aggregate indicators during bad times. For the AEI, a coefficient of 0.61 during periods of stress compares to a coefficient of 0.13 in good times. Similarly, for the

AFI, the coefficient of 1.00 during bad times is much higher than the coefficient of 0.20 in good times. All parameters are different from 0 at a 1% level of significance. Coefficients of determination are also higher in bad times: 12.8% and 36.9% for the AEI and AFI, respectively, vs. 2.4% and 6.9% in good times. In sum, the slope of the term structure does not only decrease in bad times, but it also becomes more sensitive to fluctuations in economic and market conditions.

Overall, we find that there is a positive relationship between the slope of the IV term structure and the state of the economy and that the sensitiveness of the slope to the indicators rises in bad times. Furthermore, we show that financial indicators are particularly relevant for understanding the dynamics of the IV term structure. Appendix A provides a robustness analysis of these results.

5 Smile/Smirk Patterns

In this section, we investigate the conditional dynamics of the volatility smile/smirk for SPX options. The option sample covers the period between January 4, 1996 and April 29, 2016. We measure the volatility skew as the IV differential between two options of different moneyness with 30 days to maturity. One-month maturity options are often used in option literature to study smile patterns - see Dennis, Mayhew, and Stivers (2006) or Yan (2011). We exploit two ranges of moneyness: 1.00 to 0.95 and 1.05 to 1.00. All implied volatilities are estimated using the PTIT. Good and bad times are defined according to the 10 indicators presented in section 3.3. Descriptive statistics and results for the conditional analysis of the skew are presented in Tables 9 and 10. First, Table 9 shows that the skew is more pronounced for 1.00-0.95 options (out-of-the money puts) than for 1.05-1.00 options (out-of-the money calls). This asymmetry confirms the typical shape of the volatility smile/smirk.

	1.00 – 0.95 Skew	1.05 – 1.00 Skew
Average	-0.0467	-0.0245
Standard Deviation	0.0248	0.0225
P-Value (≠0)	0.0000	0.0000

Table 9: Descriptive Statistics for the Skew
This table reports descriptive statistics for the skew of S&P 500 options. Statistics include the average skew, the standard deviation of the skew, and the p-value of the data set for a one-sample t-test. Statistics are reported for two sets of moneyness: 1.00-0.95 and 1.05-1.00. All IV values are computed using the Proximal Trilinear Interpolation Technique. The sample covers the period between January 4, 1996 and April 29, 2016.

For the 1.05-1.00 specification, the average skew is significantly lower during periods of stress for all indicators (between -3.55% and -2.79%) than during good times (between -2.37% and -2.17%). With regards to the regression analysis, the constant coefficient (i.e., average skew) is negative and displays a high t-statistics in all cases. We also observe significant slope coefficients for all indicators. The coefficients are positive for the ADS, FRB and YOY indicators (with bad times corresponding to the lowest values of the times series), and negative for the NBER, OECD, HGR, TED, BAML, VIX and EBP indicators (with bad times corresponding to the highest values of the time series). Therefore, results suggest a decrease in the skew in periods of stress. For the 1.00-0.95 specification, the average skew is higher during periods of stress for most indicators (between -4.33% and -4.03% in bad times, and between -4.83% and -4.73% in good times).[7] Overall, the opposite variations for the 1.05-1.00 and 1.00-0.95 specifications confirm a more pronounced smile in good times than in bad times.

6 The Model

In this section, we describe the economy, we solve for the European put option written on the equilibrium market price, and choose model parameters that match the key moments of aggregate consumption and dividend growth rates.

6.1 The Economic Framework

We consider a pure-exchange economy (Lucas, 1978) populated by a representative investor with recursive preferences (Epstein and Zin, 1989). The investor's utility function is defined by

$$U_t \equiv \left[(1-\delta^{dt})C_t^{\frac{1-\gamma}{\theta}} + \delta^{dt}\mathbb{E}_t\left(U_{t+dt}^{1-\gamma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}},$$

where C_t is consumption, δ is the subjective discount factor per unit of time, γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution (EIS), $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and dt is a time interval.

There are three risky securities. The first, in supply of one unit, pays the aggregate output of the economy which is equal to the investor's aggregate consumption, C_t, in equilibrium.

[7] Additional regression results for the smile are presented in Appendix C. We find significant slope coefficients for all indicators, except for the TED and VIX indicators (positive for the NBER, OECD, HGR, BAML and EBP indicators, with bad times corresponding to the highest values, and negative for the ADS, FRB and YOY indicators, with bad times correspond to the lowest values).

	(1) Unconditional	(2) Good Times	(3) Bad Times	(4) Difference (2-3)	(5) P-Value
Panel 1: 1.05-1.00 Skew					
NBER	-0.0245	-0.0235	-0.0327	0.0092	0.0000
OECD	-0.0245	-0.0224	-0.0290	0.0066	0.0000
HGR	-0.0245	-0.0235	-0.0287	0.0052	0.0000
ADS	-0.0245	-0.0236	-0.0279	0.0043	0.0000
FRB	-0.0245	-0.0224	-0.0327	0.0102	0.0000
YOY	-0.0245	-0.0236	-0.0281	0.0045	0.0000
TED	-0.0245	-0.0223	-0.0330	0.0107	0.0000
BAML	-0.0245	-0.0237	-0.0290	0.0053	0.0000
VIX	-0.0245	-0.0217	-0.0355	0.0138	0.0000
EBP	-0.0245	-0.0232	-0.0295	0.0062	0.0000
Panel 2: 1.00-0.95 Skew					
NBER	-0.0467	-0.0473	-0.0413	-0.0060	0.0000
OECD	-0.0467	-0.0482	-0.0433	-0.0050	0.0000
HGR	-0.0467	-0.0479	-0.0419	-0.0060	0.0000
ADS	-0.0467	-0.0482	-0.0408	-0.0074	0.0000
FRB	-0.0467	-0.0477	-0.0426	-0.0051	0.0000
YOY	-0.0467	-0.0480	-0.0416	-0.0064	0.0000
TED	-0.0467	-0.0470	-0.0454	-0.0016	0.0639
BAML	-0.0467	-0.0476	-0.0412	-0.0064	0.0000
VIX	-0.0467	-0.0469	-0.0458	-0.0011	0.2005
EBP	-0.0467	-0.0483	-0.0403	-0.0080	0.0000

Table 10: Conditional Descriptive Statistics of the S&P 500 IV Skew – Maturity of 30 Days

This table reports descriptive statistics of the volatility skew for various economic and financial conditions. The skew is defined as the difference between the estimated implied volatilities of options with distinct moneyness, at a maturity of 30 days. Panel 1 and 2 show the results for moneyness specifications of 1.05-1.00 and 1.00-0.95, respectively. Columns (1) to (3) show average IV term structure slopes for the entire period, good times and bad times (as defined by each indicator), respectively. Column (4) presents the difference between (2) and (3), while Column (5) displays p-values for the two-sample t-test. Section 3.3 defines the various indicators and the indicator-specific periods of good and bad times. The sample covers the period between January 4, 1996 and April 29, 2016.

The second, in zero net supply, is a stock that pays a dividend, D_t. The third, in zero net supply, is a European put option written on the stock. There is also one risk-free asset, in zero net supply, paying the risk-free rate r_{ft} to be determined in equilibrium.

Aggregate consumption, C_t, and the dividend paid by the stock, D_t, are characterized as follows:

$$\begin{aligned} \log C_t &= x_t + z_t, \\ \log D_t &= \alpha(x_t + z_t), \end{aligned} \tag{1}$$

with

$$\begin{aligned} dx_t =&(\mu_t - \tfrac{1}{2}\sigma_x^2)dt + \sigma_x dB_{xt}, \\ dz_t =& -\phi_z z_t dt + \xi_t dN_{zt}, \\ d\lambda_t =&\phi_\lambda(\bar{\lambda} - \lambda_t)dt + \sigma_\lambda\sqrt{\lambda_t}dB_{\lambda t}, \\ d\mu_t =&\phi_\mu(\bar{\mu} - \mu_t)dt + \sigma_\mu dB_{\mu t}, \end{aligned}$$

where $(B_{xt}, B_{\lambda t}, B_{\mu t})^\top$ is a standard Brownian motion and N_{zt} is a Poisson process with stochastic intensity λ_t. The jump size ξ_t follows a negative time invariant exponential distribution with parameter η. That is, the jump size is negative and characterized by the following moment-generating function:

$$\varrho(u) \equiv \mathbb{E}_t\left(e^{u\xi_t}\right) = \frac{1}{1+\frac{u}{\eta}}.$$

The log-consumption process is a sum of two terms. The first term, x_t, is the consumption growth rate had there been no disasters and consequently no recoveries either. The aim of the second term, z_t, is to model disasters and recoveries. Conditional on the occurrence of a disaster ($dN_t = 1$), the log-consumption drops instantaneously by an amount ξ_t. Following the drop, the process z_t reverts back to zero at speed ϕ and therefore implies a consumption recovery. If the mean-reversion speed ϕ is equal to zero, there are no recoveries and disasters are permanent. The dividend process in (1) is a levered claim of aggregate consumption (Abel, 1999). The parameter α determines the severity of leverage. Indeed, $\alpha\xi_t$ represents the size of the drop in log-dividend conditional on the occurrence of a disaster, while $\alpha\sigma_x$ is (the continuous part of) the volatility of the log-dividend.

6.2 Characterization of Asset Prices

We follow Eraker and Shaliastovich (2008) to solve for asset prices. The representative investor's log wealth-consumption ratio, wc_t is characterized in Proposition 1 below.

Proposition 1. *In equilibrium, the investor's log wealth-consumption ratio satisfies*

$$wc_t \equiv \log\frac{W_t}{C_t} = A + B^\top Y_t, \tag{2}$$

where "$\top$" is the transpose operator, and $Y_t = (x_t, \mu_t, z_t, \lambda_t)^\top$ is the vector of state variables.

The coefficients $A \in \mathbb{R}$ and $B \in \mathbb{R}^4$ solve a system of equations provided in Appendix D.

Proof. See Appendix D.

The risk-free rate r_{ft}, market price of continuous risk vector Λ_t^c, and market price of jump risk vector Λ_t^d are defined in Proposition 2.

Proposition 2. *The dynamics of the state-price density M_t are written*

$$\frac{dM_t}{M_t} = -r_{ft}dt - (\Lambda_t^c)^\top dB_t - \sum_{i=1}^{4}\left(\Lambda_t^{d^i} dN_t^i - \mathbb{E}_t\left(\Lambda_t^{d^i}\right) l^i(Y_t)dt\right),$$

where the risk-free rate r_{ft}, the market price of continuous risk Λ_t^c, and the market price of jump risk Λ_t^d satisfy

$$\begin{aligned} r_{ft} &= \Phi_0 + \Phi_1^\top Y_t, \\ \Lambda_t^c &= \Sigma(Y_t)^\top \Omega, \\ \Lambda_t^{d^i} &= 1 - e^{\Omega^i J_t^i}, \qquad \text{where } i = 1, \ldots, 4 \text{ stands for the } i\text{-th coordinate.} \end{aligned} \tag{3}$$

The vector of jump sizes J_t, the vector of jump intensities $l(Y_t)$, the vector $\Omega \in \mathbb{R}^4$, and the matrix $\Sigma(Y_t) \in \mathbb{R}^{4\times 4}$ are provided in Appendix D. The coefficient $\Phi_0 \in \mathbb{R}$ and $\Phi_1 \in \mathbb{R}^4$ solve a system of equations provided in Appendix E.

Proof. See Appendix E.

The log price-dividend ratio, pd_t, is characterized in Proposition 3 below.

Proposition 3. *The equilibrium log price-dividend ratio of the stock satisfies*

$$pd_t \equiv \log\frac{P_t}{D_t} = A_d + B_d^\top Y_t. \tag{4}$$

The coefficients $A_d \in \mathbb{R}$ and $B_d \in \mathbb{R}^4$ solve a system of equations provided in Appendix F.

Proof. See Appendix F.

The price of a European put option, put_t, written on the stock is characterized in Proposition 4 below.

Proposition 4. *The price of a European put option written on the stock satisfies*

$$\begin{aligned} put_t &= \mathbb{E}_t^{\mathbb{Q}}\left(e^{-\int_t^T r_s ds}\max\left(0; K - P_T\right)\right) \\ &= KG_t(\log K - A_p; \tau, 0_{4\times 1}, B_p) - e^{A_p} G_t(\log K - A_p; \tau, B_p, B_p), \end{aligned}$$

where K is the strike price, $\mathbb{Q}$ is the risk-neutral measure characterized in Appendix G, and $\tau = T - t$ is the time to maturity. The coefficients $A_p \in \mathbb{R}$, $B_p \in \mathbb{R}^4$, and the function $G_t(.; ., ., .) \in \mathbb{R}$ are provided in Appendix H.

Proof. See Appendix H.

The implied-volatility of the European put option, σ_t^{BS}, is provided in Definition 1 below.

Definition 1. *The implied-volatility of the European put option, σ_t^{BS}, is such that*

$$put_t = put_t^{BS} = e^{-r_{ft}\tau} K \Phi\left(-d_2(\sigma_t^{BS})\right) - P_t e^{-g_t \tau} \Phi\left(-d_1(\sigma_t^{BS})\right),$$

where $g_t \equiv e^{-pd_t}$ is the dividend yield of the stock and $\Phi(.)$ is the Gaussian cumulative distribution function with mean 0 and variance 1. The functions $d_1(.)$ and $d_2(.)$ satisfy

$$d_1(\sigma_t^{BS}) = \frac{\log\left(P_t/K\right) + \left(r_{ft} - g_t + \frac{1}{2}(\sigma_t^{BS})^2\right)\tau}{\sigma_t^{BS}\sqrt{\tau}},$$
$$d_2(\sigma_t^{BS}) = d_1(\sigma_t^{BS}) - \sigma_t^{BS}\sqrt{\tau}.$$

6.3 Calibration

Table 11 provides the parameters of the model. The disaster intensity parameters as well as the mean and volatility of consumption growth in normal times (i.e, in a world without disasters) are borrowed from Wachter (2013). The disaster size parameter implies that the average jump in log consumption conditional on a disaster is -25%, which is consistent with the estimation of Barro and Ursua (2008). The speed of recovery parameter implies that a 50% recovery from the disaster occurs in about 3.5 years, which matches the number estimated in Gourio (2008) and Nakamura et al. (2013). The mean-reversion speed and volatility of expected consumption growth as well as the leverage and preference parameters are in line with the long-run risk literature (Bansal and Yaron, 2004).

7 Theoretical Results

In this section we investigate the sensitivity of the implied-volatility surface to changes in economic conditions. We show that the slope of the term structure of implied-volatility is pro-cyclical, being positive in good times (when the disaster intensity is close to zero) and negative in bad times (when the disaster intensity is high). In good times, the model yields an implied-volatility smile, whereas it generates an implied-volatility smirk otherwise.

7.1 Model-Implied Volatility Surface

Figure 5 depicts the term structure of implied volatility for different moneyness and states of the economy. When there are no recoveries (lower panels), the term structures of implied

Parameter	Symbol	Value
Volatility of consumption growth (normal times)	σ_x	0.02
Long-term expected growth (normal times)	$\bar{\mu}$	0.0252
Volatility of expected growth (normal times)	σ_μ	0.01
Reversion speed of expected growth (normal times)	ϕ_μ	0.2
Reversion speed of jump intensity	ϕ_λ	0.08
Long-term jump intensity	$\bar{\lambda}$	0.0355
Volatility of jump intensity	σ_λ	0.067
Disaster size parameter	η	4
Speed of recovery	ϕ_z	0.2
Leverage parameter	α	2
Relative risk aversion	γ	10
Elasticity of intertemporal substitution (EIS)	ψ	1.75
Subjective discount factor	δ	0.96

Table 11: Parameter Values

volatility are upward-sloping irrespective of the moneyness and state of the economy. The reason is that a time-varying disaster intensity and expected dividend growth implies that long-term dividend growth rates are more risky than short-term ones (Hasler and Marfè, 2016), which translates into more risky stock returns in the long term than in the short term when the agent has a preference for early resolution of uncertainty ($\gamma > 1/\psi$ with $\psi > 1$). By definition, the existence of recoveries dampens the risk of long-term dividend growth rates relative to short-term ones. This implies that the risk of long-term stock returns relative to short-term ones tends to decrease, thereby generating a more hump-shaped or even downward-sloping term structure of implied volatility (upper panels). While the latter effect remains relatively weak when economic conditions are good, it becomes stronger as economic conditions deteriorate. The reason is that a high disaster intensity (high risk) today forecasts a lower intensity (lower risk) in the future due to mean reversion. As a result, the implied volatility tends to be lower in the long term than in the short term.

Figure 6 depicts the model-implied volatility smirk/smile under different economic conditions. Good, normal, and bad times are defined as periods in which the disaster intensity is low, moderate, and high, respectively. When there are no post-disaster recoveries (lower panels), we observe an implied-volatility smirk in each state of the world. It is the existence of disasters that explains why out-of-the-money puts have a larger implied volatility than at-the-money and in-the-money puts (Benzoni et al., 2011). When there are post-disaster recoveries (upper panels), the implied-volatility smirk is particularly severe in bad times, it is moderate in normal times, and it transforms into an implied-volatility smile in good times. While the possible occurrence of disasters tends to increase the implied volatility

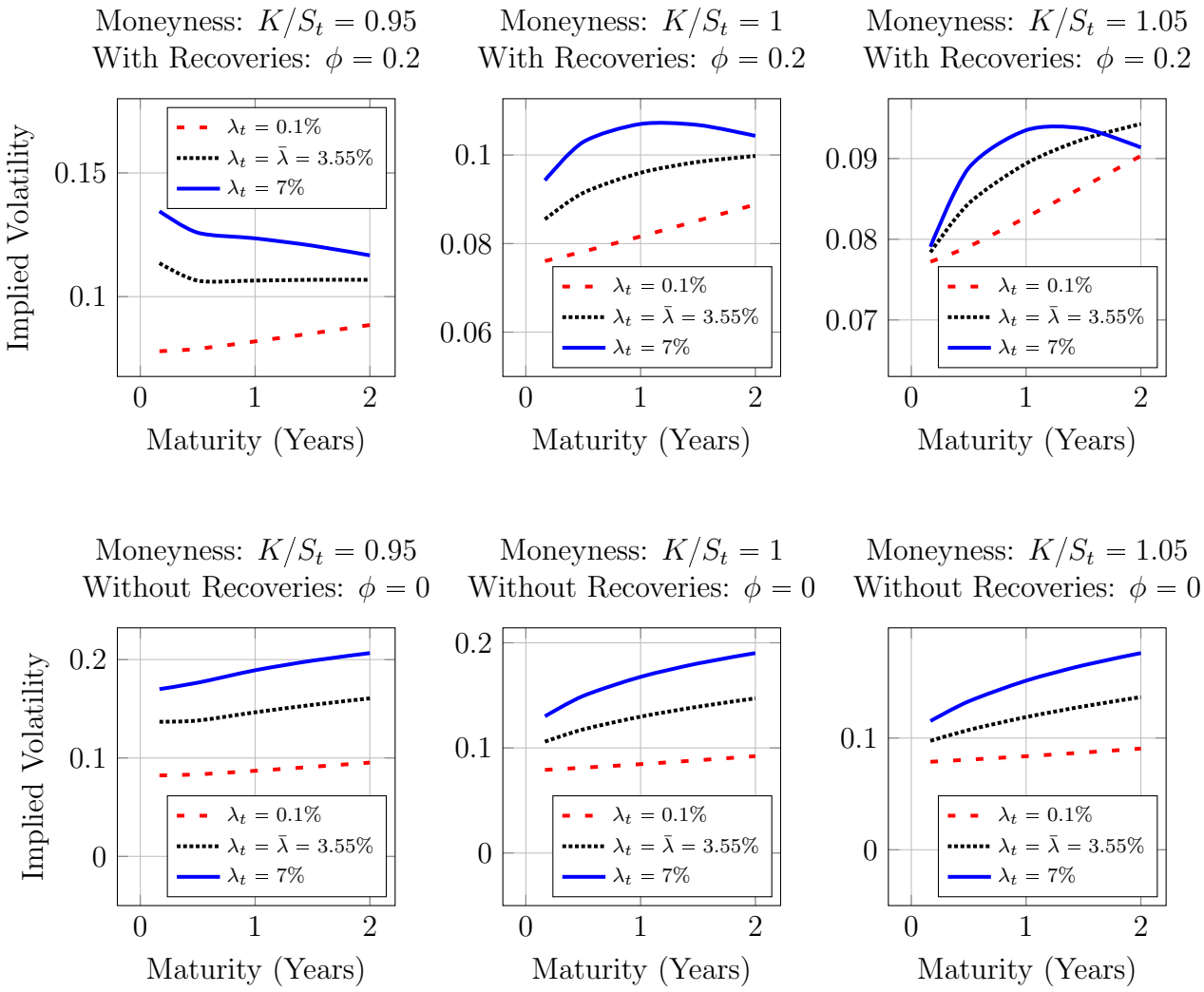

Figure 5: Term Structure of Implied Volatility vs. Economic Conditions
The upper panels depict the term structures of implied volatility for different moneyness when the model features post-disaster recoveries. The lower panels depict the term structures of implied volatility for different moneyness when the model does not feature post-disaster recoveries. Each curve represents a different economic condition, as measured by the disaster intensity. Note that we set $\gamma = 1.5$ in the no recovery case to insure finiteness of prices. State variables are $z_t = 0$ and $\mu_t = \bar{\mu}$, and the calibration is provided in Table 11.

of out-of-the-money puts relative to that of at-the-money and in-the-money puts, the existence of recoveries—abnormally high growth rates—generates the exact opposite effect. It increases the implied-volatility of in-the-money puts relative to that of at-the-money and out-of-the-money puts. While the former effect dominates when the intensity of disasters is sufficiently large, both effects are of the same order of magnitude when the disaster intensity is low, thereby creating an implied-volatility smile.

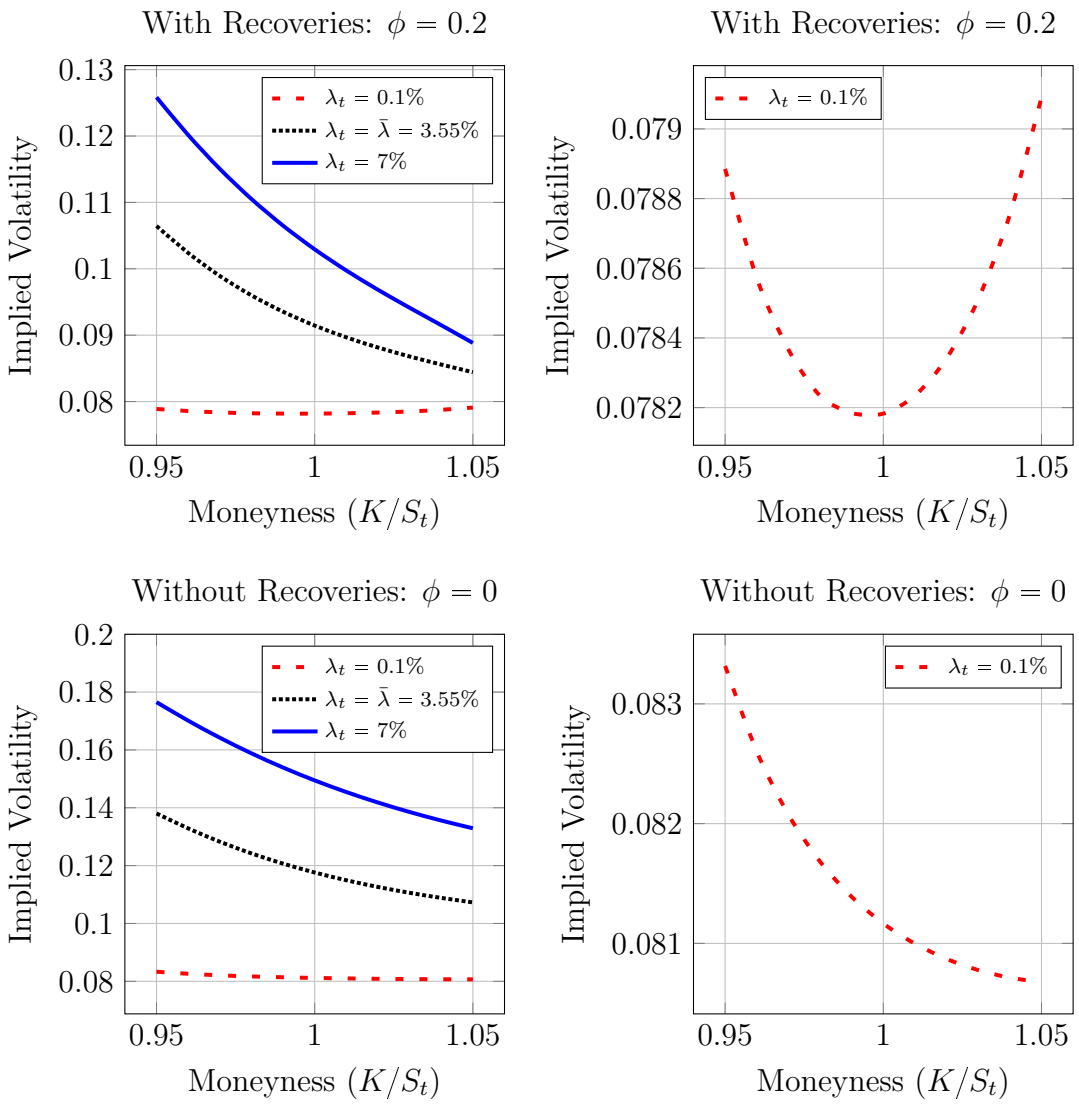

Figure 6: Implied-Volatility Smirk/Smile vs. Economic Conditions

The upper panels depict the 6-month implied-volatility smirk/smile for different economic conditions, as measured by the value of the disaster intensity, when the model features post-disaster recoveries. The lower panels depict the 6-month implied-volatility smirk/smile for different economic conditions when the model does not feature post-disaster recoveries. The right panels zooms on the 6-month implied-volatility smirk/smile in good times. Note that we set $\gamma = 1.5$ in the no recovery case to insure finiteness of prices. State variables are $z_t = 0$ and $\mu_t = \bar{\mu}$, and the calibration is provided in Table 11.

8 Conclusion

We empirically show using options written on the U.S. equity market that the implied volatility increases monotonically with the option's maturity in good and normal times, but becomes decreasing when the economy enters a bad state. In addition, the implied volatility

is skewed in bad and normal times, but resembles more and more to a smile as economic conditions improve. We build an asset pricing model in which rare disasters are followed by recoveries, and show that the predictions of the model are in line with our empirical findings.

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Appendix

A Robustness Analysis (IV Term Structure)

In this section, we provide additional results to confirm the robustness of our prior findings. First, we replicate our analysis by applying four different models for the estimation of the IVS, in addition to the Proximal Trilinear Interpolation Technique used in previous sections. Second, we analyze the movements of the IV term structure for different option specifications. Third, we investigate the conditional patterns of the IV term structure for five alternative equity indices, in the U.S. and across the world.

A.1 Alternative Estimation Models

We now investigate the robustness of our empirical results to four additional estimation methods. Model 1 reproduces the nearest-neighbor interpolation method of the PTIT without extrapolation - therefore restricting the estimation of IV values within the bounds of the "traded" IVS. Model 2 stands in-between the PTIT and Model 1, allowing for extrapolation on the moneyness axis. Model 3 incorporates a liquidity factor in the selection of the "nearest neighbors" - i.e., moneyness and maturity ranges are specified around the target identity, and the four most traded options within these limits are selected for the estimation of implied volatility. Lastly, Model 4 uses cubic spline interpolation to estimate the IVS.[8] Appendix B offers a detailed description of each model. Empirical results are reported in Appendix B.3.

For the 120-60 term structure specification at a moneyness level of 1, we find lower (and typically negative) average slopes in bad times vs. average slopes in good times, across all estimation methods. In addition, the results of (unreported) regressions are similar for all approaches: 1) Statistically significant slope coefficients indicate a positive relationship between business conditions and term structure slopes; 2) The strength of the relationship is generally higher for financial indicators than for economic indicators (R^2 between 17% and 52% for financial indicators, and between 8% and 18% for economic indicators); 3) Constant coefficients are higher during good times than during periods of stress; and 4) Slope coefficients are larger during bad times, indicating that the sensitivity of the term structure to variations in business conditions is more pronounced during periods of stress. We note that the basic PTIT and both the Models 1 and 2 display very similar results, slightly superior to Models 3 and 4 in terms of R^2 values, slope coefficients and t-statistics.

[8] The *fit* MATLAB function is employed for the application of the cubic spline interpolation method.

Altogether, the results of this paper appear to be consistent across a variety of IVS estimation methods.

A.2 Alternative Specifications

We now verify the robustness of the results by looking at option pairs with 90 and 30 days to maturity.

Table A.1 shows that, in most cases, the slope of the term structure is significantly lower during bad times, which confirm our previous findings. Overall, we confirm that the IV slope is cyclical under various term structure specifications, with a slightly more pronounced distinction between good and bad times at a moneyness level of 1 (average difference of 1.6% between good and bad times for the 90-30 specification, and 1.2% for the 120-60 term structure specification).

Regression results for all options sets also align with previous findings for both maturity sets (90-30 and 120-60). Unreported results indicate that the slope coefficients of the univariate regressions are almost always statistically significant and thus confirm a positive relationship between the slope of the IV term structure and business conditions. When considering the financial indicators, the slope coefficients are statistically significant at a 1% level significance for all indicators across all specifications, except for the TED spread. Also, empirical results show higher slope coefficients and R^2 values during bad times - indicating a stronger relationship between these indicators and the term structure slope in periods of stress. Overall, the findings of the paper are robust across various moneyness and maturity levels, with limited discrepancies between the different specifications considered.

	(1) Unconditional	(2) Good Times	(3) Bad Times	(4) Difference (2-3)	(5) P-Value
Panel 1: Moneyness of 0.95					
NBER	-0.0130	-0.0115	-0.0257	0.0142	0.0000
OECD	-0.0130	-0.0104	-0.0189	0.0086	0.0000
HGR	-0.0130	-0.0115	-0.0192	0.0076	0.0000
ADS	-0.0130	-0.0108	-0.0217	0.0109	0.0000
FRB	-0.0130	-0.0108	-0.0218	0.0110	0.0000
YOY	-0.0130	-0.0101	-0.0246	0.0145	0.0000
TED	-0.0130	-0.0122	-0.0162	0.0040	0.0000
BAML	-0.0130	-0.0092	-0.0256	0.0164	0.0000
VIX	-0.0130	-0.0086	-0.0309	0.0223	0.0000
EBP	-0.0130	-0.0102	-0.0243	0.0141	0.0000
Panel 2: Moneyness of 1.00					
NBER	0.0065	0.0085	-0.0100	0.0185	0.0000
OECD	0.0065	0.0104	-0.0020	0.0125	0.0000
HGR	0.0065	0.0089	-0.0031	0.0119	0.0000
ADS	0.0065	0.0097	-0.0061	0.0158	0.0000
FRB	0.0065	0.0097	-0.0061	0.0159	0.0000
YOY	0.0065	0.0101	-0.0077	0.0178	0.0000
TED	0.0065	0.0078	0.0014	0.0064	0.0000
BAML	0.0065	0.0107	-0.0095	0.0202	0.0000
VIX	0.0065	0.0115	-0.0134	0.0249	0.0000
EBP	0.0065	0.0104	-0.0088	0.0192	0.0000
Panel 3: Moneyness of 1.05					
NBER	0.0080	0.0092	-0.0022	0.0114	0.0000
OECD	0.0080	0.0104	0.0027	0.0078	0.0000
HGR	0.0080	0.0095	0.0019	0.0075	0.0000
ADS	0.0080	0.0103	-0.0013	0.0116	0.0000
FRB	0.0080	0.0098	0.0008	0.0090	0.0000
YOY	0.0080	0.0105	-0.0020	0.0125	0.0000
TED	0.0080	0.0076	0.0094	-0.0018	0.0282
BAML	0.0080	0.0112	-0.0039	0.0151	0.0000
VIX	0.0080	0.0110	-0.0039	0.0149	0.0000
EBP	0.0080	0.0106	-0.0025	0.0131	0.0000

Table A.1: Conditional Descriptive Statistics of the S&P 500 IV Term Structure – Maturities of 90 and 30 Days

This table reports descriptive statistics of the term structure slope for various economic and financial conditions. The term structure slope is defined as the difference between the estimated implied volatilities of options with 90 and 30 days to maturity. Panel 1 to 3 show the results for moneyness levels of 0.95, 1.00 and 1.05, respectively. Columns (1) to (3) show average IV term structure slopes unconditionally, in good times, and in bad times, respectively. Column (4) presents the difference between (2) and (3), while Column (5) displays p-values for the two-sample t-test. Section 3.3 defines the various indicators and the indicator-specific periods of good and bad times. The sample covers the period between January 4, 1996 and April 29, 2016.

A.3 Alternative Securities

The analysis of the S&P 500 option market constitutes the core of this study. We now consider options on alternative indices and explore the robustness of our conclusions for 5 Exchange-Traded Funds (ETF) - namely the iShares Russell 2000 Index ETF (ticker IWM), the iShares MSCI Brazil ETF (EWZ), the iShares China Large-Cap ETF (FXI), the iShares EAFA Index ETF (EFA) and the iShares Emerging Markets Index ETF (EEM). ETF options are selected because they display higher trading volumes than their direct index counterparts (notably due to their presence on U.S. exchanges), as discussed in Kelly, Pástor, and Veronesi (2016). Option data are retrieved from the OptionMetrics Ivy database and adjusted according to the parameters discussed in section 3.1. Descriptive statistics are presented in Tables A.2 to A.6.

	# Obs.	Median	Average	Std. Dev.	Min.	Max.
Implied Volatility	365,062	22.1%	23.1%	7.2%	8.6%	89.9%
Moneyness Level	365,062	97.7%	97.4%	8.2%	80.0%	120.0%
Maturity (Days)	365,062	68.0	94.1	80.6	6.0	365.0
Daily Volume, Total	1,702	279,761	309,650	149,170	68,283	1,195,188
Daily Volume, Per Option Average	1,702	1,449	1,640	860	337	6,461
Daily Open Interest, Total	1,702	2,531,081	2,476,284	569,007	822,924	4,279,559
Daily Open Interest, Per Option Average	1,702	12,187	13,662	5,934	3,298	36,319
Options Traded per Day	1,702	183	214	97	67	507
Available Maturities per Day	1,702	13.0	12.8	3.3	6.0	20.0
Av. # of Mon. Levels per Maturity, per Day	1,702	15.1	16.0	3.9	8.6	30.4

Table A.2: Descriptive Statistics of the Sample of iShares Russell 2000 Index ETF (IWM) options

This table reports the descriptive statistics for the implied volatility, the moneyness level (defined as the ratio of the strike price to the value of the underlying asset), the maturity of the options (in days), the daily trading volume (number of traded contracts) for all available options, the daily average trading volume per option, the daily open interest (number of contracts outstanding) for all options, the daily average open interest per option, the number of options traded per day, the number of available maturities per day, and the average number of moneyness levels per available maturity, per day. The iShares Russell 2000 Index ETF reproduces the Russell 2000 Index, an index of small capitalization companies in the U.S. (bottom 2000 of the Russell 3000 Index, with 8% market coverage). It is a total return Exchange-Traded Fund. American-style IWM options (SECID 106445) trade on the CBOE. The data set covers the period between July 28, 2009 and April 29, 2016.

	# Obs.	Median	Average	Std. Dev.	Min.	Max.
Implied Volatility	134,913	30.9%	32.7%	9.7%	11.6%	90.0%
Moneyness Level	134,913	99.6%	99.4%	9.4%	80.0%	120.0%
Maturity (Days)	134,913	53.0	82.6	71.6	6.0	365.0
Daily Volume, Total	1,702	37,194	43,989	31,687	1,811	350,561
Daily Volume, Per Option Average	1,702	493	547	323	35	3,959
Daily Open Interest, Total	1,702	541,178	569,457	249,843	119,228	2,301,171
Daily Open Interest, Per Option Average	1,702	7,150	7,266	2,335	1,427	17,387
Options Traded per Day	1,702	75	79	25	35	227
Available Maturities per Day	1,702	5.0	6.6	2.5	3.0	12.0
Av. # of Mon. Levels per Maturity, per Day	1,702	13.4	13.1	4.2	4.3	27.3

Table A.3: Descriptive Statistics of the Sample of iShares MSCI Brazil ETF (EWZ) options

This table reports the descriptive statistics for the implied volatility, the moneyness level (defined as the ratio of the strike price to the value of the underlying asset), the maturity of the options (in days), the daily trading volume (number of traded contracts) for all available options, the daily average trading volume per option, the daily open interest (number of contracts outstanding) for all options, the daily average open interest per option, the number of options traded per day, the number of available maturities per day, and the average number of moneyness levels per available maturity, per day. The iShares MSCI Brazil ETF reproduces an index of large and mid-capitalizations in Brazil (with 85% free float-adjusted coverage of Brazilian equities). It is a total return Exchange-Traded Fund. American-style EWZ options (SECID 106416) trade on the CBOE. The data set covers the period between July 28, 2009 and April 29, 2016.

	# Obs.	Median	Average	Std. Dev.	Min.	Max.
Implied Volatility	121,727	25.9%	27.4%	7.0%	10.1%	87.7%
Moneyness Level	121,727	99.7%	99.5%	8.7%	80.0%	120.0%
Maturity (Days)	121,727	71.0	93.9	76.7	6.0	365.0
Daily Volume, Total	1,702	45,106	55,996	41,460	3,865	404,730
Daily Volume, Per Option Average	1,702	683	813	521	50	4,868
Daily Open Interest, Total	1,702	770,423	804,406	271,862	146,461	1,724,847
Daily Open Interest, Per Option Average	1,702	11,395	12,615	5,608	1,380	33,439
Options Traded per Day	1,702	63	72	32	30	233
Available Maturities per Day	1,702	7.0	7.3	2.6	3.0	13.0
Av. # of Mon. Levels per Maturity, per Day	1,702	9.8	10.0	2.5	4.4	21.0

Table A.4: Descriptive Statistics of the Sample of iShares China Large-Cap ETF (FXI) options

This table reports the descriptive statistics for the implied volatility, the moneyness level (defined as the ratio of the strike price to the value of the underlying asset), the maturity of the options (in days), the daily trading volume (number of traded contracts) for all available options, the daily average trading volume per option, the daily open interest (number of contracts outstanding) for all options, the daily average open interest per option, the number of options traded per day, the number of available maturities per day, and the average number of moneyness levels per available maturity, per day. The iShares China Large-Cap ETF reproduces an index of the 50 largest and most liquid Chinese equities. It is a total return Exchange-Traded Fund. American-style FXI options (SECID 122017) trade on the CBOE. The data set covers the period between July 28, 2009 and April 29, 2016.

	# Obs.	Median	Average	Std. Dev.	Min.	Max.
Implied Volatility	94,003	20.6%	22.0%	8.3%	4.7%	89.2%
Moneyness Level	94,003	98.9%	98.4%	7.7%	80.0%	120.0%
Maturity (Days)	94,003	71.0	99.7	84.8	6.0	365.0
Daily Volume, Total	1,702	28,507	35,815	31,291	1,504	346,927
Daily Volume, Per Option Average	1,702	520	640	550	47	7,826
Daily Open Interest, Total	1,702	507,669	510,943	187,331	79,004	1,169,386
Daily Open Interest, Per Option Average	1,702	9,175	9,532	3,577	2,171	22,845
Options Traded per Day	1,702	53	55	14	25	134
Available Maturities per Day	1,702	7.0	7.8	2.1	3.0	13.0
Av. # of Mon. Levels per Maturity, per Day	1,702	7.3	7.5	2.1	3.0	15.2

Table A.5: Descriptive Statistics of the Sample of iShares EAFA Index ETF (EFA) options.
This table reports the descriptive statistics for the implied volatility, the moneyness level (defined as the ratio of the strike price to the value of the underlying asset), the maturity of the options (in days), the daily trading volume (number of traded contracts) for all available options, the daily average trading volume per option, the daily open interest (number of contracts outstanding) for all options, the daily average open interest per option, the number of options traded per day, the number of available maturities per day, and the average number of moneyness levels per available maturity, per day. The iShares EAFA Index ETF reproduces an index of large and mid-capitalizations in developed markets, excluding the U.S. and Canada (with 85% free float-adjusted coverage). It is a total return Exchange-Traded Fund. American-style EFA options (SECID 116908) trade on the CBOE. The data set covers the period between July 28, 2009 and April 29, 2016.

	# Obs.	Median	Average	Std. Dev.	Min.	Max.
Implied Volatility	145,421	23.6%	25.0%	7.5%	6.9%	89.7%
Moneyness Level	145,421	98.9%	98.7%	8.5%	80.0%	120.0%
Maturity (Days)	145,421	80.0	106.5	87.2	6.0	365.0
Daily Volume, Total	1,702	142,119	168,824	104,753	17,199	1,022,509
Daily Volume, Per Option Average	1,702	1 739	1 981	1 080	316	11,248
Daily Open Interest, Total	1,702	2115,034	2230,941	650,081	713,770	4,775,355
Daily Open Interest, Per Option Average	1,702	26,665	27,057	7,110	8,756	53,332
Options Traded per Day	1,702	85	85	24	36	177
Available Maturities per Day	1,702	9.0	8.7	2.9	3.0	16.0
Av. # of Mon. Levels per Maturity, per Day	1,702	10.4	10.3	2.3	4.1	18.6

Table A.6: Descriptive Statistics of the Sample of iShares Emerging Markets Index ETF (EEM) options

This table reports the descriptive statistics for the implied volatility, the moneyness level (defined as the ratio of the strike price to the value of the underlying asset), the maturity of the options (in days), the daily trading volume (number of traded contracts) for all available options, the daily average trading volume per option, the daily open interest (number of contracts outstanding) for all options, the daily average open interest per option, the number of options traded per day, the number of available maturities per day, and the average number of moneyness levels per available maturity, per day. The iShares Emerging Markets Index ETF reproduces an index of large and mid-capitalizations across 23 emerging markets. It is a total return Exchange-Traded Fund. American-style EEM options (SECID 116959) trade on the CBOE. The data set covers the period between July 28, 2009 and April 29, 2016.

We investigate the patterns of the IV term structure between July 28, 2009 and April 29, 2016 for at-the-money options with 120 and 60 days to maturity. All IV values are computed using the first derivation of the PTIT that excludes extrapolation (see Model 1 in Appendix B.1).[9] We consider four indicators to distinguish between good and bad times, namely the year-on-year returns of the underlying asset, the Aggregate Economic Indicator and Financial Indicators described in Section 4.3, as well as the term structure time-series of S&P 500 index options (for the same period and option characteristics). We always define bad times as the bottom quintile of the indicator.

The results are consistent across securities, as reported in Tables A.7 and A.8. Term structure slopes are lower in bad times than in good times, and display a positive linear relationship with each indicator. Only the results with the Aggregate Economic Indicator

[9]Model 1 is used to estimate the implied volatilities (IV) for securities other than the S&P 500 (SPX), as opposed to the basic PTIT used throughout this paper. The PTIT allows for extrapolation when using available option data to estimate implied volatility values at given maturity and moneyness coordinates. The number of options traded daily for non-SPX securities is significantly inferior compared to SPX options. As such, using extrapolation to estimate IV values would yield inaccurate results in many cases.

are less conclusive. In this case, the subsample analysis indicates that the slopes are not significantly different between good and bad times for the Russell 2000 Index, the EAFA Index and the Emerging Markets Index, at a 5% level of significance (see Table A.8). The results suggest that IV term structure patterns for securities other than the S&P 500 Index are more sensitive to the state of the U.S. financial market than to U.S. economic conditions.

Notably, we find a strong relationship between the term structure slope of the considered securities and the slope in the U.S. options market. Unreported tables suggest strong adjusted R^2 values (61% for the Russell 2000 Index, 20% for Brazil equities, 43% for the FXI, 52% for the EAFA Index, and 56% for the EEM). In addition, for all securities except the EWZ, R^2 values for linear regressions against the year-on-year returns of the underlying assets are lower than the R^2 values for the regressions against the Aggregate Financial Indicator - showing again the strong dependence of non-SPX IVS dynamics on the U.S. financial market. Hence, the slopes of the IV term structure appear to strongly co-move around the world.

Overall, the slope of the term structure computed for other stock indices and countries appear to be cyclical, as for U.S. S&P500 options, and vary particularly strongly with changes in global financial conditions.

	Panel A: 90-30 Term Structure			Panel B: 120-60 Term Structure		
	0.95 Mon.	1.00 Mon.	1.05 Mon.	0.95 Mon.	1.00 Mon.	1.05 Mon.
1) IWM						
Average Slope	0.0002	0.0109	0.0158	0.0027	0.0086	0.0126
Standard Deviation	0.0168	0.0157	0.0129	0.0101	0.0096	0.0088
P-Value (≠0)	0.6915	0.0000	0.0000	0.0000	0.0000	0.0000
2) EWZ						
Average Slope	-0.0126	-0.0028	0.0010	-0.0055	-0.0006	0.0019
Standard Deviation	0.0339	0.0356	0.0356	0.0219	0.0217	0.0216
P-Value (≠0)	0.0000	0.0014	0.2798	0.0000	0.2376	0.0003
3) FXI						
Average Slope	-0.0051	-0.0001	-0.0018	0.0008	0.0019	n/a
Standard Deviation	0.0203	0.0185	0.0177	0.0129	0.0122	n/a
P-Value (≠0)	0.0000	0.8608	0.0001	0.0124	0.0000	n/a
4) EFA						
Average Slope	-0.0079	0.0081	0.0169	-0.0014	0.0062	0.0106
Standard Deviation	0.0196	0.0177	0.0173	0.0131	0.0117	0.0102
P-Value (≠0)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5) EEM						
Average Slope	-0.0026	0.0085	0.0121	0.0001	0.0059	0.0085
Standard Deviation	0.0183	0.0168	0.0164	0.0119	0.0110	0.0106
P-Value (≠0)	0.0000	0.0000	0.0000	0.8153	0.0000	0.0000

Table A.7: Slope of the Term Structure by Moneyness – Options on Alternative Indices

This table reports descriptive statistics for the slope of the term structure for: iShares Russell 2000 Index ETF (IWM) options, iShares MSCI Brazil ETF (EWZ) options, iShares China Large-Cap ETF (FXI) options, iShares EAFA Index ETF (EFA) options, and iShares Emerging Markets Index ETF (EEM) options. Statistics include the average term structure slope, the standard deviation of the term structure, and the p-value of the data set for a one-sample t-test. Panel A considers the slope between implied volatility (IV) observations for options with 90 and 30 days to maturity. Panel B considers options with 120 and 60 days to maturity. Each panel reports statistics for three moneyness levels (defined as the strike price of the option to the price of the underlying asset): 0.95, 1.00, and 1.05. IV values are computed using the Model 1 approach (see Appendix B.2). The sample covers the period between July 28, 2009 and April 29, 2016.

	(1) Unconditional	(2) Good Times	(3) Bad-Times	(4) Difference (2-3)	(5) P-Value
Panel 1: IWM					
YOY	0.0086	0.0096	0.0044	0.0051	0.0000
AEI	0.0086	0.0087	0.0081	0.0006	0.3266
AFI	0.0086	0.0108	-0.0005	0.0113	0.0000
SPX TS	0.0086	0.0114	-0.0030	0.0144	0.0000
Panel 2: EWZ					
YOY	-0.0006	0.0009	-0.0066	0.0075	0.0000
AEI	-0.0006	0.0001	-0.0036	0.0037	0.0050
AFI	-0.0006	0.0019	-0.0108	0.0127	0.0000
SPX TS	-0.0006	0.0035	-0.0169	0.0204	0.0000
Panel 3: FXI					
YOY	0.0019	0.0035	-0.0045	0.0080	0.0000
AEI	0.0019	0.0027	-0.0014	0.0041	0.0000
AFI	0.0019	0.0039	-0.0061	0.0100	0.0000
SPX TS	0.0019	0.0049	-0.0102	0.0151	0.0000
Panel 4: EFA					
YOY	0.0062	0.0075	0.0012	0.0063	0.0000
AEI	0.0062	0.0060	0.0070	-0.0010	0.1612
AFI	0.0062	0.0086	-0.0032	0.0117	0.0000
SPX TS	0.0062	0.0095	-0.0067	0.0162	0.0000
Panel 5: EEM					
YOY	0.0059	0.0076	-0.0009	0.0084	0.0000
AEI	0.0059	0.0061	0.0048	0.0013	0.0521
AFI	0.0059	0.0083	-0.0040	0.0123	0.0000
SPX TS	0.0059	0.0091	-0.0070	0.0161	0.0000

Table A.8: Conditional Descriptive Statistics of the IV Term Structure Slope for Alternative Indices – Maturities of 120 and 60 Days

This table reports descriptive statistics of the term structure slope for various economic and financial conditions. The term structure slope is defined as the difference between the estimated implied volatilities of ATM options with 120 and 60 days to maturity. Panel 1 relates to iShares Russell 2000 Index ETF (IWM) options, Panel 2 to iShares MSCI Brazil ETF (EWZ) options, Panel 3 to iShares China Large-Cap ETF (FXI) options, Panel 4 to iShares EAFA Index ETF (EFA) options, and Panel 5 to iShares Emerging Markets Index ETF (EEM) options. The four indicators are the year-on-year returns of the underlying index (YOY), the Aggregate Economic Indicator (AEI), the Aggregate Financial Indicator (AFI) and the time series of the IV term structure for S&P 500 options (for at-the-money option pairs with 120 and 60 days to maturity) (SPX TS). Section 3.3 defines the periods of good and bad times for each indicator. Columns (1) to (3) show average IV term structure slopes unconditionally, in good times, and in bad times, respectively. Column (4) presents the difference between (2) and (3), while Column (5) displays p-values for two-sample t-test. IV values are estimated using the Model 1 approach (see Appendix B.2). The sample covers the period between July 28, 2009 and April 29, 2016.

B Methodology

B.1 Main IV Estimation Technique

Option contracts are traded for a finite number of unevenly distributed maturities and moneyness levels. Consequently, the IV surface has a degenerated design. Multiple estimation techniques are employed in this paper to compute daily IV values at any given maturity or moneyness level.

The Proximal Trilinear Interpolation Technique (PTIT) serves as the central IV estimation method throughout this study. The approach can be described as follows. For a target time to maturity T, we choose the closest surrounding maturities available on the market. At each maturity, we linearly interpolate between traded options to estimate volatility metrics at the desired moneyness level M. Lastly, we interpolate between the two estimated values to obtain $IV(M,T)$. The trilinear approach can be expressed as follows:

$$\begin{aligned}
\overline{IV}_t^1(M,T_1) &= (IV_t(M_1,T_1) \times (M_2 - M) + IV_t(M_2,T_1) \times (M - M_1))/(M_2 - M_1) \\
\overline{IV}_t^2(M,T_2) &= (IV_t(M_3,T_2) \times (M_4 - M) + IV_t(M_4,T_2) \times (M - M_3))/(M_4 - M_3) \\
\overline{IV}_t(M,T) &= (\overline{IV}_t^1(M,T_1) \times (T_2 - T) + \overline{IV}_t^2(M,T_2) \times (T - T_1))/(T_2 - T_1)
\end{aligned}$$

where $\overline{IV}_t(M,T)$ is the estimated implied volatility at the target time to maturity T and target moneyness level M. T_1 and T_2 correspond to the closest available maturities below and above T, respectively. M_1 and M_2 are the closest available moneyness levels below and above M, for options with maturity T_1. M_3 and M_4 are the closest available moneyness levels below and above M, for options with maturity T_2. IV_t denotes the implied volatility value of an existing (i.e., traded) option, in contrast to $\overline{IV}_t$ for estimated implied volatility values. Refer to Figure B.1 for a visual representation of the PTIT.

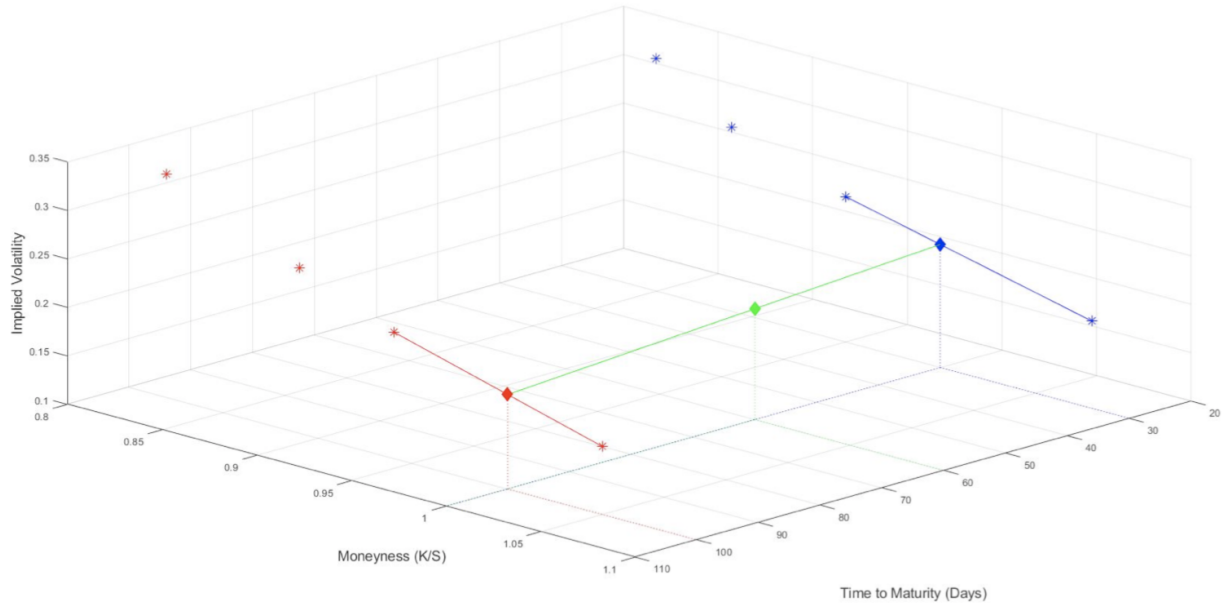

Figure B.1: Visual Representation of the Proximal Trilinear Interpolation Technique (PTIT)
The figure displays an example of the PTIT for a target moneyness level of 1 (computed as the strike price of an option divided by the price of the underlying asset) and a target maturity of 60 days. Available and estimated option metrics are displayed with star and diamond markers, respectively. The values in blue correspond to the closest available maturity below the target of 60 days (i.e., 30 days). The values in red correspond to the closest available maturity above the target of 60 days (i.e., 100 days). The blue and red lines depict the two linear interpolations implemented to estimate the volatility metrics at the desired moneyness level (i.e., the blue and red diamond markers). Lastly, the green line depicts the final linear interpolation used to estimate the IV value at the target maturity and moneyness coordinates (i.e., the green diamond marker).

Specific situations arise when traded options do not provide the required information to execute an approach based entirely on linear interpolation, especially for the less liquid securities employed in this study. In the original PTIT, which is the main approach that we use throughout this paper, we allow for linear extrapolation of implied volatility values, when market data is insufficient to interpolate, on both the moneyness and maturity axis (see Figure B.2). Specific limits are applied to ensure the validity of the extrapolated results at each step of the PTIT, in line with the filters described in section 3.1. For each extrapolation, we only keep the estimate if the obtained IV is above 4% and below 90%. In addition, we only keep the estimate if the difference between the extrapolated IV and the closest available IV (i.e., the IV of the closest available option used for the extrapolation) is inferior to 10%.

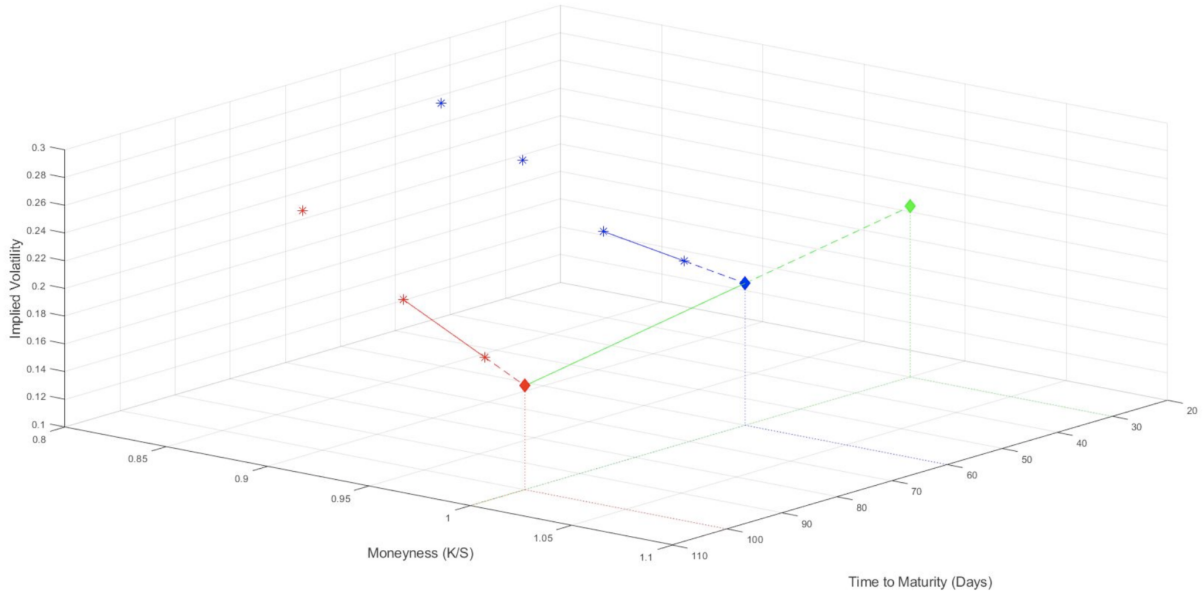

Figure B.2: Visual Representation of the Basic Proximal Trilinear Interpolation Technique (PTIT), using Extrapolation
The figure displays an example of the basic PTIT for a target moneyness level of 1 (computed as the strike price of an option divided by the price of the underlying asset) and a target maturity of 30 days, assuming that no options are available for a maturity below 30 or a moneyness above 1. Available and estimated option metrics are displayed with star and diamond markers, respectively. The values in blue and red correspond to the two closest available maturities above the target of 30 days (i.e., 60 and 100 days, respectively). The blue and red lines depict the two linear extrapolations implemented to estimate the volatility metrics at the desired moneyness level (i.e., the blue and red diamond markers). Lastly, the green line depicts the final linear extrapolation used to estimate the IV value at the target maturity and moneyness coordinates (i.e., the green diamond marker).

The application of the PTIT is an iterative process. For instance, when calculating $\overline{IV}_t^1(M, T_1)$ as per the equations above, with T_1 being the closest available maturity below the target maturity T, it is possible that no option trades with a moneyness level below the target moneyness M (i.e., M_1 does not exist). In this case, two options with maturity T_1 and moneyness levels above M could be used to extrapolate $\overline{IV}_t^1(M, T_1)$. If the extrapolation is not applicable (i.e., the data set is insufficient) or does not respect the above-described filters, T_1 would be replaced by the next closest maturity below T. If the calculation of $\overline{IV}_t^1(M, T_1)$ is still not realizable, the iteration continues with the next closest maturity. The same process is applied when computing $\overline{IV}_t^2(M, T_2)$, where T_2 is effectively the closest maturity above T for which the computation of $\overline{IV}_t^2(M, T_2)$ is achievable and in line with the applied filters. If the computation of neither $\overline{IV}_t^1(M, T_1)$ nor $\overline{IV}_t^2(M, T_2)$ is applicable given the data set, $\overline{IV}_t(M, T)$ can still be extrapolated. For instance, if the computation of $\overline{IV}_t^1(M, T_1)$

is impossible, $\overline{IV}_t(M,T)$ can be extrapolated using $\overline{IV}_t^2(M,T_2)$ and $\overline{IV}_t^3(M,T_{2*})$), with T_2 and T_{2*} the two closest maturities above T for which the implied volatility can be estimated at the desired moneyness level (see Figure B.2 for visual representation). Following the full iteration process, if the available option data set does not allow for IV estimation at the desired maturity and moneyness coordinates, the sought data point (i.e., the IV value and the corresponding date) is simply removed from the sample. For S&P 500 options and term structure slopes defined as the IV differential between at-the-money options with maturities of 120 and 60 days, the PTIT achieves a 100% success rate (i.e., no data point is removed from the sample).

B.2 Alternative Techniques

Three derivations of the basic PTIT are introduced in Section A, namely Model 1 to 3. The Model 1 excludes extrapolation. As such, after completion of the iteration process, T_1 and T_2 correspond to the closest maturities below and above T for which the computation of $\overline{IV}_t^1(M,T_1)$ and $\overline{IV}_t^2(M,T_2)$ is achievable without extrapolation, respectively. If the computation of $\overline{IV}_t^1(M,T_1)$ or $\overline{IV}_t^2(M,T_2)$ is not achievable, the sought data point is removed from the sample. For S&P 500 options and term structure slopes defined as the IV differential between at-the-money options with maturities of 120 and 60 days, Model 1 achieves a 99.9% success rate (i.e., 3 data points are removed on a total of 5,113 days). Model 2 allows for extrapolation, but on the moneyness axis only. After completion of the iteration process, T_1 and T_2 correspond to the closest maturities below and above T for which the computation of $\overline{IV}_t^1(M,T_1)$ and $\overline{IV}_t^2(M,T_2)$ is achievable (allowing for extrapolation when computing the IV at a fixed maturity), respectively. If the computation of $\overline{IV}_t^1(M,T_1)$ or $\overline{IV}_t^2(M,T_2)$ is not applicable, the sought data point is removed from the sample. For S&P 500 options and term structure slopes defined as the IV differential between at-the-money options with maturities of 120 and 60 days, Model 2 achieves a 100% success rate (i.e., no data point is removed from the sample).

Model 3, which is the third alternative of the PTIT, incorporates a liquidity factor in the estimation of implied volatility. In this model, we first identify the most traded options surrounding the target moneyness level M and target maturity T:

$$
\begin{aligned}
IV_t^A(M^+, T^-) &= \text{ most traded option with } M \leqslant M^+ \leqslant M + 0.2 \text{ and } T - 200 \leqslant T^- \leqslant T \\
IV_t^B(M^-, T^-) &= \text{ most traded option with } M - 0.2 \leqslant M^- \leqslant M \text{ and } T - 200 \leqslant T^- \leqslant T \\
IV_t^C(M^+, T^+) &= \text{ most traded option with } M \leqslant M^+ \leqslant M + 0.2 \text{ and } T \leqslant T^+ \leqslant T + 200 \\
IV_t^D(M^+, T^+) &= \text{ most traded option with } M - 0.2 \leqslant M^- \leqslant M \text{ and } T \leqslant T^+ \leqslant T + 200
\end{aligned}
$$

$\overline{IV}_t(M, T)$ is computed using a three-step process. First, $\overline{IV}_t^1(M, T^-)$ is calculated through linear interpolation between $IV_t^A(M^+, T^-)$ and $IV_t^B(M^-, T^-)$. Similarly, we compute $\overline{IV}_t^2(M, T^+)$ through linear interpolation between $IV_t^C(M^+, T^+)$ and $IV_t^D(M^+, T^+)$. Finally, we obtain $\overline{IV}_t(M, T)$ using linear interpolation between $\overline{IV}_t^1(M, T^-)$ and $\overline{IV}_t^2(M, T^+)$. See Figure B.3 for visual representation.

Lastly, Model 4 uses cubic spline interpolation to estimate the IVS. In Model 4, the fit MATLAB function is used, with piecewise cubic interpolation.

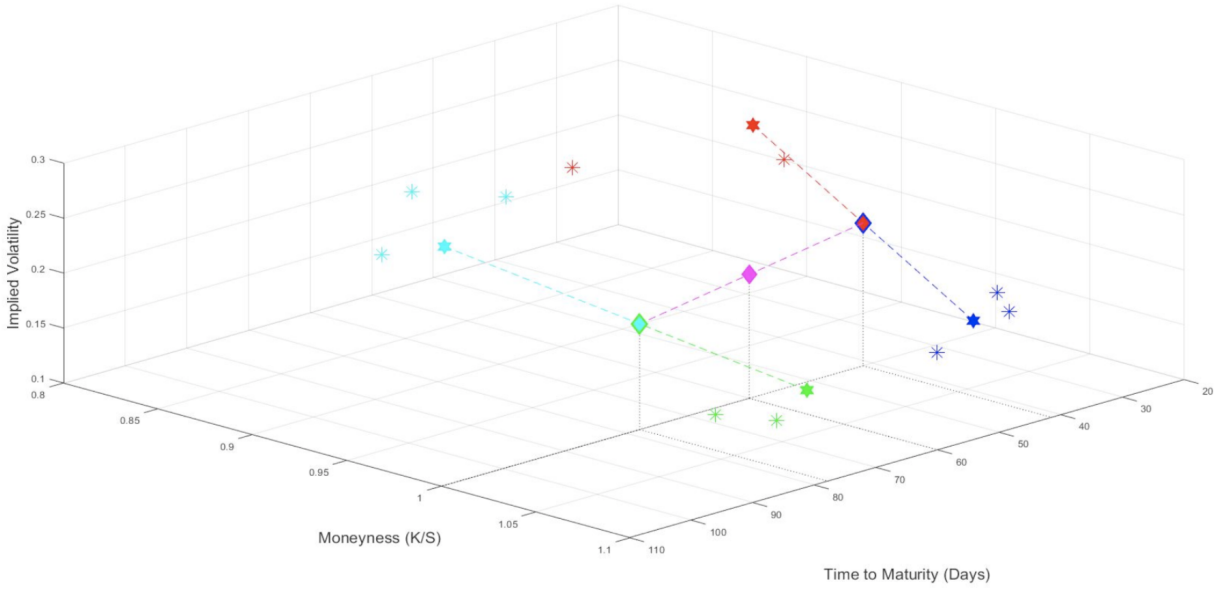

Figure B.3: Visual Representation of the Model 3 Estimation Approach
The figure displays an example of the Model 3 estimation approach for a target moneyness level of 1 (computed as the strike price of an option divided by the price of the underlying asset) and a target maturity of 60 days. Available option metrics are displayed with star markers (6 or 8 pointed). Estimated metrics are displayed with diamond markers. The values in dark blue correspond to all available options with maturity between 5 and 60 days, as well as moneyness between 1 and 1.2. The values in red correspond to all available options with maturity between 5 and 60 days, as well as moneyness between 0.8 and 1. The values in green correspond to all available options with maturity between 60 and 260 days, as well as moneyness between 1 and 1.2. The values in light blue correspond to all available options with maturity between 60 and 260 days, as well as moneyness between 0.8 and 1. Within each color, the six-pointed star denotes the most traded option. The lines and corresponding diamond markers show the linear interpolations implemented to estimate implied volatility metrics.

B.3 Results with Alternative Estimation Models

	Panel A: 90-30 Term Structure			Panel B: 120-60 Term Structure		
	0.95 Mon.	1.00 Mon.	1.05 Mon.	0.95 Mon.	1.00 Mon.	1.05 Mon.
1) Model 1						
Average Slope	-0.0118	0.0049	0.0069	-0.0035	0.0043	0.0087
Standard Deviation	0.0186	0.0183	0.0168	0.0115	0.0116	0.0114
P-Value (≠0)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2) Model 2						
Average Slope	-0.0119	0.0048	0.0066	-0.0036	0.0042	0.0087
Standard Deviation	0.0188	0.0186	0.0173	0.0117	0.0117	0.0114
P-Value (≠0)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3) Model 3						
Average Slope	-0.0124	0.0028	-0.0003	-0.0042	0.0031	0.0051
Standard Deviation	0.0177	0.0158	0.0168	0.0126	0.0108	0.0112
P-Value (≠0)	0.0000	0.0000	0.2115	0.0000	0.0000	0.0000
4) Model 4						
Average Slope	-0.0115	0.0051	0.0074	-0.0032	0.0044	0.0093
Standard Deviation	0.0189	0.0190	0.0177	0.0118	0.0121	0.0120
P-Value (≠0)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.1: Slope of the Term Structure by Moneyness, for various implied volatility estimation techniques
This table reports descriptive statistics for the slope of the term structure for S&P 500 options. Statistics include the average term structure slope, the standard deviation of the term structure, and the p-value of the data set for a one-sample t-test. Panel A considers the slope between implied volatility (IV) observations for options with 90 and 30 days to maturity. Panel B considers options with 120 and 60 days to maturity. Each panel reports statistics for three moneyness levels (defined as the strike price of the option to the price of the underlying asset): 0.95, 1.00, and 1.05. All IV values are computed using the Proximal Trilinear Interpolation Technique. The sample covers the period between January 4, 1996 and April 29, 2016. Appendix B.2 defines the alternative IV estimation techniques.

	(1) Unconditional	(2) Good Times	(3) Bad-Times	(4) Difference (2-3)	(5) P-Value
NBER	0.0043	0.0057	-0.0078	0.0135	0.0000
OECD	0.0043	0.0070	-0.0018	0.0088	0.0000
HGR	0.0043	0.0059	-0.0027	0.0087	0.0000
ADS	0.0043	0.0065	-0.0048	0.0113	0.0000
FRB	0.0043	0.0064	-0.0044	0.0108	0.0000
YOY	0.0043	0.0070	-0.0067	0.0137	0.0000
TED	0.0043	0.0054	-0.0002	0.0056	0.0000
BAML	0.0043	0.0074	-0.0077	0.0152	0.0000
VIX	0.0043	0.0076	-0.0092	0.0168	0.0000
EBP	0.0043	0.0070	-0.0068	0.0139	0.0000

Table B.2: Descriptive Statistics of the S&P 500 IV Term Structure Slope, for ATM Options with Maturities of 120 and 60 Days, using the Model 1 Estimation Approach

This table reports descriptive statistics of the term structure slope with regards to economic and financial conditions. The term structure slope is defined as the difference between the estimated implied volatilities of options with 120 and 60 days to maturity. IV values are estimated using the Model 1 approach (see Appendix B.2). The table covers all economic and financial indicators (see section 3.3 for a description of the indicators and the definitions good and bad times). Columns (1) to (3) show average IV term structure slopes for the entire period, good times and bad times, respectively. Column (4) presents the difference between (2) and (3). Column (5) displays p-values for two-sample t-tests (for the two data sets used to compute (2) and (3)). The sample covers the period between January 4, 1996 and April 29, 2016.

	(1) Unconditional	(2) Good Times	(3) Bad-Times	(4) Difference (2-3)	(5) P-Value
NBER	0.0042	0.0057	-0.0079	0.0135	0.0000
OECD	0.0042	0.0069	-0.0018	0.0087	0.0000
HGR	0.0042	0.0059	-0.0028	0.0087	0.0000
ADS	0.0042	0.0065	-0.0048	0.0113	0.0000
FRB	0.0042	0.0064	-0.0044	0.0108	0.0000
YOY	0.0042	0.0070	-0.0068	0.0137	0.0000
TED	0.0042	0.0054	-0.0002	0.0055	0.0000
BAML	0.0042	0.0074	-0.0078	0.0152	0.0000
VIX	0.0042	0.0076	-0.0093	0.0169	0.0000
EBP	0.0042	0.0070	-0.0068	0.0138	0.0000

Table B.3: Descriptive Statistics of the S&P 500 IV Term Structure Slope, for ATM Options with Maturities of 120 and 60 Days, using the Model 2 Estimation Approach

This table reports descriptive statistics of the term structure slope with regards to economic and financial conditions. The term structure slope is defined as the difference between the estimated implied volatilities of options with 120 and 60 days to maturity. IV values are estimated using the Model 2 approach (see Appendix B.2). The table covers all economic and financial indicators (see section 3.3 for a description of the indicators and the definitions good and bad times). Columns (1) to (3) show average IV term structure slopes for the entire period, good times and bad times, respectively. Column (4) presents the difference between (2) and (3). Column (5) displays p-values for two-sample t-tests (for the two data sets used to compute (2) and (3)). The sample covers the period between January 4, 1996 and April 29, 2016.

	(1) Unconditional	(2) Good Times	(3) Bad-Times	(4) Difference (2-3)	(5) P-Value
NBER	0.0031	0.0042	-0.0062	0.0104	0.0000
OECD	0.0031	0.0051	-0.0016	0.0067	0.0000
HGR	0.0031	0.0043	-0.0021	0.0065	0.0000
ADS	0.0031	0.0048	-0.0040	0.0088	0.0000
FRB	0.0031	0.0047	-0.0035	0.0082	0.0000
YOY	0.0031	0.0052	-0.0057	0.0109	0.0000
TED	0.0031	0.0039	-0.0003	0.0042	0.0000
BAML	0.0031	0.0056	-0.0066	0.0121	0.0000
VIX	0.0031	0.0058	-0.0078	0.0135	0.0000
EBP	0.0031	0.0053	-0.0060	0.0113	0.0000

Table B.4: Descriptive Statistics of the S&P 500 IV Term Structure Slope, for ATM Options with Maturities of 120 and 60 Days, using the Model 3 Estimation Approach

This table reports descriptive statistics of the term structure slope with regards to economic and financial conditions. The term structure slope is defined as the difference between the estimated implied volatilities of options with 120 and 60 days to maturity. IV values are estimated using the Model 3 approach (see Appendix B.2). The table covers all economic and financial indicators (see section 3.3 for a description of the indicators and the definitions good and bad times). Columns (1) to (3) show average IV term structure slopes for the entire period, good times and bad times, respectively. Column (4) presents the difference between (2) and (3). Column (5) displays p-values for two-sample t-tests (for the two data sets used to compute (2) and (3)). The sample covers the period between January 4, 1996 and April 29, 2016.

	(1) Unconditional	(2) Good Times	(3) Bad-Times	(4) Difference (2-3)	(5) P-Value
NBER	0.0044	0.0058	-0.0076	0.0134	0.0000
OECD	0.0044	0.0072	-0.0017	0.0089	0.0000
HGR	0.0044	0.0061	-0.0024	0.0084	0.0000
ADS	0.0044	0.0067	-0.0047	0.0114	0.0000
FRB	0.0044	0.0066	-0.0043	0.0110	0.0000
YOY	0.0044	0.0072	-0.0066	0.0138	0.0000
TED	0.0044	0.0056	-0.0000	0.0056	0.0000
BAML	0.0044	0.0077	-0.0076	0.0153	0.0000
VIX	0.0044	0.0078	-0.0090	0.0168	0.0000
EBP	0.0044	0.0072	-0.0068	0.0140	0.0000

Table B.5: Descriptive Statistics of the S&P 500 IV Term Structure Slope, for ATM Options with Maturities of 120 and 60 Days, using the Model 4 Estimation Approach

This table reports descriptive statistics of the term structure slope with regards to economic and financial conditions. The term structure slope is defined as the difference between the estimated implied volatilities of options with 120 and 60 days to maturity. IV values are estimated using the Model 4 approach (see Appendix B.2). The table covers all economic and financial indicators (see section 3.3 for a description of the indicators and the definitions good and bad times). Columns (1) to (3) show average IV term structure slopes for the entire period, good times and bad times, respectively. Column (4) presents the difference between (2) and (3). Column (5) displays p-values for two-sample t-tests (for the two data sets used to compute (2) and (3)). The sample covers the period between January 4, 1996 and April 29, 2016.

C Additional Results on the Smile/Skew Pattern

	(1) NBER	(2) OECD	(3) HGR	(4) ADS	(5) FRB	(6) Full
NBER	0,1864***					-0,0312
	(3,1601)					(0,2461)
OECD		0,2309***				0,1580**
		(4,0980)				(2,0441)
HGR			0,2259***			0,1278
			(4,2890)			(0,8105)
ADS				-0,2282***		-0,0937
				(4,2084)		(0,7544)
FRB					-0,1664***	0,0610
					(3,2384)	(0,7516)
Constant	-4,6693***	-4,6693***	-4,6693***	-4,6693***	-4,6693***	-4,6693***
	(81,3327)	(81,8547)	(81,7703)	(81,7501)	(81,1260)	(82,3261)
Observations	4907	4907	4907	4907	4907	4907
Adj. R-Squared	1%	1%	1%	0%	0%	1%

Table C.1: Relation between Economic Conditions and the Implied Volatility Skew – 1.00 and 0.95 Moneyness Levels

This table reports the results of the regression analysis of the volatility skew of S&P 500 options on various economic indicators. The dependent variable is the implied volatility skew, which is computed as the IV differential between options with moneyness levels of 1.00 and 0.95, at a maturity of 30 days. All IV values are computed using the Proximal Trilinear Interpolation Technique. The independent variables are the NBER Recession Indicator (NBER), the OECD Recession Indicator (OECD), Hamilton's GDP-based Recession Indicator Index (HGR), the Aruoba- Diebold-Scotti Business Conditions Index (ADS), and the FRB Diffusion Index for Current General Activity (FRB). We compute t-statistics (reported below the coefficients) using Newey-West standard errors with 10 lags. See section 3.3 for a definition of the economic indicators. The sample covers the period between January 4, 1996 and April 29, 2016.

	(1) NBER	(2) OECD	(3) HGR	(4) ADS	(5) FRB	(6) Full
NBER	-0,2824***					-0,1390
	(5,1388)					(1,3105)
OECD		-0,3057***				-0,1053
		(5,1035)				(1,3788)
HGR			-0,2723***			0,0791
			(4,8615)			(0,5820)
ADS				0,2418***		-0,1292
				(4,1962)		(1,2259)
FRB					0,4231***	0,4120***
					(7,1328)	(4,0252)
Constant	-2,4482***	-2,4482***	-2,4482***	-2,4482***	-2,4482***	-2,4482***
	(37,5388)	(37,6586)	(37,4771)	(37,3302)	(38,5971)	(38,7420)
Observations	4900	4900	4900	4900	4900	4900
Adj. R-Squared	2%	2%	1%	0%	4%	4%

Table C.2: Relation between Economic Conditions and the Implied Volatility Skew – 1.05 and 1.00 Moneyness Levels

This table reports the results of the regression analysis of the volatility skew of S&P 500 options on various economic indicators. The dependent variable is the implied volatility skew, which is computed as the IV differential between options with moneyness levels of 1.05 and 1.00, at a maturity of 30 days. All IV values are computed using the Proximal Trilinear Interpolation Technique. The independent variables are the NBER Recession Indicator (NBER), the OECD Recession Indicator (OECD), Hamilton's GDP-based Recession Indicator Index (HGR), the Aruoba- Diebold-Scotti Business Conditions Index (ADS), and the FRB Diffusion Index for Current General Activity (FRB). We compute t-statistics (reported below the coefficients) using Newey-West standard errors with 10 lags. See section 3.3 for a definition of the economic indicators. The sample covers the period between January 4, 1996 and April 29, 2016.

	(1) YOY	(2) TED	(3) BAML	(4) VIX	(5) EBP	(6) Full
YOY	-0,2230***					-0,0506
	(3,7862)					(0,5157)
TED		0,0661				0,0673
		(1,0551)				(0,9226)
BAML			0,2293***			0,3371***
			(4,4451)			(2,6002)
VIX				0,1108**		-0,2376***
				(2,1874)		(2,7249)
EBP					0,2154***	0,0174
					(4,1562)	(0,1287)
Constant	-4,6693***	-4,6693***	-4,6360***	-4,6693***	-4,6693***	-4,6360***
	(81,7068)	(80,4627)	(80,9557)	(80,7224)	(81,6533)	(81,5711)
Observations	4907	4907	4669	4907	4907	4669
Adj. R-Squared	1%	0%	1%	0%	1%	1%

Table C.3: Relation between Financial Conditions and the Implied Volatility Skew – 1.00 and 0.95 Moneyness Levels

This table reports the results of the regression analysis of the volatility skew of S&P 500 options on various financial indicators. The dependent variable is the implied volatility skew, which is computed as the IV differential between options with moneyness levels of 1.00 and 0.95, at a maturity of 30 days. All IV values are computed using the Proximal Trilinear Interpolation Technique. The independent variables are the year-on-year returns of the S&P 500 Index (YOY), the TED spread (TED), the BofA Merrill Lynch US High Yield Option-Adjusted Spread (BAML), the CBOE Volatility Index (VIX), and the Excess-Bond Premium (EBP). We compute t-statistics (reported below the coefficients) using Newey-West standard errors with 10 lags. See section 3.3 for a definition of the financial indicators. The sample covers the period between January 4, 1996 and April 29, 2016 (except for columns (3) and (6), covering the period between January 2, 1997 and April 29, 2016).

	(1) YOY	(2) TED	(3) BAML	(4) VIX	(5) EBP	(6) Full
YOY	0,2731***					-0,0679
	(5,4773)					(0,7540)
TED		-0,4052***				0,0186
		(4,6154)				(0,1723)
BAML			-0,4419***			0,4591***
			(6,9223)			(3,4260)
VIX				-0,7559***		-1,0050***
				(9,5186)		(9,7445)
EBP					-0,4808***	-0,2184*
					(7,8034)	(1,7973)
Constant	-2,4482***	-2,4482***	-2,4717***	-2,4482***	-2,4482***	-2,4717***
	(37,3494)	(38,5652)	(37,4671)	(42,7879)	(39,0986)	(42,3243)
Observations	4900	4900	4660	4900	4900	4660
Adj. R-Squared	1%	3%	4%	11%	5%	13%

Table C.4: Relation between Financial Conditions and the Implied Volatility Skew – 1.05 and 1.00 Moneyness Levels
This table reports the results of the regression analysis of the volatility skew of S&P 500 options on various financial indicators. The dependent variable is the implied volatility skew, which is computed as the IV differential between options with moneyness levels of 1.05 and 1.00, at a maturity of 30 days. All IV values are computed using the Proximal Trilinear Interpolation Technique. The independent variables are the year-on-year returns of the S&P 500 Index (YOY), the TED spread (TED), the BofA Merrill Lynch US High Yield Option-Adjusted Spread (BAML), the CBOE Volatility Index (VIX), and the Excess-Bond Premium (EBP). We compute t-statistics (reported below the coefficients) using Newey-West standard errors with 10 lags. See section 3.3 for a definition of the financial indicators. The sample covers the period between January 4, 1996 and April 29, 2016 (except for columns (3) and (6), covering the period between January 2, 1997 and April 29, 2016).

D Proof of Proposition 1

Following Eraker and Shaliastovich (2008), the discrete time continuously compounded log-return, $\log R_{c,t}$, on the representative investor's wealth, W_t, satisfies

$$\log R_{c,t+1} = \log \frac{W_{t+1} + C_{t+1}}{W_t} = \log\left(e^{wc_{t+1}} + 1\right) - wc_t + \log \frac{C_{t+1}}{C_t}, \tag{5}$$

where $wc_t = \log(W_t/C_t)$. Log-linearing the first term in (5) around the mean log wealth-consumption ratio yields

$$\log R_{c,t+1} \approx k_0 + k_1 wc_{t+1} - wc_t + \log \frac{C_{t+1}}{C_t}, \tag{6}$$

where the constants k_0 and k_1 satisfy

$$k_0 = -\log\left((1-k_1)^{1-k_1}k_1^{k_1}\right), \qquad k_1 = e^{\mathbb{E}(wc_t)}/\left(1+e^{\mathbb{E}(wc_t)}\right).$$

The continuous-time counterpart of (6) is

$$d\log R_{c,t} = k_0 dt + k_1 dwc_t - (1-k_1)wc_t dt + d\log C_t,$$

and the Euler equation defining the state-price density, M_t, is written

$$\mathbb{E}_t\left[\exp\left(\log\frac{M_{t+\tau}}{M_t} + \int_t^{t+\tau} d\log R_{c,s}\right)\right] = 1,$$

where

$$d\log M_t = \theta\log\delta dt - \frac{\theta}{\psi}d\log C_t - (1-\theta)d\log R_{c,t}.$$

The state variables belong to the affine class and their dynamics can be written as:

$$\begin{aligned} dY_t &= \mu(Y_t)dt + \Sigma(Y_t)dB_t + J_t \cdot dN_t \\ \mu(Y_t) &= \mathcal{M} + \mathcal{K}Y_t \\ \Sigma(Y_t)\Sigma(Y_t)^\top &= h + \sum_{i=1}^{4} H^i Y_t^i \\ l(Y_t) &= l_0 + l_1 Y_t, \end{aligned}$$

where $\mathcal{M} \in \mathbb{R}^4$, $\mathcal{K} \in \mathbb{R}^{4\times4}$, $h \in \mathbb{R}^{4\times4}$, $H \in \mathbb{R}^{4\times4\times4}$, $l_0 \in \mathbb{R}^4$, and $l_1 \in \mathbb{R}^{4\times4}$, $B_t = (B_{xt}, B_{\mu t}, B_{zt}, B_{\lambda t})^\top$ is a standard Brownian motion; $N_t = (N_{xt}, N_{\mu t}N_{zt}, N_{\lambda t})^\top$ is a vector of independent pure jump processes; $l(Y) \in \mathbb{R}^4$ is the corresponding vector of jump intensities; $J_t \in \mathbb{R}^4$ is the vector of jump sizes; and "$\cdot$" denotes element-by-element multiplication.

The state-price density satisfies

$$d\log M_t = (\theta\log\delta - (\theta-1)\log k_1 + (\theta-1)(k_1-1)B'(Y_t - \mu_Y)dt - \Omega' dY_t,$$

where $Y_t = (x_t, \mu_t, z_t, \lambda_t)^\top$, $\mu_Y = (0, \bar{\mu}, 0, \bar{\lambda})^\top$, $\Omega = \gamma(1,0,1,0)^\top + (1-\theta)k_1 B$, and the coefficients $A \in \mathbb{R}$ and $B \in \mathbb{R}^4$ are defined in (2).

The coefficients $A \in \mathbb{R}$, $B \in \mathbb{R}^4$, and $k_1 \in \mathbb{R}$ solve the following system of equations:

$$0 = \mathcal{K}^\top \chi - \theta(1-k_1)B + \frac{1}{2}\chi^\top H \chi + l_1^\top(\varrho(\chi) - 1),$$
$$0 = \theta(\log\delta + k_0 - (1-k_1)A) + \mathcal{M}^\top \chi + \frac{1}{2}\chi^\top h \chi + l_0^\top(\varrho(\chi) - 1),$$

and

$$\theta \log k_1 = \theta(\log\delta + (1-k_1)B^\top \mu_Y) + \mathcal{M}^\top \chi + \frac{1}{2}\chi^\top h \chi + l_0^\top(\varrho(\chi) - 1),$$

where $\chi = \theta\left((1-\frac{1}{\psi})(1,0,1,0)^\top + k_1 B\right)$.

□

E Proof of Proposition 2

Following Eraker and Shaliastovich (2008), the coefficients $\Phi_0 \in \mathbb{R}$ and $\Phi_1 \in \mathbb{R}^4$ defined in (3) solve the following system of equations:

$$\Phi_1 = (1-\theta)(k_1 - 1)B + \mathcal{K}^\top \Omega - \frac{1}{2}\Omega^\top H \Omega - l_1^\top(\varrho(-\Omega) - 1),$$
$$\Phi_0 = -\theta\log\delta + (\theta-1)(\log k_1 + (k_1-1)B^\top \mu_Y) + \mathcal{M}^\top \Omega - \frac{1}{2}\Omega^\top h \Omega - l_0^\top(\varrho(-\Omega) - 1).$$

□

F Proof of Proposition 3

By log-linearizing the return on the stock, Eraker and Shaliastovich (2008) show that the coefficients $A_d \in \mathbb{R}$ and $B_d \in \mathbb{R}^4$ defined in (4) and the associated coefficient $k_{1d} \in \mathbb{R}$ solve the following system of equations:

$$\begin{aligned}
0 =& \mathcal{K}^\top \chi_d + (\theta-1)(k_1-1)B + (k_{1d}-1)B_d + \frac{1}{2}\chi_d^\top H \chi_d + l_1^\top(\varrho(\chi_d) - 1),\\
0 =& \theta\log\delta - (\theta-1)(\log k_1 + (k_1-1)B^\top\mu_Y) - (\log k_{1d} + (k_{1d}-1)B_d^\top \mu_Y)\\
&+ \mathcal{M}^\top \chi_d + \frac{1}{2}\chi_d^\top h \chi_d + l_0^\top(\varrho(\chi_d) - 1),\\
A_d =& \log\left(\frac{k_{1d}}{1-k_{1d}}\right) - B_d^\top \mu_Y
\end{aligned}$$

where $\chi_d = (\alpha, 0, \alpha, 0)^\top + k_{1d}B_d - \Omega$.

□

G Characterization of the Risk Neutral Measure

Eraker and Shaliastovich (2008) show that, under the risk neutral measure $\mathbb{Q}$, the vector of state variables Y_t has the following dynamics

$$dY_t = (\mathcal{M}^{\mathbb{Q}} + \mathcal{K}^{\mathbb{Q}} Y_t)dt + \Sigma(Y_t)dB_t^{\mathbb{Q}} + J_t^{\mathbb{Q}} \cdot dN_t^{\mathbb{Q}},$$

where $B_t^{\mathbb{Q}} = B_t + \int_0^t \Lambda_s^c ds$ is a $\mathbb{Q}$-Brownian motion, $N_t^{\mathbb{Q}}$ is 4-dimensional Poisson process with $\mathbb{Q}$-jump intensity $l^{\mathbb{Q}}(Y_t) \equiv l_0^{\mathbb{Q}} + l_1^{\mathbb{Q}} Y_t$, and $J_t^{\mathbb{Q}}$ is a 4-dimensional $\mathbb{Q}$-jump size vector with moment-generating function $\varrho^{\mathbb{Q}}(u) \equiv \mathbb{E}_t\left(e^{uJ_t^{\mathbb{Q}}}\right)$.

The coefficients $\mathcal{M}^{\mathbb{Q}} \in \mathbb{R}^4$, $\mathcal{K}^{\mathbb{Q}} \in \mathbb{R}^{4\times 4}$, $l_0^{\mathbb{Q}} \in \mathbb{R}^4$, $l_1^{\mathbb{Q}} \in \mathbb{R}^{4\times 4}$, and $\varrho^{\mathbb{Q}}(.) \in \mathbb{R}^4$ satisfy

$$\begin{aligned}
\mathcal{M}^{\mathbb{Q}} &= \mathcal{M} - h\Omega, \\
\mathcal{K}^{\mathbb{Q}} &= \mathcal{K} - H\Omega, \\
l^{\mathbb{Q}}(Y_t) &\equiv l_0^{\mathbb{Q}} + l_1^{\mathbb{Q}} Y_t = l(Y_t) \cdot \varrho(-\Omega), \\
\varrho^{\mathbb{Q},i}(u^i) &= \varrho(u^i - \Omega^i)/\varrho(-\Omega^i), \qquad \text{where } i = 1, \ldots, 4 \text{ stands for the } i\text{-th coordinate.}
\end{aligned}$$

H Proof of Proposition 4

Following Chapter 8 in Duffie (2001), the price of a European put option written on the stock satisfies

$$\begin{aligned}
put_t &= \mathbb{E}_t^{\mathbb{Q}}\left(e^{-\int_t^T r_s ds} \max\left(0; K - P_T\right)\right) \\
&= KG_t(\log K - A_p; \tau, 0_{4\times 1}, B_p) - e^{A_p} G_t(\log K - A_p; \tau, B_p, B_p),
\end{aligned}$$

where $\tau = T - t$. Because the stock price is defined as follows

$$P_t \equiv e^{A_p + B_p^\top Y_t} = e^{A_d + (\alpha + B_{1d})x_t + B_{2d}\mu_t + (\alpha + B_{3d})z_t + B_{4d}\lambda_t},$$

the coefficients $A_p \in \mathbb{R}$ and $B_p \in \mathbb{R}^4$ satisfy

$$\begin{aligned}
A_p &= A_d, \\
B_{1p} &= \alpha + B_{1d}, & B_{2p} &= B_{2d}, \\
B_{3p} &= \alpha + B_{3d}, & B_{4p} &= B_{4d}.
\end{aligned}$$

The function $G_t(.;.,.,.) \in \mathbb{R}$ is defined as follows

$$G_t(q;\tau,d,\delta) \equiv \mathbb{E}_t^{\mathbb{Q}}\left(e^{-\int_t^T r_s ds} e^{d\cdot Y_T} \mathbf{1}_{\delta\cdot Y_T \leqslant q}\right) = \frac{\widehat{G}_t(0;\tau,d,\delta)}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{v}\mathrm{Im}\left(e^{-\imath v q}\widehat{G}_t(v;\tau,d,\delta)\right) dv,$$

where $\mathrm{Im}(.)$ stands for the imaginary part, $\imath$ is the imaginary number such that $\imath^2 = -1$, and the Fourier transform $\widehat{G}_t(.;.,.,.)$ of the function $G_t(.;.,.,.)$ satisfies

$$\widehat{G}_t(v;\tau,d,\delta) = \int_{-\infty}^{+\infty} e^{\imath v q} dG_t(q;\tau,d,\delta) = \mathbb{E}_t^{\mathbb{Q}}\left(e^{-\int_t^T r_s ds} e^{d\cdot Y_T} e^{\imath v \delta\cdot Y_T}\right) = e^{a(\tau)+b(\tau)^\top Y_t}.$$

The functions $a(\tau) \in \mathbb{R}$ and $b(\tau) \in \mathbb{R}^4$ satisfy the following system of ordinary differential equations

$$\begin{aligned} b'(\tau) &= -\Phi_1 + \mathcal{K}^{\mathbb{Q}^\top} b(\tau) + \frac{1}{2} b(\tau)^\top H b(\tau) + l_1^{\mathbb{Q}^\top}\left(\varrho^{\mathbb{Q}}(b(\tau)) - 1\right), \\ a'(\tau) &= -\Phi_0 + \mathcal{M}^{\mathbb{Q}^\top} b(\tau) + \frac{1}{2} b(\tau)^\top h b(\tau) + l_0^{\mathbb{Q}^\top}\left(\varrho^{\mathbb{Q}}(b(\tau)) - 1\right), \end{aligned}$$

subject to $a(0) = 0$ and $b(0) = d + \imath v\delta$.

□