Reporting Discretion, Market Discipline, and Panic Runs

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Abstract

This paper investigates the economic consequences of a financial institution’s reporting discretion in the context of bank runs. A fundamental-based run imposes market discipline on insolvent institutions, but a panic-based run shuts down institutions that could have survived with better coordination among investors. We augment a bank-run model with the financial institution’s discretion over reporting to investors. We show that reporting discretion reduces panic runs, but excessive reporting discretion weakens the market discipline. Moreover, one institution’s opportunistic use of reporting discretion exerts a negative externality on others.

JEL classification:

Key Words: Disclosure, Discretion, Market Discipline, Bank Runs, Banking Regulation
1 Introduction

Considerable amount of reporting discretion is built into banking regulations and accounting rules, such as in the area of loan loss provision, impairment, valuation for level 2 and 3 assets, securitization, and deferred tax assets. Empirical evidence has accumulated that financial institutions use such reporting discretion to overstate earnings and/or capital levels, especially in bad times. After reviewing the discretion inherent in fair value accounting rules and the empirical evidence, Laux and Leuz (2010) conclude that “banks used accounting discretion to overstate the value of their assets substantially” during the 2008 financial crisis.

What are the economic consequences of financial institutions’ reporting discretion? How does it affect the institution’s stakeholders and other financial institutions? Do stakeholders see through the opportunistic use of reporting discretion and fully undo the bias? What determines the optimal level of reporting discretion to be built into accounting rules and regulations for financial institutions? The answers to these questions have direct implications for such issues as incurred loss versus expected loss models, fair value versus historical cost accounting, and the simple method versus the advanced internal-rating based (IRB) approach to determine the risk-weighted assets under Basel II.

Two streams of existing literatures help us understand the economic consequences of financial institutions’ reporting discretion. One is a literature on the economic consequences of transparency for banks. Goldstein and Sapra (2013) review the benefits and costs of bank transparency in the context of the disclosure of stress test results. While transparency improves market and supervisory discipline, Goldstein and Sapra (2013) highlight four channels through which transparency could generate endogenous costs. Disclosure can destroy risk sharing among banks, induce ex ante sub-optimal behavior, amplify the noise in the public signal, and reduce information productions by traders. Most studies in this area, as surveyed by Goldstein and Sapra (2013), are concerned with the issue of transparency or ex ante commitment to disclosure. They do not directly address the issue of reporting discretion.

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1 For banks’ reporting discretion over loan loss provision, see, e.g., Beatty, Chamberlain, and Magliolo (1995), Liu and Ryan (2006), and Bushman and Williams (2012); for impairment, see, e.g., Vyas (2011) and Huizinga and Laeven (2012); for valuation for level 2 and 3 assets, see e.g., Kolev (2009) and Song, Thomas, and Yi (2010); for securitization, see, e.g., Dechow, Myers, and Shakespeare (2010); and for deferred tax assets, see, e.g., Skinner (2008). For surveys, see Laux and Leuz (2010) and Beatty and Liao (2013).
which relates to ex post choices. In other words, transparency is about a signal’s noise while discretion is about its bias.

The other stream is the literature on the economic consequences of discretionary disclosure, as surveyed by Verrecchia (2001), Dye (2001), Leuz and Wysocki (2007), Beyer, Cohen, Lys, and Walther (2010), Stocken (2013), and Ewert and Wagenhofer (2012). As reflected in the large number of surveys, this literature is vast and well developed. It provides a number of insights regarding the equilibrium consequences of a manager’s reporting/disclosure discretion and thus has implications for banks’ reporting discretion. However, this literature has focused mainly on non-financial firms and paid limited attention to banks’ unique features that may alter the consequences of reporting discretion.

In this paper we investigate the economic consequences of financial institutions’ reporting discretion. While financial institutions vary in a number of aspects, in our model they are characterized by a maturity mismatch between its assets and liabilities. A financial institution finances its long-term assets (loans, or other illiquid assets) with short term instruments (demand deposits, commercial papers, repos, or redeemable equity shares). This characterization seems descriptive of commercial banks, investment banks, as well as investment funds. For simplicity we call all these institutions banks and their investors creditors.

It is well-known that the maturity mismatch exposes a bank to the possibility of massive withdrawals, or runs, due to the “strategic complementarity” among creditors’ withdrawal decisions. A creditor’s benefit to withdraw earlier increases with the number of other creditors who are withdrawing. Diamond and Dybvig (1983) show that there are two equilibria if the bank’s fundamental is certain and common knowledge among creditors. In a “good” equilibrium, creditors choose not to run, believing that others are not running. As a result, all creditors enjoy the higher returns on the long-term assets. However, the model has another “bad”, panic run equilibrium in which all creditors rush to withdraw because they fear that other creditors are going to do the same and that the bank will fail. As a result, the bank is forced to liquidate its long-term assets at their liquidation value and indeed fails, self-fulfilling the creditors’ initial fear.

The multiple equilibria make it difficult to conduct comparative statics and draw empirical
predictions. Using the techniques from the global games literature (e.g., Carlsson and van Damme (1993), Morris and Shin (1998), Goldstein and Pauzner (2005) and Morris and Shin (2000)) demonstrate that the bank-run model in Diamond and Dybvig (1983) has a unique equilibrium when creditors receive noisy private signals about the bank’s fundamental, regardless of how small the noise is. In the unique equilibrium, a creditor runs if and only if her signal falls below a threshold. Their analysis reveals that bank runs can be either fundamental-based or panic-based. The former liquidates insolvent banks efficiently, but the latter shuts down solvent but illiquid banks. In other words, while bank runs impose market disciplines, they can also be excessive.

Against this backdrop we introduce into a bank-run model a manager of the bank. The manager prefers less withdrawal and has discretion to misreport to creditors. A key element of the resulting equilibrium is that the manager’s misreporting incentive is not monotonic in the bank’s fundamentals. For a bank whose fundamental is just below the run threshold, a small shift of creditors’ beliefs can switch the run equilibrium to a no-run equilibrium and such a switch yields a jump in the manager’s payoff. Thus, the manager’s incentive to influence the creditors’ beliefs is strong. In contrast, for a bank with extremely good or extremely bad fundamentals, its creditors’ withdrawal decisions are less sensitive to the change of their beliefs and thus the manager’s incentive to influence their beliefs is also weak.

This non-monotonicity of discretionary bias impedes creditors’ ability to undo the bias. Since the manager’s bias is not monotonic in the fundamental, creditors cannot distinguish between the distribution of signals of a weaker bank with a larger bias and the distribution of signals of a stronger bank with a smaller bias. Therefore, reporting discretion results in a partial pooling of banks with different fundamentals. Moreover, the bank mix in this pool is such that creditors do not run on the pool. In equilibrium, the worst fraction of banks report truthfully and fail, the best fraction of banks report truthfully and survive, while the banks in between misreport to be pooled together and survive.

With the partial pooling equilibrium characterized, we conduct comparative statics to examine the effect of reporting discretion on the probability and efficiency of bank runs. Without reporting discretion, the equilibrium converges to that in Morris and Shin (2000) in
which bank runs can be either fundamental-based or panic-based. With reporting discretion we show three results. First, the reporting discretion reduces panic-based runs. Reporting discretion enables banks around the run threshold to be pooled with stronger ones. Such pooling is sustained by the manager’s non-monotonic misreporting incentive.

Second, as the discretion increases further, the run probability is reduced further to the point that even the insolvent banks can survive with inflated reports and pooling. Therefore excessive reporting discretion impedes fundamental-based runs and weakens the market discipline on banks.

Third, one bank’s opportunistic reporting imposes a negative externality on other banks. As reporting discretion increases, the set of banks who inflate reports in equilibrium contains not only banks that are vulnerable to panic-based runs but also banks that could have survived in absence of reporting discretion. In other words, the access to reporting discretion forces even solvent and liquid banks to inflate their reports in equilibrium as well.

Our paper makes two contributions to the literature. First, it contributes to the literature on banking and bank runs. We demonstrate that panic bank runs can be alleviated by giving banks more reporting discretion. Panic bank runs often emerge as an equilibrium phenomenon when banks perform the function of liquidity transformation (Diamond and Dybvig (1983), Goldstein and Pauzner (2005)) or when banks use demand deposit as a commitment to pay out to creditors (Diamond (1984) and Diamond and Rajan (2001)). A number of costly measures to mitigate panic bank runs have been proposed in the literature, such as deposit insurance (Bryant (1980), Diamond and Dybvig (1983)) or lenders of last resort (Bagehot (1906), Rochet and Vives (2004)). Our paper shows that a bank’s reporting discretion reduces the incidence of panic bank runs, albeit at the cost of potentially weakening the market discipline and generating negative externality on other banks.

Second, the paper also compliments the literature on reporting discretion by examining the economic consequences of banks’ reporting discretion. Banks are characterized by a significant maturity mismatch of its assets and liabilities. They are often more prone to panic runs that result from the coordination failure among creditors. Reporting discretion can mitigate panic bank runs by improving the coordination of creditors’ beliefs and decisions.
The rest of the paper are organized as follows. Section 2 describes the model, Section 3 presents the benchmark without reporting discretion, Section 4 solves for the equilibrium with reporting discretion, Section 5 examines the effects of reporting discretion on the incidence and efficiency of bank runs, Section 6 discusses the empirical and policy implications of our results, and section 7 concludes. The appendix contains all the proofs.

2 The model

Our model is built on Morris and Shin (2000) (hereafter MS). We first describe their setting and then add to their model a manager of the bank who prefers less withdrawal and has discretion over the reporting to creditors.

Consider a risk neutral economy with no discounting, one consumption good, three dates, \( t = 0, 1, 2 \), and a continuum \([0, 1]\) of creditors. At date 0, each creditor receives an endowment of one unit of consumption good and invests it in a bank. The bank is characterized by the mismatch of its assets and liabilities. On its asset side, it has an exclusive access to a long-term illiquid investment technology that converts one unit of consumption good at \( t = 0 \) to \( R \) units at \( t = 2 \). The technology, however, is illiquid. If proportion \( l \) of consumption good invested in the technology are withdrawn at \( t = 1 \), then the remaining investment generate \( R e^{-\delta l} \) per unit. In other words, for one unit investment at \( t = 0 \) and early withdrawal \( l \) at \( t = 1 \), the technology yields \( l \times 1 \) at \( t = 1 \) and \((1 - l) \times R e^{-\delta l} \) at \( t = 2 \). \( \delta \geq 0 \) captures the costs of premature liquidation. Defining \( r = \ln R \), we can rewrite the gross rate of return with early withdrawal \( l \) as \( e^{r-\delta l} \).

On its liability side, the bank permits creditors to withdraw their investment at either \( t = 1 \) or \( t = 2 \). If withdrawn at \( t = 1 \), a creditor receives her one unit consumption good back. If withdrawn at \( t = 2 \), the creditor receives the random output of the technology. Each creditor’s utility over consumption at \( t = 1 \) and \( t = 2 \), \( c_1 \) and \( c_2 \), is \( \ln(c_1 + c_2) \).

The return \( r \), which we call the bank’s fundamental, has an improper prior over the real line. Before making the withdrawal decision at \( t = 1 \), each creditor receives a private signal \( x_i \) about \( r \) (to be described later).
This completes the description of the model in MS. On top of their model, we introduce the bank's manager. The manager observes the bank's fundamental $r$ perfectly and has reporting discretion. Specifically, the manager can add a bias $m(r)$ to the distribution of creditors’ private signals:

$$x_i = r + m(r) + \frac{1}{\sqrt{\beta}} \varepsilon_i, \quad (1)$$

where $\varepsilon_i$ is a standard normal random variable (with mean 0 and variance 1) and independent across creditors. The bias costs the manager privately $km$, $k > 0$ measures the cost of misreporting or the amount of discretion the manager enjoys. MS is thus a special case of our model with $k = \infty$.

Finally, the manager is assumed to prefer less withdrawal, regardless of the bank's fundamentals. Specially, the manager's payoff with fundamental $r$ is

$$w(m, r) = 1 - l - km. \quad (2)$$

For any given fundamental $r$, the manager benefits from a smaller fraction of withdrawal, $l$. Note the simplicity of the functional form of $w(m, r)$ is only for convenience. For example, a more general objective function in the form of $w(m, r) = 1 - f(l) + h(r) - km$ with $f' > 0$ and $h' > 0$ does not affect the main results qualitatively.

The timeline of the model is as follows:

- At $t = 0$, the bank offers a short-term contract, receives 1 unit of consumption good, and invests it in an illiquid project.

- At $t = 1$, the manager observes $r$ privately and chooses a bias $m(r)$. Each creditor observes a private signal $x_i = r + m(r) + \frac{1}{\sqrt{\beta}} \varepsilon_i$ and decides whether to withdraw.

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2Following the global games literature, we assume an improper prior for $r$. As a result, we assume reporting discretion affects agents’ private signals and don’t consider the presence of public signals. Our information structure can be interpreted alternatively to incorporate the institutional feature that banks’ reporting is public. For example, suppose the bank issues a public report $x = r + m(r) + \eta$ and each agent receives a private signal $z = \eta - \varepsilon_i$ that helps them understand or interpret the public report better. $\eta$ and $\varepsilon_i$ are independent. Thus, the agent’s belief about $r$ is summarized by $x_i = r + m(r) + \varepsilon_i$, the same as our information structure. See Kendall and Pearson (1995) for a similar interpretation as well as empirical evidence in support of the interpretation. We speculate our results will not qualitatively change even in the presence of an additional public signal so long as we are in the unique equilibrium regime.
• At $t = 2$, the remaining investment, if any, pays out.

Before proceeding, we discuss two implicit assumptions of the model. First, we take the bank’s maturity mismatch as given. From a theoretical point view, it can be micro-founded through creditors’ demand for insurance against idiosyncratic liquidity shocks as in Diamond and Dybvig (1983). Goldstein and Pauzner (2005) show that while such demandable contracts, i.e., short-term debt, are a cause of bank runs, they can still be desirable even when their destabilizing effect is taken into account. From an empirical perspective, such maturity mismatch and its consequences for possible runs seem descriptive of financial institutions. For example, in the 2008 financial crisis, financial institutions that financed their assets with asset-backed commercial papers and repos suffered enormous withdrawals that further reduced the returns of their remaining investment (Acharya, Gale, and Yorulmazer (2011), Gorton and Metrick (2012)).

Second, we also take the conflict of interests between the investors and the manager as given. While investors benefit from state-contingent withdrawals, the manager prefers less withdrawal regardless of the financial institution’s fundamentals. One interpretation is that the manager’s compensation, power, job security, reputation, and human capital are all tied to the financial institution’s size and decrease as the number of creditors who run increases. For example, fees charged by most investment funds increase in the size of assets under management. For another example, that managers enjoy private benefit from controlling a larger firm has received ample empirical support (e.g., Zingales (1995) and Dyck and Zingales (2004)). Thus, we implicitly assume that while financial institutions may try to mitigate this conflict of interest, they will not be able to eliminate it due to contracting frictions.

3 The equilibrium without reporting discretion

We briefly discuss the solution to the bank run model in the absence of reporting discretion. We restrict attention to a threshold equilibrium in which a creditor runs if and only if her signal falls below a common threshold. MS contain the proof that any equilibrium has to be a threshold equilibrium.
The model is characterized by the strategic complementarity among creditors’ run decisions. If a creditor withdraws, she guarantees her utility of 0 (= \ln 1), which is independent of others’ choices. If she stays, her utility is \( r - \delta l = \ln e^{r - \delta l} \) when a proportion \( l \) of creditors withdraw. Thus, her utility differential between late withdrawal at \( t = 2 \) (i.e., wait) and early withdrawal at \( t = 1 \) (i.e., run) is \( v(r, l) = r - \delta l. \)

She runs if and only if \( v < 0 \). As a tie breaker, we assume that a creditor indifferent between early and late withdraw chooses to wait.\footnote{Assuming otherwise will not change any of the results as such creditors are of measure zero.} Two properties of \( v(r, l) \) are useful. First, \( v(r, l) \) increases with \( r \). A creditor’s payoff to wait increases in the bank’s fundamental. Second, \( v(r, l) \) decreases with \( l \). A creditor’s payoff to wait also increases as the number of creditors who wait increases. This is the strategic complementarity among creditors’ withdrawal decisions.

Diamond and Dybvig (1983) show that this strategic complementarity can lead to multiple equilibria. Consider the case in which the fundamental \( r \in [0, \delta) \) is certain and common knowledge among creditors. While creditors are certain about the fundamental, they are not sure about others’ decisions. In fact, creditors’ beliefs about other creditors’ decisions can be arbitrary, resulting in the multiple equilibria. In a “good” equilibrium in which all creditors believe that others are waiting, \( i.e., l = 0 \), each creditor finds it optimal to wait because late withdrawal generates higher expected utility than early withdrawal, that is, \( v(r, 0) = r \geq 0 \).

In equilibrium, creditors’ beliefs about others’ actions are confirmed. However, there is also a “bad” equilibrium in which all creditors are panic and believe that others are going to run, \( i.e., l = 1 \). Since \( v(r, 1) = r - \delta < 0 \), all creditors find it optimal to run, forcing the bank to liquidate the entire investment at a loss. In equilibrium, creditors’ initial panic becomes self-fulfilling.

This multiplicity of equilibria with self-fulfilling beliefs is not conducive to comparative statics because the occurrence of each equilibrium is completely dictated by creditors’ arbitrary beliefs (about others’ actions) that are not anchored in the model’s parameters. MS, following Goldstein and Pauzner (2005), use the global games techniques to obtain a unique
equilibrium of the bank run model.

The main technique is to inject a small amount of independent noise into creditors’ private beliefs about the fundamental. When each creditor receives a noisy private signal \( x_i \) about the fundamental \( r \), \( r \) is no longer common knowledge among creditors. The resulting equilibrium summarized below is unique even as the noise in private beliefs approaches 0.

**Lemma 1** In the absence of reporting discretion, the bank-run model has a unique equilibrium in which a creditor runs if and only if she observes a signal below threshold \( x^{MS} = \delta \Phi(0) = \frac{\delta}{2} \). As \( \beta \) approaches \( \infty \), a bank with fundamental \( r \) suffers a run if and only if \( r < x^{MS} \). That is, \( l^{MS}(r) = 1 \) if \( r < x^{MS} \) and \( l^{MS}(r) = 0 \) if \( r \geq x^{MS} \).

The proof is essentially the same as in MS. We explain the intuition here to help understand the results later. Consider a creditor’s expected utility differential between late and early withdrawal upon receiving a signal \( x_i \),

\[
\Delta(x_i, l(.)) = E_{r, \varepsilon_i}[v(r, l|x_i) = E_{r, \varepsilon_i}[r - \delta l|x_i] = x_i - \delta E_{r, \varepsilon_i}[l|x_i].
\]

She runs if and only if \( \Delta(x_i, l) < 0 \). When the fundamental is uncertain, a creditor’s private signal affects her decision through two channels. The first is the usual role of the signal in informing her of the bank’s fundamental, i.e., \( E_{r, \varepsilon_i}[r|x_i] = x_i \). A higher signal indicates higher profitability of the investment and leads to her greater incentive to wait. The other channel is that the creditor also uses the private signal to infer other creditors’ signals and actions, i.e., \( E_{r, \varepsilon_i}[l|x_i] \). This is the complicated part of the proof, but we will show that \( E_{r, \varepsilon_i}[l|x_i] \) increases in \( x_i \). Thus, a higher signal makes the creditor more opportunistic about other creditors’ probability to wait. Combining the two channels, a higher private signal improves the creditor’s belief about both the fundamental and other creditors’ decisions. Thus, the creditor’s decision rule must be monotonic in her signal and the threshold can be tied down by \( \Delta(x^{MS}, l(.), x^{MS}) = 0 \).

Now we show that \( E_{r, \varepsilon_i}[l|x_i] \) increases in \( x_i \). Conditional on \( x_i \), creditor \( i \) believes that \( r \) is normally distributed with mean \( x_i \) and precision \( \beta \), i.e., \( r|x_i \sim N(x_i, 1/\beta) \). Moreover, she believes that creditor \( j \)'s signal \( x_j \) is normally distributed with mean \( x_i \) and precision \( \frac{\beta}{2} \). This
is because \( x_j|x_i = r|x_i + \varepsilon_j N(x_i, \frac{\sigma}{\sqrt{n}}) \). By the law of large numbers, the expected fraction of creditors who run is the same as the probability that creditor \( j \) runs. Therefore, from \( i \)'s perspective, the fraction of creditors who run is

\[
E_{r,\varepsilon_i}[l|x_i] = \Pr(x_j < x^{MS}) = \Pr(\sqrt{\frac{\beta}{2}}(x_j-x_i) < \sqrt{\frac{\beta}{2}}(x^{MS}-x_i)) = \Phi(\sqrt{\frac{\beta}{2}}(x^{MS}-x_i)).
\]  

Eqn. \([4]\) indicates that it is indeed true that \( E_{r,\varepsilon_i}[l|x_i] \) increases in \( x_i \).

4 The equilibrium with reporting discretion

We now introduce reporting discretion. At cost \( km \), the manager can add a bias \( m \) and each creditor receives a signal \( x_i = r + m(r) + \varepsilon_i \). We first focus on a threshold equilibrium characterized by \( \hat{x} \), that is, a creditor withdraws if and only if her private signal \( x_i < \hat{x} \). Later we show that such a threshold equilibrium is optimal among all equilibria. It takes three steps to solve for the threshold equilibrium. First, taking the common threshold \( \hat{x} \) as given, the manager chooses a best response of misreporting \( m^*(r; \hat{x}) \). Second, taking other creditors’ common threshold \( \hat{x} \) and the manager’s best response as given, each creditor determines a withdrawal strategy \( x^*(m^*(r; \hat{x}); \hat{x}) \). Finally, rational expectations require that \( x^*(m^*(r; \hat{x}); \hat{x}) = \hat{x} \), which determines \( x^* \) and \( m^*(r; x^*) \).

We start with the manager’s reporting strategy, which is summarized below. As proved in the Appendix, when \( k \geq \sqrt{\frac{\beta}{2}} \), there is no misreporting and the equilibrium is the same as MS. Thus, in the text we focus on the interesting case with \( k < \sqrt{\frac{\beta}{2}} \).

**Lemma 2** Suppose a creditor withdraws at \( t = 1 \) if and only if her signal is smaller than \( \hat{x} \). For any given threshold \( \hat{x} \), the manager’s reporting strategy is not monotonic in the fundamental. Specifically,

\[
m^*(r; \hat{x}) = \begin{cases} 
    r_2 - r & \text{if } r \in [r_1, r_2] \\
    0 & \text{if } r \notin [r_1, r_2]
\end{cases}
\]
with \( r_2 > \hat{x} \) and \( r_1 \) determined by

\[
\sqrt{\beta} \phi(\sqrt{\beta}(\hat{x} - r_2)) = k \quad \text{(5)}
\]

\[
\Phi(\sqrt{\beta}(\hat{x} - r_1)) - \Phi(\sqrt{\beta}(\hat{x} - r_2)) = k(r_2 - r_1) \quad \text{(6)}
\]

The manager’s reporting strategy is illustrated in Figure [1]. The red solid line represents the manager’s reporting bias as a function of \( r \), i.e., \( m^*(r) \), while the blue dash line is the mean of creditors’ signals, i.e., \( r + m^*(r) \). The figure’s most important feature is that the reporting bias is not monotonic in the fundamental. The red line is 0 until point \( r = r_1 \), at which it jumps to \( r_2 - r_1 \). Then it decreases along the 45 degree line, reaches 0 at point \( r = r_2 \), and stays there afterwards. The manager does not misreport when the fundamental is either sufficiently low (i.e., \( r < r_1 \)) or sufficiently high (i.e., \( r > r_2 \)). When the fundamental is in the between with \( r \in [r_1, r_2] \), the manager manipulates the report to the same level characterized by \( r_2 \).

To see the intuition for Lemma[2] we first examine the effect of misreporting on creditors’ withdrawal decision under a threshold strategy \( \hat{x} \). From the manager’s perspective, creditor \( i \)’s signal \( x_i \) is normally distributed with mean \( r + m(r) \) and precision \( \beta \). The expected withdrawal is equal to the probability that \( x_i \) is smaller than \( \hat{x} \) (by the law of large numbers) and can be written as

\[
l(m(r; \hat{x}), \hat{x}) = \Pr(x_i < \hat{x}) = \Phi(\sqrt{\beta}(\hat{x} - r - m(r))). \quad \text{(7)}
\]

Two features of \( l \) deserve attention. First, \( l \) is decreasing in \( m \), because \( \Phi \), the CDF of a normal distribution, is an increasing function. For any given threshold strategy of creditors, misreporting reduces withdrawals by shifting the distribution of creditors’ signals towards the right.

Second, the marginal benefit of misreporting on reducing withdrawal, i.e., \( \phi(\sqrt{\beta}(\hat{x} - r - m(r))) \), is peaked at \( r + m = \hat{x} \). The manager’s marginal incentive to misreport is stronger when the mean of the distribution of signals is closer to \( \hat{x} \). This feature results from the creditors’ threshold strategy. By definition a threshold threshold entails discontinuity in the
Figure 1: Non-monotonic Reporting Bias
relation between a creditor’s belief and decision. To illustrate the intuition, consider the case with \( \beta \) approaching \( \infty \) and thus each creditor receiving signal \( r + m \). Suppose a bank has fundamental \( r = \hat{x} - \eta \) where \( \eta \) is an arbitrarily small positive number. Without manipulation, all creditors receive \( r < \hat{x} \) and run. With manipulation \( 2\eta \), all creditors receive \( \hat{x} + \eta \) and stay. Thus, a bias of \( 2\eta \) moves the bank out of a run equilibrium to a no-run equilibrium. In contrast for \( r << \hat{x} \) or \( r >> \hat{x} \), a small bias, such as \( 2\eta \), does not change investors’ decisions at all. This non-monotonicity of the marginal benefit of misreporting plays a key role in characterizing the equilibrium.

Substituting eqn. \( (7) \) into the manager’s payoff \( w(m, r) \), the manager’s reporting decision can be written as

\[
\max_m w(m; r) \equiv 1 - \Phi(\sqrt{\beta}(\hat{x} - r - m(r))) - km
\]

\[
\text{s.t. } m \geq 0
\]

Writing out the Lagrangian of the problem and denoting \( \mu \) as the multiplier, we have the first order conditions:

\[
0 = \sqrt{\beta} \phi(\sqrt{\beta}(\hat{x} - r - m^*(r))) - k - \mu
\]

\[
\mu \geq 0, m^*(r) \geq 0, \text{ and } \mu m^*(r) = 0
\]  

As we focus on the case with \( k < \sqrt{\frac{\beta}{2\pi}} \), eqn. \( (8) \) has at least one root. If \( m^* > 0 \), which implies \( \mu > 0 \), Eqn. \( (8) \) suggests that \( r + m^*(r) \) is a constant, because \( \hat{x} \) and \( k \) are constants independent of \( r \). Define this constant as \( r_2 = r + m^*(r) \). The second order condition, \( -\sqrt{\beta} \phi'(\sqrt{\beta}(\hat{x} - r_2)) < 0 \), further requires that \( r_2 > \hat{x} \). Finally, since \( \phi(\sqrt{\beta}(\hat{x} - r - m^*(r))) \) is decreasing in \( r \) (when \( r_2 > \hat{x} \)) and \( m^* > 0 \), \( m^*(r) \) decreases in \( r \). As \( r \) approaches \( r_2 \), \( m^*(r) \) approaches 0. This yields eqn. \( 5 \) in Lemma \( 2 \).

If the manager misreports, he does so to the level of \( r + m^*(r) = r_2 \) and obtains utility

\[
w(m^*; r) = 1 - \Phi(\sqrt{\beta}(\hat{x} - r_2)) - k(r_2 - r)
\]

He can also choose \( m^*(r) = 0 \) to obtain utility

\[
w(0; r) = 1 - \Phi(\sqrt{\beta}(\hat{x} - r))
\]

The comparison of \( w(m^*; r) \) and \( w(0; r) \) yields eqn. \( 6 \) in Lemma...
We now turn to the second step that determines the creditors’ strategies. Taking other creditors’ threshold \( \hat{x} \) and the manager’s best response \( m^*(r; \hat{x}) \) as given, a creditor receiving \( x_i \) decides whether to withdraw at \( t = 1 \). She calculates her expected utility differential with late versus early withdrawal as follows:

\[
\Delta(x_i, l(., \hat{x})) = E[r|x_i, m^*(r; \hat{x})] - \delta E[l|x_i, m^*(r; \hat{x})].
\]  

A creditor withdraws if and only if \( \Delta(x_i, l(., \hat{x})) < 0 \). Like eqn. \( \text{(3)} \) in the benchmark, the creditor uses her private signal to forecast both the fundamental and other creditors’ decisions. Unlike the benchmark, the creditor now has to take into account the effect of the manager’s potential misreporting on her inference problem. That is, both expectations, \( E[r|x_i, m^*(r; \hat{x})] \) and \( E[l|x_i, m^*(r; \hat{x})] \), are conditioned not only on \( x_i \), but also on the manager’s misreporting strategy \( m^*(r; \hat{x}) \). This additional layer complicates the calculation and makes it difficult to get a closed-form expression of both conditional expectations. However, for our purpose it suffices to show that \( E[r|x_i, m^*(r; \hat{x})] \) is increasing and that \( E[l|x_i, m^*(r; \hat{x})] \) is decreasing in \( x_i \), which implies a unique threshold strategy for creditor \( i \).

**Lemma 3** For any threshold \( \hat{x} \) and the manager’s best response \( m^*(r; \hat{x}) \), a creditor withdraws if and only if her signal is lower than a threshold. That is, \( \Delta(x_i, l(., \hat{x})) < 0 \) if and only if \( x_i < x^*(m^*(r; \hat{x}), \hat{x}) \). \( x^*(m^*(r; \hat{x}), \hat{x}) \) is the unique solution to equation \( \Delta(x^*(m^*(r; \hat{x}), \hat{x}), l(., \hat{x})) = 0 \).

The key step to prove the lemma is the observation that under the misreporting strategy characterized in Lemma \( \text{(2)} \), misreporting does not change the ordering of the signals: if \( r_a < r_b \), then \( r_a + m_a^* \leq r_b + m_b^* \). This property implies that even in the presence of misreporting the distributions of \( r \) and \( x_j \) conditional on \( x_i \) still satisfy the monotone likelihood ratio property (MLRP). As a result, a higher signal improves the creditor’s beliefs about both the fundamental and other creditors’ signals, that is, \( E[r|x_i, m^*(r; \hat{x})] \) is increasing and \( E[l|x_i, m^*(r; \hat{x})] \) decreasing in \( x_i \), just as in the benchmark. Therefore, \( \Delta(x_i, l(., \hat{x})) \) is strictly increasing in \( x_i \) and \( x^* \) characterized in the lemma is unique.
Finally, rational expectations require that the manager and creditors’ strategies are consistent with each other. That is, \( x^*(m^*(r; \hat{x}), \hat{x}) = \hat{x} \). Imposing this condition gives us an equation that characterizes \( x^* \) and we prove that \( x^* \) is unique.

**Proposition 1** The unique equilibrium consists of creditors’ withdrawal threshold \( x^* \) and the manager’s reporting thresholds \( r_1 \) and \( r_2 \) such that

\[
\sqrt{\beta} \phi(\sqrt{\beta}(x^* - r_2)) = k \tag{10}
\]

\[
\Phi(\sqrt{\beta}(x^* - r_1)) - \Phi(\sqrt{\beta}(x^* - r_2)) = k(r_2 - r_1) \tag{11}
\]

and

\[
x^* + \frac{k}{\beta} - \frac{1}{\sqrt{\beta}} \phi(\sqrt{\beta}(r_1 - x^*)) + k(r_2 - r_1)[\left(\frac{r_2 + r_1}{2}\right) - x^*] = \frac{\delta}{2} \tag{12}
\]

As we discussed above, the global game technique is to inject a small amount of noise into creditors’ private signals to break the common knowledge about the fundamentals. What matters for the uniqueness is the existence, rather than the amount, of noise. Thus, the equilibrium analysis in the literature focuses on the limiting case (as the noise approaches 0) to highlight the coordination issues (e.g., Goldstein and Pauzner (2005), Plantin, Sapra, and Shin (2008)). We present the limiting case below.

**Proposition 2** As \( \beta \) tends to \( \infty \), the unique equilibrium consists of creditors’ withdrawal threshold \( x^* \) and the manager’s reporting thresholds \( r_1 \) and \( r_2 \) such that

\[
x^* = r_2 = x^{MS} + \frac{1}{2k},
\]

\[
r_1 = x^{MS} - \frac{1}{2k}.
\]

Recall that \( x^{MS} = \frac{\delta}{2} \) is creditors’ run threshold in the benchmark without reporting discretion. The intuition behind Proposition 2 is as follows. Proposition 2 introduces the manager’s misreporting into the equilibrium. Suppose we add a tiny bit of reporting discretion but creditors still use the same threshold \( x^{MS} \), that is, they believe that the manager does not
engage in misreporting. Then the manager with \( r = x^{MS} - \eta \) finds it profitable to add a bias of \( \eta \) if \( 1 > k\eta \), which contradicts the creditors’ beliefs. Thus, \( x^{MS} \) is no longer an equilibrium. As \( k \) decreases and thus reporting discretion increases, more banks find it optimal to misreport, which implies a smaller \( r_1 \). On the other hand, creditors rationally understand the manager’s misreporting incentive, discount their signals, and move the run threshold toward the right, implies a large \( r_2 \). The misreporting thresholds \( r_1 \) and \( r_2 \) eventually satisfy two conditions. First, the manager with \( r = r_1 \) is indifferent between misreporting or not, that is, \( (r_2 - r_1)k = 1 \). Second, conditional on \( x_i = r_2 \), creditor \( i \) is indifferent between early and late withdrawal. From the manager’s equilibrium reporting strategy characterized in Proposition 1, the creditor understands that the average quality of the pool reporting \( r_2 \) is \( \frac{r_2 + r_1}{2} \). The net benefit of waiting even if all others are running is \( \frac{r_2 + r_1}{2} - \delta = 0 \). Combining these two conditions gives us \( r_2 = \frac{\delta}{2} + \frac{1}{2k} \) and \( r_1 = \frac{\delta}{2} - \frac{1}{2k} \).

5 Analysis of the equilibrium

In this section, we analyze the effects of reporting discretion on the incidence and efficiency of banks runs. We first establish the benchmarks in the absence of coordination failure and/or reporting discretion.

We first examine the first best benchmark with neither coordination failure nor reporting discretion. Suppose there is only one creditor who learns \( r \) perfectly. She chooses \( l \) to maximize the expected utility \( E[l \ln 1 + (1 - l) \ln e^{(r - \delta l)}] = (1 - l)(r - \delta l) \). Thus, \( l^{FB}(r) = 1 \) if and only if \( r < x^{FB} \equiv 0 \). A single creditor withdraws from the bank if and only if the bank’s continuation value is lower than its liquidation value. Runs are all fundamental based and serves as market discipline on the insolvent banks (banks with fundamental \( r < 0 \)). This result is shown in Figure 2. The case is labelled “FB.” The dash line represents bank runs. Since it stops at \( r = 0 \), all runs are fundamental based. Figure 2 also contains other cases that we will discuss in detail below.

The coordination failure among creditors generates panic bank runs. According to Lemma 1 (as \( \beta - > \infty \)), creditors run \( (i.e., l^{MS}(r) = 1) \) if and only if \( r < x^{MS} = \frac{\delta}{2} \). Banks with
intermediate fundamentals (i.e., $r \in [0, \frac{\delta}{2}]$), that is, solvent but illiquid banks, suffer runs because creditors fear that other creditors are running. The set of these banks is increasing as the coordination failure increases ($\delta$ increases). These runs are socially inefficient and could have been avoided with better coordination among creditors. Hence, they are labeled as panic runs. They are illustrated in the case "MS" in Figure 2. Recall the dash line represents bank runs. It extends from $r = 0$ in the first best case further to the right to $r = x^{MS}$. All the runs in this extension are panic-based.

These benchmark results are summarized in the following Lemma. The proof omitted as it is contained in the argument above.

**Lemma 4** In the first best case, bank runs discipline the insolvent banks. In MS, bank runs bring down not only insolvent banks but also some solvent but illiquid banks.

Now we turn to the economic consequences of reporting discretion. Comparing Proposition 2 with Lemma 1, reporting discretion alter the run thresholds. We conduct comparative statics to examine its efficiency consequences.
Proposition 3 Reporting discretion reduces panic bank runs. That is, \( r_1 < x^{MS} \), increases in \( k \), and approaches \( x^{MS} \) from below as \( k \) approaches infinity.

Proposition 3 shows that reporting discretion mitigates panic bank runs. With reporting discretion, all banks with \( r \geq r_1 \) do not suffer bank runs. Since \( r_1 < x^{MS} \), the set of banks suffering panic runs is reduced from \( r \in [0, x^{MS}) \) to \( \max[0, r_1] \), by \( \max[x^{MS}, r_1 - x^{MS}] \). In other words, banks with fundamental \( r \in [r_1, x^{MS}] \) are rescued from panic bank runs in the presence of reporting discretion. As shown in figure 2 labelled “\( k > 1/\delta \)”, the region of panic-based run shrinks relative to the MS benchmark case. This benefit is decreasing in \( k \) and thus increasing in reporting discretion. The reduction of panic runs is illustrated in Figure 2.

The intuition for Proposition 3 hinges on two components. First, reporting discretion gives banks a powerful tool to coordinate creditors’ beliefs. Since runs for banks with \( r \in [r_1, x^{MS}] \) result from creditors’ failure to coordinate, rather than from their fundamentals, reporting discretion can benefit these banks if it succeeds in influencing creditors’ beliefs. Second, that reporting discretion indeed influences creditors’ beliefs in equilibrium relies on the pooling nature of banks’ reporting strategies. Since the coordination failure varies with and is not monotonic in the banks’ fundamentals, banks’ incentives to use reporting discretion to influence creditors’ beliefs are not monotonic in their fundamentals. This non-monotonicity sustains a pooling equilibrium in which banks with weak fundamentals but larger biases are pooled with banks with stronger fundamentals but smaller biases. That is, \( r + m^*(r) = r_2 \) for \( r \in [r_1, r_2] \). Pooling of banks with different fundamentals is essential for reducing panic-based runs because a fully-revealing equilibrium would result in investors completely undoing the bias and we are then back to the MS case.

Proposition 4 Excessive reporting discretion weakens market discipline. That is, \( r_1 < x^{FB} = 0 \) if and only if \( k < \frac{1}{5} \).

Proposition 4 shows that excessive reporting discretion leads to inefficient continuation of insolvent banks. While solvent but illiquid banks (i.e., those with \( r \) between 0 and \( x^{MS} \)) can use costly reporting bias to pool with stronger banks to avoid panic runs, insolvent banks (i.e., those with \( r \) below 0) can also manipulate creditors’ signals up to pool with those stronger
banks. The same non-monotonicity of misreporting incentive that sustains the pooling for solvent but illiquid banks also sustains the pooling of insolvent banks with stronger one. As more discretion is granted to the manager (\(k\) decreases), insolvent banks find it cheaper to influence creditors’ beliefs (a lower \(r_1\)). Proposition 4 shows that \(r_1\) is smaller than 0, the first best run threshold, if and only if \(k < \frac{1}{3}\). The weakened market discipline is illustrated in Figure 2.

An interesting implication is that there exists an intermediate degree of discretion (i.e., \(k^* = \frac{1}{3}\)) such that banks are liquidated according to the first best criterion. When the coordination issue is more important for the bank (a larger \(\delta\)), the optimal discretion is larger (a lower \(k^*\)). Reporting discretion serves as an effective tool to coordinate creditors’ beliefs.

**Proposition 5** Reporting discretion generates negative externality. That is, \(r_2 > x^{MS}\), decreases in \(k\), and exceeds \(\delta\) if and only if \(k < \frac{1}{3}\).

Proposition 5 shows that misreporting by banks with \(r \in [r_1, x^{MS}]\) generates negative externality on banks with \(r \in [x^{MS}, r_2]\). Note that none of the banks with \(r \in [x^{MS}, r_2]\) suffer panic runs in MS. Moreover, banks with \(r \in [r_2, \delta]\), which is non-empty when \(k < \frac{1}{3}\), never suffer any runs even in Diamond and Dybvig (1983). Nevertheless, Proposition 5 shows that to survive, all banks with \(r \in [x^{MS}, r_2]\) engage in costly misreporting \(m^*(r) = r_2 - r > 0\). As shown in figure 2 labelled “\(k < 1/3\)”, when \(k\) is too small, we have both a region (\(r \in [r_1, 0]\)) where market discipline is weakened and a region (\(r \in [x^{MS}, r_2]\)) where banks that can survive without manipulation in the absence of discretion has to involve in costly manipulation to survive. The set of banks engaging in misreporting is illustrated in Figure 2.

The intuition is as follows. Given the equilibrium in Proposition 1, the manager with \(r \in [x^{MS}, r_2]\) who manipulates less than \(r_2 - r\) suffers runs (i.e., \(l = 1\)) and can avoid this by engaging in a bias of \(r_2 - r\). Since \(k(r_2 - r) < k(r_2 - r_1) = 1\), the manager finds it optimal to manipulate. The access to reporting discretion by other managers thus dilutes the information content of the signals of the stronger banks and forces these banks to engage in costly manipulation as well. Thus, reporting discretion makes them strictly worse off.
In sum, the economic consequences of reporting discretion is mixed. It reduces the incidence of panic bank runs, but at the same time it can also weaken the market disciplines on insolvent banks and force strong banks to engage in costly misreporting.

6 Empirical and policy implications

Our results have several empirical and policy implications. First, our model provides three directly testable implications. Reporting discretion, by enabling banks to hide bad information at some cost, can reduce the possibility of panic-based runs on vulnerable banks, which is our first implication (Proposition 3). Second, excessive reporting discretion by banks can weaken the market discipline on weak banks and allow weak banks to be inefficiently continued (Proposition 4). Third, strong banks object to such access to reporting discretion (Proposition 5).

These predictions, if confirmed, have implications for policy making. For example, in the midst of the recent financial crisis, FASB issued new guidance to allow banks more discretion in implementing mark-to-market accounting rules. The popular press has alleged that the additional reporting discretion enables managers to "fudge the truth." (e.g., Barr (2009), Bigman and Desmond (2009), Scannell (2009)) Moreover, as discussed in the first paragraph of Introduction, a large empirical literature has confirmed those allegations that banks exploit report discretion to boost their earnings and capital levels during the recent financial crisis. However, it is still not clear how to evaluate the economic consequences of reporting discretion that enables banks to hide bad information. Our model provides one testable mechanism for the economic consequences of banks’ reporting discretion. Since panic-based runs are prominent in the banking industry and banks are especially prone to such runs in the crisis period, this mechanism might be of first-order importance. Thus, the economic consequences of FASB’s guidance might be more nuanced and complicated than the popular press alleged.

Second, our paper predicts that reporting discretion results in a kink in the empirical distribution of reports and concentration of reports on a threshold. This "manipulation to
meet-or-beat phenomena” has been well documented in the earnings management literature for firms in other industries (e.g., Burgstahler and Dichev (1997), Degeorge, Patel, and Zeckhauser (1999)). We show that similar phenomena exists for banks despite the differential role banks and other industries play in the economy. The discontinuous drop in the manager’s payoff of missing market expectations is often offered as an explanation to the meet-or-beat phenomena (e.g., Bartov, Givoly, and Hayn (2002), Beyer (2008), Guttman, Kadan, and Kandel (2006)). Our model provides one setting in which the discontinuous drop in the manager’s payoff of missing a benchmark arises endogenously. Even though the manager’s payoff is continuous in the portion of early withdrawal, it becomes discontinuous in the creditors’ beliefs as creditors optimally choose a threshold strategy.

7 Conclusion

We study the economic consequences of banks’ reporting discretion in a setting in which provision of risk-sharing exposes banks to creditor runs. When no discretion is allowed, panic-based runs bring down solvent banks, as in Goldstein and Pauzner (2005) and Morris and Shin (2000). Granting discretion to managers allow vulnerable banks to pool together with strong banks. On one hand, panic-based runs on solvent banks are reduced. On the other hand, managers of weak banks may also be able to use discretion to misreport and pool together with strong banks, resulting in a new type of inefficiency that weak banks that should be terminated remains in operation. Our results suggest that the economic consequences of allowing banks more reporting discretion may be more complicated than what has been discussed in popular press.

8 Appendix

Proof of Lemma 1

Proof. Consider a candidate threshold \( \hat{x} \) such that creditor \( i \) runs if and only if \( x_i < \hat{x} \).

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Equation (13) characterizes creditor $i$’s utility differential between running and not running:

$$\Delta(x_i, l(., \hat{x})) = E[r|x_i] - \delta E[l|x_i]$$

Since $x_j = r + \frac{1}{\sqrt{\beta}} \varepsilon_j$ and $r$ has an improper prior, $r|x_i$ is normal with $E[r|x_i] = x_i$ and $\text{Var}[r|x_i] = \frac{1}{\beta}$. Moreover, creditor $i$ uses her signal to forecast other creditors’ withdrawal decisions. By the law of large numbers, the mass of creditors who run, i.e., $l$, is the probability that creditor $j$ has a signal $x_j < \hat{x}$. Recall that $x_j = r + \frac{1}{\sqrt{\beta}} \varepsilon_j$. Thus, $x_j|x_i$ is normal with $E[x_j|x_i] = E[r|x_i] = x_i$ and $\text{Var}[x_j|x_i] = \text{Var}[r|x_i] + \text{Var}[\frac{1}{\sqrt{\beta}} \varepsilon_j] = \frac{1}{\beta} + \frac{1}{\beta} = \frac{2}{\beta}$.

Thus,

$$E[l|x_i] = \Pr(x_j < \hat{x} | x_i) = \Phi\left(\frac{\hat{x} - E[x_j|x_i]}{\sqrt{\text{Var}[x_j|x_i]}}\right) = \Phi\left(\sqrt{\frac{\beta}{2}} (\hat{x} - x_i)\right).$$

Insert $E[r|x_i]$ and $E[l|x_i]$ into equation (13), we have

$$\Delta(x_i, l(., \hat{x})) = x_i - \delta \Phi\left(\sqrt{\frac{\beta}{2}} (\hat{x} - x_i)\right)$$

$\Delta(x_i, l(., \hat{x}))$ is strictly increasing in $x_i$, $\Delta(x_i \rightarrow -\infty, l(., \hat{x})) < 0$, and $\Delta(x_i \rightarrow \infty, l(., \hat{x})) > 0$. Thus, for any given $\hat{x}$, there is one unique $x_{i}^{MS}(\hat{x})$ such that $\Delta(x_{i}^{MS}(\hat{x}), l(., \hat{x})) = 0$. Finally, for $\hat{x}$ to be an equilibrium, rational expectations require that $x_{i}^{MS}(\hat{x}) = \hat{x}$. Thus, the equilibrium $x^{MS}$ is determined by

$$\Delta(x^{MS}, l(., x^{MS})) = x^{MS} - \frac{\delta}{2} = 0$$

Note also that $x^{MS}$ is unique. In addition, $\Delta(x_i, l(., x^{MS})) > 0$ if and only if $x_i > x^{MS}$, consistent with the threshold strategy. The total withdrawal for bank $r$ is then $l^{MS}(r) = \Pr(x_i < x^{MS}) = \Phi(\sqrt{\beta}(x^{MS} - r))$. As $\beta$ approaches $\infty$, $l^{MS}(r) = 1$ if $r \leq x^{MS}$ and 0 if $r > x^{MS}$.

Proof of Lemma 2

Proof. Suppose creditors follow a threshold strategy $\hat{x}$, i.e., they run if and only if $x_i < \hat{x}$. From the manager’s perspective, the expected withdrawal $l$, i.e., equation (7), is reproduced.
here:

\[ l(r, \hat{x}) = \Pr(x_i < \hat{x} | r; m) = \Phi(\sqrt{\beta}(\hat{x} - (r + m))]. \]

Knowing \( r \) and taking \( \hat{x} \) as given, the manager solves the following problem:

\[
\max_m w(m; r) \equiv 1 - \Phi(\sqrt{\beta}(\hat{x} - r - m(r)) - km
\]

\[ s.t. \ m \geq 0 \]

Note this maximization problem is equivalent to manager choosing \( m \) to solve the following minimization problem

\[
\min_m W(r, m, \hat{x}) \equiv 1 - w(r, m, \hat{x}) = \Phi(\sqrt{\beta}(\hat{x} - (r + m)) + km
\]

subject to \( m \geq 0 \)

Writing the Lagrangian as

\[ L(m, \mu) = \Phi(\sqrt{\beta}(\hat{x} - (r + m)) + km - \mu m \]

where \( \mu \) is the Lagrangian multiplier on the constraint \( m \geq 0 \).

The Kuhn-Tucker conditions with respect to \( m \) are

\[
L_m = -\sqrt{\beta} \phi(\sqrt{\beta}(\hat{x} - (r + m)) + k - \mu = 0
\]

\[ m \geq 0, \mu \geq 0, \mu m = 0 \]

If \( \mu > 0 \), then \( m = 0 \) and

\[ W(r, 0, \hat{x}) = \Phi(\sqrt{\beta}(\hat{x} - r)) \]

If \( \mu = 0 \), then \( m \) is determined by the FOC.

\[ \phi(\sqrt{\beta}(\hat{x} - (r + m)) = \frac{k}{\sqrt{\beta}}. \]
This equation has no solution if $\frac{k}{\sqrt{\beta}} > \frac{1}{\sqrt{2\pi}}$. Otherwise, it has two symmetric solutions, one with $\hat{x} - (r + m) > 0$ and the other $\hat{x} - (r + m) < 0$. The SOC requires that $\phi'(\sqrt{\beta}(\hat{x} - (r + m))) > 0$ and thus we keep only the solution $\hat{x} - (r + m) < 0$. The optimal solution $m^*$ is

$$m^* = \hat{x} - r + \frac{1}{\sqrt{\beta}} h(\frac{k}{\sqrt{\beta}}).$$

where $h(\frac{k}{\sqrt{\beta}}) \geq 0$ and satisfies $\phi(h(\frac{k}{\sqrt{\beta}})) = \frac{k}{\sqrt{\beta}}$. In other words, $h(\frac{k}{\sqrt{\beta}})$ is the non-negative root of the equation $\phi(t) = \frac{k}{\sqrt{\beta}}$ and $-h(\frac{k}{\sqrt{\beta}})$ is the non-positive root. The manager’s expected cost at $m^*$ is

$$W(r, m^*, \hat{x}) = \Phi(\sqrt{\beta}(\hat{x} - (r + m^*))) + km^* = \Phi(-h(\frac{k}{\sqrt{\beta}})) + km^*.$$

Thus, the optimal $x^*$ is chosen by comparing $W(r, m^*, \hat{x})$ with $W(r, 0, \hat{x})$. Denote the difference between $W(r, m^*, \hat{x})$ and $W(r, 0, \hat{x})$ as $\Lambda(r)$, we have

$$\Lambda(r) \equiv W(r, m^*, \hat{x}) - W(r, 0, \hat{x})$$

$$= \Phi(\sqrt{\beta}(\hat{x} - (r + m^*))) + km^* - \Phi(\sqrt{\beta}(\hat{x} - r))$$

$$= \Phi(-h(\frac{k}{\sqrt{\beta}})) + km^* - \Phi(\sqrt{\beta}(\hat{x} - r))$$

Clearly there is no manipulation when $\Lambda(r) > 0$ and there is manipulation when $\Lambda(r) < 0$ and $m^* > 0$.

The derivative of $\Lambda(r)$ with respect to $r$ is

$$\frac{d\Lambda(r)}{dr} = -k + \sqrt{\beta} \phi'(\sqrt{\beta}(\hat{x} - r))$$

For any fixed $k$, since $\frac{k}{\sqrt{\beta}} \leq \frac{1}{\sqrt{2\pi}}$, there are (at most) two solutions $r_a \leq \hat{x} \leq r_b$ that satisfies $-k + \sqrt{\beta} \phi'(\sqrt{\beta}(\hat{x} - r)) = 0$, or $\phi(\sqrt{\beta}(\hat{x} - r)) = \frac{k}{\sqrt{\beta}}$. Note that $\phi(\sqrt{\beta}(\hat{x} - r))$ is increasing in $r$ when $r \leq \hat{x}$ and decreasing in $r$ when $r \geq \hat{x}$ and reaches its maximum when $r = \hat{x}$. Thus, $\phi(\sqrt{\beta}(\hat{x} - r)) \geq \frac{k}{\sqrt{\beta}}$ when $r_a \leq r \leq r_b$ and $\phi(\sqrt{\beta}(\hat{x} - r)) \leq \frac{k}{\sqrt{\beta}}$ when $r \leq r_a$ or $r \geq r_b$ where $r_a = \hat{x} - \frac{1}{\sqrt{\beta}} h(\frac{k}{\sqrt{\beta}})$ and $r_b = \hat{x} + \frac{1}{\sqrt{\beta}} h(\frac{k}{\sqrt{\beta}})$. This results in $\frac{d\Lambda(r)}{dr} < 0$ when
r < r_a or r > r_b and \( \frac{d\Lambda(r)}{dr} \geq 0 \) when \( r_a \leq r \leq r_b \).

Note that \( \Lambda(r_b) = km^*(r_b) = k[-r_b + \hat{x} + \frac{1}{\sqrt{\beta}} h(\frac{k}{\sqrt{\beta}})] \) = 0. Since \( \frac{d\Lambda(r)}{dr} < 0 \) when \( r > r_b \), we have \( \Lambda(r) < 0 \) when \( r > r_b \equiv r_2 \) where \( \hat{x} - r_2 = -\frac{1}{\sqrt{\beta}} h(\frac{k}{\sqrt{\beta}}) \), or equivalently, \( \sqrt{\beta} \phi(\sqrt{\beta}(r_2 - \hat{x})) = k \), which is equation (5). While it may seem to imply that manipulation is optimal, the optimal manipulation amount \( m^* = r_b - r_2 < 0 \) contradicts with the constraint \( m \geq 0 \).

Thus, there is no manipulation when \( r > r_b \). Since \( \frac{d\Lambda(r)}{dr} \geq 0 \) when \( r_a \leq r \leq r_b \), we also have \( \Lambda(r) \leq 0 \) when \( r_a \leq r \leq r_b \). Thus, there will be manipulation when \( r \) is between \( r_a \) and \( r_b \).

On the other hand, when \( r \to -\infty \), \( \Lambda(r) = 1 + \Phi(-h(\frac{k}{\sqrt{\beta}})) + k(\hat{x} - r + \frac{1}{\sqrt{\beta}} h(\frac{k}{\sqrt{\beta}})) \to +\infty > 0 \). Since \( \Lambda(r_a) \leq \Lambda(r_b) \leq 0 \) and \( \frac{d\Lambda(r)}{dr} < 0 \) when \( r < r_a \), there will be a \( r_1 \leq r_a \) such that \( \Lambda(r_1) = \Lambda(r_b) = 0 \). It follows that when \( r < r_1, \Lambda(r) > 0 \), resulting in no manipulation.

When \( r_1 \leq r \leq r_2 \), \( \Lambda(r) \geq 0 \) and there will be manipulation with \( m^* = r_2 - r \). Note that \( \Lambda(r_1) = 0 \) is equivalent to \( \Phi(-h(\frac{k}{\sqrt{\beta}})) + k(r_2 - r_1) - \Phi(\sqrt{\beta}(\hat{x} - r_1)) = 0 \), or equivalently, \( k(r_2 - r_1) = \Phi(\sqrt{\beta}(\hat{x} - r_1)) - \Phi(-h(\frac{k}{\sqrt{\beta}})) = \Phi(\sqrt{\beta}(\hat{x} - r_1)) - \Phi(\sqrt{\beta}(\hat{x} - r_2)) \), which is equation (6). The lemma is thus proved.

Proof of Lemma 3

Proof. Taking other creditors threshold strategies at \( \hat{x} \) and the manager’s best response as given, creditor \( i \) has to find it optimal to use a threshold strategy. Creditor \( i \)'s utility differential between staying and running, i.e., equation (9), is reproduced here

\[
\Delta(x_i, l(., \hat{x})) \equiv E[r|x_i, m^*] - \delta E[l|x_i, m^*].
\]

We need to show that \( \Delta(x_i, l(., \hat{x})) \) is monotonically increasing in \( x_i \), positive as \( x_i \to +\infty \) and negative as \( x_i \to -\infty \). This would imply a unique threshold \( x^*(m^*(\hat{x})) \) that results in \( \Delta(x_i, l(., \hat{x})) < 0 \) if and only if \( x_i < x^*(m^*(\hat{x})) \), which is Lemma 3.

Before we proceed to do the relevant calculations, note that the law of large numbers implies that \( l(., \hat{x}|x_i, m^*) = \Pr(x_j < \hat{x}|x_i, m^*) \). Conditional on \( x_i \), creditor \( i \) conjectures the distribution of \( r + m^*(r, \hat{x}) \equiv y(r, \hat{x}) \) using Bayes’ rule and taking into account manager’s manipulation strategy \( m^*(r, \hat{x}) \). Since there is no manipulation when \( r < r_1 \) or \( r > r_2 \), \( y(r, \hat{x}) = r \). Thus \( y(r, \hat{x}) \) has an (improper) prior density of 1 whenever \( y(r, \hat{x}) = r < r_1 \) or
y(r, \hat{x}) = r > r_2$. When $r \in [r_1, r_2]$, manager’s manipulation results in $y(r, \hat{x}) = r_2$ for all $r \in [r_1, r_2]$. Thus $y(r, \hat{x})$ has an (improper) prior density of $r_2 - r_1$ when $y(r, \hat{x}) = r_2$ and a prior density of 0 whenever $y(r, \hat{x}) \in [r_1, r_2)$. Also note $x_i = y(r, \hat{x}) + \frac{1}{\sqrt{\beta}} \varepsilon_i$, resulting in the conditional distribution of $x_i$ on $y(r, \hat{x})$ to be normal with mean $y(r, \hat{x})$ and precision $\beta$. We denote the conditional distribution of $x_i$ on $y$ by $g(x_i | y(r, \hat{x}))$, the (improper) prior density function (cumulative distribution function) of $y$ by $j(y(r, \hat{x})) \cdot (J(y(r, \hat{x}))$ and the posterior distribution of $y(r, \hat{x})$ conditional on $x_i$ by $f(y(r, \hat{x})|x_i)$. Then, by Bayes’ rule,

$$f(y(r, \hat{x})|x_i) = \frac{g(x_i | y(r, \hat{x}))j(y(r, \hat{x}))}{\int_{-\infty}^{+\infty} g(x_i | y(r, \hat{x}))dJ(y(r, \hat{x}))}$$

$$= \begin{cases} 
\sqrt{\frac{\beta}{2\pi}} e^{-\frac{\beta}{2} (y(r, \hat{x}) - r_1)^2} & \text{if } y(r, \hat{x}) < r_1 \text{ or } y(r, \hat{x}) > r_2 \\
\sqrt{\frac{\beta}{2\pi}} e^{-\frac{\beta}{2} (y(r, \hat{x}) - r_2 + r_1)^2} & \text{if } y(r, \hat{x}) = r_2 \\
0 & \text{if } y(r, \hat{x}) \in [r_1, r_2)
\end{cases}$$

(14)

From the discussion above, we can write out explicitly the expression $\int_{-\infty}^{+\infty} g(x_i | y(r, \hat{x}))dJ(y(r, \hat{x}))$ as

$$\int_{-\infty}^{+\infty} g(x_i | y(r, \hat{x}))dJ(y(r, \hat{x})) = \int_{-\infty}^{r_1} \sqrt{\frac{\beta}{2\pi}} e^{-\frac{\beta}{2} (y(r, \hat{x}) - r_1)^2} dr + \sqrt{\frac{\beta}{2\pi}} e^{-\frac{\beta}{2} (y(r, \hat{x}) - r_2)^2} (r_2 - r_1) + \int_{r_2}^{+\infty} \sqrt{\frac{\beta}{2\pi}} e^{-\frac{\beta}{2} (y(r, \hat{x}) - r_2)^2} dr$$

$$= 1 - \Phi(\sqrt{\beta}(x_1 - r_1)) + \sqrt{\beta} \phi(\sqrt{\beta}(x_1 - r_2))(r_2 - r_1) + \Phi(\sqrt{\beta}(x_1 - r_2)).$$

Insert into equation (14), we get the expression of the density of $y(r, \hat{x})$ conditional on $x_i$ as

$$f(y(r, \hat{x})|x_i) = \begin{cases} 
\sqrt{\frac{\beta}{2\pi}} e^{-\frac{\beta}{2} (y(r, \hat{x}) - r_1)^2} & \text{if } y(r, \hat{x}) < r_1 \text{ or } y(r, \hat{x}) > r_2 \\
\sqrt{\frac{\beta}{2\pi}} e^{-\frac{\beta}{2} (y(r, \hat{x}) - r_2 + r_1)^2} & \text{if } y(r, \hat{x}) = r_2 \\
0 & \text{if } y(r, \hat{x}) \in [r_1, r_2)
\end{cases}$$
We can now calculate the cumulative distribution of \( y(r, \hat{x}) \) conditional on \( x_i, F \), as

\[
F(a) \equiv \Pr(y(r, \hat{x}) < a | x_i)
\]

\[
= \int_{-\infty}^{a} f(t|x_i)dt
\]

\[
= \begin{cases}
\frac{1 - \Phi(\sqrt{\beta(x_i - a)})}{1 - \Phi(\sqrt{\beta(x_i - r_1)}) + \sqrt{\beta} \Phi(\sqrt{\beta(x_i - r_2)}) (r_2 - r_1) + \Phi(\sqrt{\beta(x_i - r_2))}} & \text{if } a \leq r_2 \\
\frac{1 - \Phi(\sqrt{\beta(x_i - r_1)}) + \sqrt{\beta} \Phi(\sqrt{\beta(x_i - r_2)}) (r_2 - r_1) + \Phi(\sqrt{\beta(x_i - r_2))}}{1 - \Phi(\sqrt{\beta(x_i - r_1)}) + \sqrt{\beta} \Phi(\sqrt{\beta(x_i - r_2)}) (r_2 - r_1) + \Phi(\sqrt{\beta(x_i - r_2))}} & \text{if } a > r_2
\end{cases}
\]

Since \( x_j = y(r, \hat{x}) + \varepsilon_j \), we have

\[
\Pr(x_j < \hat{x}|x_i) = \Pr(y(r, \hat{x}) + \varepsilon_j < \hat{x}|x_i) = \Pr(y(r, \hat{x}) < \hat{x} - \varepsilon_j|x_i)
\]

\[
= \int_{-\infty}^{+\infty} \int_{-\infty}^{\hat{x} - r_2} \frac{\sqrt{\beta}}{2\pi} e^{-\frac{\beta}{2} \varepsilon_j^2} \Pr(y(r, \hat{x}) < \hat{x} - \varepsilon_j|x_i) d\varepsilon_j
\]

\[
= \int_{-\infty}^{\hat{x} - r_2} \frac{\sqrt{\beta}}{2\pi} e^{-\frac{\beta}{2} \varepsilon_j^2} \frac{\Phi(\sqrt{\beta(x_i - r_1)}) + \sqrt{\beta} \Phi(\sqrt{\beta(x_i - r_2)}) (r_2 - r_1) + \Phi(\sqrt{\beta(x_i - r_2))}}{1 - \Phi(\sqrt{\beta(x_i - r_1)}) + \sqrt{\beta} \Phi(\sqrt{\beta(x_i - r_2)}) (r_2 - r_1) + \Phi(\sqrt{\beta(x_i - r_2))}} d\varepsilon_j
\]

After some algebra, we have

\[
\Pr(x_j < \hat{x}|x_i, m^*(r, \hat{x}))
\]

\[
= \frac{1 - [\Phi(\sqrt{\beta(x_i - r_1)}) - \sqrt{\beta} \Phi(\sqrt{\beta(x_i - r_2)}) (r_2 - r_1) - \Phi(\sqrt{\beta(x_i - r_2)))] \Phi(\sqrt{\beta}(\hat{x} - r_2))}{1 - \Phi(\sqrt{\beta(x_i - r_1)}) + \sqrt{\beta} \Phi(\sqrt{\beta(x_i - r_2)}) (r_2 - r_1) + \Phi(\sqrt{\beta(x_i - r_2))}}
\]

\[
\int_{-\infty}^{+\infty} \frac{\sqrt{\beta}}{2\pi} e^{-\frac{\beta}{2} \varepsilon_j^2} \Phi(\sqrt{\beta(x_i - (\hat{x} - \varepsilon_j))}) d\varepsilon_j
\]

\[
= \frac{1 - [\Phi(\sqrt{\beta(x_i - r_1)}) - \sqrt{\beta} \Phi(\sqrt{\beta(x_i - r_2)}) (r_2 - r_1) - \Phi(\sqrt{\beta(x_i - r_2)))] \Phi(\sqrt{\beta}(\hat{x} - r_2))}{1 - \Phi(\sqrt{\beta(x_i - r_1)}) + \sqrt{\beta} \Phi(\sqrt{\beta(x_i - r_2)}) (r_2 - r_1) + \Phi(\sqrt{\beta(x_i - r_2))}}
\]

The expression above gives \( l(.| \hat{x}, x_i, m^*) \). To calculate \( \Delta(x_i, l(., \hat{x})) \equiv E[r|x_i, m^*(r, \hat{x})] - \delta E[l|x_i, m^*(r, \hat{x})] \), we need to also calculate \( E[r|x_i, m^*(r, \hat{x})] \). To calculate \( E[r|x_i, m^*(r, \hat{x})] \), we need to know the distribution of \( r \) conditional on creditor’s own signal \( x_i \) and manager’s
manipulation strategy \( m^*(r, \hat{x}) \) that the creditor will infer. We now calculate the cumulative distribution function of \( r \) conditional on \( x_i \) and \( m^*(r, \hat{x}) \).

By Bayes’ rule, we have

\[
\Pr(r < a | x_i, m^*(r, \hat{x})) = \frac{\int_{-\infty}^{a} f(x_i | r, m^*(r, \hat{x}))dr}{\int_{-\infty}^{+\infty} f(x_i | r, m^*(r, \hat{x}))dr}
\]

\[
= \frac{\int_{-\infty}^{a} \sqrt{\beta}(\sqrt{\beta}(x_i - r - m^*(r, \hat{x})))dr}{\int_{-\infty}^{+\infty} \sqrt{\beta}(\sqrt{\beta}(x_i - r - m^*(r, \hat{x})))dr}
\]

Note that \( r + m^*(r, \hat{x}) = r \) when \( r < r_1 \) and \( r > r_2 \) and \( r + m^*(r, \hat{x}) = r_2 \) when \( r \in [r_1, r_2] \).

The denominator can thus be written as

\[
\int_{-\infty}^{+\infty} \sqrt{\beta}(\sqrt{\beta}(x_i - r - m^*(r, \hat{x})))dr
\]

\[
= \int_{-\infty}^{r_1} \sqrt{\beta}(\sqrt{\beta}(x_i - r))dr + \int_{r_1}^{r_2} \sqrt{\beta}(\sqrt{\beta}(x_i - r_2))dr + \int_{r_2}^{+\infty} \sqrt{\beta}(\sqrt{\beta}(x_i - r))dr
\]

\[
= 1 - \Phi(\sqrt{\beta}(x_i - r_1)) + \sqrt{\beta}(\sqrt{\beta}(x_i - r_2))(r_2 - r_1) + \Phi(\sqrt{\beta}(x_i - r_2))
\]

Similarly, the numerator can be calculated as follows.

When \( a \leq r_1 \),

\[
\int_{-\infty}^{a} \sqrt{\beta}(\sqrt{\beta}(x_i - r - m^*(r, \hat{x})))dr = \int_{-\infty}^{a} \sqrt{\beta}(\sqrt{\beta}(x_i - r))dr = 1 - \Phi(\sqrt{\beta}(x_i - a))
\]

When \( r_1 < a \leq r_2 \),

\[
\int_{-\infty}^{a} \sqrt{\beta}(\sqrt{\beta}(x_i - r - m^*(r, \hat{x})))dr = \int_{-\infty}^{r_1} \sqrt{\beta}(\sqrt{\beta}(x_i - r))dr + \int_{r_1}^{a} \sqrt{\beta}(\sqrt{\beta}(x_i - r_2))dr
\]

\[
= 1 - \Phi(\sqrt{\beta}(x_i - r_1)) + \sqrt{\beta}(\sqrt{\beta}(x_i - r_2))(a - r_1)
\]

When \( a > r_2 \),

\[
\int_{-\infty}^{a} \sqrt{\beta}(\sqrt{\beta}(x_i - r - m^*(r, \hat{x})))dr = \int_{-\infty}^{r_1} \sqrt{\beta}(\sqrt{\beta}(x_i - r))dr + \int_{r_1}^{r_2} \sqrt{\beta}(\sqrt{\beta}(x_i - r_2))dr + \int_{r_2}^{a} \sqrt{\beta}(\sqrt{\beta}(x_i - r))dr
\]

\[
= 1 - \Phi(\sqrt{\beta}(x_i - r_1)) + \sqrt{\beta}(\sqrt{\beta}(x_i - r_2))(r_2 - r_1) + \Phi(\sqrt{\beta}(x_i - r_2)) - \Phi(\sqrt{\beta}(x_i - a))
\]

We now have the expression for the cumulative distribution of \( r \) conditional on \( x_i \) and \( m^*(r, \hat{x}) \). When taking derivatives with respect to \( a \), we have the probability density of \( r \)
conditional on \( x_i \) and \( m^*(r, \hat{x}) \), denoted as \( \psi(r| x_i, m^*(r, \hat{x})) \), as

\[
\psi(r| x_i, m^*(r, \hat{x})) = \begin{cases} 
\frac{\sqrt{\beta} \phi(\sqrt{\beta}(x_i-r))}{1-\Phi(\sqrt{\beta}(x_i-r_1))+\sqrt{\beta} \phi(\sqrt{\beta}(x_i-r_2))(r_2-r_1)+\Phi(\sqrt{\beta}(x_i-r_2))} & \text{if } r < r_1 \\
\frac{\sqrt{\beta} \phi(\sqrt{\beta}(x_i-r_2))}{1-\Phi(\sqrt{\beta}(x_i-r_1))+\sqrt{\beta} \phi(\sqrt{\beta}(x_i-r_2))(r_2-r_1)+\Phi(\sqrt{\beta}(x_i-r_2))} & \text{if } r_1 < r \leq r_2 \\
\frac{\sqrt{\beta} \phi(\sqrt{\beta}(x_i-r))}{1-\Phi(\sqrt{\beta}(x_i-r_1))+\sqrt{\beta} \phi(\sqrt{\beta}(x_i-r_2))(r_2-r_1)+\Phi(\sqrt{\beta}(x_i-r_2))} & \text{if } r > r_2 
\end{cases}
\]

Note that the cumulative distribution function of \( r \) conditional on \( x_i \) and \( m^*(r, \hat{x}) \) is differentiable except at a single point \( r = r_1 \), which does not affect the calculation of conditional expectation of \( r \).

Given the density function of \( r \) conditional on \( x_i \) and \( m^*(r, \hat{x}) \), we can calculate \( E[r| x_i, m^*(r, \hat{x})] \) as

\[
E[r| x_i, m^*(r, \hat{x})] = \int_{-\infty}^{r_1} r \sqrt{\beta} \phi(\sqrt{\beta}(x_i-r)) dr + \int_{r_1}^{r_2} \frac{\sqrt{\beta} \phi(\sqrt{\beta}(x_i-r_2))}{1-\Phi(\sqrt{\beta}(x_i-r_1))+\sqrt{\beta} \phi(\sqrt{\beta}(x_i-r_2))(r_2-r_1)+\Phi(\sqrt{\beta}(x_i-r_2))} dr + \int_{r_2}^{\infty} \frac{\sqrt{\beta} \phi(\sqrt{\beta}(x_i-r))}{1-\Phi(\sqrt{\beta}(x_i-r_1))+\sqrt{\beta} \phi(\sqrt{\beta}(x_i-r_2))(r_2-r_1)+\Phi(\sqrt{\beta}(x_i-r_2))} dr
\]

Equation (15)

Note that the denominator is independent of \( r \). We now calculate the numerator of each term in equation (15).

The first term,

\[
\int_{-\infty}^{r_1} r \sqrt{\beta} \phi(\sqrt{\beta}(x_i-r)) dr = \int_{-\infty}^{r_1} \frac{r \sqrt{\beta}}{\sqrt{2\pi}} e^{-\frac{1}{2}(x_i-r)^2} dr = \int_{-\infty}^{r_1} \frac{r}{\sqrt{2\pi}} e^{-\frac{1}{2}(x_i-r)^2} dr 
\]

When \( r_1 - x_i \leq 0 \),

\[
\int_{-\infty}^{r_1-x_i} (r - x_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(r-x_i)^2} d(r - x_i) = \frac{1}{2} \int_{+\infty}^{(r_1-x_i)^2} \frac{\sqrt{\beta}}{\sqrt{2\pi}} e^{-\frac{1}{2}t} dt = -\frac{1}{\sqrt{2\pi} \sqrt{\beta}} e^{-\frac{1}{2}((r_1-x_i)^2 + t)}
\]

Similarly, when \( r_1 - x_i > 0 \),

\[
\int_{0}^{r_1-x_i} (r - x_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(r-x_i)^2} d(r - x_i) = \frac{1}{\sqrt{2\pi} \sqrt{\beta}} e^{-\frac{1}{2}((r_1-x_i)^2 + t)}.
\]

Thus, \( \int_{-\infty}^{r_1} r \sqrt{\beta} \phi(\sqrt{\beta}(x_i-r)) dr = x_i[1 - \Phi(\sqrt{\beta}(x_i-r_1))] - \frac{1}{\sqrt{\beta} \sqrt{2\pi}} e^{-\frac{1}{2}((r_1-x_i)^2 + t)} \phi(\sqrt{\beta}(x_i-r_1)) \).

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We can similarly calculate the third term as
\[
\int_{r_2}^{+\infty} \sqrt{\beta}(\sqrt{\beta}(x_i - r))rdr = x_i \Phi(\sqrt{\beta}(x_i - r_2)) + \frac{1}{\sqrt{\beta}}\phi(\sqrt{\beta}(x_i - r_2)).
\]
The second term,
\[
\int_{r_1}^{r_2} \sqrt{\beta}(\sqrt{\beta}(x_i - r_2))rdr = \frac{1}{2} \sqrt{\beta}\phi(\sqrt{\beta}(x_i - r_2))(r_2^2 - r_1^2) = \frac{1}{2} \sqrt{\beta}\phi(\sqrt{\beta}(x_i - r_2))(r_2^2 - r_1^2)
\]
Insert those expressions into (15),
\[
E[r|x_i, m^*(r, \hat{x})] = \frac{1}{1 - \Phi(\sqrt{\beta}(x_i - r_1)) + \sqrt{\beta}\phi(\sqrt{\beta}(x_i - r_2))(r_2 - r_1) + \Phi(\sqrt{\beta}(x_i - r_2))} \times
\{ x_i[1 - \Phi(\sqrt{\beta}(x_i - r_1))] - \frac{1}{\sqrt{\beta}}\phi(\sqrt{\beta}(x_i - r_1)) \\
+ \frac{1}{2} \sqrt{\beta}\phi(\sqrt{\beta}(x_i - r_2))(r_2^2 - r_1^2) + x_i\Phi(\sqrt{\beta}(x_i - r_2)) + \frac{1}{\sqrt{\beta}}\phi(\sqrt{\beta}(x_i - r_2)) \}
\]
We now have the expression for both \(E[r|x_i, m^*(r, \hat{x})]\) and \(E[l|x_i, m^*(r, \hat{x})]\) and we can calculate the expected payoff differential of staying versus running as
\[
\Delta(x_i, l(., \hat{x})) = E[r|x_i, m^*(r, \hat{x})] - \delta E[l|x_i, m^*(r, \hat{x})]
\]
\[
= E[r|x_i, m^*(r, \hat{x})] - \delta \Pr(x_j < \hat{x}|x_i, m^*(r, \hat{x}))
\]
\[
= \frac{1}{1 - \Phi(\sqrt{\beta}(x_i - r_1)) + \sqrt{\beta}\phi(\sqrt{\beta}(x_i - r_2))(r_2 - r_1) + \Phi(\sqrt{\beta}(x_i - r_2))} \times
\{ x_i[1 - \Phi(\sqrt{\beta}(x_i - r_1))] - \frac{1}{\sqrt{\beta}}\phi(\sqrt{\beta}(x_i - r_1)) \\
+ \frac{1}{2} \sqrt{\beta}\phi(\sqrt{\beta}(x_i - r_2))(r_2^2 - r_1^2) + x_i\Phi(\sqrt{\beta}(x_i - r_2)) + \frac{1}{\sqrt{\beta}}\phi(\sqrt{\beta}(x_i - r_2)) \}
\]
\[
- \delta \Pr(x_j < \hat{x}|x_i, m^*(r, \hat{x}))
\]
where
\[
r_2 = \hat{x} - \frac{1}{\sqrt{\beta}}h(\frac{k}{\sqrt{\beta}})
\]
and
\[
\Phi(\sqrt{\beta}(\hat{x} - r_1)) = \Phi(-h(\frac{k}{\sqrt{\beta}})) = k(r_2 - r_1)
\]
follows from the optimal solution of manager’s manipulation problem.
It can be shown that \(\Delta(x_i, l(., \hat{x}))\) is strictly increasing in \(x_i\). We leave the proof of this
property to the very end.

Since $\Delta(x_i, l(., \hat{x}))$ is continuous and strictly monotone in $x_i$ and that when $x_i \to +\infty$, $\Delta(x_i, l(., \hat{x})) \to x_i \to +\infty > 0$ and when $x_i \to -\infty$, $\Delta(x_i, l(., \hat{x})) \to x_i - 1 \to -\infty < 0$. Thus there is a unique $x^*(m^*(r, \hat{x}))$ s.t. $\Delta(x^*(m^*(r, \hat{x})), l(., \hat{x})) = 0$ and $\Delta(x_i, l(., \hat{x})) < 0$ (i.e., creditor chooses to run) if and only if $x_i < x^*(m^*(r, \hat{x}))$. The lemma is thus proved.

We now complete the proof by showing that $\Delta(x_i, l(., \hat{x}))$ is strictly increasing in $x_i$. Note that since $\Delta(x_i, l(., \hat{x})) = E[r|x_i, m^*(r, \hat{x})] - \delta E[l|x_i, m^*(r, \hat{x})]$, if we can show that $E[r|x_i, m^*(r, \hat{x})]$ is weakly increasing in $x_i$ and that $E[l|x_i, m^*(r, \hat{x})]$ is strictly increasing in $x_i$ then we are done. The proof below shows that this is indeed the case.

We first show that $E[r|x_i, m^*(r, \hat{x})]$ is weakly increasing in $x_i$.

Denote the distribution of $r$ conditional on $x_i$ and $m^*(r, \hat{x})$ as $\Psi(r|x_i, m^*(r, \hat{x}))$. Note that if we can show that $\forall x_1 < x_2, \Psi(r|x_2, m^*(r, \hat{x}))$ first order stochastically dominates $\Psi(r|x_1, m^*(r, \hat{x}))$ then we are done. A sufficient condition for first order stochastic dominance is maximum likelihood ratio property (MLRP). We now show that the density $\psi(r|x_i, m^*(r, \hat{x})$ satisfies MLRP for all $r \neq r_1$.

Note that in order to show MLRP we need to show that $\forall x_1 < x_2$, $\frac{\partial}{\partial r} \left( \frac{\psi(r|x_2, m^*(r, \hat{x}))}{\psi(r|x_1, m^*(r, \hat{x}))} \right) \geq 0$.

When $r < r_1$, $\frac{\partial}{\partial r} \left( \frac{\psi(r|x_2, m^*(r, \hat{x}))}{\psi(r|x_1, m^*(r, \hat{x}))} \right) = \frac{1-\Phi(\sqrt{\beta(x_1-r_1)})+\Phi(\sqrt{\beta(x_2-r_1)})}{1-\Phi(\sqrt{\beta(x_2-r_1)})+\Phi(\sqrt{\beta(x_1-r_1)})} \frac{\partial}{\partial r} \left( \frac{\sqrt{\beta(x_1-r_1)}}{\sqrt{\beta(x_2-r_1)}} \right) > 0$ as normal density satisfies MLRP.

When $r_1 < r \leq r_2$, $\frac{\partial}{\partial r} \left( \frac{\psi(r|x_2, m^*(r, \hat{x}))}{\psi(r|x_1, m^*(r, \hat{x}))} \right) = \frac{1-\Phi(\sqrt{\beta(x_1-r_1)})+\Phi(\sqrt{\beta(x_2-r_1)})}{1-\Phi(\sqrt{\beta(x_2-r_1)})+\Phi(\sqrt{\beta(x_1-r_1)})} \frac{\partial}{\partial r} \left( \frac{\sqrt{\beta(x_1-r_1)}}{\sqrt{\beta(x_2-r_1)}} \right) = 0$.

When $r > r_2$, $\frac{\partial}{\partial r} \left( \frac{\psi(r|x_2, m^*(r, \hat{x}))}{\psi(r|x_1, m^*(r, \hat{x}))} \right) = \frac{1-\Phi(\sqrt{\beta(x_1-r_1)})+\Phi(\sqrt{\beta(x_2-r_1)})}{1-\Phi(\sqrt{\beta(x_2-r_1)})+\Phi(\sqrt{\beta(x_1-r_1)})} \frac{\partial}{\partial r} \left( \frac{\sqrt{\beta(x_1-r_1)}}{\sqrt{\beta(x_2-r_1)}} \right) > 0$ as normal density satisfies MLRP.

Thus, we know that $\psi(r|x_i, m^*(r, \hat{x})$ satisfies MLRP for all $r \neq r_1$. This implies that $\Psi(r|x_2, m^*(r, \hat{x})$ first order stochastically dominates $\Psi(r|x_1, m^*(r, \hat{x})$ for all $r \neq r_1$. We now show that $\Psi(r_1|x_2, m^*(r, \hat{x})$ first order stochastically dominates $\Psi(r_1|x_1, m^*(r, \hat{x})$, or, more generally, $\Psi_x(r_1|x_1, m^*(r, \hat{x}) \leq 0$. This will ensure that $\Psi(r|x_2, m^*(r, \hat{x})$ first order stochastically dominates $\Psi(r|x_1, m^*(r, \hat{x})$ for all $r$. 31
Note that \( \Psi(r_1 | x, m^*(r, \hat{x})) = \frac{1 - \Phi(\sqrt{B}(x-r_1))}{1 - \Phi(\sqrt{B}(x-r_1)) + \int_{r_1}^{\infty} \sqrt{B} \phi(\sqrt{B}(x-r-m^*(r, \hat{x})))dr} \)

\[= \frac{\Phi(\sqrt{B}(r_1-x)) + \int_{r_1}^{\infty} \sqrt{B} \phi(\sqrt{B}(x-r-m^*(r, \hat{x})))dr}{\Phi(\sqrt{B}(r_1-x)) + \int_{r_1}^{\infty} \sqrt{B} \phi(\sqrt{B}(x-r-m^*(r, \hat{x})))dr} \]

To show that \( \Psi_x(r_1 | x, m^*(r, \hat{x})) \leq 0 \), since \( A(r_1-x) \geq 0 \) and \( B(x) \geq 0 \), we need \( A(r_1-x)B'(x) + B(x)A'(r_1-x) \leq 0 \), or \( \frac{A'(r_1-x)}{A(r_1-x)} \geq - \frac{B'(x)}{B(x)} \)

Note that \( \frac{A'(r_1-x)}{A(r_1-x)} = \frac{\sqrt{B} \phi(\sqrt{B}(r_1-x))}{\Phi(\sqrt{B}(r_1-x))} = \frac{\sqrt{B} \phi(\sqrt{B}(x-r_1))}{1 - \Phi(\sqrt{B}(x-r_1))} \geq - \frac{B'(x)}{B(x)} = \frac{\int_{r_1}^{\infty} \sqrt{B} \phi(\sqrt{B}(x-r-m^*(r, \hat{x})))dr}{\int_{r_1}^{\infty} \sqrt{B} \phi(\sqrt{B}(x-r-m^*(r, \hat{x})))dr} \)

Insert \( \phi'(\sqrt{B}(x-r-m^*(r, \hat{x}))) = -\sqrt{B}(x-r-m^*(r, \hat{x})) \phi(\sqrt{B}(x-r-m^*(r, \hat{x}))) dr \) into the expression above, we have

\[\frac{\phi(\sqrt{B}(x-r_1))}{1 - \Phi(\sqrt{B}(x-r_1))} \geq \frac{\int_{r_1}^{\infty} \sqrt{B}(x-r-m^*(r, \hat{x})) \phi(\sqrt{B}(x-r-m^*(r, \hat{x})))dr}{\int_{r_1}^{\infty} \phi(\sqrt{B}(x-r-m^*(r, \hat{x})))dr} \]

Now define a density \( \varphi(r|x) \equiv \frac{\phi(\sqrt{B}(x-r-m^*(r, \hat{x})))}{\int_{r_1}^{\infty} \phi(\sqrt{B}(x-r-m^*(r, \hat{x})))dr} \) for \( r \geq r_1 \).

We will have that \( \frac{\phi(\sqrt{B}(x-r_1))}{1 - \Phi(\sqrt{B}(x-r_1))} \geq \frac{\int_{r_1}^{\infty} \sqrt{B}(x-r-m^*(r, \hat{x})) \phi(\sqrt{B}(x-r-m^*(r, \hat{x})))dr}{\int_{r_1}^{\infty} \phi(\sqrt{B}(x-r-m^*(r, \hat{x})))dr} \) is equivalent to

\[\frac{\phi(\sqrt{B}(x-r_1))}{1 - \Phi(\sqrt{B}(x-r_1))} \geq \frac{\int_{r_1}^{\infty} \sqrt{B}(x-r_1+r_1-r-m^*(r, \hat{x})) \phi(\sqrt{B}(x-r-m^*(r, \hat{x})))dr}{\int_{r_1}^{\infty} \phi(\sqrt{B}(x-r-m^*(r, \hat{x})))dr} = \sqrt{B}(x-r_1) + \sqrt{B}r_1 - \int_{r_1}^{\infty} \frac{1}{\phi(\sqrt{B}(x-r-m^*(r, \hat{x})))} \sqrt{B}(r+m^*(r, \hat{x})) \phi(\sqrt{B}(x-r-m^*(r, \hat{x})))dr \]

which is equivalent to

\[\frac{\phi(\sqrt{B}(x-r_1))}{1 - \Phi(\sqrt{B}(x-r_1))} - \sqrt{B}(x-r_1) \geq \sqrt{B}(r_1) - \int_{r_1}^{\infty} (r+m^*(r, \hat{x})) \varphi(r|x) dr. \]

Note that the left hand side is positive because \( \frac{\phi(\sqrt{B}(x-r_1))}{1 - \Phi(\sqrt{B}(x-r_1))} \) is the hazard ratio at \( \sqrt{B}(x-r_1) \) which is larger than \( \sqrt{B}(x-r_1) \).

The right hand side is negative as \( \int_{r_1}^{\infty} (r+m^*(r, \hat{x})) \varphi(r|x) dr \geq r_1 + m^*(r_1, \hat{x}) > r_1. \)

Thus, we established that \( \forall x_1 < x_2, \Psi(r|x_2, m^*(r, \hat{x})) \) first order stochastically dominates \( \Psi(r|x_1, m^*(r, \hat{x})) \). This results in \( E[r|x_i, m^*(r, \hat{x})] \) weakly increasing in \( x_i \).

We now show that \( \Pr(x_j < \hat{x}|x_i, m^*(r, \hat{x})) \) strictly decreasing in \( x_i \) thus resulting in
\[ \Delta(x_i, l(\cdot, \hat{x})) \text{ strictly increasing in } x_i. \]

To see this, note that
\[
\Pr(x_j < \hat{x}|x_i, m^*(r, \hat{x})) = \int_{-\infty}^{\infty} \frac{\sqrt{3}}{\sqrt{2\pi} \epsilon^j} e^{-\frac{3}{2} \epsilon^j^2} \Pr(y(r, \hat{x}) < \hat{x} - \epsilon_j|x_i) d\epsilon_j.
\]

Thus, to show that \( \Pr(x_j < \hat{x}|x_i, m^*(r, \hat{x})) \) is strictly decreasing in \( x_i \), it is sufficient to show that \( \Pr(y(r, \hat{x}) < \hat{x} - \epsilon_j|x_i) \) is decreasing in \( x_i \) and strictly decreasing in \( x_i \) for a region of non-zero measure. This is equivalent to show that \( F(y(r, \hat{x}) < \hat{x} - \epsilon_j|x_i) \equiv \Pr(y(r, \hat{x}) < \hat{x} - \epsilon_j|x_i) \) first order stochastically dominates \( F(y(r, \hat{x}) < \hat{x} - \epsilon_j|x_2) \) and that the domination is strict on a region of non-zero measure. But this is straightforward as the density distribution of \( y(r, \hat{x}) \) conditional on \( x_i, f(y(r, \hat{x})|x_i) \), satisfies MLRP and strict MLRP whenever \( y(r, \hat{x}) < r_1 \) or \( y(r, \hat{x}) > r_2 \) which is of course a non-zero measure. The proof that \( \Delta(x_i, l(\cdot, \hat{x})) \) is strictly increasing in \( x_i \) is now finished and so is the proof of Lemma 3. ■

Proof of Proposition 1

**Proof.** Lemma 3 states that there is a unique \( x^*(m^*(\hat{x})) \) such that \( \Delta(x^*(m^*(\hat{x})), l(\cdot, \hat{x})) = 0 \).

From now on we denote \( x^*(m^*(\hat{x})) \) as \( x^*(\hat{x}) \) and write the equation as \( \Delta(x^*(\hat{x}), l(\cdot, \hat{x})) = 0 \).

We need to prove that \( x^*(\hat{x}) = \hat{x} \equiv x^* \) has one and only one fixed point.

Inserting the expression of \( E[r|x_i, m^*(r, \hat{x})] \) and \( E[l|x_i, m^*(r, \hat{x})] \) into the equation, we have

\[
\frac{1}{1 - \Phi(\sqrt{3}(x^*(\hat{x}) - r_1)) + \sqrt{3}\phi(\sqrt{3}(x^*(\hat{x}) - r_2))(r_2 - r_1) + \Phi(\sqrt{3}(x^*(\hat{x}) - r_2))} 
\times 
\{ x^*(\hat{x})[1 - \Phi(\sqrt{3}(x^*(\hat{x}) - r_1))] - \frac{1}{\sqrt{3}}\phi(\sqrt{3}(x^*(\hat{x}) - r_1)) + \frac{1}{2}\sqrt{3}\phi(\sqrt{3}(x^*(\hat{x}) - r_2))(r_2 - r_1)^2 
+ x^*(\hat{x})\Phi(\sqrt{3}(x^*(\hat{x}) - r_2)) + \frac{1}{\sqrt{3}}\phi(\sqrt{3}(x^*(\hat{x}) - r_2)) \} 
\]

\[
- \delta \times \frac{1}{1 - \Phi(\sqrt{3}(x^*(\hat{x}) - r_1)) + \sqrt{3}\phi(\sqrt{3}(x^*(\hat{x}) - r_2))(r_2 - r_1) - \Phi(\sqrt{3}(x^*(\hat{x}) - r_2))] \Phi(\sqrt{3}(\hat{x} - r_2)) 
\times 
\int_{-\infty}^{+\infty} \frac{\sqrt{3}}{\sqrt{2\pi} \epsilon^j} e^{-\frac{3}{2} \epsilon^j^2} \Phi(\sqrt{3}(x^*(\hat{x}) - (\hat{x} - \epsilon_j))) d\epsilon_j 
\]

\[
- \frac{1}{1 - \Phi(\sqrt{3}(x^*(\hat{x}) - r_1)) + \sqrt{3}\phi(\sqrt{3}(x^*(\hat{x}) - r_2))(r_2 - r_1) + \Phi(\sqrt{3}(x^*(\hat{x}) - r_2))} \}
\]

\[= 0 \]
Or equivalently,

\[
\begin{align*}
\{x^*(\hat{x})[1 - \Phi(\sqrt{\beta}(x^*(\hat{x}) - r_1))] - &\frac{1}{\sqrt{\beta}} \phi(\sqrt{\beta}(x^*(\hat{x}) - r_1)) + \frac{1}{2} \sqrt{\beta} \phi(\sqrt{\beta}(x^*(\hat{x}) - r_2))(r_2^2 - r_1^2) \\
+ &x^*(\hat{x})\Phi(\sqrt{\beta}(x^*(\hat{x}) - r_2)) + \frac{1}{\sqrt{\beta}} \phi(\sqrt{\beta}(x^*(\hat{x}) - r_2)) \} \times \\
\{ -1 + [\Phi(\sqrt{\beta}(x^*(\hat{x}) - r_1)) - \sqrt{\beta} \phi(\sqrt{\beta}(x^*(\hat{x}) - r_2))(r_2 - r_1) - \Phi(\sqrt{\beta}(x^*(\hat{x}) - r_2))] \Phi(\sqrt{\beta}(\hat{x} - r_2)) \\
+ &\int_{-\infty}^{+\infty} \frac{\sqrt{\beta}}{\sqrt{2\pi}} e^{-\frac{\beta}{2} \xi^2} \Phi(\sqrt{\beta}(x^*(\hat{x}) - (\hat{x} - \xi))d\xi \}
\end{align*}
= 0
\]

where

\[
r_2 = \hat{x} - \frac{1}{\sqrt{\beta}} h\left(\frac{k}{\sqrt{\beta}}\right)
\]

and

\[
\Phi(\sqrt{\beta}(\hat{x} - r_1)) - \Phi(-h\left(\frac{k}{\sqrt{\beta}}\right)) = k(r_2 - r_1)
\]

which follow from manager’s optimal manipulation decision.

Rational expectations require that \(x^*(\hat{x}) = \hat{x} \equiv x^*\). Insert this condition into \(\Delta(x^*(\hat{x}), l(., \hat{x})) = 0\), we have

\[
\begin{align*}
\{x^*[1 - \Phi(\sqrt{\beta}(x^* - r_1))] - &\frac{1}{\sqrt{\beta}} \phi(\sqrt{\beta}(x^* - r_1)) + \frac{1}{2} \sqrt{\beta} \phi(\sqrt{\beta}(x^* - r_2))(r_2^2 - r_1^2) \\
+ &x^*\Phi(\sqrt{\beta}(x^* - r_2)) + \frac{1}{\sqrt{\beta}} \phi(\sqrt{\beta}(x^* - r_2)) \} \times \\
\{ -1 + [\Phi(\sqrt{\beta}(x^* - r_1)) - \sqrt{\beta} \phi(\sqrt{\beta}(x^* - r_2))(r_2 - r_1) - \Phi(\sqrt{\beta}(x^* - r_2))] \Phi(\sqrt{\beta}(x^* - r_2)) \\
+ &\int_{-\infty}^{+\infty} \frac{\sqrt{\beta}}{\sqrt{2\pi}} e^{-\frac{\beta}{2} \xi^2} \Phi(\sqrt{\beta}\xi)d\xi \}
\end{align*}
= 0
\]

We will also have

\[
r_2 = x^* + \frac{1}{\sqrt{\beta}} h\left(\frac{k}{\sqrt{\beta}}\right)
\]

and

\[
\Phi(\sqrt{\beta}(x^* - r_1)) - \Phi(-h\left(\frac{k}{\sqrt{\beta}}\right)) = k(r_2 - r_1)
\]
Insert the above two equations into the indifference condition and use the algebraic fact that
\[
\int_{-\infty}^{+\infty} \frac{\sqrt{\beta}}{2\pi} e^{-\frac{1}{2}\sigma^2} \Phi(\sqrt{\beta} \varepsilon_j) d\varepsilon_j = \int_{-\infty}^{+\infty} \sqrt{\beta} \phi(\sqrt{\beta} \varepsilon_j) \Phi(\sqrt{\beta} \varepsilon_j) d\varepsilon_j \\
= \Phi^2(\sqrt{\beta} \varepsilon_j)|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} \sqrt{\beta} \phi(\sqrt{\beta} \varepsilon_j) \Phi(\sqrt{\beta} \varepsilon_j) d\varepsilon_j
\]
would result in \[
\int_{-\infty}^{+\infty} \frac{\sqrt{\beta}}{2\pi} e^{-\frac{1}{2}\sigma^2} \Phi(\sqrt{\beta} \varepsilon_j) d\varepsilon_j = \frac{1}{2}
\]
We then have
\[
x^* \left[ 1 - \Phi(\sqrt{\beta}(x^* - r_1)) \right] - \frac{1}{\sqrt{\beta}} \Phi(\sqrt{\beta}(x^* - r_1)) + \frac{1}{2} k(r_2^2 - r_1^2) \\
x^* \Phi(h(\frac{k}{\sqrt{\beta}})) + \frac{k}{\beta} \\
+ \delta(\Phi(\sqrt{\beta}(x^* - r_1)) - k(r_2 - r_1) - \Phi(h(\frac{k}{\sqrt{\beta}})) \Phi(h(\frac{k}{\sqrt{\beta}})) = \frac{\delta}{2} \tag{16}
\]
with
\[
r_2 = x^* + \frac{1}{\sqrt{\beta}} h(\frac{k}{\sqrt{\beta}})
\]
and
\[
\Phi(\sqrt{\beta}(x^* - r_1)) - \Phi(-h(\frac{k}{\sqrt{\beta}})) = k(r_2 - r_1)
\]
To show that there is a fixed point and the fixed point is unique, we need to show that equation (16) has a unique solution. Note that the right hand side of equation (16) is independent of \(x^*\). Thus we show the existence of a unique fixed point by proving that the left hand side of equation (16) is strictly increasing in \(x^*\) and is smaller than the right hand side when \(x^* \to -\infty\) and greater than the right hand side when \(x^* \to +\infty\).

Note that from equations (6) and (5) we know that \(r_2\) and \(r_1\) are all functions of \(x^*\) and differentiating those two equations with respect to \(x^*\) would result in \(\frac{\partial r_2}{\partial x^*} = \frac{\partial r_1}{\partial x^*} = 1\). This implies that \(r_2 - r_1\) and \(x^* - r_1\) does not vary with \(x^*\). Thus the derivative of the left hand side of equation (16) with \(x^*\) is \([1 - \Phi(\sqrt{\beta}(x^* - r_1))] + k(r_2 - r_1) + \Phi(h(\frac{k}{\sqrt{\beta}})) > 0\), implying that the left hand side is strictly increasing in \(x^*\). On the other hand, note that from equations (6) and (5), when \(x^* \to -\infty, r_2 \to -\infty\) and \(r_1 \to -\infty\). From above, \(r_2 - r_1\) and \(x^* - r_2\) is a constant of \(x^*\) and is thus bounded. This results in the left hand side approaching \(-\infty\) and thus smaller than \(\frac{\delta}{2}\). Similarly when \(x^* \to -\infty, r_2 \to -\infty\) and \(r_1 \to -\infty\), resulting in
the right hand side approaching $+\infty$ and is larger than $\frac{\delta}{2}$. Thus, there exists a unique fixed point that solves equation (16) and the proof is complete. ■

Proof of Proposition 2

Proof. When $\beta \to +\infty$, $\frac{1}{\sqrt{\beta^2}} h\left(\frac{k}{\sqrt{\beta}}\right)$ can be shown to approach zero, resulting in $r_2 = x^* > r_1$. Meanwhile, equation (11) becomes $k(x^* - r_1) = 1$. Equation (16), the indifference condition, becomes $x^* + k\left[\frac{1}{2}(x^2 - r_1^2) - x^*(x^* - r_1)\right] = \frac{\delta}{2}$, which is equivalent to $x^* - \frac{1}{2} k(x^* - r_1)^2 = \frac{\delta}{2}$. Substituting in $k(x^* - r_1) = 1$, we have $x^* = r_2 = \frac{\delta}{2} + \frac{1}{2k}$ and $r_1 = \frac{\delta}{2} - \frac{1}{2k}$. ■

Proof of Proposition 3

Proof. $\frac{\partial r_1}{\partial k} = \frac{1}{2k^2} > 0$ thus $r_1$ is increasing in $k$. When $k \to +\infty$, $r_1 = \frac{\delta}{2} = x^{MS}$. ■

Proof of Proposition 4

Proof. Since $r_1 = \frac{\delta}{2} - \frac{1}{2k}$, $r_1 < 0$ if and only if $\frac{\delta}{2} - \frac{1}{2k} < 0$, or $k < \frac{\delta}{2}$. ■

Proof of Proposition 5

Proof. Since $r_2 = \frac{\delta}{2} + \frac{1}{2k}$, $r_2 > \frac{\delta}{2}$ for any $k$. $r_2 > \delta$ if and only if $\frac{\delta}{2} + \frac{1}{2k} > \delta$, or $k < \frac{\delta}{3}$. ■

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