# A Benchmark for Collateralized Loan Obligations \*

Redouane Elkamhi<sup>†</sup>

Ruicong Li<sup>‡</sup>

Yoshio Nozawa<sup>§</sup>

Toronto

CAIFC

Toronto

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#### Abstract

We build a benchmark for AAA-rated tranches of Collateralized Loan Obligations (CLOs) using Business Development Companies (BDCs), which hold a diversified portfolio of loans as CLOs do. However, BDCs are publicly listed, and their share price, equity volatility, and borrowing cost are observable. Furthermore, BDCs' debt is not rated as AAA. Applying a structural model to BDCs, we extract market-implied correlation in their loan portfolio, compare spreads on CLO tranches and BDC-implied benchmark, and find that observed large credit spreads on CLO senior tranches after the financial crisis are a fair reflection of the systematic risk of correlated loan defaults.

JEL Classification: G12, G13

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<sup>&</sup>lt;sup>†</sup>Rotman School of Management, Toronto, ON M5S 3E6, Canada; redouane.elkamhi@rotman.utoronto.ca. <sup>‡</sup>China-ASEAN Institute of Financial Cooperation, Guangxi University, Nanning, Guangxi, China; rliah@connect.ust.hk

<sup>&</sup>lt;sup>§</sup>Rotman School of Management and UTSC, Toronto, ON M5S 3E6, Canada; yoshio.nozawa@rotman.utoronto.ca

# 1 Introduction

The market for the U.S. leveraged loans — syndicated loans for borrowers with low credit quality — grew rapidly after the financial crisis in  $2008.^1$  As of 2020, the outstanding amount reached \$1.2 trillion, which is about the same size as the high-yield corporate bond market. The growth is supported by an increased investment through Collateralized Loan Obligations (CLO), a form of shadow banks that invest in leveraged loans and fund such investment by issuing debts with various seniority, called tranches. At the end of 2020, CLOs were the largest class of institutional investors of leveraged loans, owning more than 50% of the market. The popularity of CLOs is driven by the demand for senior tranches that are protected by capital buffer from junior tranches. Payoffs to senior tranche holders decline only when defaults in the underlying loan portfolio exceed the protection offered by junior tranches. As a result, most senior tranches are rated AAA by major rating agencies. Indeed, as of 2020, there has not been any default of U.S. CLO senior tranches, even during the financial crisis in 2008 and the pandemic-driven recession in 2020. On the other hand, credit spreads on these senior tranches have been relatively high, with an average spread of 1.34%after the financial crisis. The safety and seemingly large credit spreads attract regulated institutional investors to purchase these securities, fueling the rapid growth in corporate loans in the U.S. However, whether such demand from regulated investors distorts credit spreads on CLOs or not remains unclear.

Evaluating whether credit spreads on senior tranches correctly reflect their risk or not poses a challenge to researchers. Investors in senior tranches lose money only when a large fraction of underlying loans default at once. Thus, correlation within the borrower's asset is a key determinant for the price of AAA tranches. However, estimating correlation using historical default data is difficult because there are few observations of systematic default

<sup>&</sup>lt;sup>1</sup>The exact definition of leveraged loan varies across data providers and government entities. Bloomberg has a definition of a leveraged loan based on credit ratings, the primary use of proceeds and credit spreads. The U.S. Federal Reserve, on the other hand, defines leveraged loans based on the use of proceeds, Debt-to-EBITDA ratio, and other criteria.

large enough to affect the payoffs on senior tranches.<sup>2</sup> Even if an accurate estimate of default correlation is available, estimating risk premiums on correlated default is an additional task, which depends on the marginal utility of investors in the worst state of the world where multiple defaults of loans occur at the same time.

To overcome this challenge, we construct a benchmark to evaluate the risk of correlated loan default by examining observed stock price, volatility, and credit spreads of publiclytraded business development companies (BDCs). Much like CLOs, BDCs invest in a portfolio of loans issued by private-sector borrowers, and such investments are funded by debt and equity. Thus, BDCs are another participant in the shadow banking system. However, there are two key differences between BDCs and CLOs: first, some BDCs are publicly listed and traded in stock exchanges, and their stock price, bond price, and financial statements are available to researchers through the public database; second, debts of BDCs are typically not rated as AAA by rating agencies, and thus the demand for the debt is not motivated by artificial demand for AAA-rated securities created by capital regulation on banks and insurance firms. Therefore, BDCs provide researchers a unique setup to study "counterfactuals" for CLO AAA-tranches: we ask, what if CLOs' senior tranches did not have an AAA rating by studying the value of BDCs' debt. By comparing CLOs and BDCs, we can quantify the effect of the so-called "reaching-for-yield" behavior of regulated institutional investors on the pricing of CLO AAA-tranches, because such behavior may affect CLO AAA-tranches but will not affect the pricing of BDC debt and equity.

To this end, we collect BDCs' loan holdings information from their 10-K filings and examine the micro-level data to ensure that our sample of BDCs is comparable to CLOs. Specifically, for our analysis, we select a subsample of BDCs that have a portfolio of corporate loans diversified across borrowers and industries. By inspecting the loan holding information of BDCs, we also find that loans provided by BDCs carry higher LIBOR spreads than those

 $<sup>^{2}</sup>$ Nickerson and Griffin (2017) address this challenge by examining the pre-crisis default data, while Griffin and Nickerson (2020) study the sample during the Covid crisis.

of CLOs, but time to maturity is similar.

We then estimate a structural credit risk model of Nagel and Purnanandam (2019) applied to the value-weighted average portfolio of BDCs. Unlike a traditional credit risk model of Merton (1974), Nagel and Purnanandam (2019) estimate the default risk of a lender who invests in a portfolio of loans exposed to downside risk. In this setup, equity holders of a lender do not enjoy upside potential because their asset payoffs are capped by the face value of underlying loans. Instead, the payoffs to lenders' equity resemble those of mezzanine debts which suffer from downside risk; thus, the downside risk is reflected in equity price and volatility. Similar to the Black-Scholes model which allows researchers to extract implied volatility from option prices, the Nagel-Purnanadam model allows us to infer the downside risks in loan portfolios from observed equity price and volatility of a lender.

In the Nagel and Purnanandam (2019) model, a debt of a lender is a senior claim which loses value only if a significant fraction of the loan portfolio is impaired. Thus, correlation in the borrower's asset value is the key parameter that determines the value of the lender's debt. All else being equal, a higher correlation reduces the value of the lender's debt due to an increased chance of multiple defaults occurring at once. As a result, we can infer a forward-looking, market-implied correlation of the borrower's asset by fitting the model to observed credit spreads on BDC's debt.

We first calibrate this model every month to observed BDC's equity price, equity volatility, and borrowing cost to estimate key parameters of the model, including correlation in the borrower's asset. We find that the market-implied correlation parameter is quite high, estimated at 0.78 on average over the sample period. This high correlation reflects the fact that the average credit spreads on BDCs' bonds are relatively high at 2.61%.

We assume that estimated correlation parameters are common across BDCs and CLOs, but we adjust other model parameters to match observable characteristics of CLOs that differ from BDCs. First, BDCs tend to invest in riskier loans than CLOs do, and thus we set a higher borrower leverage parameter for BDCs than for CLOs. Second, BDCs are less levered than CLOs are, so we set the lender's leverage higher for CLOs than for BDCs.

After calibrating the model parameters to CLOs, we compute the model-implied credit spreads on CLO senior tranches. We find that the model-implied credit spreads on CLO AAA tranches are on average 1.24% from January 2010 to June 2020. Comparing the model-based tranches and credit spreads in the data, the model-implied benchmark spreads are about the same as the average spreads on CLOs in the data over the same period (1.34%). Therefore, we find that the CLO AAA tranches are fairly valued and their credit spreads are comparable to the benchmark. To generate credit spreads large enough to justify spreads on CLO tranches, the model requires a high value of an estimated correlation. If we instead restrict the model with a correlation of 0.5 (as Nagel and Purnanandam (2019) do), then the model significantly underestimates the credit spreads on CLO AAA tranches.

The fact that our model matches credit spreads on CLO senior tranches implies that a relatively large credit spread on those securities does not necessarily represent an attractive investment opportunity. Credit spreads on CLO senior tranches are higher than AAA-rated corporate credit spreads, and this reflects the difference in systematic risk exposure rather than mispricing. While a default of an individual AAA-rated firm is an idiosyncratic event, a default of a CLO senior tranche is a systematic event that occurs only in the worst state of the world, commanding large risk premiums. In our setup, this large risk premium is embedded in the correlation parameter implied by asset prices. Rather than estimating the P-measure correlation using realized comovement in the unobservable borrower's asset values and then adjusting for risk premiums, we directly obtain the market-implied correlation parameter from comparable assets.

Our analysis shows that CLO investors fairly price the systematic risk of loan defaults relative to the benchmark. This finding is interesting for two reasons: first, there is extensive literature documenting the demand for safe assets (e.g., Krishnamurthy and VissingJorgensen 2012) which arguably distorts the price of privately created safe assets such as CLO senior tranches<sup>3</sup>; second, given the well-known regulatory arbitrage motive to purchase securitized products. Since capital regulations are imposed based on the total risk of a security rather than its systematic risk exposure, regulated investors such as commercial banks and insurance firms have the motivation to purchase AAA-rated securities which carry greater systematic risk exposure than those with lower systematic risk. Due to a shortage of arbitrage capital or short-sale constraints, such buying pressure can lead to credit spreads on CLOs that are lower than fundamentals. Empirically, however, we do not find evidence supporting the price impact of buying pressures on CLO senior tranches in our sample after the financial crisis.

In contrast, we find that AAA-rated CLO tranches were overvalued before the financial crisis. Our model suggests that credit spreads on AAA-rated CLO tranches should have been on average 1.34% before the crisis, about the same as the post-crisis average. However, the actual average credit spreads on CLO tranches were only 0.28% during the pre-crisis period. The striking difference in credit spreads on senior tranches before and after the financial crisis may reflect two changes that happened in the market: first, post-crisis regulations on banks and insurance firms might have forced these investors to accurately recognize the systematic nature of risks in CLO senior tranches; second, the collapse of Collateralized Debt Obligations (CDOs) during the financial crisis raised investor awareness about the risk of structured finance products in general, and thus investors updated their belief about the chance of systematic default of underlying loans. Both channels are consistent with our finding that CLO senior tranches were overvalued before the crisis, but not after the crisis.

Our results are not driven by mispricing of BDC equities or insufficient diversification of BDC loan portfolios. Since our idea hinges on the accuracy of BDC stock and bond prices, we examine how BDC debt and equity are priced. We find that BDC stocks are reasonably

<sup>&</sup>lt;sup>3</sup>Foley-Fisher et al. (2020) argue that CLO senior tranches changed from "information insensitive" securities to "information sensitive" securities during the COVID-19 crisis.

priced such that the three-factor model of Fama and French (1993) can explain their average excess returns well. We also find that the liquidity of BDC's bonds is about the same as other comparable corporate bonds.

To study the effect of insufficient diversification of BDCs, we modify the model of Nagel and Purnanandam (2019) to incorporate idiosyncratic shocks to the borrower's asset and find that the portfolio diversification for the average BDC is sufficient in producing estimation outcomes close enough to the base model of perfect diversification. Furthermore, we also extend the model to examine the effect of leverage constraints imposed on CLOs due to the contracts between CLOs and investors. We confirm that our main results do not significantly change when accounting for more realistic features of structured finance products.

Finally, we conduct an out-of-sample test of the model by pricing the credit spreads on CLO junior tranches. Ensuring that the model prices both senior and junior tranches is important for the following reason: suppose that credit spreads on CLO senior tranches are lower than fundamentals due to buying pressure from regulated institutional investors, while credit spreads on junior tranches are free from such bias; then, the model that prices CLO senior tranches should not be able to price junior tranches. We find that the Nagel and Purnanandam (2019) model calibrated to BDC data matches observed credit spreads on CLO junior tranches as well as it does for senior tranches. This result provides additional evidence that this model provides a useful benchmark for CLO tranches.

This paper contributes to a strand of literature that examines the price of correlation risk, including Longstaff and Rajan (2008), Coval, Jurek, and Stafford (2009), Collin-Dufresne, Goldstein, and Yang (2012), Culp, Nozawa, and Veronesi (2018), and Driessen, Maenhout, and Vilkov (2009). Coval, Jurek, and Stafford (2009) and Collin-Dufresne, Goldstein, and Yang (2012) study stock index option prices to infer correlation risk premiums in CDOs. Drawing inference about correlation in underlying debt returns from correlation in stock returns is a valid approach so long as one factor (such as borrower's asset value) drives both stock price and loan values. Our paper does not rely on such an assumption that ties equity and debt, as we compare an entity that invests in a portfolio of loans (CLO) with other entities also investing in loans (BDCs). This similarity in the benchmark allows us to directly infer the price of correlated default risk in loans. Furthermore, listed options are short-term derivatives, and their expiry is at most 3 years, which is shorter than typical debt issued by CDOs or CLOs. In our framework, the maturity of BDC debt is comparable to CLO tranches, alleviating the concern about maturity mismatch.

This paper also relates to the growing literature on shadow banks and the role of CLOs in the syndicated loan market. Irani and Meisenzahl (2017), Irani et al. (2020), Kundu (2020a,b,c), and Elkamhi and Nozawa (2021) report evidence of fire sale in the loan market by banks and CLOs using different identification strategies. Loumioti and Vasvari (2018, 2019) study the effect of portfolio constraints on CLO performance. Munday et al. (2018), Loumioti (2019), Chernenko, Erel, and Prilmeier (2021), and Davydiuk, Marchuk, and Rosen (2020) examine the characteristics and performance of nonbank lending. Importantly, Cordell, Roberts, and Schwert (2020) study the performance of CLOs by examining the historical payout data. Our study complements theirs by studying forward-looking information contained in asset prices.

Finally, this paper relates to the literature on the effect of the reaching-for-yield behavior of institutional investors on fixed income securities. Becker and Ivashina (2015), Choi and Kronlund (2017), Choi and Chen (2019) and Acharya and Naqvi (2019) report evidence for the reaching for yield behavior affecting prices for a variety of asset classes. Since these papers do not study CLOs, our paper complements them by studying the potential effect of the reaching-for-yield behavior on CLO AAA tranches.

The remainder of the paper is organized as follows. In Section 2, we discuss the background information on BDCs and CLOs, and describe data. In Section 3, we explain the model and report calibration results. In Section 4, we extend the baseline model and conduct a series of robustness tests to address potential concerns about the model estimate. Section 5 concludes.

# 2 Background and Data

# 2.1 Institutional Background for Business Development Companies

A business development company (BDC) is a U.S. domestic closed-end fund that invests mostly in certain securities specified in Section 55(a) of the 1940 Act and elects BDC status (by submitting Form N-6 and Form N-54A). Specifically, BDCs must allocate at least 70% of their investment in securities issued by private firms or public firms with a market capitalization less than \$250 million. In our sample of loan-oriented BDCs, on average 90.3% of BDC assets consist of loans, and the rest consist of equity investments, warrants, and equity tranches of CLOs. BDCs usually "self-originate" loans from corporate restructuring deals such as leveraged buyouts rather than purchasing syndicated loans arranged by commercial banks.

The initial idea behind BDCs is to help finance small and medium-sized businesses as well as to provide managerial consulting to the borrowers. This idea of a BDC looks much like today's private equity firm. However, in practice, much of today's BDC assets consist of loans rather than equity investment. Thus, despite the spirit of the law, some BDCs become an investment vehicle that is not actively involved with the management of borrowers but offers investors exposure to a diversified portfolio of loans.

BDCs are required to pay out more than 90% of their income to shareholders. The BDCs that satisfy the payout requirement are exempt from corporate income tax. This tax benefit is often cited by practitioners as a motivation for establishing a BDC for loan investments.

The payout requirement also leads to a relatively high dividend yield on BDC equity; on average, the dividend yield is as high as 10% in our sample.

BDCs issue equities either in the public or private market. In addition to equity issuance, BDCs can use debt to borrow from outside. Generally, BDCs are required to maintain the ratio of debt to asset below 50%, but such a ratio can be increased up to 66.7% under certain conditions.<sup>4</sup> Much of the debt is loans from banks, with a fraction of BDCs issuing bonds in the public market. Despite low leverage, no BDC bonds are rated AAA in our sample.

Though BDCs were enabled by law in 1980, we see a sizeable increase in incorporation only in the late 1990s. The number of publicly-listed loan-oriented BDCs in our sample, shown in Panel A of Figure 1, is 2 in 2005, and increases to 27 in 2020, reflecting the growing interest in investing in corporate loans for small and medium-sized borrowers. The total market value of equity was around \$20 billion in June 2020, which is relatively small compared with the outstanding collateralized loan obligation (CLO) of about \$600 billion in 2020.

We report the value-weighted average of the book-to-market ratio, leverage ratio, and dividend yield in Figure 1. The average book-to-market ratio was generally below one before the financial crisis in 2008, but close to one after the crisis, suggesting that the market value is close to the fair value of the portfolio of loans. We also observe that BDCs issue non-trivial amounts of debt, leading to an average debt-to-asset ratio of between 0.3 and 0.4. However, this leverage ratio is still lower than typical AAA-rated CLO tranches, for which leverage is 0.66 in our sample. Consistent with the statutory requirement, the dividend yield of BDCs is high.

<sup>&</sup>lt;sup>4</sup>The limit on leverage can be increased with either i) approval of a majority of the BDC's board of directors and a majority of disinterested directors, or ii) approval of a majority of the BDC's stockholders.

#### 2.2 Institutional Background for CLOs

CLOs are special purpose vehicle that specializes in loan investment. They hold a diversified portfolio of syndicated loans and fund their investment by issuing debt securities with various seniorities. A debt security with different seniority is called a tranche. CLO managers actively manage CLO portfolios to strike a balance between safety for senior tranches and higher returns on junior tranches. From the launch to the reinvestment date of CLOs, CLO managers reinvest proceeds from loan investments in other loans to maintain the size of the balance sheet. After the reinvestment period, CLO tranches are amortized as CLOs receive loan repayments. As a result, CLO tranches are usually redeemed before the legal maturity date.

A CLO's most senior tranche is often rated AAA by major rating agencies, and these senior tranches on average account for 66% of the total assets in our sample. An AAA tranche loses money only after junior tranches are wiped out due to multiple defaults across many borrowers. Since multiple defaults occur only if default events are highly correlated, valuation for an AAA tranche crucially depends on the correlation among borrowers' fundamentals.

Though it is difficult to price CLO senior tranches, they have become increasingly popular among regulated institutional investors.<sup>5</sup> AAA-rated CLO senior tranches offer a higher yield (with the post-crisis average of LIBOR + 1.34%) at issue than AAA-rated corporate bonds (0.65%) do. Furthermore, for the U.S. CLOs, there was no default of AAA-rated senior tranches during the financial crisis in 2008 and its aftermath, providing comfort to the investors.

The regulatory treatment of investment in CLO tranches changed dramatically after the financial crisis, which led to different generations of CLOs. Those issued before the crisis are CLO1.0, while CLOs issued afterward are called CLO2.0 and CLO3.0. Compared with

<sup>&</sup>lt;sup>5</sup>According to Deutsche Bank Research, banks hold 59% of AAA tranches, followed by asset managers (19%) and insurance firms (17%).

CLO1.0, CLO2.0 has higher levels of subordination, tighter collateral eligibility requirements, and shorter reinvestment periods. Thus, CLO2.0 is generally regarded as safer than CLO1.0. CLO3.0, which was launched after 2014, further improved safety by adhering to the Volcker rule and restricting collateral more tightly than its predecessors.<sup>6</sup>

In Basel III, capital charges against banks' investment in CLO tranches are no longer evaluated based solely on the credit rating of the tranche. Rather, the capital charge is based on an estimate for the charges on the underlying pool of assets as if held directly on the balance sheet, adjusted for the degree of subordination. More senior tranches require less capital, subject to a 20% risk-weight floor for U.S. banks. Furthermore, CLO senior tranches are not treated as High-Quality Liquid Assets for the liquidity coverage ratio<sup>7</sup> calculation for banks, differentiating senior tranches from Treasury securities. Finally, under Basel III, banks are required to examine and evaluate the risk of the underlying loans for CLOs.<sup>8</sup>

CLO tranches are illiquid and not actively traded in the secondary market.<sup>9</sup> Furthermore, CLOs do not value loans using the market price: they are required to evaluate the loan holding at the fair value only for defaulted loans and CCC-rated loans in excess of the predetermined threshold (typically 7.5% of the asset). This feature of CLOs creates opacity in evaluating the fair value of CLO tranches. Because of this limitation, we focus on the primary market credit spreads on CLO tranches throughout the article.

<sup>&</sup>lt;sup>6</sup>Specifically, CLO3.0 is not allowed to invest in high-yield bonds.

<sup>&</sup>lt;sup>7</sup>Banks are required to maintain a certain level of the liquidity coverage ratio — the ratio of High-Quality Liquid Assets such as Treasury securities to short-term liabilities — under Basel III.

<sup>&</sup>lt;sup>8</sup>Another notable development regarding regulations on CLOs is the risk retention rule proposed by the Dodd-Frank Act which requires CLO managers to hold a certain fraction of CLO debt securities. The rule took effect in December 2016 but was subsequently withdrawn in May 2018, after the industry organization, the Loan Syndication and Trading Association, successfully sued the Federal Reserve and Securities Exchange Commission.

<sup>&</sup>lt;sup>9</sup>According to Bloomberg (2019), the secondary market transaction volume of CLO tranches via Bid Wanted in Competition (BWIC) auction was \$26.1 billion in 2018. Given the outstanding amount of \$616.9 billion (SIFMA), the annual turnover rate was 4.2%, which was lower than the annual turnover of 34.23% for corporate bonds (Schestag, Schuster, and Uhrig-Homburg (2016)).

#### 2.3 Data

We use BDCs' monthly stock prices from Compustat from January 2005 to June 2020, and balance sheet data from Compustat Fundamentals Quarterly. The start date is determined by the year we find qualified BDCs in the selection process described below. We merge the accounting information in year q to stock price information in year q + 1.

In order to identify BDCs, we obtain a list of BDCs on the Securities and Exchange Commission's website. The website provides the complete list since 2012. We augment this list by manually searching for Forms N-6 and N54-A filings in the Electronic Data Gathering, Analysis, and Retrieval system (EDGAR). To ensure that our sample of BDCs is comparable to CLOs, we select a subsample of BDCs that resemble CLOs. To this end, we hand-collect loan holding information for BDCs from their 10-K filings.

Specifically, we scrape the 10-K files in EDGAR and obtain lists of loans held by BDCs. The loan information typically includes borrower names; collateral, seniority, maturity of loans; interest rates; and the dollar amount of the loan held in the portfolio at the end of the fiscal year. We then select a subset of BDCs based on three criteria: i) a BDC must invest at least 80% of their investments in loans; ii) a BDC must hold at least 30 loans; iii) the Herfindahl index for industry shares of BDC's portfolio is below 0.1. We include a BDC in our sample once it meets these three criteria and keep it in the sample as long as the three-year moving average satisfies the criteria. The third condition on the Herfindahl index requires BDCs to be reasonably well-diversified across industries. This condition removes BDCs that focus on a specific business sector, such as Hercules Capital Incorporated and Horizon Technology Finance Corporation that invests primarily in loans issued by technology companies.

After the selection process, we end up with the final sample of 39 BDCs from January 2005 to June 2020. Appendix A explains this data collection process in detail and provides a list of BDCs with a CRSP permno identifier and firm-level summary statistics.

For data on CLO tranches and holdings, we use the CLO-i data provided by Acuris company. This database provides information on CLO tranches, including issue dates, issue amount, and LIBOR spreads at issuance from January 2005 to June 2020 as well as detailed loan holding information from January 2007 to June 2020.

We obtain transaction prices for the BDCs' bonds from standard TRACE. Specifically, we follow Bessembinder et al. (2009), use transactions with a volume above \$100,000, and compute the volume-weighted average price to construct daily bond prices. We then use the last date with non-missing observations in a month as a month-end price.

## 2.4 Comparing BDCs and CLOs

Table 1 compares the summary statistics of BDCs and CLOs. Though the aggregate market size of CLOs is greater than BDCs, each CLO tends to be smaller than a BDC. The average CLO holds \$364 million of assets under management, which is less than the average BDC whose asset size is \$948 million. CLOs are on average more highly levered than BDCs. The ratio of senior debt to an asset is 0.66 for the average CLO, while it is 0.34 for the average BDC. The effective debt maturity is similar: the effective maturity for CLO senior tranche is 4.93 years, while the average BDC bond in our sample has 4.90 years to maturity. To compute the effective maturity for CLOs, we take the cash-flow weighted average of the time of redemption since CLO issuance.

Now we compare the loan holdings of CLOs with those of BDCs. Panel C of Table 1 reports that loans held by BDCs are on average riskier than those held by CLOs: the average loan credit spreads over LIBOR are 350 bps for CLOs while they are 747 bps for BDCs. Many of the loans held by CLOs are rated by rating agencies with an average rating of 15 (corresponding to B), while most of the loans held by BDCs are unrated. Thus, when we calibrate a credit risk model later, we account for the difference in riskiness of the loans held by BDCs and CLOs.

Panel C of Table 1 sheds light on the degree of portfolio diversification for BDCs and CLOs. On average, a CLO holds 232 loans while a BDC invests in 107 loans. We further dissect the degree of portfolio diversification across the borrower's industry. To classify loans into industries, we use Moody's 35 industry classification. For each month and lender, we compute the fraction of loans provided to borrowers in each of the 35 industries.

Panel C reports the share of the largest industry in the lender's portfolio. The average CLO has 14.6% exposure to the largest industry in the portfolio, while the average BDC has 16.2%, which is very similar to the average CLO. We also compute the Herfindahl index using the industry shares and find that the average CLO has an index of 7.85% while the average BDC has 8.75%. Therefore, the degree of BDCs' portfolio diversification is similar to that of the CLOs'. However, the fewer number of loans for BDCs may give rise to bias due to the idiosyncratic risks of borrowers. To evaluate the magnitude of the bias, we analyze the effect of the number of loans on credit spreads for a lender's debt in Section 4.2, and show that quantitatively, the effect on debt valuation is negligible.

# 3 Model and Results

# 3.1 Structural Default Risk Model of Nagel and Purnanandam (2019)

In this section, we describe a structural credit risk model of Nagel and Purnanandam (2019). In this model, a lender invests in a pool of loans to a large number of borrowers. The lender finances such investments by issuing equity and a zero-coupon bond. The payoffs for the lender's equity and debt are capped by the face value of the underlying pool of loans. Thus, unlike the Merton model, the payoffs are not log-normally distributed. Instead, the asset value of the lender is left-skewed due to the downside risk of underlying loans. In deriving the value of lender's debt and equity, the model accounts for this nature of lender's assets. Let borrower *i*'s asset value follow the geometric Brownian motion under the Q-measure.

$$\frac{dA_t^{\tau,i}}{A_t^{\tau,i}} = (r-\delta)dt + \sigma(\sqrt{\rho}dW_t + \sqrt{1-\rho}dZ_t^{\tau,i})$$
(1)

where r is a risk-free rate,  $\delta$  is the payout rate of borrowers' asset,  $\sigma$  is asset volatility,  $\rho$  is a correlation parameter,  $dW_t$  is a systematic shock,  $dZ_t$  is borrower-specific shock, and  $\tau$  is the age of the loan (i.e. the lender provided the loan at  $t = -\tau$ ). We form cohorts of borrowers based on  $\tau$ .

A lender provides a zero-coupon loan to borrower i who sets initial loan-to-value ratio l. At maturity of the loan, the borrower pays the face value back to the lender:

$$F_1(\mu) = \bar{A} l e^{\mu T}$$

where  $\bar{A}$  is the initial asset value common for all firms and cohorts,  $\mu$  is the promised yield for the loan.

We solve for  $\mu$  using the Merton (1974) model. The payoff at the maturity of the loan  $t = T - \tau$  is:

$$L_{T-\tau}^{\tau,i} = \min\left[A_{T-\tau}^{\tau,i}, F_1(\mu)\right]$$
(2)

Therefore,  $\mu$  must solve:

$$\bar{A}l = e^{-rT} E^Q_{-\tau} [L^{\tau,i}_{T-\tau}(\mu)].$$
(3)

This solution for the loan yield,  $\mu$ , does not depend on  $\overline{A}$  because it scales both borrower's asset and the face value of the loan. Instead, the yield reflects the riskiness of each loan defined by l and  $\sigma$ . Later, we calibrate the value of l such that  $\mu - r$  equals the observed credit spreads on loans in the data.

To derive the payoffs to the loan portfolio, we use the following aggregate asset value and

log asset value computed by averaging across firms, conditional on realization of aggregate shock,  $W_{T-\tau}$ :

$$A_{T-\tau}^{\tau} = \int_{0}^{1} A_{T-\tau}^{\tau,j} dj,$$
  
=  $\bar{A} \exp\left\{ (r-\delta)T - \frac{1}{2}\rho\sigma^{2}T + \sigma\sqrt{\rho}(W_{T-\tau} - W_{-\tau}) \right\},$  (4)

and

$$a_{T-\tau}^{\tau} = \int_{0}^{1} \log A_{T-\tau}^{\tau,j} dj,$$
  
=  $\log \bar{A} + \left\{ (r-\delta)T - \frac{1}{2}\sigma^{2}T + \sigma\sqrt{\rho}(W_{T-\tau} - W_{-\tau}) \right\}.$  (5)

A lender invests in a diversified portfolio of loans. The payoff to the lender from all borrowers in cohort  $\tau$  is

$$L_{T-\tau}^{\tau} = \int_{0}^{1} L_{T}^{j} dj$$
  
=  $\int_{0}^{1} A_{T}^{j} dj - \int_{0}^{1} \max(A_{T}^{j} - F, 0) dj$   
=  $A_{T} \Phi(d_{1}) + F \Phi(d_{2}).$  (6)

where

$$d_2 = -\frac{\log F_1(\mu) - a_{T-t}^{\tau}}{\sqrt{1 - \rho}\sqrt{T}\sigma}$$

$$\tag{7}$$

$$d_1 = -d_2 - \sqrt{1 - \rho}\sqrt{T}\sigma. \tag{8}$$

The portfolio payoff in (6) shows that idiosyncratic risk  $dZ_t$  is diversified away in a loan

portfolio, and thus it affects the fraction of defaulting loans to the portfolio but does not induce any uncertainty to the portfolio payoffs. Instead, shocks to the portfolio come only from the systematic shock,  $dW_t$ .

In the model, changing the correlation parameter,  $\rho$ , does not change the probability of default for individual loans; instead, the average fraction of loans defaulting only depends on  $\sigma$ . However, changing  $\rho$  changes the magnitude of the systematic shock, and affects the uncertainty of bank assets. All else being equal, a higher value of  $\rho$  leads to higher volatility and lower skewness of the lender's assets.

At the maturity of a loan, the lender reinvests the proceeds into a new cohort. We assume that the (ex-ante) riskiness of the new set of loans is the same as the initial set of loans. Therefore, the borrower's asset value is reset at:

$$A_{(T-\tau)+}^{\tau,i} = \frac{L_{T-\tau}^{\tau}}{l},$$
(9)

and the face value of the new loan is given by:

$$F_2(\mu) = A^{\tau,i}_{(T-\tau)+} l e^{\mu T}.$$
(10)

Since the riskiness for a loan at issuance is fixed, we use a constant value of yield,  $\mu$ , for all cohorts, before and after refinancing. Nagel and Purnanandam (2019) show that this feature of the model in which loans rollover to a new generation is quantitatively important in generating realistic credit spreads on lender's debt in good times. After a positive shock, the borrower's asset value rises and provides a buffer for the loan. However, at the refinance point, the borrower takes away excess collateral to keep the constant loan-to-value ratio l, which reduces the protection available for the lender. Thus, the loan rollover helps generate a greater default risk of a lender in good times than the Merton model does.

Applying (10) to (6), we obtain the payoff to the second generation of loans,  $L_{2T-\tau}^{\tau}$ . Let

*H* be the maturity of the lender's debt, such that H < T. The value of the lender's assets is the average of all *N* cohorts:

$$V_{H} = \frac{1}{N} \left( \sum_{\tau < H} e^{-r(\tau + T - H)} E_{H}^{Q} [L_{2T - \tau}^{\tau}] + \sum_{\tau \ge H} e^{-r(\tau - H)} E_{H}^{Q} [L_{T - \tau}^{\tau}] \right).$$
(11)

The first term on the right-hand side of (11) is the loan value for cohorts that refinance before H, while the second term represents the loan value for the cohorts that do not refinance before H.

Following Nagel and Purnanandam (2019), we assume that there is a dividend paid to the lender's shareholders prior to the debt payoff. For simplicity, we assume that the dividend is paid in a lump sum just before the bond maturity H:

$$Div_H = V_H (1 - e^{-\gamma H}).$$

The payoffs to the debt (with face value D) and equity of the lender are:

$$B_H = \min(D, V_H - Div_H), \tag{12}$$

$$S_{H} = \max(V_{H} - Div_{H} - D, 0) + Div_{H}.$$
(13)

This expression reflects the fact that the lender's shareholders receive a stream of dividends before debt repayment.

The price of debt and equity is computed by the risk-neutral expected payoff discounted at the risk-free rate,  $B_0 = e^{-rH} E^Q[B_H]$  and  $S_0 = e^{-rH} E^Q[S_H]$ . To compute these values, we simulate paths of  $dW_t$  from normal distribution 5,000 times and take the average across paths.

Since the payoffs to the lender's assets are capped by the face value of the loan portfolios, the lender's equity holders do not enjoy the upside potential of the assets. Therefore, the payoffs to the lender's equity holders resemble those of mezzanine debt that is sensitive to the downside risk of the asset. Furthermore, we compute credit spreads of the lender's debt by:

$$s = -\frac{1}{H}\log\frac{B_0}{D} - r.$$
(14)

Finally, we compute instantaneous equity volatility implied by the model. To compute volatility, we suppose that there is a one-time shock  $dW_0$  to the borrower's asset at time 0. More specifically, we set the shock for each of the existing cohorts (as of time t) equal to the product of the fraction of the loan's life  $\tau/T$  and  $dW_0$ . For each value of  $dW_0$ , we compute aggregate asset value  $A_0$  and lender's stock price  $S_0$ . We then compute the numerical derivative of log  $S_0$  with respect to log  $A_0$ . The instantaneous equity volatility is the product of the derivative and  $\sigma\sqrt{\rho}$ .

In the model,  $dW_0$  measures a shock to the underlying loans that are issued before time 0. By changing  $dW_0$ , we change the lender's asset value  $A_0$  as well as the volatility of equity. Figure 2 shows how lender's equity value, volatility and risk-neutral probability of default depend on  $dW_0$ . As  $dW_0$  increases, the value of equity rises, volatility falls and the probability of default falls.<sup>10</sup> Later, we calibrate  $dW_0$  to match observable quantities in the data.

# 3.2 Calibration Method

For each month, we calibrate the model by matching three objectives, the ratio of equity value to an asset (the sum of the book value of debt and the market value of equity), equity volatility and credit spreads on BDC's debt for an average BDC. We estimate equity variance using daily stock returns over the rolling 3-month window for each BDC, and compute value-weighted equity variance. We then run an AR(1) regression to predict the next quarter

<sup>&</sup>lt;sup>10</sup>In practice, we can increase the equity volatility by increasing the asset volatility  $\sigma$ . However, this will also change the riskiness of each loan at the same time. By letting  $dW_0$  vary rather than  $\sigma$ , we ensure the constant riskiness of borrowers but generate time-varying lender's equity volatility.

variance with a lagged quarterly variance. The fitted value provides the measure of BDC equity variance and thus volatility.

For credit spreads on BDC's debt, we have data on secondary market credit spreads on BDC's bonds from April 2014. To match the maturity of CLO senior tranches, we only use bonds with a remaining time to maturity of more than 3 years. There are 13 bonds and the average credit spreads over LIBOR are 233bps. Before April 2014, we extrapolate the credit spreads based on the credit spreads on AAA-rated corporate bonds. Specifically, we take the average of the ratio of credit spreads on BDC's bonds to AAA-rated corporate bonds and multiply it by AAA spreads before April 2014.<sup>11</sup>

To match three observable quantities, we have three free parameters in the model: the size of loan portfolio  $F_1$ , initial shock  $dW_0$ , and correlation  $\rho$ . The first two were the free parameters used in Nagel and Purnanandam (2019), while the last one is added to match credit spreads. To fit the data every month, we search for the values of  $F_1$ ,  $dW_0$ , and  $\rho$  that match the equity volatility, market value of equity and credit spreads. A lower value of  $dW_0$  reduces asset value and increases the riskiness of a lender's assets, while a lower value of  $F_1$  mainly reduces asset values.

We use the set of other parameters for the model in Panel A, Table 2. Borrower's asset depreciation rate  $\delta$  is set at 0.5% per year, which is the same value as Nagel and Purnanandam (2019). BDC's payout rate  $\gamma$  is estimated at 2.83% using the average payout to asset ratio in our sample. This value is much higher than Nagel and Purnanandam (2019) who set  $\gamma = 0.2\%$  for their sample of commercial banks, reflecting the fact that BDCs are required to distribute at least 90% of their earnings to shareholders. For CLOs,  $\gamma$  is estimated at 1.63% using the average of the payout to equity tranches to the asset value.

In the data, the average loan maturity for BDCs and CLOs is 4.4 and 4.0 years, respec-

<sup>&</sup>lt;sup>11</sup>To verify the validity of this extrapolation, in Appendix Figure A1, we plot the ratio of total interest expense to the total liability of the BDCs in our sample. The figure shows that the level of borrowing cost for BDCs is similar between the pre-and post-crisis period, which is consistent with our finding that our model-based CLO spreads are similar between the pre-and post-crisis period.

tively. Since this is the remaining time to maturity, the initial loan maturity must be longer than these values. Thus, we set the loan maturity parameter T to 5 years for both BDCs and CLOs. We set the maturity of the lender's debt H to 3 years. In the main results, we assume those debts are not callable but relax this assumption in an extension of the model considered in Section 4.4. In the model, the lender's debt is assumed to be a zero-coupon bond, while in practice it is a coupon-bearing bond. To account for the difference, we multiply quasi-market leverage with  $e^{(r_t+s_t)H}$  to calculate D, where  $r_t$  is H-period risk-free rate and  $s_t$  is the observed credit spreads in month t.

Panel B of Table 2 shows targets for model calibration. Comparing BDCs and commercial banks, BDCs tend to have much lower leverage than commercial banks do, while BDC's equity volatility is similar to commercial banks. For example, the average annualized equity volatility for BDCs in our sample is 32%, while Nagel and Purnanandam (2019) report commercial banks' equity volatility is 29%. To generate comparable equity volatility with lower leverage, BDC's asset has to be riskier than that of banks. Thus, we set the borrower's asset volatility to be 100% per year, much higher than the value used for commercial banks. High borrower's asset volatility should not be confused with the high volatility of lender's assets, as bank's asset is a loan on the borrower, and idiosyncratic risk is diversified away. To justify the choice of volatility parameter, we examine the relationship between asset volatility and equity volatility in the model in Appendix B.

Since not all BDCs issue corporate bonds, we further split the sample of BDCs into two groups: bond issuers and non-bond issuers. For each group, we form a value-weighted portfolio and compute the average leverage and equity volatility. We find that these two groups of BDCs share similar values of fundamentals. The average leverage is 32% for bond issuers while it is 37% for non-bond issuers. Equity volatility is also similar; it is 30% for bond issuers and 33% for non-bond issuers. Therefore, we use the average leverage and equity volatility across all BDCs, while taking the average of credit spreads across available bonds for the inputs to the model. In order to account for the difference in loan quality when calibrating the model, we set the loan-to-value ratio l such that credit spreads on the loan are equal to the average LIBOR spreads for BDCs and CLOs in the data. Since loans provided by CLOs have lower credit spreads than those by BDCs, we use a lower value of l for CLOs than for BDCs.

#### **3.3** Estimation Results

In this section, we report the fit of the model to the average BDC, and then apply the estimated parameters,  $F_1$ ,  $dW_0$ , and  $\rho$  to derive model-implied credit spreads on the average CLO's AAA-tranche.

Figure 3 shows the fit of the model to the BDC data. We calibrate the model to equity value, equity volatility and credit spreads on BDC's debt using three free parameters ( $F_1$ ,  $dW_0$  and  $\rho$ ). The model matches the data well, and thus the two lines mostly overlap with each other.

Panel C of Table 2 reports the average of the estimated parameters using BDC data, separately for different subperiods in our sample. The estimated correlation coefficient  $\rho$  is quite high throughout the sample period, ranging from 0.65 to 0.89 with a peak right after the financial crisis. We emphasize that the estimated value is for risk-neutral correlation in borrower's asset values rather than for correlation in default under the P-measure. Just like we apply the Black-Scholes formula to options to back out implied volatility, our measure of asset correlation is implied from the market price and can be higher or lower than the actual correlation. Parameter  $dW_0$  measures shock to asset values of the lender. Consistent with Nagel and Purnanadam (2019), we see large negative values of  $dW_0$  during the financial crisis in 2008 and its aftermath in 2009, while  $dW_0$  is near zero in other periods.

To examine the effect of high correlation, we follow Nagel and Purnanandam (2019) and estimate a restricted model by setting  $\rho = 0.5$  for all months. We then calibrate the model every month using equity value and volatility and plot the model-implied credit spreads at the bottom panel of Figure 3. For the non-crisis period, the restricted model with  $\rho = 0.5$  constantly underpredicts the credit spreads in the data. At the end of the sample period (June 2020), the model-based credit spread is 4.02% which is non-trivial given the modest leverage of BDCs. However, the credit spread in the data is higher at 4.78%. Therefore, the model requires the correlation parameter to be greater than 0.5 to match credit spreads in the data.

We next take the estimated values of  $F_1$ ,  $dW_0$ , and  $\rho$  given and generate the modelimplied credit spreads for CLO's AAA-rated tranche. We use the summary statistics in Table 2 to guide our choice of other parameters. For CLOs, we set the lender's leverage to be fixed at 0.66, which is higher than the leverage for BDCs. Thus, accounting for coupons, the input to the model is  $D = 0.66e^{(r_t+s_t)H}$ , where we use the average corporate AAA spreads for  $s_t$ . We use a fixed value of leverage over time because we are interested in constructing the benchmark for the CLO primary market. At issuance, most CLOs have very similar leverage across entities and over time, and thus we use a constant value of leverage in our calibration. Furthermore, we use the payout rate  $\gamma = 1.63\%$  and set borrower's leverage lsuch that credit spreads on underlying loans match the data in Table 1.

In Figure 4, we compare equity volatility, equity value and credit spreads between BDCs and CLOs generated by the model. In the model, there are two key differences between BDCs and CLOs. First, the underlying assets are riskier for BDCs than for CLOs, suggesting that CLOs should be safer than BDCs. Second, BDCs are less levered than CLOs are, which implies that CLOs should be riskier than BDCs. Which effect dominates the other depends on the choice of parameters.

Figure 4 shows that equity volatility and credit spreads for BDCs are higher than those for CLOs. Thus, the effect of lower risk of underlying assets for CLOs dominates the effect of higher leverage, making CLO's debt safer than that of BDCs. The equity value (relative to asset) is lower for CLOs than BDCs except during the financial crisis. The different time-series behavior in equity value reflects the fact that we hold CLO's leverage constant while BDC's leverage varies over time. This apparent discrepancy is due to the fact that we aim to price credit spreads on CLO's tranches at issuance while we learn about the model parameters from the secondary market price of BDC's debt and equities.

We next compare the model-implied credit spreads on CLO's senior tranches and actual credit spreads. Figure 5 presents the main result of the paper: it shows the AAA-rated CLO tranche credit spreads at CLO's issuance, model-based credit spreads, and AAA-rated corporate credit spreads from the ICE BofA ML index.<sup>12</sup> The issue spreads on CLO senior tranches are low at around 0.3% before the crisis, and increase after the crisis, staying mostly above 1.0%. There are no new CLOs issued in 2009 and 2010, and thus the credit spreads are missing on those years. On the other hand, corporate AAA spreads are slightly lower than CLO spreads before the crisis, and stay substantially below the CLO spreads after the crisis.

Panel A of Figure 5 presents benchmark credit spreads based on the unrestricted model, in which correlation is treated as a free parameter. We first focus on the post-crisis period. After accounting for higher leverage of CLOs, the model-based credit spreads are quite high: with the BDC-implied correlation parameters, the average benchmark credit spread is 1.62% from 2010 to 2013, 1.02% from 2014 to 2017, and 0.83% from 2018 to 2020, which are comparable to the average CLO AAA-rated spreads of 1.43% from 2010 to 2013, 1.09% from 2014 to 2017, and slightly lower than 1.18% from 2018 to 2020. Panel D of Table 2 presents the credit spreads on CLO senior tranches averaged over various subperiods in our sample. The lower model-based credit spreads from 2018 to 2020 correspond to lower values of correlation and slightly higher values of  $dW_0$  than those from 2010 to 2017 (Panel C). Still, the post-crisis average (2010-2020) of 1.24% for the model is close to that for the data (1.34%).

<sup>&</sup>lt;sup>12</sup>ICE BofA ML index provides an option-adjusted spread over Treasuries. We subtract the difference between the 10-year swap rate and 10-year Treasury yield to convert the spreads such that they are LIBOR spreads.

Overall, we do not find compelling evidence for the potential buying pressure from regulatory-constrained institutional investors which leads to overvaluation of CLO senior tranches. The large credit spreads on CLO senior tranches for CLO2.0 (2010-2013) and CLO3.0 (2014-) reflect the risk premiums required to bear the inherent systematic risk of correlated defaults among underlying loans.

We next discuss our findings for CLO1.0, which is issued in the pre-crisis period. In contrast to the post-crisis sample, Panel A of Figure 5 shows that the model-based credit spreads are higher than those on CLO AAA-rated tranches before the crisis. Panel D of Table 2 reports credit spreads on CLO senior tranches in the data and in the model. The average credit spreads before December 2007 are 0.28% in the data, while they are 1.34% in the calibrated model. These estimates suggest that CLO senior tranches are overvalued during the pre-crisis period.

This difference in credit spreads between the pre-and post-crisis is the opposite of what is predicted from the improved safety of post-crisis CLOs (2.0 and 3.0) relative to the precrisis CLOs (1.0). A potential reason for the difference is learning by investors. After the financial crisis, investors learned from the collapse of Collateralized Debt Obligations (CDO) and updated their belief about the tail risk in diversified portfolios of defaultable securities. As a result, the credit spreads on CLO senior tranches after the crisis increased to the level comparable to our benchmark based on BDCs. Benzoni, Collin-Dufresne, and Goldstein (2011) make a similar observation on investors' learning in the options market before and after Black Monday in 1987. However, it is also possible that changes in banking sector regulations affected the capital charges on holding CLO tranches, leading to the difference in credit spreads before and after the financial crisis.

To overcome the limitation of the sample period of BDC's corporate bonds, we also conduct the analysis using credit default swap (CDS) spreads on banks, in which we calibrate the model to equity values, equity volatility and CDS spreads for banks from January 2005 to June 2020. These results are reported in Appendix C. In short, we find that the qualitatively same conclusion holds when we construct the benchmark for CLOs using the data on banks.

To understand the role of asset correlation on CLO credit spreads, we examine the restricted model with  $\rho = 0.5$ . Panel B of Figure 5 presents benchmark credit spreads when we estimate the model by restricting  $\rho = 0.5$ . As expected from the fit to the BDC credit spreads, a low correlation leads to low model-based credit spreads for CLOs. In this calibration, the average benchmark spread after the crisis is 0.39%, which is much lower than the average CLO AAA-rated spreads. If we take this assumption of loan correlation, then we would conclude that the credit spreads on the CLO AAA-rated tranche after the financial crisis are in fact too high. This argument is implausible since there is no portfolio constraint that prevents investors from buying CLOs. There is a short-sale constraint, but it would give rise to the overvaluation of CLOs rather than undervaluation. In contrast, in this calibration, the CLO tranche spreads before the financial crisis are about the same as the benchmark. Thus, the CLO senior tranches during the pre-crisis periods are priced as if the correlation were 0.5.

Finally, we discuss the potential bias due to the slightly higher industry concentration of BDC's portfolio than that of CLO's portfolios. If this difference in industry concentration leads to a difference in correlation parameter  $\rho$ , then we should use a lower value of  $\rho$  to value CLOs than for BDCs. A lower value of  $\rho$  will decrease our estimates for the benchmark credit spreads on CLO senior tranches, which strengthens our argument that CLO senior tranches are not overvalued relative to the benchmark after the financial crisis.

# 3.4 Role of Borrower's Asset Correlation and Comparison Between BDCs and CLOs

The previous section shows that the Nagel and Purnanandam (2019) model matches the credit spreads of CLO senior tranches during the post-crisis period, but with a high value of

borrower's asset correlation.

A common approach to pin down the correlation parameter is to measure comovement in stock returns (e.g. Coval, Jurek, and Stafford (2009)). To compute stock return correlation in our sample period, we compute correlation parameter  $\rho$  using daily stock returns for all stocks whose market value is below the NYSE 50th percentile every month and take the average across stocks.<sup>13</sup> The time-series average from January 2005 and June 2020 for the correlation parameter is 0.20, much lower than the value implied by BDC's asset prices reported in Panel D of Table 2.

The high correlation values backed out from observed BDC credit spreads are partly due to the well-known credit spread puzzle of Huang and Huang (2012). More recently, Bai, Goldstein, and Yang (2018) show that one needs large jump risk premiums in a structural model of debt to match downside risk priced in investment-grade corporate bonds. Since the Nagel and Purnanandam (2019) model only includes diffusion shocks in asset value dynamics with no jumps, the model instead assigns a large value of  $\rho$  to generate large skewness in the lender's asset return distribution that matches observed credit spreads. In Appendix D, we compare the skewness of asset values generated from the jump-diffusion model and from the Nagel and Purnanandam (2019) model and find that even with a large value of  $\rho$ , the Nagel and Purnanandam (2019) model does not generate skewness in asset value as negative as that of the jump-diffusion model.

The large gap in the Q-measure correlation backed out from asset prices and the Pmeasure correlation in daily stock price movements highlights the importance of our approach in evaluating CLO senior tranches. By comparing an asset price with another comparable asset price, we keep the model simple and transparent. If instead one wishes to write down a

$$\hat{\rho}_{it} = \left(\frac{Cov_t(R_{id}, R_{mkt,d})}{\sigma_t(R_{id})\sigma_t(R_{mkt,d})}\right)^2$$

<sup>&</sup>lt;sup>13</sup>For every stock below the NYSE 50th percentile, we use daily returns in each month and compute squared correlation coefficient,

where  $R_{mkt,d}$  is a daily return on the CRSP value-weighted market portfolio.

model that matches P-measure and Q-measure correlation in a unified framework, one would have to impose more restrictive assumptions on dynamics of correlation and correlation risk premiums. We avoid this complication by focusing on the Q-measure estimates.

Still, the high asset price-implied correlation estimates are striking. Thus, we further discuss three potential concerns below, including the quality of BDC's bond price data, portfolio constraints imposed on CLOs, and the loss-given default of BDCs' debt.

One may be concerned about the estimates for the cost of debt for BDCs. For the postcrisis data, credit spreads on BDCs are taken from transaction prices in TRACE that are more reliable than quotes. However, given there are only 13 bonds, they may not fully reflect the borrowing cost for a typical BDC. If anything, the true borrowing cost may be higher, because typically only large BDCs publicly issue bonds, and smaller BDCs rely on bank lending.

On the other hand, since BDCs rarely issue corporate bonds, those bonds may therefore be illiquid and their credit spreads may contain liquidity premiums. This observation implies that the true borrowing cost which reflects BDC's default risk should be lower than observed credit spreads. In Section 4.1, we examine the role of liquidity in more detail.

Another factor that might affect credit spreads on the CLO AAA tranche is portfolio constraints imposed on CLOs. CLO AAA-rated tranches are like bonds with a variety of maintenance covenants, which require CLOs to keep leverage below a threshold and restrict CLOs from investing too much in risky loans. Taking the purpose of the constraints at face value, the existence of constraints on CLOs should help reduce credit spreads on their debt. Because such constraints do not exist for BDCs, these portfolio constraints may bias the estimated model-based spreads on AAA-rated CLO tranches.

However, Loumioti and Vasvari (2018) show that, contrary to their original intention, portfolio constraints can have negative effects on the performance of CLOs. Instead of providing safety for CLO investors, portfolio constraints distort investment behavior away from the optimal strategy, leading to lower returns on CLO tranches. However, the empirical evidence in Loumioti and Vasvari (2018) is mainly concerned about lower payoffs to equity tranches, and thus senior tranches are likely to be less sensitive to the negative side effect of portfolio constraints. To quantify the effect of portfolio constraints on the credit spreads of senior tranches, we extend the Nagel and Purnanandam (2019) model to include leverage constraint on the lender, and present the results in Section 4.4. In short, we find that including portfolio constraints does not change our conclusion significantly.

Finally, in Appendix E, we discuss the estimated loss given default for debts issued by BDCs and CLOs. Our model generates reasonable estimates for loss given default for CLOs relative to Moody's historical recovery rate for first-lien loans. BDC's loss given default is somewhat higher, reflecting the riskier nature of their assets.

# 4 Discussion and Extensions

In this section, we address potential concerns in the main results. First, since we effectively compare CLOs with BDCs through the lens of the model, it is important to ensure that stocks and bonds for BDCs in the data are reasonably priced. Second, in the main analysis, we assume that a lender is perfectly diversified such that the idiosyncratic risk of borrowers does not affect the risk of the lender. We relax this assumption and verify that results do not change with imperfect diversification. Third, we examine whether the calibrated model also prices CLO junior tranches correctly or not. Finally, we extend the Nagel-Purnanandam model and price CLO credit spreads when CLOs' leverage is constrained based on the prespecified contractual agreement between CLOs and investors.

## 4.1 Are BDC's Bonds and Stocks Fairly Priced?

Since we use the observed market price of BDC's securities to back out the fair value of CLO senior tranches, it is important to ensure that equities of BDCs are fairly priced. To this end, we report summary statistics of the value-weighted average portfolio of BDC stocks in Table 3. The average excess return is 0.09% per month with the annualized Sharpe ratio of 0.08, which is low because the sample ends in the middle of a recession. Since these are small firms, the stock returns are serially correlated, with the AR(1) coefficient of 0.13. The serial correlation is statistically significantly different from zero for up to 4 months, and thus we adjust for standard error estimates in the following analysis.

We also regress BDC portfolio returns in excess of T-bill rates on the three-factor model of Fama and French (1993), and find that the market beta is close to one, while size and value betas are positive. The positive size factor loading is hardly surprising since BDCs provide funds to small- and medium-sized private firms. BDC's stocks load positively on the value factor, suggesting that BDCs offer funds to value firms rather than growth firms. These three factors explain the average returns on BDC's equity well, and the estimate for alpha is -0.37% per month which is statistically indistinguishable from zero. The negative alpha is mostly due to observations in 2020, as the alpha estimated until 2019 is -0.07%. These results show that BDC's stock seems to be fairly priced, despite some evidence for stale pricing.

Next, we examine whether BDC's bonds are fairly priced or not. On average, these bonds have a credit rating of BBB, and thus we compare credit spreads on BDC's bonds with the average BBB credit spreads (over LIBOR). Table 4 presents the credit spreads and other characteristics of bonds for BDCs and BBB-rated bonds.

During the sample period from April 2014 to June 2020, the average credit spreads on BDCs are 261 bps, 84 bps higher than the average BBB-rated bonds. The higher credit spreads potentially reflect the higher risk premiums due to the exposure to the correlated default risk of BDCs, or to some other factors affecting credit spreads such as maturity and liquidity.

The average maturity of BDC bonds is 4.90 years, only slightly shorter than the average BBB-rated bonds of 5.62 years. The liquidity of BDC bonds is similar to those for BBB-rated bonds: the Roll measure of transaction costs proposed by Bao, Pan, and Wang (2011) and the imputed round-trip costs (IRCs) proposed by Feldhutter (2012) are similar for BDCs and CLOs, though IRCs are slightly higher for BDCs than for average BBB-rated bonds. On the other hand, bonds for BDCs have a somewhat higher turnover (12% per month) than for average BBB-rated bonds (8% per month). Therefore, the difference in maturity or liquidity is unlikely to explain the 80 bps difference in credit spreads between BDCs and average BBB bonds. Thus, the plausible explanation is the difference in exposure to systematic risk: since BDC's debt is more exposed to systematic risk than an individual BBB-rated firm's debt, BDC's bond should command higher credit spreads than the average BBB-rated corporate bond.

Finally, we compare the liquidity of BDC bonds with that of CLO senior tranches. CLO tranches are traded through an auction initiated by selling institutions (Bid Wanted In Competition, BWIC). Hendershott et al. (2020) show that based on successful BWIC, bid-ask spreads appear to be very small, often less than 10bps for senior tranches. However, once they account for the fact that BWIC often ends up unsuccessful, then the effective transaction costs are higher. Specifically, they estimate transaction costs of CLO senior tranches in the range between 10 and 50 bps (see their Figure 6), which is similar to IRCs (a measure of bid-ask spreads) for BDC bonds in Table 4. The estimated liquidity of CLO and BDC's debt security provides comfort to our approach in creating a benchmark for CLOs.

#### 4.2 Effect of Insufficient Portfolio Diversification

The Nagel-Purnanandam model assumes perfect diversification of idiosyncratic risk. However, we find that not all BDCs have a perfectly diversified portfolio of loans. Furthermore, CLOs tend to own more loans than BDCs do, which potentially affects the comparability between the debt of BDCs and that of CLOs. Therefore, in this section, we modify the model to examine how different degrees of diversification would affect the price of a lender's debt.

Specifically, we simulate idiosyncratic shocks for 30, 100, and 500 borrowers for each lender, evaluate integrals in (4), (5) and (6) by summing over borrowers, and examine the resulting credit spreads on lender's debt. For this exercise, we take the parameter values of Nagel and Purnanandam (2019) given and vary the number of loans in lender portfolios. For each choice of the number of loans, we fix parameters including the dollar value of the size of the portfolio and compute the spreads. We also vary  $dW_0$  to see if the effect of diversification for healthy lenders (who have a large value of  $dW_0$ ) is different from that for depressed lenders (who have a small value of  $dW_0$ ).

Figure 6 presents credit spreads for lenders with the three levels of diversification. With N = 100 and N = 500, credit spreads are quite similar to each other, and close to the Nagel and Purnanandam (2019) model's assumption of the infinite number of loans. This result is encouraging given that the average number of loans for BDCs and CLOs is 107 and 232, respectively (Panel C, Table 1). For the lender with 30 loans, the model generates slightly lower credit spreads than the true model does. Therefore, in selecting BDCs in our sample, we set 30 loans as the minimum number of loans that a BDC holds.

#### 4.3 Does the Model Also Price Junior Tranches?

To verify the accuracy of the calibrated Nagel and Purnanandam (2019) model, we conduct an out-of-sample test using credit spreads on CLO junior tranches. This test is important in assessing the potential buying pressure of regulated institutional investors on AAA-rated senior tranches. In our main results, the calibrated model fits the credit spreads on senior tranches well. Suppose that our model fits the biased spreads on senior tranches that are potentially distorted by the "reaching-for-yield" motives of regulated investors. Then the same model should not fit credit spreads on junior tranches well, because junior tranches do not have AAA ratings and are not widely held by institutional investors subject to regulatory capital constraints.

To price junior tranches, we modify the model of Nagel and Purnanandam (2019) as follows: let  $D_1$  be the face value of a senior tranche, and  $D_2$  be the sum of the face values of senior and junior tranches. Then, the payoffs to each tranche holder are given by:

$$B_H(S) = \min(D_1, V_H - Div_H) \tag{15}$$

$$B_H(J) = \min(D_2, V_H - Div_H) - \min(D_1, V_H - Div_H)$$
(16)

$$S_H = \max(V_H - Div_H - D_2, 0) + Div_H$$
(17)

Then, we can compute the credit spreads on the junior tranche as:

$$s(J) = -\frac{1}{H} \log \frac{B_0(J)}{D_2 - D_1} - r \tag{18}$$

where  $B_0(J) = e^{-rH} E^Q[B_H(J)].$ 

In the data, we define junior tranches as the ones that have a credit rating below BBBat issuance. The average ratio of senior and junior tranches to the portfolio value is 0.88. Thus, we use the value of leverage  $D_2 = 0.88e^{(r_t + s_t^{BBB})H}$  where  $s_t^{BBB}$  is LIBOR spreads on BBB corporate bonds. For other parameters, we use the same value as in the main results in Section 3. With this set of parameters, we compute credit spreads in (18).

Figure 7 shows the average primary-market credit spreads on CLO junior tranches and BDC-based benchmark computed using (18). Unlike credit spreads on senior tranches, credit spreads on CLO junior tranches are similar in levels before and after the crisis. Before the crisis, the model credit spreads are somewhat higher than the data, but the gap mostly disappears in 2007. In the post-crisis period, CLO junior spreads become higher than the benchmark after 2014. On average, the CLO junior spreads are 5.50% between 2010 and 2013, and 6.28% between 2014 and 2017. In contrast, the benchmark is 5.86% and 5.05% over the same period. Therefore, the model does a reasonable job in matching CLO junior tranches between 2010 and 2013 but it underpredicts the credit spreads in the more recent sample, consistent with the main results for senior tranches.

#### 4.4 Effect of Leverage Constraint

A CLO's investment and financial decisions are constrained by contracts between investors and CLO managers. Most notably, the ratio of CLO's asset to senior tranche, called the overcollateralization (OC) ratio, has a pre-specified lower bound. The OC ratio is monitored regularly, and if it falls below the threshold level, then the CLO has to stop paying dividends and coupons to equity and junior tranches and pay down a senior tranche. From the model's perspective, when a negative shock hits borrowers and the lender's asset value falls, then the lender is asked to prepay debt by selling their assets. All else being equal, such restriction makes debt issued by the lender safer and decreases credit spreads on senior tranches.

On the other hand, 56% of CLOs in our sample issue callable debt, which enables CLOs to redeem the debt at the face value after a certain non-call period. The median non-call period is 3.11 years. Since CLO's debt is variable-rate securities, there is little interest rate risk that triggers the call. However, a significant improvement in the credit quality of the

underlying asset may encourage CLOs to redeem the debt early and resolve themselves, so that the CLO manager can launch a new CLO with more attractive funding conditions. This callability of CLO tranches hurts payoffs to investors in a good state of the world and makes CLO's credit spreads larger.

In this section, we extend the model of Nagel and Purnanandam (2019) to account for these two features of CLOs and check if financial covenants on CLOs may affect the modelbased credit spreads significantly. For simplicity, we consider one point in time  $H_0 < H$  in which CLOs (investors) decide to exercise the call (put) based on the ratio of asset value in  $H_0$  to the face value of debt,

$$B_{H_0} = \begin{cases} e^{-r(H-H_0)}D & \text{if } E^Q[V_H|H_0] > \bar{c}D \text{ or } E^Q[V_H|H_0] < \underline{c}D, \\ e^{-r(H-H_0)}E^Q[B_H|H_0] & \text{otherwise.} \end{cases}$$
(19)

where  $\underline{c}$  is the lower bound for the OC ratio, and  $\overline{c}$  is the upper bound. In our sample, a median CLO has the cutoff value for a senior OC ratio test of 110%, and thus we set  $\underline{c} = 1.1$ . We also set  $\overline{c} = 1.6$  which is the 95th percentile of the senior OC ratio in our sample.

Finally, the value of CLO's debt under the constraint is given by,

$$B_0 = e^{-rH_0} E_0^Q [B_{H_0}]. (20)$$

The bond value in (19) reflects the fact that CLO tranche holders are somewhat protected from the downside risk by the OC ratio test, while they also give up upside potentials due to the callability of CLO tranches. We use the same parameter values as in the main results in Section 3, but apply the new formula in (20) with  $H_0 = 1$  to compute credit spreads on CLO senior tranches. Specifically, we simulate 100 paths of systematic shocks dW from t = 0 to  $t = H_0$ , and from each path, we further simulate 100 paths from  $t = H_0$  to t = T to compute the nested expectation  $E^Q[V_H|H_0]$  in (19). Figure 8 shows the model-based credit spreads on CLO senior tranches with constraints. The main results of the paper mostly remain unchanged with the portfolio constraints on CLOs. Most notably, after the financial crisis, the model generates credit spreads on CLO senior tranches that are comparable to the data, while the model generates credit spreads greater than the data before the crisis.

The only notable difference between the main results in Figures 4 and 8 is the model-based credit spreads during the financial crisis in 2009. In Figure 4, the model-based credit spreads increase significantly during the financial crisis, while they do not increase in Figure 8. The reason is that, under a significant deterioration of credit quality of borrowers, the value of the downside protection offered by the OC ratio test far exceeds the cost of callability, and thus the extended model generates credit spreads that are lower than the main results during the crisis. Due to the lack of the new CLO issue data during the crisis, we cannot judge whether the performance of one model is better than the other based on the difference in model prediction. However, we could confirm that the main theme of the paper, in which CLO senior tranches are fairly priced after the crisis but not before, remains largely unchanged under the extended model.

# 5 Conclusion

In this paper, we apply a structural credit risk model of Nagel and Purnanandam (2019) to BDCs and estimate market-implied correlated loan default risk priced in debt and equity of BDCs. We apply the estimation results to evaluate credit spreads on AAA-rated CLO tranches and find that these tranches are overpriced before the financial crisis, but seem to be fairly priced after the crisis.

Our results do not suggest that the systematic risk in the loan portfolios is unimportant. On the contrary, the estimated value of correlation in borrower's asset is very high, suggesting that indeed CLO senior tranches are exposed to a tail event that has a small chance of occurring but creates hazardous effects for investors. Though the default probability for AAA-rated CLO tranches is low, the CLO tranches would default in a bad state of the world with high marginal utility, commanding a large risk premium. Elkamhi and Nozawa (2021) dissect the source of such systemic risk involving CLOs.

Our results *do* suggest that investors seem to be aware of this tail risk after the financial crisis. Before the financial crisis, credit spreads on AAA-rated tranches were too low compared with our BDC-based benchmark, but this gap narrowed significantly after the crisis. One explanation for this difference is changes in investors' perception of rare events. Though Residential Mortgage-Backed Securities that underlie CDOs and syndicated loans that underlie CLOs are different, the collapse of CDOs during the crisis apparently helped investors realize the risk of investing in senior tranches of structured products such as CLOs. The other explanation is that the post-crisis banking regulation raises capital charges for regulated investors for risk exposure to structured finance products, and these investors now demand higher risk premiums. Identifying the reason for the change in CLO credit spreads is left for future research projects.

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Figure 1: Market Size and Average BDC Characteristics

This figure shows the total number of firms, total market value of equity, a value-weighted average of the book-to-market ratio, book leverage ratio, and dividend yield for BDCs in the sample.



Figure 2: Lender's Equity Value, Volatility and Default Probability

We compute the equity value, volatility, and probability of default for a lender using the parameters of Nagel and Purnanandam (2019) with different values of  $dW_0$ . For each value of  $dW_0$ , we compute aggregate borrower asset value  $A_0$  as well as model-implied equity value  $S_0$ ,  $\sigma_E$  and the probability that the lender defaults.



Figure 3: BDC's Equity Volatility, Equity Value and Credit Spreads

The figure shows equity volatility, equity values relative to the asset, and credit spreads of bonds issued by BDCs. To estimate equity volatility, we first compute the 3-month rolling window realized variance. We then regress the realized variance next 3 months on the past 3 months to obtain the forward-looking measure of equity variance and volatility. The black line is the data, the blue line is the calibrated model, and the orange line is the output of the model when borrower asset correlation is restricted to 0.5 as in Nagel and Purnanandam (2019).

Figure 4: Equity Volatility, Equity Value and Credit Spreads: Comparing BDCs and CLOs

![](_page_44_Figure_1.jpeg)

The figure shows equity volatility, equity values relative to the asset, and credit spreads of bonds issued by BDCs and CLOs implied by the estimated Nagel-Purnanandam model. We first calibrate the model to the value-weighted portfolio of BDCs (the blue line) and then modify the lender's and borrower's leverage ratio to match the characteristics of average CLOs. The orange line presents the model-implied credit spreads issued by the average CLO.

![](_page_45_Figure_0.jpeg)

Figure 5: Credit Spreads on CLO AAA-Rated Tranche and Model-Based Spreads

The black line shows the simple average of LIBOR spreads on CLO AAA tranches newly issued over the past 3 months, and the gray area presents the 10th and 90th percentiles. There were no new CLO issues from September 2008 to March 2010. The pink dash line shows the benchmark credit spreads estimated using the Nagel and Purnanandam (2019) model. The blue dash line shows the option-adjusted spreads of AAA-rated corporate bonds (measured against LIBOR swap rate).

![](_page_46_Figure_0.jpeg)

Figure 6: Effect of Diversification on Credit Spreads on Lender's Debt

The figure plots credit spreads on lenders with the number of loans N = 30, 100, 500 against the benchmark case of the infinite number of loans in Nagel and Purnanadam (2019). The parameter values are taken from Table 2.

![](_page_47_Figure_0.jpeg)

Figure 7: Credit Spreads on CLO Junior Tranche and Model-Based Spreads

The black line shows the simple average of LIBOR spreads on CLO junior tranches (rated BB+ or below at issuance) newly issued over the past 3 months. The gray area shows the 10th and 90th percentiles for LIBOR spreads. There were no new CLO issues from September 2008 to March 2010. The dashed line shows the benchmark credit spreads estimated using the Nagel and Purnanandam (2019) model.

Figure 8: Model-Based Credit Spreads on CLOs with Overcollateralization Ratio Constraint

![](_page_48_Figure_1.jpeg)

The black line shows the simple average of LIBOR spreads on CLO AAA tranches newly issued over the past 3 months, and the gray area presents the 10th and 90th percentiles. There were no new CLO issues from September 2008 to March 2010. The pink dash line shows the benchmark credit spreads estimated using the Nagel and Purnanandam (2019) model. The blue dash line shows the option-adjusted spreads of AAA-rated corporate bonds (measured against LIBOR swap rate).

	CLO	BDC
Panel A. Market data (end of 2019)		
Total asset values (\$ billions)	510	24
Panel B. Entity-level data		
Asset value (\$ millions)	364	948
Senior debt-to-asset ratio	0.66	0.34
Debt time to maturity (years)	4.93	4.90
Panel C. Loan-level data		
Number of loans	232	102
Size of loan holding (\$ millions)	1.74	11.96
LIBOR spreads on loans (bps)	350	747
Average loan maturity (years)	4.40	4.27
Top 1 Industry Share $(\%)$	14.58	16.19
Herfindahl Index $(\%)$	7.85	8.75
Credit rating of loans	В	unrated
Portfolio turnover ( $\%$ per year)	37.38	n.a.

Table 1: Comparing BDCs and CLOs

Panel A is the sum of all CLOs and BDCs at the end of 2019, while Panels B and C are the time-series averages of the cross-sectional average from 2007 to 2019. Senior debt to asset ratio for CLOs is the ratio of AAA-rated tranche to the total asset at CLO issuance, while senior debt to asset ratio for BDCs is quasi-market leverage (book value of debt divided by the sum of the book value of debt and market value of equity) averaged over the sample period. Debt time to maturity for CLOs is the realized maturity for senior tranches of CLOs computed by

$$\hat{\tau} = \frac{\sum_{t} CF_t \tau_t}{\sum_{t} CF_t}$$

where  $CF_t$  is the dollar amount of repayments of the tranche in month t,  $\tau_t$  is the time since CLO's closing date in month t. Debt time to maturity for BDCs is the simple average of time to maturity of bonds issued by BDCs.

To compute industry share, we classify loans holdings based on Moody's 35 industries. We compute the share of the largest industry and the Herfindahl index for each lender. We then take the average across lenders to compute the Top 1 Industry Share and Herfindahl Index in Panel C.

Table 2:	Parameters	and	Inputs
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Parameter	Name	Banks	BDC	CLO
$F_1 \\ dW_0$	Size of lender's portfolio A shock to borrower's asset	Market-Implied Market-Implied		
ρ	Borrower's asset correlation	0.50	Market-	Implied
$\delta$	Borrower asset depreciation	0.005	0.005	0.005
$\gamma$	Bank payout rate	0.20%	2.83%	1.63%
T	Bank loan maturity	10 years	5 years	5 years
H	Bank debt maturity	5 years	3 years	3 years
l	Loan-to-value ratio	0.66	Matched to	Loan Data
σ	Borrower asset volatility	0.20	1.00	1.00

#### Panel A. Model Parameters

#### Panel B. Calibration Targets (Average)

Input names	Banks	BDC			CLO AAA
		All	with bonds	w/o bonds	
Average market leverage	0.87	0.34	0.32	0.37	0.66
Average equity volatility	0.29	0.32	0.30	0.33	n.a.

	Average for Subsamples						
	2005-2007	2008	2009	2010-2013	2014-2017	2018-2020	Sample
$F_1$	1.59	2.10	2.10	1.38	1.36	1.52	1.53
$dW_0$	-0.32	-1.73	-1.96	0.04	0.38	0.05	-0.18
ho	0.78	0.78	0.80	0.89	0.74	0.65	0.78

#### Panel C. Calibrated Parameters to BDCs

Panel D. Calibration Results for CLO Credit Spreads (%)

	Average for Subsamples						
	2005-2007	2008	2009	2010-2013	2014-2017	2018-2020	Sample
Model	1.34	1.82	1.93	1.62	1.02	0.83	1.32
CLO	0.28	1.12	n.a.	1.43	1.09	1.18	1.03
Corp AAA	0.12	1.62	1.76	0.61	0.70	0.68	0.70

Panel A reports the pre-determined parameters for the model. Panel B reports the time-series average of the calibration target in the data. The values for banks are taken from Nagel and Purnanandam (2019). The value for BDC is computed by taking the average over time for the value-weighted portfolio of BDCs. BDC with bonds are a subsample of BDCs that issue corporate bonds, and BDCs w/o bonds are those that do not issue bonds. CLO AAA leverage is computed as the ratio of AAA-tranche to the total tranches at issuance. Panel C presents the calibrated model parameters using the BDC data. Panel D reports the credit spreads for CLO senior tranches based on the model and the data on newly issued CLOs each year. There is no CLO issues in 2009. Corp AAA is LIBOR spreads on corporate bonds issued by AAA-rated corporations.

	Mean	Std	$\operatorname{SP}$	AR(1)	AR(12)
Estimates	0.19	7.90	0.08	0.13	0.06
	α	RMRF	SMB	HML	$R^2$
b	-0.37	1.15	0.36	0.83	0.69
t(b)	(-1.28)	(9.39)	(2.26)	(3.91)	

Table 3: Summary Statistics for the Value-Weighted Portfolio of BDC Stocks:January 2005-June 2020

The table reports the value-weighted average monthly returns in excess of T-bill rate on the portfolio of BDC stocks in percent. SP is the annualized Sharpe ratio. The bottom panel reports the estimated time-series regression on the three factors of Fama and French (1993):

$$R_t^e = \alpha + b_1 RMRF_t + b_2 SMB_t + b_3 HML_t + \varepsilon_t,$$

where RMRF is the stock market factor, SMB is the size factor, and HML is the value factor. Values in parentheses are t-statistics adjusting for Newey-West 6 lags.

	Avg LIBOR	Average	Liq	uidity M	easures
	$\begin{array}{c} \text{Spreads} \\ \text{(bps)} \end{array}$	$\begin{array}{c} \text{Maturity} \\ \text{(yrs)} \end{array}$	Roll (%)	$\frac{IRC}{(\%)}$	Turnover
BDC	261	4.90	0.92	0.36	0.12
BBB Firms Difference	$\begin{array}{c} 177\\ 84 \end{array}$	5.62 -0.72	1.12 -0.20	$\begin{array}{c} 0.21 \\ 0.15 \end{array}$	$\begin{array}{c} 0.08 \\ 0.05 \end{array}$

Table 4: Credit Spreads and Characteristics of BDC and BBB-rated CorporateBonds

BDC represents corporate bonds issued by BDCs: there are 13 bonds issued by 8 BDCs. We take the average across corporate bonds every month from April 2014 to June 2020. We use corporate bonds with more than three years to maturity. BBB Firms is LIBOR credit spreads for BBB-rated corporate bonds over the same period. Roll and IRC are the illiquidity measures of Roll (1984) and Feldhutter (2012), respectively.

# A Data on BDCs

# A.1 Identifying BDCs Based on SEC Filings

We identify BDCs by the form types N-54A and N-54C that they filed to the Electronic Data Gathering, Analysis, and Retrieval system used at the U.S. Securities and Exchange Commission (SEC EDGAR). According to EDGAR Filer Manual – Volume II <sup>14</sup>, the submission types for BDCs are N-6F (Notice of intent by business development companies to elect to be subject to Sections 55 through 65 of the 1940 Act), N-54A (Notification of election), and N-54C (Notification of withdrawal). For the period between the filing dates of N-54A and N-54C, the firm is identified as BDC. By 31 December 2018, there are 262 firms (by unique CIK used in EDGAR system immune to name changes) that filed N-54A in which 125 filed N-54C later. Then we build an index of annual reports (form types 10-K, 10KSB, 10-K405) filed by those BDCs within their identified periods and get 204 unique firms and 983 firm-year observations (in total 1150 flings including amendments).

## A.2 Selecting Publicly Traded BDCs

BDCs issue equities either in the public or private market. We only focus on the publiclytraded ones so we drop those CIKs that cannot be linked point-in-time to a permno using the link table from CRSP/Compustat Merged database. This step results in 62 unique BDCs (a totally 582 filings including amendments) with stock price information in CRSP. We then download all the 582 filings in case BDCs report refined information on their loan portfolios in amendments.

 $<sup>^{14} \</sup>rm June~2019$  version, more information can be found at https://www.sec.gov/info/edgar/edmanuals.htm

## A.3 Collecting Loan Portfolio Data from 10-K

We collect information on loan portfolios (all debt holdings including loan, debt, note, bond, revolver, mortgage, etc.) from the Consolidated Schedule of Investments in the downloaded annual reports. While we use textual programming and web scraping techniques to accelerate the data collection process, we manually check for exceptions file by file. There are 4 basic steps to process one single 10-K filing.

First, we check the availability of target information and extract qualified tables from the complete submission text file to a local csv file. Firms may file an N-54A but fail to maintain their status as a business development company. (For example, FRESHSTART VENTURE CAPITAL CORP, CIK 818897, filed N-54A in 1998 and N-54C in 2018, but no information related to BDC is disclosed in 10-K filed in 1999 and 2000.) BDCs may be loan-oriented or equity-oriented, concentrate in one industry or invest in a wide range of industries. We only collect those BDCs which have less than 30% investment in Equity/Warrants/Member Units etc. and no obvious dominating industry by Moody's 35 Classification. We also compare information in 10-K and 10-K/A and collect 10-K/A when the Schedule of Investments part is amended. If equity investments are reported in different panels from debt investments, we drop equity panels for efficiency.

Second, we manually check the csv file, correct for idiosyncratic errors, and launch different subroutines adapted to this file. The subroutines are designed to clean special characters, drop duplicated titles/headers, align distorted columns and rows, restore negative values in broken parentheses, recode N/A, drop subtotal rows, merge broken cells, generate new variables from subtitles, and reshape the tables, and so on. The output of this step is a panel dataset with raw information rearranged so that each row is an investment record with all its attributes.

Third, we separate key information from long descriptive text columns. For example, when loan type, maturity date, par value, or interest rate are contained in a single column called investment details, we use a regular expression to parse them out and create new variables.

Lastly, we standardize variable names, data types, date formats, and industry classification. Since BDCs often assign ad-hoc industry descriptions to companies they invest in, it is difficult to compare the level of industry concentration between BDCs and CLOs. Thus, we map all industry types reported by BDCs to Moody's 35 classifications which are used by CLOs in our sample. The task is done with a two-step classifier. The first step is simply to match keywords and select Moody's class with the highest matching rate. If the vocabulary of the 10-K raw text is too rich to fall into Moody's industry description word lists, or more than one Moody's classes return the same maximum or zero score, all the candidate Moody's classes enter the second step comparison.

In the second step, we use a simple knowledge graph technology based on WordNet<sup>15</sup> to deal with both the richness and the vagueness of the word choices in BDC's 10-K files. WordNet is a large lexical database of English where nouns, verbs, adjectives, and adverbs are grouped into sets of cognitive synonyms (Synsets), each expressing a distinct concept. Synsets are interlinked by means of conceptual-semantic and lexical relations. The resulting network of meaningfully related words and concepts can be navigated and analyzed with graph theories. With the help of WordNet, a graph-based measure of conceptual similarity can be built between any formal English word. We calibrated the conceptual similarity scoring machine by adjusting both the hyperparameters in the function and the keyword list depicting each of Moody's 35 industries. Each 10-K raw industry is mapped deterministically to one single class of Moody's 35 no matter how vague it is. (For example, if 10-K says Service = \$1 million dollars, then we should assume that all of such loans are invested in (1) SERVICES; CONSUMER (rather than splitting it 50:50 into (1) SERVICES; CONSUMER and (2) SERVICES; BUSINESS in Moody's classification). Thus, we have a conservative re-classification that would not underestimate the industry concentration of

<sup>&</sup>lt;sup>15</sup>https://wordnet.princeton.edu/

BDC loan portfolios.

## A.4 Selecting BDCs into the Final Sample

We summarize the loan portfolio and select BDCs into our final sample based on the following criteria: the number of loans no less than 30, the fraction of loans no less than 0.8, and Moody's 35 industry Herfindahl index no larger than 0.1. We start including a firm once it meets the criteria, and maintain the firm unless the 3-year moving average fails to satisfy the criteria. Table A1 reports the list of BDCs that are in our final sample, as well as their firm-level summary statistics, which is the average of values over time. The blank cell indicates that these values are missing in 10-K filings.

# B Borrower's Asset Volatility and Lender's Equity Volatility

In this section, we examine the relationship between borrower's asset volatility  $\sigma$  and lender's equity volatility. In the Merton model, equity is a levered claim on an asset, and thus equity volatility can be given as a simple increasing function of asset volatility and leverage. In contrast, the link between *borrower's* asset volatility and *lender's* equity volatility in the Nagel-Purnanandam model is more complicated because the lender holds a loan to the borrower, and the lender's portfolio is diversified.

In our calibration, we fix the credit spreads on the loan to each borrower, and thus an increase in the borrower's asset volatility corresponds to lower leverage of the borrower. Still, a change in borrower's asset volatility affects lender's equity volatility by changing the shape of the distribution of loan payoffs. Therefore, an increase in asset volatility does not necessarily increase equity volatility, depending on the state of the economy.

Figure A2 plots the model's equity volatility against asset volatility parameter,  $\sigma$  using the set of parameters for CLOs in Table 2. Panel A is an example of the bad state, with  $dW_0 = -1$ . As  $\sigma$  increases from 20% (as in Nagel and Purnanandam (2019)) to 150%, the lender's equity volatility increases but only mildly. Even with  $\sigma = 1.5$  and  $\rho = 0.8$ , the lender's equity volatility is only around 40%. The link between  $\sigma$  and the lender's equity volatility is state-dependent. In Panel B, we show the relationship between  $\sigma$  and equity volatility is flat or even slightly decreasing.

# C Applying the Nagel-Purnanandam Model to the Sample of Banks

In this section, we apply the structural credit risk model of Nagel and Purnanandam (2019) to the value-weighted portfolio of banks that have debt covered by Credit Default Swap (CDS) contracts. The advantage of applying the model to bank CDS spreads is that we have a longer time series of credit spreads for banks than for BDCs. However, there are also disadvantages in using banks to calibrate the model: first, bank's operation involves a variety of asset classes and services including loans to retail customers such as residential mortgages and transaction services in bank branches, making them less comparable to CLOs; second, the price of bank's debt is likely to be affected by the possibility of bank-runs and deposit insurance; third, unlike BDCs, we have little information about the riskiness of loans provided by banks; fourth, banks may have off-balance-sheet assets and liabilities which are difficult to account for. Therefore, we focus on the sample of BDCs in the main results of the paper.

Nonetheless, it is interesting to apply the model to banks, back out the implied parameters  $(F_1, dW_0 \text{ and } \rho)$ , and apply it to CLO senior tranches. In the remainder of the section, we explain the data for this analysis, our sample of banks, and the calibration results.

We use Compustat for the balance sheet information, CRSP for stock prices, and Markit for CDS spreads. The sample period is monthly from January 2005 to December 2018 to be consistent with the analysis on BDCs. First, we select firms in Compustat that have SIC codes of 6020, 6021, 6022, and 6211, and CDS contracts in Markit that have a document clause of XR or XR14, and are in North America. We then merge Compustat data to Markit CDS data based on firm names. Lastly, we select the subsample of banks that have nonmissing 5-year CDS spreads. This selection process left us 27 banks with 2,668 bank-month observations, as reported in Table A2. We then form a value-weighted portfolio of banks every month to compute the average quasi-market leverage and equity volatility, as we do for BDCs.

Next, every month, we calibrate the model to match three objectives; bank equity values relative to an asset, bank equity volatility, and average 5-year CDS spreads. In order to match CDS spreads, we depart from Nagel and Purnanandam (2019) in two ways: first, we set a lower value of  $\sigma = 0.1$  as we do not find the solution to match our objectives with  $\sigma = 0.2$  used in Nagel and Purnanandam (2019); second, we let  $\rho$  be a free parameter which is implied by observed asset prices rather than fixing it to  $\rho = 0.5$ .

Panel A of Table A3 reports the calibrated parameters to the sample of banks. In order to generate observed large equity volatility with asset volatility parameter  $\sigma = 0.1$ , the model requires negative shocks to banks' assets throughout the sample, with the average  $dW_0 = -8.39$ . The model-implied correlation parameter is high, with a full-sample average of 0.70.

Panel B of Table A3 reports the model-implied credit spreads on CLO senior tranches, in which we use the set of parameters backed out from banks. The post-crisis average credit spreads based on the model is 1.04%, which is somewhat lower than the BDC-based model of 1.24% and actual CLO spreads of 1.34%. The pre-crisis average is 0.89% for the bankbased model, 1.54% for the BDC-based model, and 0.30% for the CLO data. Therefore, the qualitative conclusion of the main analysis of the paper still holds when we calibrate the model to the sample of banks. Namely, before the financial crisis, credit spreads on CLOs are too low relative to the model-implied benchmark. Since 2010, the credit spreads on CLO senior tranche are not lower than the benchmark created from similar assets.

# D Skewness in Asset Values and Correlation Parameter

Our calibration of the Nagel-Purnanandam model to the BDC data implies that credit spread-implied correlation is larger than what is expected from stock return comovements, stemming from the large risk premiums on BDC's bonds. These large risk premiums can be justified by two factors: first, from the literature on the credit spread puzzle, we know that corporate credit spreads are generally too high compared with the model based on diffusion shocks. As the Nagel-Purnanandam model does not include jumps in borrowers' asset dynamics, we need a large value of correlation to generate negative skewness in lenders' asset values. Second, since BDC's bond is systematic, in a sense that it only defaults when multiple borrowers in the underlying pool of loans default. This systematic nature of the risk implies that risk premiums on BDC's bonds should be higher than the average BBB-rated corporate bonds with idiosyncratic default risk.

To evaluate the magnitude of the first factor, we use the standard jump-diffusion model used in Bai, Goldstein, and Yang (2018) calibrated to match BBB-rated corporate CDS spreads, and compare the distribution of asset values with that of the Nagel-Purnanandam model, calibrated to match BDC's credit spreads.

The jump-diffusion model of Bai, Goldstein, and Yang (2018) is devised to measure default risk of an individual firm. The payoff of a CDS contract on a firm depends on the

distribution of the firm's asset, which follows the risk-neutral dynamics:

$$\frac{dA_t}{A_t} = (r-\delta)dt + \sigma dZ_t^Q + (e^y - 1)dq_t - \lambda^Q \xi^Q dt$$
(21)

where  $dq_t$  is a Poisson process with risk-neutral intensity  $\lambda^Q$ , and y follows the exponential distribution:

$$\pi^Q(y=Y) = \eta e^{\eta Y} 1_{Y<0} \tag{22}$$

$$\xi^Q = E^Q[e^y - 1] = \frac{\eta}{\eta + 1} - 1 \tag{23}$$

The distribution in (22) implies that the model rules out positive jumps and only has negative ones.

Bai, Goldstein, and Yang (2018) calibrate this model to CDS spreads and find that the model with the following set of parameters matches observed spreads well:  $\sigma = 0.21$ ,  $\lambda^Q = 0.20$ ,  $\eta = 2.53$ . Furthermore, they report the average payout ratio,  $\delta$ , for BBB firms is 0.051. Using these parameters, we let  $A_0 = 1$  and simulate (21) 1,000 times from t = 0 to T = 3 years to obtain the distribution of log  $A_3$ .

To understand the role of correlation in the Nagel-Purnanandam model, we use the set of parameters calibrated to BDC data, as reported in Table 2. For time-varying parameters  $F_1$  and  $dW_0$ , we use the time-series average over the full sample. Holding all the other parameters fixed, we let correlation parameter in (1) to vary from 0.1 to 0.9. For each value of  $\rho$ , we compute the summary statistics of the logarithm of the lender's asset, log  $V_3$ .

Figure A3 compares the log asset value of a firm in the jump-diffusion model and that of a lender in the Nagel-Purnanandam model. For comparison, we also include the results using the standard Merton model by shutting down the jump risk in the jump-diffusion model. The top panel plots the standard deviation of log assets for each model. Because of the jump risk, the standard deviation in the jump-diffusion model is higher than other models. An increase in correlation in the Nagel-Purnanandam model is associated with higher volatility of the lender's asset, but even with  $\rho = 0.9$ , the standard deviation is still below the jump-diffusion benchmark.

The middle panel plots skewness from the three models. The diffusion model generates skewness close to zero, while the jump-diffusion model generates negative skewness below -5. In the Nagel-Purnanandam model, a higher value of  $\rho$  leads to lower skewness, but it still generates skewness higher than the jump-diffusion model even with  $\rho = 0.9$ . The Nagel-Purnanandam model generates a higher risk-neutral probability of default than the jump-diffusion model because BDCs are more levered than the average BBB firm is. However, Figure A3 shows that even with a high value of  $\rho$ , it is not easy to generate skewness in distribution as much as the standard jump-diffusion model does, and it is not surprising that our calibration of the Nagel-Purnanandam model requires a large value of  $\rho$ .

# E Role of Recovery Rate

A crucial input in evaluating the price of debt securities is loss given default. Nagel and Purnanandam (2019) model does not differentiate the probability of default and loss given default, as the goal of the model is to price debt issued by a lender, and it is sufficient to know the product of the probability of default and loss given default. Still, it is important to ensure that the model-implied loss given default is reasonable relative to the historical data. In principle, we can do this by comparing model-implied loss given default and historical loss given default of corporate loans. This comparison assumes that BDCs and CLOs are passthrough entities, and thus the recovery of their debt depends on the recovery of underlying loans.

However, a direct comparison between the loss given default of lender's debt and historical loss given default for corporate loans is difficult for two reasons: i) the model generates riskneutral loss given default implied by asset prices, and ii) when AAA-rated senior tranche defaults, the economy is likely to be in a deep recession in which loss given default is higher than the historical average. With this caveat in mind, we examine the model-implied recovery rate (i.e. one minus loss given default) given by,

$$Recovery = 1 - \frac{E^Q[B_H|V_H - Div_H < D]}{D}.$$

In Figure A4, the model-implied recovery rate for CLO-senior tranches fluctuates between 0.40 and 0.65, with the lowest value recorded at the end of 2008. In contrast, in Moody's (2017), the issuer-weighted market recovery for first-lien bank loans is 0.67 from 1983-2017. Moody's average recovery averaged within the recession period (1992, 2002, 2008, and 2009) is 0.57. Therefore, our estimates are somewhat lower than the historical average recovery rate but reasonably similar to the loss given default during the recession, when CLO senior tranche may default.

We also find that model-implied recovery for BDC is lower than that for CLOs. This difference likely reflects the riskiness of underlying loans between these two entities.

![](_page_63_Figure_0.jpeg)

![](_page_63_Figure_1.jpeg)

This figure plots the estimates for the overall cost of borrowing for BDCs by year. Each year, we scale interest expense with total liability for each BDC and report the mean, median, the 10th-(P10) and 90th-(P90) percentiles of the cross-sectional distribution.

![](_page_64_Figure_0.jpeg)

Figure A2: Relationship Between Borrower's Asset Volatility  $\sigma$  and Lender's Equity Volatility

This figure plots the model-implied lender's equity volatility against borrower's asset volatility parameter  $\sigma$ . The other parameters in the model are taken from the average CLOs in Table 2. For each value of  $\sigma$ , we choose the borrower's leverage l to match the average credit spreads on the loan held by CLOs. The top panel plots the case when  $dW_0 = -1$  while the bottom panel is the case with  $dW_0 = 0$ .

![](_page_65_Figure_0.jpeg)

Figure A3: Comparing the Jump-Diffusion Model and the Gaussian Copula

![](_page_66_Figure_0.jpeg)

Figure A4: Model-Implied Recovery Rate for BDCs and CLOs

 $Recovery = 1 - \frac{E^Q[B_H|V_H - Div_H < D]}{D}.$ 

The model parameters for BDCs and CLOs are reported in Table 2.

permno	Name	Sample	Sample	# loans	Fraction	Herfindahl
		begins	ends		of loans	Index
12309	Full Circle Capital Corp	2013	2016	34.8	0.89	0.10
12528	Medley Capital Corp	2012	2018	68.3	0.91	0.10
12584	Solar Senior Capital Ltd	2013	2016	41.0	0.86	0.10
12693	PennantPark Floating Rate Capital	2011	2018	67.0	0.96	0.09
12747	New Mountain Finance Corp	2013	2018	108.2	0.92	0.20
12885	Fidus Investment Corp	2013	2018	50.2	0.88	0.13
13380	TCP Capital Corp	2014	2018	111.3	0.91	0.11
13650	Monroe Capital Corp	2013	2018	77.2	0.98	0.10
13692	Stellus Capital Investment Cor	2015	2018	49.0	0.96	0.10
13698	OFS Capital Corp	2013	2018	50.8	0.91	0.13
13714	WhiteHorse Finance Inc	2015	2018	34.0	0.93	0.11
13826	Garrison Capital Inc	2014	2018	67.0	0.96	0.13
13927	Harvest Capital Credit Corp	2016	2018	30.7	0.93	0.10
14050	Oaktree Strategic Income Corp	2015	2018	87.0	0.98	0.14
14151	Capitala Finance Corp	2015	2018	53.3	0.88	0.10
14418	American Capital Senior Floating Ltd	2015	2015	132.0	0.81	0.08
14520	TPG Specialty Lending Inc	2014	2018	41.8	1.00	0.12
14522	TriplePoint Venture Growth BDC	2015	2018	45.0	0.87	0.16
14565	FS Investment Corp	2015	2018	137.0	0.93	0.14
16780	TCG BDC Inc	2018	2018	91.0	0.90	0.08
58836	Newtek Business Services Corp	2015	2018	$1,\!157.8$	0.87	0.10
86161	Ameritrans Capital Corp	2009	2011	43.7	0.84	0.11
89103	Gladstone Capital Corp	2005	2018	65.3	0.94	0.12
89210	MCG Capital Corp	2005	2007	100.7	0.81	0.15
90121	Apollo Investment Corp	2007	2018	98.0	0.84	0.09
90291	Prospect Capital Corp	2009	2018	112.2	0.91	0.12
90401	Ares Capital Corp	2006	2018	223.5	0.89	0.12
90729	Gladstone Investment Corp	2007	2010	51.5	0.84	0.11
90817	Patriot Capital Funding Inc	2007	2009	62.3	0.93	0.12
91670	Kohlberg Capital Corp	2008	2008	127.0	0.82	0.08
91834	Triangle Capital Corp	2009	2018	65.7	0.90	0.08
91858	Saratoga Investment Corp	2008	2016	40.3	0.85	0.13
91966	Pennantpark Investment Corp	2007	2018	53.1	0.89	0.08
92090	BlackRock Capital Investment Corp	2008	2018	49.3	0.89	0.08
92309	Main Street Capital Corp	2011	2018	160.4	0.82	0.08
92694	Oaktree Specialty Lending Corp	2008	2018	106.0	0.95	0.13
93267	Solar Capital Ltd	2010	2013	37.0	0.83	0.13
93352	Golub Capital BDC Inc	2010	2018	272.4	0.93	0.13
93357	THL Credit Inc	2013	2016	48.5	0.89	0.13

Table A1: List of BDCs

permno	Name	Share of	Asset	Avg. Loan	LIBOR	Loan
		largest	Value	Size	Spread	Maturity
		industry	(\$ mil.)	(\$ mil.)	(%)	(years)
12309	Full Circle Capital Corp	0.17	130	3.5	11.85	
12528	Medley Capital Corp	0.19	914	12.1	9.14	
12584	Solar Senior Capital Ltd	0.17	306	6.3	5.87	
12693	PennantPark Floating Rate Capital	0.16	321	5.0	5.95	
12747	New Mountain Finance Corp	0.29	1,409	12.7	7.51	4.69
12885	Fidus Investment Corp	0.21	416	7.6		3.56
13380	TCP Capital Corp	0.21	1,232	11.2	8.82	3.79
13650	Monroe Capital Corp	0.17	306	4.2	8.71	4.01
13692	Stellus Capital Investment Cor	0.20	354	7.2	8.79	3.76
13698	OFS Capital Corp	0.27	274	5.3	7.03	
13714	WhiteHorse Finance Inc	0.21	428	12.4	9.44	3.54
13826	Garrison Capital Inc	0.23	431	6.3	7.90	3.59
13927	Harvest Capital Credit Corp	0.16	134	4.3	9.76	3.02
14050	Oaktree Strategic Income Corp	0.26	603	7.4	5.98	4.78
14151	Capitala Finance Corp	0.18	498	8.4	9.47	3.06
14418	American Capital Senior Floating Ltd	0.18	282	1.8	4.62	
14520	TPG Specialty Lending Inc	0.23	1,336	35.2	8.19	3.28
14522	TriplePoint Venture Growth BDC	0.27	355	6.6		
14565	FS Investment Corp	0.25	$3,\!985$	27.3	7.87	4.55
16780	TCG BDC Inc	0.13	1,971	19.8	6.47	4.22
58836	Newtek Business Services Corp	0.20	256	0.2		13.14
86161	Ameritrans Capital Corp	0.20	29	0.6		1.49
89103	Gladstone Capital Corp	0.20	354	5.7		3.27
89210	MCG Capital Corp	0.25	1,086	10.0		4.33
90121	Apollo Investment Corp	0.17	2,960	28.2	7.53	3.97
90291	Prospect Capital Corp	0.23	3,703	27.5	8.47	3.96
90401	Ares Capital Corp	0.23	$5,\!384$	33.1	6.50	4.14
90729	Gladstone Investment Corp	0.19	300			4.02
90817	Patriot Capital Funding Inc	0.22	340	5.1		4.21
91670	Kohlberg Capital Corp	0.15	498	3.3		
91834	Triangle Capital Corp	0.17	671	8.9	8.34	3.68
91858	Saratoga Investment Corp	0.22	197	4.4		3.73
91966	Pennantpark Investment Corp	0.15	950	15.8	7.44	4.16
92090	BlackRock Capital Investment Corp	0.14	1,072	20.8	7.11	4.05
92309	Main Street Capital Corp	0.16	1,224	6.0	6.91	
92694	Oaktree Specialty Lending Corp	0.24	1,465	13.6	7.00	3.86
93267	Solar Capital Ltd	0.23	$1,\!178$	26.4		4.06
93352	Golub Capital BDC Inc	0.23	$1,\!161$	4.9	5.68	
93357	THL Credit Inc	0.21	646	12.6	9.05	4.16

Table A1, Continued

Table A2: List of Banks with CDS

permno	name
59176	AMERICAN EXPRESS CO
90880	AMERIPRISE FINANCIAL INC
59408	BANK OF AMERICA CORP
49656	BANK OF NEW YORK MELLON CORP
68304	BEAR STEARNS COMPANIES INC
70519	CITIGROUP INC
25081	COMERICA INC
83862	E TRADE FINANCIAL CORP
34746	FIFTH THIRD BANCORP
86868	GOLDMAN SACHS GROUP INC
47896	JPMORGAN CHASE & CO
64995	KEYCORP
65330	LEGG MASON INC
80599	LEHMAN BROTHERS HOLDINGS INC
76557	MBNA CORP
59379	MELLON FINANCIAL CORP
69032	MORGAN STANLEY
56232	NATIONAL CITY CORP
85073	PROVIDIAN FINANCIAL CORP
35044	REGIONS FINANCIAL CORP
72726	STATE STREET CORP
68144	SUNTRUST BANKS INC
11786	SVB FINANCIAL GROUP
71563	TRUIST FINANCIAL CORP
66157	U S BANCORP
36469	WACHOVIA CORP
38703	WELLS FARGO & CO

		Full				
	2005-2007	2008	2009	2010-2013	2014-2018	Sample
$F_1$	1.70	1.98	1.67	1.28	1.25	1.43
$dW_0$	-13.82	-15.90	-8.60	-5.71	-5.81	-8.39
ho	0.46	0.72	0.93	0.78	0.73	0.70

# Table A3: CLO Credit Spreads Based on Banks

Panel A. Model Parameters Calibrated to Banks

Panel B. Model-Based Credit Spreads, Credit Spreads on CLOs and Corporate Bonds

	Average for Subsamples					Post	Full
	2005-2007	2008	2009	2010-2013	2014-2018	Crisis	Sample
Model(Banks)	0.89	1.37	0.96	0.93	1.13	1.04	1.03
Model(BDC)	1.54	2.85	1.94	1.53	1.00	1.24	1.47
CLO	0.30	1.17	n.a.	1.49	1.23	1.34	1.09
Corp AAA	0.12	1.62	1.76	0.61	0.68	0.65	0.69

Panel A presents the calibrated model parameters using the bank data. Panel B reports the credit spreads for CLO senior tranches based on the model and the data on newly issued CLOs each year. There is no CLO issues in 2009. Corp AAA is LIBOR spreads on corporate bonds issued by AAA-rated corporations.