Customer Needs and Solutions (2022) 9:74–94
https://doi.org/10.1007/s40547-022-00131-6

RESEARCH ARTICLE

Business Expansion Through Acquisition

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Accepted: 19 September 2022 / Published online: 27 October 2022

Abstract
My objective is to better understand how a business should expand through acquisition. In a differentiated market where firms first choose quality and then compete in prices, the idea is to analyze acquisition as an expansion strategy. The specific questions I consider are the following: (a) is it better to acquire a direct or an indirect competitor? (b) how are the quality levels affected by acquisition and does it matter whether the path to increase quality is fixed costs or costs that depend on the volume of sales? (c) how are profits affected by acquisition? (d) how are prices affected by acquisition? and (e) what are the welfare effects of acquisition? To study these questions, I employ a spatial model in which each attractive location in the market is occupied by a business. The analysis shows that a firm enjoys superior profitability by acquiring a direct competitor. This obtains because independent of quality, the ability to coordinate prices with the acquisition of a direct competitor is strong: this reduces the intensity of price competition. Second, the model shows that the synergy created by direct mergers is inversely related to the cost of building quality when higher quality comes from fixed investments and is unaffected by the cost of quality when the costs depend on the volume of sales. Third, the model shows that post-acquisition, the merged firm implements reductions in quality when higher quality comes from fixed investments but chooses the same quality when higher quality is delivered by higher variable costs. Competitors respond by increasing quality in the first case and by leaving quality unchanged in the second case. In addition, when higher quality comes from fixed investment, direct acquisitions create a market outcome where price and quality are negatively correlated. Finally, the model shows that the effect of acquisition on total welfare is ambiguous in the case of fixed investment; however, it is unambiguously lower when higher quality comes from higher variable costs.

Keywords Coordination · Quality and price competition · Horizontal differentiation

1 Introduction

1.1 Background

Business expansion is an important goal for many businesses and acquisitions are pervasive. One might ask why would a firm choose acquisition instead of creating a new business to cater to customers it does not yet have? At least two perspectives should lead a firm to prefer acquisition. First, one disadvantage of creating a business to serve customers already served by a competitor is that the new business will intensify competition for the customers the competitor has. Second, many horizontally differentiated markets are crowded: a majority of the viable locations for businesses are already taken. Often, existing businesses meet consumer needs effectively. In such situations, a better approach to expand one's presence in a category may be that of acquiring a competitor (and its location) in the battlefield.

Due to the ubiquity of mergers and acquisitions, they are of significant interest to practitioners and academics. Atmar et al. [2] propose that acquisitions (or mergers) can be classified into three categories: (1) snapping up challengers, (2) expanding the portfolio and (3) betting on adjacent industries. Not surprisingly, the category of acquisitions that exhibits the worst performance is "betting on adjacent industries"; only 22% of these mergers deliver total shareholder return (TSR) which exceeds the median. This is perhaps unsurprising given that the acquired firms operate in markets outside the acquirer's core categories. The first two categories exhibit superior TSR. Here, the new owner of the acquired firm has knowledge and skill about the category and this makes it a safer bet. In addition, there are often significant economies of scale when competitors supply products or services made in the same way.

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Table 1 Recent acquisitions

Merger/Acquisition	Date	Category	Acquisition type	Details*
Empire Inc. acquires Longo's	May 2021	Grocery retailing in the Greater Toronto Area (GTA)	Direct	Empire stores (Sobeys and Freshco) are direct competitors of Longo's throughout the GTA
Estee Lauder (EL) acquires Deciem	February 2021	Cosmetics	Indirect	EL (mainstream) and Deciem (antithesis cosmetics: white packaging, scientific names, inexpensive)
Purina Acquires Lily's Kitchen (LK)	June 2020	Dog Food in the UK (focus on the wholesome ingredient segment)	Direct	Competition between Beneful and LK can be managed
Puig acquires Charlotte Tilbury (CT)	June 2020	Cosmetics	Indirect	Puig (sustainable cosmetics) and CT (Luxury)
Hostess Brands acquires Voortmans	December 2019	Sweet Packaged Snacks (Canada)	Direct	Hostess Brands (snacking cakes) and Voortmans (premium, branded wafers and specialty cookies
Burger King acquires Tim Hortons	December 2014	Quick Service Restaurant (Canada)	Indirect	Burger King (burgers, hot fast food) and Tim Hortons (donuts, coffee, sandwiches)
L'Oréal acquires Cadum	April 2012	Personal care products (France)	Indirect	L'Oréal adult PC products and Cadum (focus on infants)

* Details for all the mergers listed in the table are available from the author.

Table 1 provides a summary of recent acquisitions ranging from grocery retailing to personal care products. The table highlights the tendency of firms to merge (or acquire) direct and indirect competitors in the same category.

The advantages of making an acquisition within the same category are clear. For example, despite Estée Lauder and Deciem serving different segments through different channels, the opportunity to realize economies in the manufacturing of cosmetics is substantial. An unanswered question is nevertheless raised by the data reported in [2]. Total Shareholder Return (TSR) is highest in the direct competitor category ("snapping up challengers"). Here, 64% of the mergers deliver TSR above the median. In contrast, for the indirect category ("expanding the portfolio"), only 50% of the mergers deliver TSR that exceeds the median. Thus, while scale economies provide strong logic for business expansion, demand-side factors may be critical for determining whether or not mergers deliver above average performance. My objective is to examine this issue and provide insight about how markets will be impacted by acquisition.

Academic papers that examine how acquisitions affect markets are primarily focussed on issues of anti-trust and how they affect total welfare [40]. As a result, when the "total welfare" implications of an acquisition are weak, our understanding of how the acquisition of a competitor affects profitability is limited.[1] In general, markets where acquisitions occur are mature (for example, all of the categories listed in Table 1 are mature) and competitors have well-established locations. In addition, consumers are informed about the offers in the market and gravitate to offers that best meet their needs. However, in these markets, firms make on-going decisions about the quality of the customer experience. These decisions create the context for pricing competition that unfolds between firms.[2]

It is in this context that my objective is to answer the following questions.

1. When a firm acquires a competitor in a horizontally differentiated market, is it better to acquire a firm that is a direct or indirect competitor?
2. How are the quality levels affected when a firm expands through acquisition and does it matter whether the path to increase quality is fixed costs or costs that depend on the volume of sales?
3. How are the profits post-acquisition affected by whether the cost to create quality comes from fixed investments or costs that depend on the volume of sales?
4. How are prices affected by whether the cost to create quality comes from fixed investments or costs that depend on the volume of sales?
5. What are the welfare effects of acquisition?

In my analysis, I abstract away from supply-side considerations such as economies of scale and complementarities

[1] Often, competition in markets is such that the acquisition of one firm by another has relatively small effects on total welfare.

[2] The model applies to markets where the level of quality can be adjusted more easily than the horizontal location. For example, it is easier for a Quick Service Restaurant to modify the service experience of customers than to change its cuisine category (to move from burgers to pizza for example).

in the supply chain. Supply-side factors certainly provide a strong basis for acquisition. However, to explain the superior performance of acquisitions of direct competitors compared to indirect competitors, it is imperative to understand the demand-side effects of acquisition. These effects are understudied and if better understood, firms can expand with a more complete picture of how acquisitions affect profitability.

The analysis identifies five key findings. The first four are managerial in nature and the final finding relates to the effect of acquisition on total welfare. First, a firm is invariably better off acquiring a direct competitor versus an indirect competitor. This obtains because independent of how the quality of the customer experience is delivered, the ability to coordinate prices with the acquisition of a direct competitor is strong. This leads to higher average prices.

Second, when the quality is delivered by fixed investment, the model shows that the merged firm implements significant reductions in the quality of the customer experience. In contrast, the level of quality provided to customers is unaffected by acquisitions when the cost to create quality is proportional to the firm's level of sales.

Third, the analysis shows that the benefits of acquisition are different depending on whether the path to increase quality is fixed costs or costs that depend on the volume of sales. When quality is increased through fixed investment, the benefits are inversely related to the cost of providing quality. That is, when quality can be added at lower cost, the benefit of an acquisition is reduced. Investments in quality are competed away and the less expensive these investments are, the more firms spend. In contrast, when the path to higher quality is to increase the variable cost to serve a customer, the benefit created by direct merger is unaffected by the cost of increasing quality. A unique characteristic of the variable cost context is that higher quality is financed by each customer who buys.

Acquisition generally leads to higher prices, yet when quality is delivered through fixed investment, the relationship between quality and price is affected substantially by whether an acquisition is direct or indirect. With a direct acquisition, consumers face a market in which lower quality products are more expensive than high quality products. With an indirect acquisition, the opposite occurs. When the cost to create quality is proportional to the firm's level of sales, only adjacent acquisitions lead to higher prices.

The fifth finding relates to the effect of the acquisition on total welfare. Typically, regulators are leery of acquisitions in markets that do not generate significant economies.[3] In my model, by construction, there are no economies. Accordingly, the welfare effects of acquisitions are entirely driven by "demand-side" factors: pricing coordination that might create welfare losses and changes to the quality of products that might generate welfare gains or losses. Without economies, the model indicates that we are more likely to observe the acquisition of a direct versus an indirect competitor. Accordingly, I focus my comments on the total welfare effect of acquiring a direct competitor.

When higher quality is delivered by fixed costs, I find that total welfare can *increase* when the cost to raise quality is less than a threshold. In this situation, the welfare gains from more efficient quality provision outweigh the welfare losses created by pricing coordination. When higher quality is delivered through higher variable costs, I find the total welfare is unambiguously reduced by acquisition.

This means that with the acquisition of a direct competitor, regulators need to be sensitive to how the quality that customers experience is delivered by firms (through fixed investments or costs related to volume), the expected changes in quality (post-acquisition) and the expected changes in pricing (which is the typical focus of regulators). I now review the literature relevant to this study.

1.2 Literature Review

In general, the literature on competitor acquisition focuses on issues of anti-trust and how acquisition (or merger) impacts total welfare [40]. The assessment typically reflects a trade-off between economies (created by the acquisition) and welfare losses due to monopoly power [29]. There is also a rich literature on how market foreclosure creates monopoly power (and losses in total welfare) through downstream or upstream integration [, pp. 194–196]. It is generally assumed that horizontal mergers (or acquisitions) are profitable for the firms involved [30, 37]. Davidson and Deneckere [9] show this is invariably the case in markets characterized by Bertrand competition. To a degree, the literature has ignored why and how a firm should choose a takeover target.

Spatial models have been used to assess both the pricing and welfare implications of mergers. In a model with price competition and quadratic travel costs, Levy and Ritzes [20] show that the incentive of neighboring firms to merge is higher than that of non-neighboring firms.[4] From a merger perspective, this implies that the closeness of competition in spatial markets should be the primary concern for anti-trust authorities. Brekke et al. [4] consider how mergers affect the provision of quality delivered by fixed investments in 3-firm spatial model setting and they find that the merging firms reduce quality and also possibly price. This model is related to my setting; however, with only three firms, the relative

[3] The economies may be in terms of manufacturing, distribution, servicing and/or administration.

[4] Similarly, Giraud-Heraud et al. [15] find that acquisitions of adjacent competitors are more profitable in horizontal markets restricted to price competition.

profitability and impact of adjacent versus indirect acquisition (by a focal firm) cannot be compared. In addition, my objective is to understand whether the impact of acquisition on quality is affected by the path firms use to create quality (fixed investment or variable costs).

Other topics in this literature include analyzing how merger activity affects the potential for entry and vice versa. Stigler [35] suggests that mergers may not be profitable as entrants sometimes derive positive externalities from merger activity within an industry. Rothschild [32] focuses on how the incentives of a potential entrant are affected when two incumbents in a spatial market merge. It is important to note that the question of whether to target a direct or an indirect competitor when firms make strategic decisions about quality before competing in prices, has not been considered.[5]

The impact of acquisitions (or mergers) has also been studied in a quantity setting (or Cournot) game [8, 12, 23]. As before the focus of this literature tends to be the welfare implications of mergers. When Cournot firms compete in a spatially differentiated market, [28] investigate how a merger affects the locational decisions of the merged firm. The authors show that acquisitions are often profitable because the merged firm coordinates the location of its plants as well as the quantity decisions. As noted earlier, mergers (or acquisitions) are assumed to be profitable for the merged firm. The challenge is to determine the overall welfare implications of mergers because at times, mergers are beneficial.

In addition, this study is related to the literature on product line management. As noted by Lilien et al. [21], when there is significant heterogeneity in the willingness to pay for products, a product line is called for. In particular, when consumers are heterogeneous in their willingness to pay for a key attribute like quality, power, or warranty length, a firm (with monopoly power) can construct a menu of offers and through second degree price discrimination increase profitability [22, 27]. The heterogeneity I consider is not based on differences in willingness to pay for an attribute that "all consumers" find valuable. The management of a product line in a horizontally differentiated characterized by price competition is considered by Giraud-Heraud et al. [16]. The authors find that a multi-product firm with adjacent products coordinates the prices of its product line and uses lower priced external brands to shield the interior of the product line from competition.

There is also research which examines how firms will manage competing product-lines. Early work considers how firms would compete when they can offer more than one product [3, 38]. More recently, researchers have studied whether firms should employ mass customization versus a fixed number of offers to compete [24], competition between specialist and generalist firms [14] and personalized pricing and quality customization [13]. Typically the research on product line management and competition assumes that firms create offers beyond a single product at low cost (or zero cost). Moreover, the notion of "taking over" another firm's offer is not on the table. The context I consider is the opposite. First, I assume that every viable segment (location) in the market is already occupied. Hence, the notion of creating a new offer is ruled out by assumption. Second, I assume that acquisition is the path that the firm chooses to expand.

There is also a rich literature which examines how firms compete in multi-stage games. One of the first models to examine a stage of quality investment where firms later compete in prices is [34].[6] Shaked and Sutton show how quality differentiation arises naturally in a market where consumers are heterogeneous in their willingness to pay for quality.[7] In contrast, Rhee [31] shows that firms may choose similar levels of quality when the heterogeneity across consumers is along a dimension that is horizontal in nature. Two-stage models of competition (with price as the final stage) has also been used to analyze advertising [10] and retail service [18]. The tendency that firms have to differentiate in terms of quality has unpredictable effects on total welfare and consumer surplus. As a result, Chioveanu [7] uses a model of quality and price competition to assess the effectiveness of minimum quality standards.

In a nutshell, the approach I propose to study the impact of acquisition builds on existing models of quality and price competition. Similar to Iyer [18], the dimension of non-price competition "quality" is assumed to affect the willingness to pay of all consumers who patronize the firm in question, by the same amount. In addition, the sequence of making a choice about quality (which leads to fixed costs or changes in the marginal cost to serve a customer) and then competing in prices reflects the strategic nature of the quality decision in the markets that provide motivation for this study.[8]

2 The Model

To address the questions described in Section 1.1, I need a framework which has the following characteristics. First, it is important to choose a framework in which the key

[5] The literature recognizes that mergers may have an impact on product quality. Willig [41] discusses the impact on total welfare of changes in quality due to a merger when the change in quality is exogenous.

[6] The model of Shaked and Sutton consists of three stages where the first stage is an entry decision. the second stage is a choice of quality and the third stage is a choice of prices.

[7] Models that look at quality competition are generally set in markets where consumers are heterogeneous in their willingness to pay for quality [1, 6, 39].

[8] The findings are affected by the sequential nature of the quality and price decisions. Contexts where quality and price are chosen simultaneously are less common.

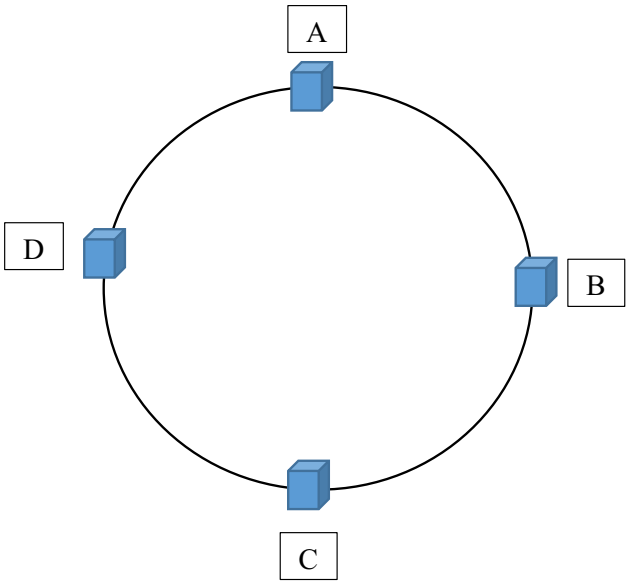

Fig. 1 Model to study business expansion by acquisition

attractive locations are occupied by competitors. Second, there should be no ex ante rationale for one of the competitors to be significantly more attractive for takeover than another (for example, a larger captive market or better managerial skill). Finally, the framework has to be sufficiently rich to reflect the choice that a firm can have between acquiring a direct competitor (with which the firm competes directly) and an indirect competitor (with which the firm competes indirectly).

The model I use to examine this issue is a circular spatial model in the spirit of Hotelling [17] and Salop [33] in which the firms are located at 12 o'clock, 3 o'clock, 6 o'clock and 9 o'clock. As shown in Fig. 1, these positions will be known as Firms A, B, C and D respectively.

Consumers are assumed to be uniformly distributed around the circular market. The cost a consumer incurs to select a product from a firm is proportional to the shortest distance along the circle from the consumer to the firm. It is important to note that each firm is a preferred choice ceteris paribus for one quarter of the market. Each location has equal potential to generate profit and the managerial capabilities at each location are assumed identical. The best location a hypothetical fifth firm could choose would make it preferred for one eighth of the market at best. This framework is designed to represent a context where acquisition is a better path to expansion than creating a new business. This could be because the fixed costs to create a new business are sufficiently high (to deter entry). But in addition, here, the crowded nature of the market means that the potential market available to a fifth entrant is significantly smaller. I assume that Firm A is the focal firm and the decision that Firm A makes is to expand by acquiring either

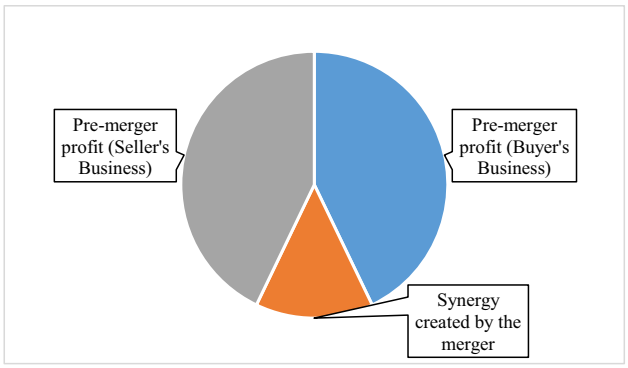

Fig. 2 A necessity for acquisition to take place

Firm B or Firm D (direct competitors) or Firm C (an indirect competitor).[9]

2.1 The Mechanics of Acquisition

I make the assumption that no owner of the four firms in the market is willing to sell his/her business for less than the profit the business currently generates. This implies that if a merger creates synergy, a zone of potential agreement is created for any owner (the buyer) who wishes to acquire the business of another (the seller). Figure 2 shows a situation where the profit of the merged firm is equivalent to the pre-merger profit of the seller plus the pre-merger profit of the buyer plus *synergy* created by the merger.

When a merger creates synergy, there exists the potential for a merger that leaves the seller and buyer better off.[10] The buyer offers the seller the gray area plus a fraction of the orange area. This makes both the seller and the buyer better off compared to the situation prior to acquisition. I assume an acquiring firm prefers the acquisition alternative that maximizes the size of the orange area.

I now move to a precise description of consumer decision making.

2.2 Consumer Decision-making

As noted earlier, consumers are uniformly distributed around a unitary circular market with a density of 1. Consumers are assumed to be informed about the attributes and prices of all firms in the market. The location of a consumer in the market is identified as x which is the distance from the

[9] Similar to Levy and Reitzes [20], the "merged" firm continues to operate from two locations.

[10] There are contexts where a firm might merge due to its profits being reduced if two competitors merge; however, that is not a factor in this framework. Even firms "on the outside" benefit from the reduced intensity of competition.

consumer to Firm A moving in a clockwise direction (this implies that 0 and 1 identify the same location which is precisely the location of Firm A). A consumer buys at most one product and will buy from the firm that provides her with the maximum utility. The utility U from Firm i ($i = A,B,C$ and D) for a consumer located at x is given by:

$$U_i(x) = q_i + v - dt - p_i \quad (1)$$

In this expression, q_i is the level of quality (or service) provided by Firm i and v is the consumer's willingness to pay for a base product that involves no transportation cost (v is identical for all companies). The term dt is the transportation cost a consumer incurs to consume Firm i's product. The variable d is the shortest distance from the consumer to Firm i and t is the transportation cost per unit distance. Without loss of generality, I normalize t to 1. Recall that a consumer chooses the route to Firm i which is shortest. This implies that the transportation cost a consumer incurs to consume Firm A's product is x if she is located on the right half of the market and $1 - x$ if she is located on the left half of the market. The corresponding transportation costs for each firm can be worked out in a similar fashion. Note that the outside option for consumers provides 0 utility so a consumer declines to buy if the highest utility she knows of is less than 0. To ensure full coverage, I assume that v is sufficiently high so that every consumer finds an alternative that yields positive utility.

It is important to note that this analysis is based on a market where consumers have heterogeneous tastes for different products but they are homogenous in terms of their valuation of quality. In many categories where the main differences between choices are horizontal in nature, this is a reasonable assumption. However, the ability to generalize the findings of this model to a market where consumer valuations of quality are heterogeneous is limited.

2.3 Realization of Demand for Each Firm

The demand for each firm is based on consumers choosing the firm that provides the maximum utility. To structure the analysis, I assume that each firm realizes positive demand from each of the segments that divides the market in four.[11] Demand for each firm is determined by finding the consumer that is indifferent between the two adjacent firms in the four line segments. The indifference point between Firms A and B is assumed to x_1, between Firms B and C is assumed to x_2, between Firms C and D is assumed to x_3 and between Firms D and A is assumed is x_4. The indifferent consumers are shown in Fig. 3.

[11] When the equilibrium is determined, the outcomes are checked to ensure that the assumption is justified.

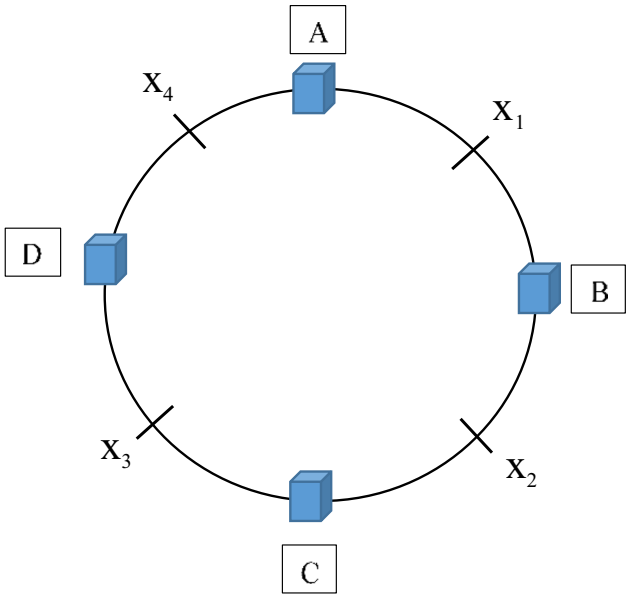

Fig. 3 Indifferent consumers between the 4 firms

Table 2 Indifferent consumers

Indifference	Point expression
x_1	$\frac{1}{2}p_B - \frac{1}{2}p_A + \frac{1}{2}q_A - \frac{1}{2}q_B + \frac{1}{8}$
x_2	$\frac{1}{2}p_C - \frac{1}{2}p_B + \frac{1}{2}q_B - \frac{1}{2}q_C + \frac{3}{8}$
x_3	$\frac{1}{2}q_C - \frac{1}{2}p_C + \frac{1}{2}p_D - \frac{1}{2}q_D + \frac{5}{8}$
x_4	$\frac{1}{2}p_A - \frac{1}{2}q_A - \frac{1}{2}p_D + \frac{1}{2}q_D + \frac{7}{8}$

The expression for the indifferent consumer between Firm A and B; B and C; C and D; and D and A are found by setting equal the surplus from each adjacent pair of firms given by Eq. 1. The expressions are shown in Table 2.

Given prices and quality levels, simple calculations imply that the demand D_i for each firm ($i = A,B,C$ or D) is $1 - x_4 + x_1$, $x_2 - x_1$, $x_3 - x_2$ and $x_4 - x_3$ respectively. I now move to the decisions that firms make prior to consumers making choices.

2.4 Firm Decisions

Prior to a possible acquisition by Firm A, the firms compete strategically by first choosing quality levels. Similar to Motta [26], I examine two different paths through which firms spend to increase the quality of the customer experience.

1. First, I consider changes to the quality of the customer experience delivered by a fixed investment, unaffected by a firm's volume of sales. Such investments include on-going efforts to improve processes, measurement of the customer experience, the quality of manufacturing

or even the consumption environment (by updating/freshening the interior of a restaurant or a retail store). Here, to choose a quality level q_i, a firm incurs a cost βq_i^2 and the convexity of costs reflects the idea that changes become increasingly expensive as a firm seeks to improve quality. As in Mussa and Rosen [27] and Iyer [18], we restrict our attention to the range of β for which the profit functions are concave.

2. Second, I consider changes to the quality of the customer experience through costs that affect the cost of every item that is produced and sold by the firm. These improvements are often related to using superior raw materials or increased labor inputs to deliver the desired outcomes. Examples of such improvements might be using higher quality produce and meat in a restaurant or hiring piece-rate workers with a higher level of training. As in Moorthy [25], to choose a quality level q_i, the firm incurs a marginal cost of βq_i^2. This means that the total cost to deliver higher quality when higher quality is delivered by better raw materials or increased labor input is $D_i \beta q_i^2$ where D_i is the quantity sold. The convexity of cost reflects the idea that firms start by making inexpensive changes to the customer experience and subsequent changes become more expensive. For example, a gourmet restaurant might start by increasing the quality of its vegetables but to move to the next level, higher qualities of meat and fish might be needed.

I assume that firms choose quality simultaneously. After qualities are chosen and observed by the four firms, the firms compete in prices. As a simplification, I assume that the per unit or marginal cost prior to quality being chosen is zero. When the path to improving quality is fixed investment, the marginal cost remains at zero throughout the analysis. When the path to improving quality is through costs proportional to the volume of sales, the marginal cost depends on the quality chosen by the firm. Prices are chosen simultaneously and then consumers make their decisions and firms realize profits. Using the expressions derived earlier, the profit functions for each firm when fixed investments are used to increase quality are:

$$\pi_i = p_i D_i - \beta q_i^2,\ i = A, B, C \text{ or } D \tag{2}$$

When increases in variable cost are used to increase quality, the profit functions are:

$$\pi_i = \left(p_i - \beta q_i^2\right) D_i,\ i = A, B, C \text{ or } D \tag{3}$$

The solution to the game is based on a Sub-game Perfect Nash Equilibrium and my focus is pure strategy equilibria. As mentioned in Section 2.2, the reservation utility v is sufficiently high such that all consumers buy. [12]

[12] In equilibrium (with t normalized to 1 and the marginal cost normalized to zero), $v \geqq \frac{3}{8}$ is sufficient to ensure coverage.

The extensive form of the game can thus be summarized by the following steps:

1. The firms simultaneously choose quality q_i ($i = A,B,C$ or D).
2. The firms simultaneously choose prices p_i ($i = A,B,C$ or D).
3. Consumers observe the quality levels of the firms and make a decision about which, if any, firm to purchase from.

Note that the timing of this game implies that firms choose quality strategically prior to setting prices. In most cases, this is reasonable. However, there may be situations because of either regulation or the nature of the business, where price and quality are set simultaneously. A model with this alternative timing is available as supplemental material (a link is provided at the end of the Appendix).

This completes the description of the pre-acquisition market. When Firm A makes an acquisition of Firm j ($j = B,C$ or D), the objective functions of the competitors of Firm A are unchanged. However, when fixed investment is the basis to increase quality, Firm A's objective function after the acquisition is:

$$\pi_A = p_A D_A + p_j D_j - \beta q_A^2 - \beta q_j^2, j = B, C \text{ or } D \tag{4}$$

When increases in variable cost are used to increase quality, Firm A's objective function after the acquisition is:

$$\pi_A = \left(p_A - \beta q_A^2\right) D_A + \left(p_j - \beta q_j^2\right) D_j, j = B, C \text{ or } D \tag{5}$$

After the acquisition, Firm A sets q_A and q_j simultaneously in the first stage and p_A and p_j simultaneously in the second stage to maximize profit. In the post-acquisition game, the three competitors observe the quality choices before choosing prices. In the next section, I present the analysis and consider both paths that firms use to increase quality.

3 Analysis

Using an example based on the Canadian Quick Service Restaurant (QSR) market, consider the acquisition strategy of Burger King. On the one hand Burger King (which I assume is at position A in Fig. 3) can acquire a competitor that has a similar menu (like Wendy's or Harvey's). This would be analogous to acquiring either Competitor B or D in Fig. 3. On the other hand, Burger King could acquire a competitor like Tim Hortons, Coffee Time or Second Cup which do not compete directly with Burger King (these restaurants specialize in coffee and baked goods like donuts). I start by considering the base case which is the benchmark to assess the potential gains created by acquisition.

3.1 Base Case when Fixed Investment Is Used to Increase Quality

Prior to acquisition, the four firms choose qualities and price independently. The firms start by making investments in quality. The firms then observe the quality decisions of their competitors and compete in prices in the final stage of the game. The outcome is summarized in Lemma 1. Here, we restrict our attention to value of β for which the profit functions are concave $\left(\beta > \frac{25}{144}\right)$.

Lemma 1 With investments in quality, the equilibrium is symmetric and firms choose quality of $\frac{5}{48\beta}$, prices of $\frac{1}{4}$ and earn profits of $\frac{144\beta-25}{2304\beta}$.

Lemma 1 shows that the prices before acquisition are unrelated to the level of quality chosen by the firms. This demonstrates that the benefits of quality are competed away and consumers are the sole beneficiaries of investments in quality.

3.2 Base Case when Higher Variable Cost Is Used to Increase Quality

Here, I examine the equilibrium outcome prior to acquisition when higher variable cost is used to increase quality. After the quality decision is made, the firms observe the quality choices of all competitors and compete in prices. The outcome is summarized in Lemma 2.

Lemma 2 With investments in quality, the equilibrium is symmetric and firms choose quality of $\frac{1}{2\beta}$, prices of $\frac{1}{4\beta}(\beta+1)$ and earn profits of $\frac{1}{16}$.

Lemma 2 shows that the equilibrium is quite different when quality improvement is obtained through higher variable costs. The level of quality delivered to the market is inversely related to β in both cases. However, when variable costs are the basis for higher quality, firms increase prices such that *consumers finance* the higher level of quality. Interestingly, the profitability of firms is unaffected by the level of quality that is ultimately delivered.

3.3 Acquisition when Improved Quality Comes from Fixed Investment

I now consider the case when Firm A acquires Firm B or D (a direct competitor), the equilibrium prices and profits are summarized in Lemma 3. Here, we restrict our attention to values of β where (a) the second order conditions (for all firms) are satisfied and (b) the Hessian matrix of the two business firm is negative semi-definite $\left(\beta > \frac{3\sqrt{43849}+2439}{8410}\right)$.

Lemma 3 With investments in quality, when Firm A acquires a direct competitor, it chooses quality of $\frac{57-290\beta}{575\beta-3625\beta^2}$, the prices are $\frac{1}{5}\frac{290\beta-57}{145\beta-23}$ and it earns profit of $\frac{(25\beta-2)(290\beta-57)^2}{625\beta(145\beta-23)^2}$. The competitors choose quality of $\frac{114-855\beta}{1150\beta-7250\beta^2}$, charge prices of $\frac{29}{10}\frac{15\beta-2}{145\beta-23}$ and the profit of each competitor is $\frac{(21025\beta-3249)(15\beta-2)^2}{2500\beta(145\beta-23)^2}$.

Lemma 3 shows that when Firm A acquires a direct competitor, it chooses lower quality than the competitors.[13] After the acquisition, consumers face firms that are different in terms of quality and prices. This leads to unequal demand for each firm and different market partitions compared to pre-acquisition. Moreover, the acquiring firm reduces quality from pre-acquisition levels. As noted in the pre-acquisition case, the main beneficiaries of quality investment are consumers. Hence, it follows that reduced investment in quality might benefit a merged firm.

I now consider the situation where Firm A acquires Firm C (the indirect competitor). The equilibrium prices and profits of this situation are summarized in Lemma 4. As before, we restrict our attention to values of β where (a) the second order conditions (for all firms) are satisfied and (b) the Hessian matrix of the two business firm is negative semi-definite $\left(\beta > \frac{1}{4}\right)$.

Lemma 4 With investments in quality, when Firm A acquires Firm C (the indirect competitor), it chooses quality of $\frac{5-18\beta}{54\beta-216\beta^2}$, the prices are $\frac{1}{18}\frac{18\beta-5}{4\beta-1}$ and it earns profit of $\frac{(9\beta-1)(18\beta-5)^2}{1458\beta(4\beta-1)^2}$. The competitors choose quality of $\frac{10-45\beta}{108\beta-432\beta^2}$, charge prices of $\frac{1}{9}\frac{9\beta-2}{4\beta-1}$ and the profit of each competitor is $\frac{(9\beta-2)^2(144\beta-25)}{11664\beta(4\beta-1)^2}$.

Lemma 4 shows that when Firm A acquires an indirect competitor, as with direct acquisitions, the acquiring firm and the competitor choose different prices and qualities versus the pre-acquisition outcome. Similar to the case of direct acquisition, the acquiring firm reduces quality compared to the pre-investment levels. A further insight of Lemma 4 is that the profits of the merged firm are less than the sum of profits earned by the two firms before the acquisition when $\beta < \beta^* = \frac{\sqrt{19}}{9} + \frac{3}{4} \approx 1.2343$. This means that unless the fixed investment cost parameter for quality exceeds β^*, an indirect acquisition does not create synergy.

The key outcomes of Lemmas 3 and 4 are summarized in Table 3 and these are used to determine the optimal strategy for the focal firm and equilibrium outcomes after acquisition takes place.

[13] Lemma 3 is conditional on β being greater than a threshold that is sufficient for the existence of a unique equilibrium. Details about the limit for β such the conditions for uniqueness are satisfied are provided in the A.

Table 3 Key results for improved quality comes from fixed investment

Context	Price at merged firm	Price at independents	Quality at merged firm	Quality at independents	Profit of the merged firm	Profit of the competitors
Direct acquisition	$\frac{1}{5}\frac{290\ \beta-57}{145\ \beta-23}$	$\frac{29}{10}\frac{15\ \beta-2}{145\ \beta-23}$	$\frac{57-290\beta}{575\ \beta-3625\ \beta^2}$	$\frac{114-855\beta}{1150\ \beta-7250\ \beta^2}$	$\frac{(25\beta-2)(290\beta-57)^2}{625\beta(145\beta-23)^2}$	$\frac{(21025\beta-3249)(15\beta-2)^2}{2500\beta(145\beta-23)^2}$
Indirect acquisition	$\frac{1}{18}\frac{18\beta-5}{4\beta-1}$	$\frac{1}{9}\frac{9\beta-2}{4\beta-1}$	$\frac{5-18\beta}{54\beta-216\beta^2}$	$\frac{10-45\beta}{108\beta-432\beta^2}$	$\frac{(9\beta-1)(18\beta-5)^2}{1458\beta(4\beta-1)^2}$	$\frac{(9\beta-2)^2(144\beta-25)}{11664\beta(4\beta-1)^2}$

Proposition 1 identifies the optimal acquisition strategy for Firm A in a market where firms first make investments in quality.

Proposition 1 With investments in quality, Firm A generates superior profitability by acquiring a direct competitor versus an indirect competition. The advantage is increasing in β. Firm A's profit is also greater than the sum of the profits of the two firms (pre-acquisition). This advantage is increasing in β.

Proposition 1 demonstrates that the main benefit of an acquisition is the ability to coordinate prices. The benefit is created because a merged firm coordinates the pricing decisions of both offers. This relaxes price competition and all firms benefit. This echoes findings from economics where the focus is strategic takeovers in horizontal markets *restricted to price competition* [20]. Of course, this model investigates the impact of acquisitions in a horizontal market where firms compete in both quality and price. Here, I show that benefits come from two sources: price coordination and lower investments in quality.

This raises two key questions. First, why does the benefit of acquiring a direct competitor versus an indirect competitor increase in β (the cost parameter for quality investment)?

The reason is, when the cost of increasing quality is low (β is small), the difference between the quality offered by the 2-product firm and the competitors is amplified. This obtains because the incentive of a firm with two adjacent offers to increase quality is lower; the incentive to increase share by capturing sales from the adjacent business is absent when that adjacent business has been acquired. However, in contrast to prices which are *strategic complements* in this model, qualities are *strategic substitutes*.[14] As a result, the best response of both competitors is to increase quality creating a significant gap between the low qualities of the merged firm and the qualities of the competitors. This leads to reduced demand for the merged firm. With low demand, the benefit of price coordination is diminished.

Second, why does the benefit of acquiring a direct competitor versus the sum of the profits pre-acquisition *increase* in β (the cost parameter for quality investment)? As noted earlier, the benefit of acquisition in a model with quality investment comes from investing less in quality and price coordination. As β increases, money lost to investments in quality are smaller and thus, the advantage created by the acquisition (versus pre-acquisition) increases.

3.4 Acquisition when Improved Quality Comes from Higher per Unit Costs

I now consider the case when Firm A acquires Firm B or D (a direct competitor), the equilibrium prices and profits are summarized in Lemma 5.

Lemma 5 When higher variable cost is the path to increase quality and Firm A acquires a direct competitor, it chooses quality of $\frac{1}{2\beta}$, the prices are $\frac{1}{20\beta}(8\beta+5)$ and it earns profit of $\frac{4}{25}$. The competitors choose quality of $\frac{1}{2\beta}$, charge prices of $\frac{1}{20\beta}(6\beta+5)$ and the profit of each competitor is $\frac{9}{100}$.

Lemma 5 shows that when Firm A acquires a direct competitor, the qualities of its offers are unaffected by acquisition. However, the acquisition incentivizes the merged firm to charge significantly higher prices. The competitors also increase prices from pre-acquisition prices but not to the same degree. Interestingly, the main beneficiaries of the acquisition are the independent competitors who continue to compete with the merged firm. Their profits increase proportionately more than the profits of the merged firm.

I now consider the situation where Firm A acquires Firm C (the indirect competitor). The equilibrium prices and profits are summarized in Lemma 6.

Lemma 6 When higher variable cost is the path to increase quality and Firm A acquires Firm C (the indirect competitor), it chooses quality of $\frac{1}{2\beta}$, the prices are $\frac{1}{4\beta}(\beta+1)$ and it earns profit of $\frac{1}{8}$. The competitors choose quality of $\frac{1}{2\beta}$, charge prices of $\frac{1}{4\beta}(\beta+1)$ and the profit of each competitor is $\frac{1}{16}$.

Lemma 6 shows that when Firm A acquires an indirect competitor, the impact on the market is negligible. Firm A doubles it profit but this is identical to what Firms A and C

[14] In competitive models, when the best responses of competitors for a key decision (like price or quantity) are positively correlated, they are known as strategic complements. When they are negative correlated, they are known as strategic substitutes [5].

Table 4 Key results for improved quality comes from higher per unit costs

Context	Price at merged firm	Price at independents	Quality at merged firm	Quality at independents	Profit of the merged firm	Profit of the competitors
Direct acquisition	$\frac{1}{20\beta}(8\beta+5)$	$\frac{1}{20\beta}(6\beta+5)$	$\frac{1}{2\beta}$	$\frac{1}{2\beta}$	$\frac{4}{25}$	$\frac{9}{100}$
Indirect acquisition	$\frac{1}{4\beta}(\beta+1)$	$\frac{1}{4\beta}(\beta+1)$	$\frac{1}{2\beta}$	$\frac{1}{2\beta}$	$\frac{1}{8}$	$\frac{1}{16}$

earned prior to acquisition. The analysis raises an important question. When higher variable cost is used to raise quality of the products, why are the qualities of products unaffected by acquisition. The explanation is that a higher price facilitated by higher quality leads to a first order change in the profit of each firm whereas the increase in demand facilitated by an increase in quality is a second order change. For each product sold by a firm, there is an incentive to increase the quality until the marginal cost of increasing quality is equal to 1. This occurs when quality is equal to $\frac{1}{2\beta}$. While increasing quality leads to higher demand at the margin, this effect is of second order.

The key outcomes of Lemmas 5 and 6 are summarized in Table 4 and these are used to determine the optimal strategy for the focal firm and equilibrium outcomes after acquisition takes place.

Proposition 2 which identifies the optimal acquisition strategy when improved quality comes from higher per unit costs.

Proposition 2 When higher variable cost is to increase quality, Firm A generates superior profitability by acquiring a direct competitor versus an indirect competition. Firm A's profit is also greater than the sum of the profits of the two firms (pre-acquisition). The benefits of a direct acquisition are unrelated to β.

Proposition 2 reinforces the logic that explains Proposition 1. The main benefit of acquiring a direct competitor is the ability to coordinate prices. In fact, when variable costs are used to increase quality, the *only* benefit delivered by acquisition is the coordination of prices because the qualities of products are unaffected by acquisition. In contrast to the case of fixed investment, the benefits of acquisition are unrelated to the cost of providing quality.

These different relationships flow from the nature of the expenditures being made. With fixed investments, the money spent on quality is sunk by the time firms set prices. As a result, the benefits of higher quality are competed away and only consumers benefit. Even a merged firm suffers significantly when the cost to create quality is lower.

In contrast, when the costs to create quality are variable in nature, a firm only pays the cost to create quality *if a sale is made*. As a result, the quality level is chosen in advance such that the marginal cost of adding quality is exactly equal to the marginal revenue. In the next section, I move to the second research question which relates to how the quality of products is affected by acquisition.

3.5 The Effect of Acquisition on Quality when Fixed Investment Is Used to Increase Quality

In this section, I focus on how acquisition affects the quality of the offerings in the market when fixed investment is used to increase quality. [15] To facilitate presentation, the quality levels are presented as q_r^f where the subscript r refers to the regime (direct acquisition, indirect acquisition and pre-acquisition) and the subscript f refers the firm (the merged firm or the competitors). The ordering of the relative quality levels post-acquisition relative to the pre-acquisition quality level are summarized in Corollary 1.

Corollary 1 Given a cost of building quality $\beta > \frac{\sqrt{2372329}+503}{5220} \approx 0.39142$, the equilibrium quality levels are ordered as follows $q^{competitor}_{direct\ acq} > q^{competitor}_{indirect\ acq} > q_{pre\ acq} > q^{merged\ firm}_{indirect\ acq} > q^{merged\ firm}_{direct\ acq}$. When $\beta \in \left(\frac{3\sqrt{43849}+2439}{8410}, \frac{\sqrt{2372329}+503}{5220}\right)$, the equilibrium quality levels are ordered as follows $q^{competitor}_{direct\ acq} > q^{competitor}_{indirect\ acq} > q^{competitor}_{direct\ acq} > q^{competitor}_{indirect\ acq} > q_{pre\ acq} > q^{merged\ firm}_{direct\ acq} > q^{merged\ firm}_{indirect\ acq}$.

Corollary 1 indicates that there is a slight reordering of the qualities for values of β at the low end of the allowable range (between ≈ 0.36471 and ≈ 0.39142). Nevertheless, the ordering of quality levels exhibits a regular pattern as shown in Fig. 4.

First, the merged firm chooses quality that is strictly less than the pre-acquisition quality and the competitors do the opposite (Bars 1 and 2 versus Bars 4 and 5 in Fig. 4). As explained earlier, the merged firm has less incentive to invest in quality than the competitors. Because the first stage is a game of strategic substitutes, the best response of the competitors is to increase quality.

Figure 4 also shows that the gap in qualities between the merged firm and its competitors in the case of direct acquisition (the difference between the blue bars) is strictly greater than for an indirect acquisition (the difference between the green bars). Corollary 2 summarizes this observation.

[15] As noted in the previous section, the quality of offers is unaffected by acquisition when costs that are proportional to the quantity sold are used to increase quality.

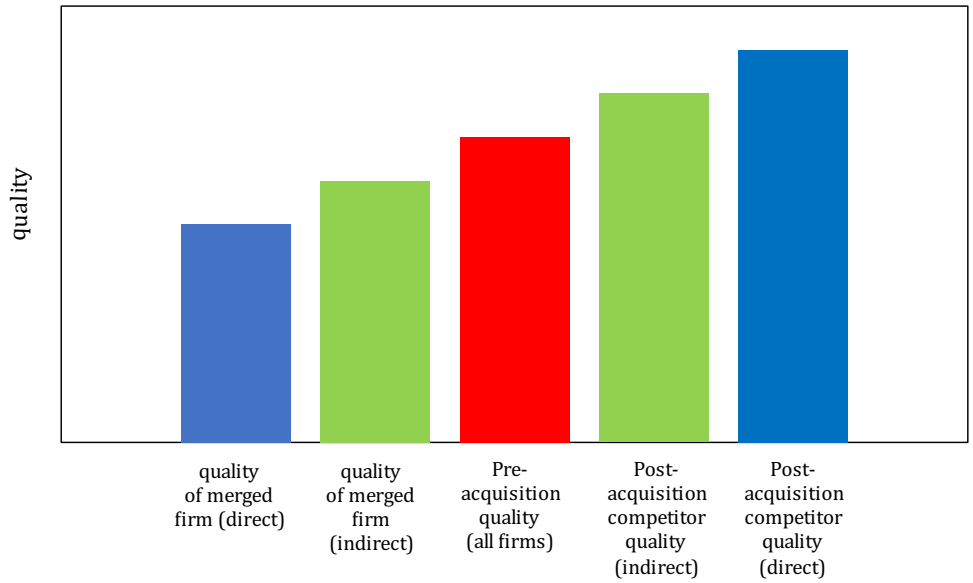

Fig. 4 Quality ordering (Fixed Investment Case) $\beta \gtrapprox 0.39142$

Corollary 2 The quality gap between the merged firm and its competitors is always larger for a direct versus an indirect acquisitions.

The explanation is that the link between a firm's choice of quality and its effect on a direct firm is stronger than its effect on an indirect firm. An indirect acquisition reduces the incentive to increase quality because of how it influences prices in the second stage. However, with a direct acquisition (Firm A acquires Firm B), a lower choice of quality for Firm A makes a lower choice of quality for Firm B optimal: this generates direct savings for the merged firm. Even without economies, when a firm makes a direct acquisition, it has an incentive to (a) reduce quality across both offers, (b) increase prices for both offers and (c) capture significantly less of the market than was captured by the two firms pre-acquisition. Because of higher prices and the savings in fixed investment, this leads to higher profits for the merged firm compared to the combined pre-acquisition profits of the two firms. The reason the "positives" of reduced quality outweigh the "negatives" of lost market share is that the main beneficiaries of quality investment are consumers not firms.

Corollary 3 summarizes the impact of how Firm A's acquisition of a competitor affects the average level of quality.

Corollary 3 The average quality after acquisition drops and it drops more for an indirect than a direct acquisition.

The key insight provided by Corollary 3 is that acquisition leads to a deterioration in the average quality of products sold in the marketplace.[16] The reduction in average quality is generated by the reduced incentive that the merged firm has to invest in quality; this creates an incentive for the competitors to ratchet up quality in response, but the competitors' increase in quality is insufficient to compensate for the reductions implemented by the merged firm. It is interesting that while the gap between the merged firm and its competitor is larger with a direct acquisition, the reduction in average quality is higher with an indirect acquisition. The reason for this is that with a direct acquisition, the merged firm also implements a price increase; this attenuates the merged firm's incentive to reduce the quality of its products further. In 2007, the European Commission raised concern that service quality would be reduced by the proposed takeover of Aer Lingus by Ryanair in 2007 [4]. [17] Because Aer Lingus and Ryanair are direct competitors; this model suggests that concerns about reductions in service quality would be higher were the merging firms indirect competitors.

3.6 The Effect of Acquisition on Prices

The analysis leads to a key observation regarding the impact of acquisition on the relationship between price and quality across the firms when fixed investment is the path to increase quality. To illustrate this observation I examine the ratio $\frac{\Delta p}{\Delta q} = \frac{p_{comp} - p_{acq}}{q_{comp} - q_{acq}}$ for direct and then indirect acquisitions. After a direct acquisition, $\frac{\Delta p}{\Delta q} = \frac{56}{55} - \frac{29}{11}\beta < 0$ (in the allowable range). This implies that with a direct acquisition, an inverse relationship between price and quality is observed; higher quality offers are less expensive. This means that acquisition has a negative effect on consumers who are in the "trading area" of the merged firm. They pay higher prices for offers that are lower quality than before the acquisition. After an

[16] In [4], acquisition leads to an average quality increase. However, in a 3 firm model, there are only two firms after an acquisition. As a result, the incentives of the sole outside firm to increase quality are amplified.

[17] The European Commission decision was rendered on June 27, 2007 (Case # COMP/M.4439).

indirect acquisition, $\frac{\Delta p}{\Delta q} = \frac{2}{3} > 0$, the opposite is observed. The merged firm serves the market with offers that are both lower quality and lower priced than the competitors (a typical positive relationship between price and quality appears). Building on the finding of Brekke et al. [4] about how a merged firm will adjust quality, this model goes further and provides insight about how price and the relationship between quality and price are affected by acquisition.

When higher variable costs are the path to increase quality, average pricing in the market is unaffected by an indirect acquisition. However, when a direct competitor is acquired, the merged firm increases its prices by $\frac{3}{20}$ and the competitors increase their prices by $\frac{1}{20}$. The actual prices charged post-acquisition depend on β (the cost of adding quality) but the magnitude of the increase is unrelated to β. This underlines the absence of a strategic effect through quality when variable costs are the path to increase quality. As shown in Table 4, the market is largely unchanged but the merger leads to a reduction in the intensity of price competition.

3.7 QSR Stylized Example

Here, I return to the example of Section 3 in which Burger King chooses between the acquisition of a direct competitor like Wendy's or Harvey's or an indirect competitor like Tim Hortons. While the model is stylized, Burger King acquired an indirect competitor in 2014: Tim Hortons.[18] The economies created by merging a restaurant chain that specializes in hamburgers and hot food and another which focuses on baked goods, donuts and coffee are minimal so the model is close to the situation being considered. A review of the popular press reveals 4 main paths to enhance quality in QSRs.[19] The 4 paths are (a) keeping the menu "fresh", (b) upgrading processes within the restaurant (such as automated menu boards), (c) ensuring that restaurant environment is "appealing and contemporary" and (d) training and educating employees to provide better customer facing service. All of these paths reflect a context where the QSR is making a fixed investment to upgrade quality.

The model provides insight regarding the expected outcomes of such a merger. First, the model predicts that the merged firm will reduce both investments in quality and prices (this will be countered by a ramping up of quality by the competitors). Second, the model predicts that the merged firm will lose market share. Third, because quality is important in the QSR market, the model suggests that the acquisition will generate minimal if any gains in terms of profit.

In Canada, the five years since the Tim Hortons acquisition have been difficult for Restaurant Brands International (RBI). I focus my comments on Tim Hortons because in Canada, Burger King is relatively small compared to its key competitors (290 outlets versus 1475 McDonalds outlets and 383 Wendy's outlets).[20] There is also information suggesting that Burger King's recent performance has been lackluster; it seems as if the main focus for Burger King after the Tim Hortons acquisition was the benefit of tax inversion obtained by shifting its head office to Canada.[21]

Returning to Tim Hortons, there is compelling evidence that investments in quality for Tim Hortons were reduced in the years following the acquisition. Since 2014, Tim Hortons, which built its reputation on good coffee, fresh donuts and tasty breakfasts, changed. Tim Hortons became a low cost QSR chain that (a) took years to upgrade coffee lids that leaked on consumers, (b) launched a loyalty program that provides a large number of coffee giveaways (a de facto price reduction), (c) ships partially frozen donuts to its franchises and (d) has a frequently changing menu that includes flavored drinks and gimmick donuts [19]. The drop in quality is reflected by Tim Hortons falling off the Reputation Institute's list of the 50 most reputable Canadian companies (in 2017, it held the 13th spot). Contrast this with McDonald's activity during the same period (McDonald's is a direct competitor of Burger King). McDonald's redesigned most of its franchises to provide a "more elevated" relaxed café experience for customers.[22]

These changes led to a loss of market share by Tim Hortons. While same store sales remained flat in the years immediately following the acquisition, they have fallen recently by as much as 4.1% [19]. The financial performance of the Tim Hortons division of RBI is not available publicly but the announcements by RBI and investors are consistent with profits that are significantly below expectations. Moreover, Tim Hortons' revenues in 2019 were just over $6.7 billion US and this represents a decrease of 1.5% from the previous

[18] The details of the acquisition are provided in https://globalnews.ca/news/1724238/its-official-tim-hortons-burger-king-become-one/

[19] Background on the challenge of improving quality in QSRs is available at https://www.paystone.com/blog/7-tips-to-improve-the-customer-experience-in-your-restaurantput, https://benbria.com/4-elements-of-cx-that-every-qsr-should-measure/ and, https://www.usashade.com/resources/articles/ways-to-enhance-your-qsr-experience.

[20] Background is available at https://www.scrapehero.com/top-fast-food-chains-in-canada/. Because Burger King is a strictly controlled global brand and its Canadian operations are relatively small, the flexibility of RBI to adjust the quality of Burger King in Canada is limited.

[21] Background on Burger King is available at https://www.eatthis.com/news-burger-king-decline/ and https://www.forbes.com/sites/jonhartley/2014/08/25/burger-kings-tax-inversion-and-canadas-favorable-corporate-tax-rates/?sh=5c4edb6c3ed7.

[22] These changes are discussed in https://business.financialpost.com/news/retail-marketing/mcdonalds-extends-its-cafe-chain-rollout-as-it-battles-tim-hortons-and-starbucks

year.[23] Because RBI earns its profit almost entirely from revenue-based royalties, it is not a stretch to say that profits have not increased. In a nutshell, the Tim Hortons acquisition has led to market changes largely consistent with the predictions of the model. Indeed, the model suggests that Burger King would have fared better were it to have acquired a direct competitor like Wendy's.

The reduction in quality at Tim Hortons since the acquisition leads to the question of how these changes affect total welfare. Returning to the model, when fixed investment is the basis for increasing quality should the predicted reduction in average product quality caused by acquisition, be a source of concern for market regulators? In addition, if quality is increased through higher variable costs, are acquisitions a cause for concern. I consider these questions in the next section.

4 The Welfare Implications of Acquisition

As noted in Section 2, I focus on situations where all consumers buy. I start by analyzing the welfare implications of acquisition when fixed investments are used to increase quality. Total welfare is given by the benefit created from consumption (B), less transportation costs (T) less investments in quality made by every firm (I).

The benefit from consumption is given by

$$B = \int_{x_4}^{x_1}\left(v+q_A\right)dx + \int_{x_1}^{x_2}\left(v+q_B\right)dx + \int_{x_2}^{x_3}\left(v+q_C\right)dx \int_{x_3}^{x_4}\left(v+q_D\right)dx \tag{6}$$

Total transportation costs are given by:

$$T = \int_0^{x_1} x dx + \int_{x_1}^{\frac{1}{2}}\left(\frac{1}{4}-x\right)dx + \int_{\frac{1}{4}}^{x_2}\left(x-\frac{1}{4}\right)dx + \int_{x_2}^{\frac{1}{2}}\left(\frac{1}{2}-x\right)dx + \int_{\frac{1}{2}}^{x_3}\left(x-\frac{1}{2}\right)dx + \int_{x_3}^{\frac{3}{4}}\left(\frac{3}{4}-x\right)dx + \int_{\frac{3}{4}}^{x_4}\left(x-\frac{3}{4}\right)dx + \int_{x_4}^{1}(1-x)dx \tag{7}$$

Total investments in quality are given by

$$I = \sum_{i=A}^{D} \beta q_i^2$$

To start analysis of the case where firms make fixed investments to increase quality, I determine the welfare maximizing outcome that would be chosen by a central planner. The welfare maximizing outcomes are summarized in Lemma 7.

Lemma 7 The welfare maximizing choice of qualities depends on β:

1. When $\beta < 1$, one firm chooses quality of $q = \frac{1}{2\beta}$, the other firms choose $q = 0$ and all consumers buy from the firm that invests in quality.
2. When $\beta > 1$, all firms choose quality of $\frac{1}{8\beta}$ and consumers buy from the firm which is the shortest distance away.

Lemma 7 demonstrates that a central planner chooses between the cost of building quality into four products at four locations against higher transportation costs and only making quality investment for one of the four products. When the cost of building quality is sufficiently low ($\beta < 1$), the benefit of being able to make investment in quality at a single firm outweighs the higher transportation costs that the market incurs to buy that product. Conversely, when the cost of building quality is high ($\beta > 1$) the central planner minimizes transportation costs by building quality (albeit at a lower level) into all four products.

The pre-acquisition competitive outcome minimizes transportation costs because every consumer buys from the firm which is closest (the price and qualities at every firm are identical). However, the firms choose a quality level that is less than the level that would be chosen by a central planner, i.e., $\frac{5}{48\beta} < \frac{1}{8\beta}$. Before acquisition, the only beneficiaries from firm investments in quality are consumers. The inability that firms have to capture value from the investments made in quality explains why pre-acquisition quality levels are less than the welfare maximizing level.

I am interested in the impact that acquisitions (direct and indirect) have on total welfare. The expressions for total welfare before the acquisition, for a direct acquisition and for an indirect acquisition are provided in the A. I use the variables $W_{pre-acquisition}$, W_{direct} and $W_{indirect}$ to represent total welfare in each of the three cases. Comparison of the expressions leads to Proposition 3.

Proposition 3 The relative order for total welfare in each situation depends on β:

1. When $\beta \in (\approx 0.36471, \approx 0.36766)$, $W_{indirect} > W_{direct} > W_{pre-acquisition}$.
2. When $\beta \in (\approx 0.36766, \approx 0.38230)$, $W_{direct} > W_{indirect} > W_{pre-acquisition}$.
3. When $\beta \in (\approx 0.38230, \approx 0.399)$, $W_{direct} > W_{pre-acquisition} > W_{indirect}$
4. When $\beta \in (0.399, \approx 0.58624)$, $W_{pre-acquisition} > W_{direct} > W_{indirect}$.
5. When $\beta \gtrapprox 0.58624)$, $W_{pre-acquisition} > W_{indirect} > W_{direct}$

Proposition 3 leads to a complex set of zones for comparisons of total welfare. However, Fig. 5, a plot of the three cases, highlights the key insight of Proposition 3.

When the cost to increase quality exceeds a threshold, $\beta \gtrapprox 0.399$, total welfare is reduced by acquisition independent of whether it is direct or indirect. In contrast, when the cost to increase quality is low $\beta \in (\approx 0.365, \approx 0.382)$, total

[23] These results are discussed in https://www.cbc.ca/news/business/rbi-tim-hortons-popeyes-burger-king-earnings-1.5458089

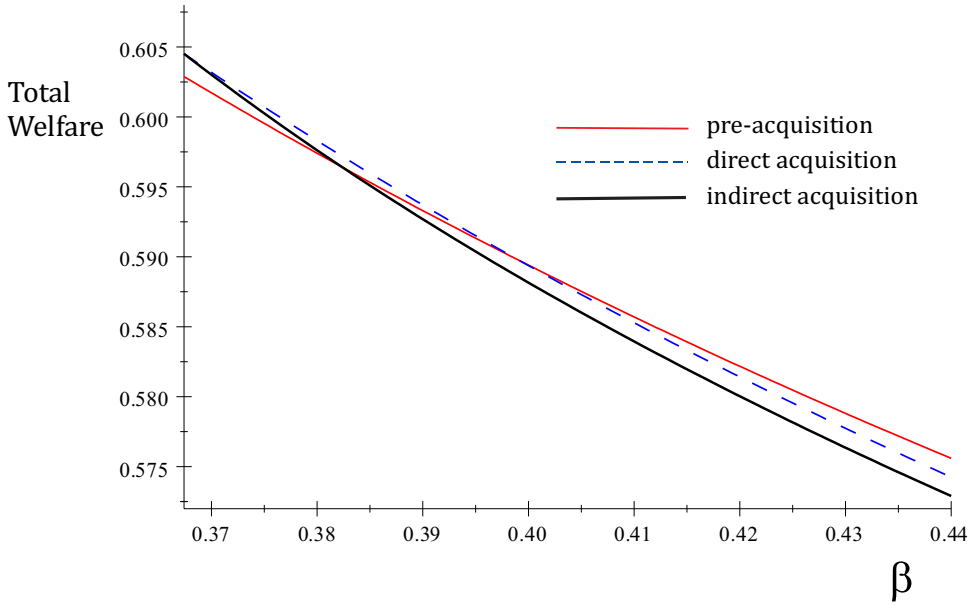

Fig. 5 Illustrative plot for total welfare for $\upsilon = \frac{1}{2}$

welfare is *increased* by acquisition. Importantly, acquisitions do not always lead to a reduction in total welfare.

Prior to the acquisition, competitive firms make equal investments in quality. Because of symmetry, the pre-acquisition equilibrium minimizes transportation costs for consumers. When the cost to build quality into products is sufficiently high, the first-best outcome (in terms of total welfare) also entails minimizing transportation costs. As a result, at high levels of β, acquisitions lead to a reduction in total welfare. Nevertheless, the pre-acquisition competitive market delivers quality levels than are less than the quality levels that maximize total welfare.

Conversely, when the cost to build quality into products is low, Proposition 3 shows that acquisitions can *enhance* total welfare.[24] As noted in Corollary 1, acquisition leads to significant differences in quality between products. From Lemma 7, we know that a central planner makes an asymmetric choice of quality to maximize total welfare. The trade-off when qualities are asymmetric is that transportation costs are higher.

With the acquisition of an indirect competitor, total welfare declines compared to the pre-acquisition level except when the cost to build quality is at the low end of the range. The reason that total welfare declines is that high quality products (sold by the competitors) are more expensive: this means that the benefit of high quality is realized by a small fraction of the market.

In contrast, two factors come into play with the acquisition of a direct competitor. First, the merged firm chooses lower quality levels: this elicits high quality from both competitors. When the cost to build quality is low ($\beta < 1$), the welfare maximizing choice of qualities (the first-best outcome) entails but *one firm* choosing a high level of quality; this does not occur with the acquisition of a direct competitor but the pattern of qualities is *closer* to the first best choice of qualities. This enhances welfare. Second, the equilibrium entails the competitors choosing lower prices than the merged firm. This increases the fraction of the market which benefits from high quality. This too enhances welfare. Of course, these effects have to be weighed against total welfare losses due to price coordination. In a nutshell, when the cost to build quality is sufficiently low, a direct acquisition leads to increases in total welfare.

The total welfare effect of acquisition in the case where variable costs are the path to increase quality is easier to determine. Because the choice of quality that each firm makes is independent of whether an acquisition has taken place, the welfare effect of acquisition is determined by transportation costs. In the case of a direct acquisition, the merged firm charges higher prices than the competitors. This means the indifference point between Firms D and A and between Firms B and C is not halfway between the firms. As a result, post-acquisition, some consumers do not patronize the firm which is closest so transportation costs are higher. Hence, direct acquisitions lead to reductions in total welfare.

These findings are important for regulators who approve and assess acquisitions. Naturally, a key consideration is whether a merger (or an acquisition) leads to significant economies.[25] However, the model I use focuses on the

[24] Brekke et al. [4] find that total welfare can increase when demand is sufficiently responsive to quality. This finding is related to my explanation; invariably, total welfare can increase when acquisition leads to more efficient investments in quality.

[25] The economies may be in terms of manufacturing, distribution, servicing and/or administration.

demand-side effects of acquisition. In the absence of supply-side economies, we are more likely to see direct acquisitions. In addition, regulators need to assess the demand-side effects on total welfare.

In a market where variable costs are used to increase quality, a total welfare loss is created with a direct acquisition because of pricing coordination. Conversely, in a market where fixed investments are the path to create quality, the demand-side effects are nuanced. Total welfare can increase if the cost to raise quality is less than a threshold. This means that with the acquisition of a direct competitor, regulators need to be sensitive to more efficient investments in quality (post-acquisition) because these may outweigh losses created by higher pricing. Recent empirical papers underline the importance of accounting for endogenous changes in decisions other than price to assess merger effects [11, 36]. When decisions such as quality are made endogenously, they can have significant impact on total welfare.

5 Conclusion

Top level, the purpose of this study is to better understand acquisition as an expansion strategy. My interest is crowded horizontal markets where firms first make decisions about quality and then compete in prices. I focus on the demand-side effects of acquisitions because their impact on profitability is important and overlooked.[26] In addition, I consider two different paths that can be used to increase quality: fixed investments or higher variable costs. In reality, a combination of both paths apply to most categories. Hence, my findings apply to situations where one path is dominant. An interesting extension to this research would be to examine fixed investment that reduces the marginal cost of increasing quality.

On a second level, I wish to provide insight into the impact that acquisitions have on the qualities of offers in the market. On a third and final level, I want to learn whether acquisitions in crowded horizontal markets are bad for society as a whole. Are there situations where acquisitions can increase total welfare?

The analysis shows that in general, a firm enjoys superior profitability by acquiring a direct competitor. This obtains because independent of how quality is affected, the ability to coordinate prices with the acquisition of a direct competitor is strong: this reduces the intensity of price competition and leads to higher prices. Because prices are strategic complements, the coordination of prices by a merged firm is a win-win for all firms in the market. A further insight provided by the model is that the benefits of a merger are positively related to the cost of increasing quality when the quality is increased by fixed investment. When quality is increased through higher variable costs, the benefit of an acquisition is unaffected by the cost a firm incurs to increase quality.

The impact of acquisition on quality depends on whether firms increase quality through fixed investment or through higher variable costs. When quality is increased by fixed investment, the quality stage (of the two-stage game) is one of strategic substitutes. Acquisition weakens the incentive to provide quality for the firm that makes an acquisition. This creates an incentive for competitors to increase their quality from pre-acquisition levels. In sum, the model shows that independent of whether an acquisition is direct or indirect, the net effect of these changes is a reduction of the average quality in the market. In contrast, when quality is increased by variable costs, acquisition does not alter the choice of quality. Independent of how many consumers a firm serves, the optimal level of quality is unaffected.

Finally, the welfare effects of acquisition depend on both the path used to increase quality and the cost of increasing quality. When quality is increased through variable costs, acquisition unambiguously leads to a reduction in total welfare. In contrast when quality is increased through fixed investment, the model shows that acquisition (or a merger) can lead to higher total welfare. When the cost to raise quality is inexpensive, competition leads firms to provide relatively high levels of quality. This leads to an inefficient pattern of quality investment. Acquisition can facilitate more efficient investment in quality and the welfare gains from more efficient investment more than offset welfare losses due to less efficient matching.

Appendix

Proof of Lemma 1 Using the demand functions provided in the main text, the following first order conditions for the final stage of the game must be satisfied.

$$\begin{aligned}
\frac{\partial \pi_A}{\partial p_A} &= \tfrac{1}{2}p_B - 2p_A + q_A - \tfrac{1}{2}q_B + \tfrac{1}{2}p_D - \tfrac{1}{2}q_D + \tfrac{1}{4} = 0\\
\frac{\partial \pi_B}{\partial p_B} &= \tfrac{1}{2}p_A - 2p_B + \tfrac{1}{2}p_C - \tfrac{1}{2}q_A + q_B - \tfrac{1}{2}q_C + \tfrac{1}{4} = 0\\
\frac{\partial \pi_C}{\partial p_C} &= \tfrac{1}{2}p_B - 2p_C - \tfrac{1}{2}q_B + q_C + \tfrac{1}{2}p_D - \tfrac{1}{2}q_D + \tfrac{1}{4} = 0\\
\frac{\partial \pi_D}{\partial p_D} &= \tfrac{1}{2}p_A + \tfrac{1}{2}p_C - \tfrac{1}{2}q_A - \tfrac{1}{2}q_C - 2p_D + q_D + \tfrac{1}{4} = 0
\end{aligned}$$

These conditions imply prices of:

$$\begin{aligned}
p_A &= \tfrac{5}{12}q_A - \tfrac{1}{6}q_B - \tfrac{1}{12}q_C - \tfrac{1}{6}q_D + \tfrac{1}{4}\\
p_B &= \tfrac{5}{12}q_B - \tfrac{1}{6}q_A - \tfrac{1}{6}q_C - \tfrac{1}{12}q_D + \tfrac{1}{4}\\
p_C &= \tfrac{5}{12}q_C - \tfrac{1}{6}q_B - \tfrac{1}{12}q_A - \tfrac{1}{6}q_D + \tfrac{1}{4}\\
p_D &= \tfrac{5}{12}q_D - \tfrac{1}{12}q_B - \tfrac{1}{6}q_C - \tfrac{1}{6}q_A + \tfrac{1}{4}
\end{aligned}$$

I substitute into each firm's objective function to obtain first order conditions for the first stage of the game.

[26] Often the focus of acquisition strategy is economies and complementarities unlocked through acquisition.

$$\frac{\partial \pi_A}{\partial q_A} = \frac{25}{72}q_A - \frac{5}{36}q_B - \frac{5}{72}q_C - \frac{5}{36}q_D - 2\beta q_A + \frac{5}{24} = 0$$
$$\frac{\partial \pi_B}{\partial q_B} = \frac{25}{72}q_B - \frac{5}{36}q_A - \frac{5}{36}q_C - \frac{5}{72}q_D - 2\beta q_B + \frac{5}{24} = 0$$
$$\frac{\partial \pi_C}{\partial q_C} = \frac{25}{72}q_C - \frac{5}{36}q_B - \frac{5}{72}q_A - \frac{5}{36}q_D - 2\beta q_C + \frac{5}{24} = 0$$
$$\frac{\partial \pi_D}{\partial q_D} = \frac{25}{72}q_D - \frac{5}{72}q_B - \frac{5}{36}q_C - \frac{5}{36}q_A - 2\beta q_D + \frac{5}{24} = 0$$

The unique solution to this game is $q_A = q_B = q_C = q_D = \frac{5}{48\beta}$. This implies equilibrium prices of $\frac{1}{4}$ and profits of $\frac{144\beta-25}{2304\beta}$ for all firms. To ensure that the equilibrium is a unique maximum for all 4 players, two conditions must be satisfied. First, the matrix of first order conditions must be of full rank. The second condition is that the second order conditions be satisfied for all 4 firms.

The matrix of first order conditions A is

$$A = \begin{bmatrix} \frac{25}{72} - 2\beta & -\frac{5}{36} & -\frac{5}{72} & -\frac{5}{36} \\ -\frac{5}{36} & \frac{25}{72} - 2\beta & -\frac{5}{36} & -\frac{5}{72} \\ -\frac{5}{72} & -\frac{5}{36} & \frac{25}{72} - 2\beta & -\frac{5}{36} \\ -\frac{5}{36} & -\frac{5}{72} & -\frac{5}{36} & \frac{25}{72} - 2\beta \end{bmatrix}$$

The determinant of A is $\frac{1}{648}\beta(18\beta - 5)(24\beta - 5)^2$. In order for A to be non-singular, $\beta \notin \left\{0, \frac{5}{24}, \frac{5}{18}\right\}$ is required and the second order conditions are satisfied when $\frac{25}{72} - 2\beta < 0 \Rightarrow \beta > \frac{25}{144}$. Hence, we restrict our attention to the parameter space $\beta > \frac{25}{144}$ where the matrix A is non-singular and the second order conditions are satisfied.

Proof of Lemma 2 Using the demand functions provided in the main text, the following first order conditions for the final stage of the game must be satisfied.

$$\frac{\partial \pi_A}{\partial p_A} = \beta q_A^2 + q_A - 2p_A + \frac{1}{2}p_B - \frac{1}{2}q_B + \frac{1}{2}p_D - \frac{1}{2}q_D + \frac{1}{4} = 0$$
$$\frac{\partial \pi_B}{\partial p_B} = \beta q_B^2 + q_B + \frac{1}{2}p_A - 2p_B + \frac{1}{2}p_C - \frac{1}{2}q_A - \frac{1}{2}q_C + \frac{1}{4} = 0$$
$$\frac{\partial \pi_C}{\partial p_C} = \beta q_C^2 + q_C + \frac{1}{2}p_B - 2p_C - \frac{1}{2}q_B + \frac{1}{2}p_D - \frac{1}{2}q_D + \frac{1}{4} = 0$$
$$\frac{\partial \pi_D}{\partial p_D} = \beta q_D^2 + q_D + \frac{1}{2}p_A + \frac{1}{2}p_C - \frac{1}{2}q_A - \frac{1}{2}q_C - 2p_D + \frac{1}{4} = 0$$

These conditions imply prices of:

$$p_A = \tfrac{7}{12}\beta q_A^2 + \tfrac{5}{12}q_A + \tfrac{1}{6}\beta q_B^2 - \tfrac{1}{6}q_B + \tfrac{1}{12}\beta q_C^2 - \tfrac{1}{12}q_C + \tfrac{1}{6}\beta q_D^2 - \tfrac{1}{6}q_D + \tfrac{1}{4}$$
$$p_B = \tfrac{1}{6}\beta q_A^2 - \tfrac{1}{6}q_A + \tfrac{7}{12}\beta q_B^2 + \tfrac{5}{12}q_B + \tfrac{1}{6}\beta q_C^2 - \tfrac{1}{6}q_C + \tfrac{1}{12}\beta q_D^2 - \tfrac{1}{12}q_D + \tfrac{1}{4}$$
$$p_C = \tfrac{1}{12}\beta q_A^2 - \tfrac{1}{12}q_A + \tfrac{1}{6}\beta q_B^2 - \tfrac{1}{6}q_B + \tfrac{7}{12}\beta q_C^2 + \tfrac{5}{12}q_C + \tfrac{1}{6}\beta q_D^2 - \tfrac{1}{6}q_D + \tfrac{1}{4}$$
$$p_D = \tfrac{1}{6}\beta q_A^2 - \tfrac{1}{6}q_A + \tfrac{1}{12}\beta q_B^2 - \tfrac{1}{12}q_B + \tfrac{1}{6}\beta q_C^2 - \tfrac{1}{6}q_C + \tfrac{7}{12}\beta q_D^2 + \tfrac{5}{12}q_D + \tfrac{1}{4}$$

I substitute into each firm's objective function to obtain first order conditions for the first stage of the game.

$$\tfrac{\partial \pi_A}{\partial q_A} = -\tfrac{1}{72}\left(10\beta q_A - 5\right)\left(-5\beta q_A^2 + 5q_A + 2\beta q_B^2 - 2q_B + \beta q_C^2 - q_C + 2\beta q_D^2 - 2q_D + 3\right) = 0$$
$$\tfrac{\partial \pi_B}{\partial q_B} = -\tfrac{1}{72}\left(10\beta q_B - 5\right)\left(2\beta q_A^2 - 2q_A - 5\beta q_B^2 + 5q_B + 2\beta q_C^2 - 2q_C + \beta q_D^2 - q_D + 3\right) = 0$$
$$\tfrac{\partial \pi_C}{\partial q_C} = -\tfrac{1}{72}\left(10\beta q_C - 5\right)\left(\beta q_A^2 - q_A + 2\beta q_B^2 - 2q_B - 5\beta q_C^2 + 5q_C + 2\beta q_D^2 - 2q_D + 3\right) = 0$$
$$\tfrac{\partial \pi_D}{\partial q_D} = -\tfrac{1}{72}\left(10\beta q_D - 5\right)\left(2\beta q_A^2 - 2q_A + \beta q_B^2 - q_B + 2\beta q_C^2 - 2q_C - 5\beta q_D^2 + 5q_D + 3\right) = 0$$

The symmetric solution to this problem is $q_A = q_B = q_C = q_D = \frac{1}{2\beta}$. The second order conditions at this fixed point are $\frac{\partial^2 \pi_A}{\partial q_A^2} = \frac{\partial^2 \pi_B}{\partial q_B^2} = \frac{\partial^2 \pi_C}{\partial q_C^2} = \frac{\partial^2 \pi_D}{\partial q_D^2} = -\frac{5}{12}\beta < 0$ implying that all firms are maximizing their choice of quality. This solution implies that $p_A = p_B = p_C = p_D = \frac{1}{4\beta}(\beta + 1)$ and $\pi_A = \pi_B = \pi_C = \pi_D = \frac{1}{16}$.

Proof of Lemma 3 Using the demand functions provided in the main text, the following first order conditions for the final stage of the game must be satisfied.

$$\frac{\partial \pi_M}{\partial p_A} = p_B - 2p_A + q_A - \frac{1}{2}q_B + \frac{1}{2}p_D - \frac{1}{2}q_D + \frac{1}{4} = 0$$
$$\frac{\partial \pi_M}{\partial p_B} = p_A - 2p_B + \frac{1}{2}p_C - \frac{1}{2}q_A + q_B - \frac{1}{2}q_C + \frac{1}{4} = 0$$
$$\frac{\partial \pi_C}{\partial p_C} = \frac{1}{2}p_B - 2p_C - \frac{1}{2}q_B + q_C + \frac{1}{2}p_D - \frac{1}{2}q_D + \frac{1}{4} = 0$$
$$\frac{\partial \pi_D}{\partial p_D} = \frac{1}{2}p_A + \frac{1}{2}p_C - \frac{1}{2}q_A - \frac{1}{2}q_C - 2p_D + q_D + \frac{1}{4} = 0$$

These conditions imply prices of:

$$p_A = \frac{64}{145}q_A - \frac{6}{145}q_B - \frac{24}{145}q_C - \frac{34}{145}q_D + \frac{2}{5}$$
$$p_B = \frac{64}{145}q_B - \frac{6}{145}q_A - \frac{34}{145}q_C - \frac{24}{145}q_D + \frac{2}{5}$$
$$p_C = \frac{57}{145}q_C - \frac{22}{145}q_B - \frac{7}{145}q_A - \frac{28}{145}q_D + \frac{3}{10}$$
$$p_D = \frac{57}{145}q_D - \frac{7}{145}q_B - \frac{28}{145}q_C - \frac{22}{145}q_A + \frac{3}{10}$$

I substitute into each firm's objective function to obtain first order conditions for the first stage of the game.

$$\frac{\partial \pi_M}{\partial q_A} = \frac{9032}{21025}q_A - \frac{5668}{21025}q_B - \frac{632}{21025}q_C - \frac{2732}{21025}q_D - 2\beta q_A + \frac{4}{25} = 0$$
$$\frac{\partial \pi_M}{\partial q_B} = \frac{9032}{21025}q_B - \frac{5668}{21025}q_A - \frac{2732}{21025}q_C - \frac{632}{21025}q_D - 2\beta q_B + \frac{4}{25} = 0$$
$$\frac{\partial \pi_C}{\partial q_C} = \frac{6498}{21025}q_C - \frac{2508}{21025}q_B - \frac{798}{21025}q_A - \frac{3192}{21025}q_D - 2\beta q_C + \frac{171}{725} = 0$$
$$\frac{\partial \pi_D}{\partial q_D} = \frac{6498}{21025}q_D - \frac{798}{21025}q_B - \frac{3192}{21025}q_C - \frac{2508}{21025}q_A - 2\beta q_D + \frac{171}{725} = 0$$

The unique solution to this game is $q_A = q_B = \frac{57-290\beta}{575\beta-3625\beta^2}$ and $q_C = q_D = \frac{114-855\beta}{1150\beta-7250\beta^2}$. This implies $p_A = p_B = \frac{1}{5}\frac{290\beta-57}{145\beta-23}$ and profit for the merged firm of $\frac{(25\beta-2)(290\beta-57)^2}{625\beta(145\beta-23)^2}$. The competitors prices are $\frac{29}{10}\frac{15\beta-2}{145\beta-23}$ and the profit of each competitor is $\frac{(21025\beta-3249)(15\beta-2)^2}{2500\beta(145\beta-23)^2}$. To ensure that the equilibrium is a unique maximum for all players, three conditions must be satisfied. First, the matrix of first order conditions must be of full rank. Second, the Hessian matrix for the two product firm must negative semi-definite. Third, the second order conditions must be satisfied for all 3 firms.

The matrix of first order conditions A is

$$A = \begin{bmatrix} \frac{9032}{21025} - 2\beta & -\frac{5668}{21025} & -\frac{632}{21025} & -\frac{2732}{21025} \\ -\frac{5668}{21025} & \frac{9032}{21025} - 2\beta & -\frac{2732}{21025} & -\frac{632}{21025} \\ -\frac{798}{21025} & -\frac{2508}{21025} & \frac{6498}{21025} - 2\beta & -\frac{3192}{21025} \\ -\frac{2508}{21025} & -\frac{798}{21025} & -\frac{3192}{21025} & \frac{6498}{21025} - 2\beta \end{bmatrix}$$

The determinant of A is $\frac{16\beta(145\beta-23)\left(-70731\beta+121945\beta^2+9576\right)}{17682025}$. There are three values of β that would lead to the determinant of A being zero. In order for the determinant of A to be non-zero, $\beta \notin \left\{0, \frac{23}{145}, \frac{2439-3\sqrt{43849}}{8410}, \frac{3\sqrt{43849}+2439}{8410}\right\}$ is required.

The Hessian matrix for the merged firm must be negative semi-definite. The Hessian matrix is:

$$\begin{bmatrix} \frac{\partial^2\pi_M}{\partial q_A^2} & \frac{\partial^2\pi_M}{\partial q_A q_B} \\ \frac{\partial^2\pi_M}{\partial q_A q_B} & \frac{\partial^2\pi_M}{\partial q_B^2} \end{bmatrix} = \begin{bmatrix} \frac{9032}{21025}-2\beta & -\frac{5668}{21025} \\ -\frac{5668}{21025} & \frac{9032}{21025}-2\beta \end{bmatrix}$$

The derivative of the principal minors and the relevant conditions are as follows:

Principal minor	Determinant	Condition	Condition on β	Numeric
1st	$\frac{9032}{21025}-2\beta$	negative	$\beta > \frac{4516}{21025}$	0.21479
2nd	$4\beta^2-\frac{36128}{21025}\beta+\frac{2352}{21025}$	positive	$\beta > \frac{294}{841}$	0.34958

The second order condition for the merged firm is the 1st principal minor of the Hessian. For the independent competitors, the second order conditions are satisfied when $\frac{\partial^2\pi_C}{\partial q_C^2} = \frac{\partial^2\pi_D}{\partial q_D^2} = \frac{6498}{21025}-2\beta \Rightarrow \beta > \frac{3249}{21025} \approx 0.15453$. Hence, we restrict our attention to the parameter space $\beta > \frac{3\sqrt{43849}+2439}{8410} \approx 0.36471$ where the matrix A is non-singular, the Hessian matrix of the two product firm is negative semi-definite and the second order conditions are satisfied.

Proof of Lemma 4 Using the demand functions provided in the main text, the following first order conditions for the final stage of the game must be satisfied.

$$\begin{aligned}
\frac{\partial\pi_M}{\partial p_A} &= \tfrac{1}{2}p_B - 2p_A + q_A - \tfrac{1}{2}q_B + \tfrac{1}{2}p_D - \tfrac{1}{2}q_D + \tfrac{1}{4} = 0 \\
\frac{\partial\pi_B}{\partial p_B} &= \tfrac{1}{2}p_A - 2p_B + \tfrac{1}{2}p_C - \tfrac{1}{2}q_A + q_B - \tfrac{1}{2}q_C + \tfrac{1}{4} = 0 \\
\frac{\partial\pi_M}{\partial p_C} &= \tfrac{1}{2}p_B - 2p_C - \tfrac{1}{2}q_B + q_C + \tfrac{1}{2}p_D - \tfrac{1}{2}q_D + \tfrac{1}{4} = 0 \\
\frac{\partial\pi_D}{\partial p_D} &= \tfrac{1}{2}p_A + \tfrac{1}{2}p_C - \tfrac{1}{2}q_A - \tfrac{1}{2}q_C - 2p_D + q_D + \tfrac{1}{4} = 0
\end{aligned}$$

These conditions imply prices of:

$$\begin{aligned}
p_A &= \tfrac{5}{12}q_A - \tfrac{1}{6}q_B - \tfrac{1}{12}q_C - \tfrac{1}{6}q_D + \tfrac{1}{4} \\
p_B &= \tfrac{5}{12}q_B - \tfrac{1}{6}q_A - \tfrac{1}{6}q_C - \tfrac{1}{12}q_D + \tfrac{1}{4} \\
p_C &= \tfrac{5}{12}q_C - \tfrac{1}{6}q_B - \tfrac{1}{12}q_A - \tfrac{1}{6}q_D + \tfrac{1}{4} \\
p_D &= \tfrac{5}{12}q_D - \tfrac{1}{12}q_B - \tfrac{1}{6}q_C - \tfrac{1}{6}q_A + \tfrac{1}{4}
\end{aligned}$$

I substitute into each firm's objective function to obtain first order conditions for the first stage of the game.

$$\begin{aligned}
\frac{\partial\pi_M}{\partial q_A} &= \tfrac{13}{36}q_A - \tfrac{1}{9}q_B - \tfrac{5}{36}q_C - \tfrac{1}{9}q_D - 2\beta q_A + \tfrac{1}{6} = 0 \\
\frac{\partial\pi_B}{\partial q_B} &= \tfrac{25}{72}q_B - \tfrac{5}{36}q_A - \tfrac{5}{36}q_C - \tfrac{5}{72}q_D - 2\beta q_B + \tfrac{5}{24} = 0 \\
\frac{\partial\pi_M}{\partial q_C} &= \tfrac{13}{36}q_C - \tfrac{1}{9}q_B - \tfrac{5}{36}q_A - \tfrac{1}{9}q_D - 2\beta q_C + \tfrac{1}{6} = 0 \\
\frac{\partial\pi_D}{\partial q_D} &= \tfrac{25}{72}q_D - \tfrac{5}{72}q_B - \tfrac{5}{36}q_C - \tfrac{5}{36}q_A - 2\beta q_D + \tfrac{5}{24} = 0
\end{aligned}$$

The unique solution to this game is $q_A = q_C = \frac{5-18\beta}{54\beta-216\beta^2}$ and $q_B = q_D = \frac{10-45\beta}{108\beta-432\beta^2}$. This implies $p_A = p_C = \frac{1}{18}\frac{18\beta-5}{4\beta-1}$ and profit for the merged firm of $\frac{(9\beta-1)(18\beta-5)^2}{1458\beta(4\beta-1)^2}$. The competitors prices are $\frac{1}{9}\frac{9\beta-2}{4\beta-1}$ and the profit of each competitor is $\frac{(9\beta-2)^2(144\beta-25)}{11664\beta(4\beta-1)^2}$. To ensure that the equilibrium is a unique maximum for all players, three conditions must be satisfied. First, the matrix of first order conditions must be of full rank. Second, the Hessian matrix for the merged firm must negative semi-definite. Third, the second order conditions must be satisfied for all 3 firms.

The matrix of first order conditions A is

$$A = \begin{bmatrix} \frac{13}{36}-2\beta & -\frac{1}{9} & -\frac{5}{36} & -\frac{1}{9} \\ -\frac{5}{36} & \frac{25}{72}-2\beta & -\frac{5}{36} & -\frac{5}{72} \\ -\frac{5}{36} & -\frac{1}{9} & \frac{13}{36}-2\beta & -\frac{1}{9} \\ -\frac{5}{36} & -\frac{5}{72} & -\frac{5}{36} & \frac{25}{72}-2\beta \end{bmatrix}$$

The determinant of A is $16\beta^4 - \frac{34}{3}\beta^3 + \frac{8}{3}\beta^2 - \frac{5}{24}\beta$. There are three values of β that would lead to the determinant of A being zero. In order for the determinant of A to be non-zero, $\beta \notin \left\{0, \frac{5}{24}, \frac{1}{4}\right\}$ is required. The Hessian matrix for the merged firm must be negative semi-definite. The Hessian matrix is:

$$\begin{bmatrix} \frac{\partial^2\pi_M}{\partial q_A^2} & \frac{\partial^2\pi_M}{\partial q_A q_C} \\ \frac{\partial^2\pi_M}{\partial q_A q_C} & \frac{\partial^2\pi_M}{\partial q_C^2} \end{bmatrix} = \begin{bmatrix} \frac{13}{36}-2\beta & -\frac{5}{36} \\ -\frac{5}{36} & \frac{13}{36}-2\beta \end{bmatrix}$$

The derivative of the principal minors and the relevant conditions are as follows:

Principal minor	Determinant	Condition	Condition on β	Numeric
1st	$\frac{13}{36}-2\beta$	negative	$\beta > \frac{13}{72}$	0.18056
2nd	$4\beta^2-\frac{13}{9}\beta+\frac{1}{9}$	positive	$\beta > \frac{1}{4}$	0.25

The second order condition for the merged firm is the 1st principal minor of the Hessian. For the independent firms, the second order conditions are satisfied when $\frac{\partial^2\pi_B}{\partial q_B^2} = \frac{\partial^2\pi_D}{\partial q_D^2} = \frac{25}{72}-2\beta \Rightarrow \beta > \frac{25}{144} \approx 0.17361$. Hence, we restrict our attention to the parameter space $\beta > \frac{1}{4}$ where the

matrix A is non-singular, the Hessian matrix of the two product firm is negative semi-definite and the second order conditions are satisfied.

Proof of Proposition 1 Using the expressions in Lemmas 3 and 4, the difference between the profit with a direct takeover and the profit with an indirect takeover is $\Delta_{direct\ vs.\ indirect} = \frac{211973828\beta - 3075376271\beta^2 + 19990524690\beta^3 - 59964200775\beta^4 + 67439790000\beta^5 - 4833836}{7290000\beta(145\beta-23)^2(4\beta-1)^2}$. Because the denominator is positive, the sign of the numerator determines the sign of $\Delta_{direct\ vs.\ indirect}$. The numerator is a polynomial of degree 5 that is positive for all values of $\beta \gtrapprox 0.30486$. The allowable zone for $\beta > \frac{3\sqrt{43849}+2439}{8410} \approx 0.36471$ (Lemma 3), hence $\Delta_{direct\ vs.\ indirect} > 0$. Next, I consider the comparative static of $\Delta_{direct\ vs.\ indirect}$ with respect to β.

$$\frac{\partial \Delta_{direct\ vs.\ indirect}}{\partial \beta} = \frac{-3436857396\beta + 43760064639\beta^2 - 301079321447\beta^3 + 1213624858065\beta^4 - 2773032376725\beta^5 + 2812775989500\beta^6 + 111178228}{7290000\beta^2(145\beta-23)^3(4\beta-1)^3}.$$

$\frac{\partial \Delta_{direct\ vs.\ pre-acq}}{\partial \beta} = \frac{(-7104242375\beta^3 + 201868275\beta^2 + 339269115\beta - 17938367)}{-720000\beta^2(145\beta-23)^3}$.
In the allowable range for β, the denominator is negative so the sign of the numerator determines the sign of $\frac{\partial \Delta_{direct\ vs.\ pre-acq}}{\partial \beta}$. The numerator is a polynomial of degree 3 that is negative for all values of $\beta \gtrapprox 0.15234$. The allowable zone for $\beta > \frac{3\sqrt{43849}+2439}{8410} \approx 0.36471$, hence $\frac{\partial \Delta_{direct\ vs.\ pre-acq}}{\partial \beta} > 0$.

Proof of Lemma 5 Using the demand functions provided in the main text, the following first order conditions for the final stage of the game must be satisfied.

$$\begin{aligned}
\frac{\partial \pi_M}{\partial p_A} &= \beta q_A^2 + q_A - \tfrac{1}{2}\beta q_B^2 - \tfrac{1}{2}q_B - 2p_A + p_B + \tfrac{1}{2}p_D - \tfrac{1}{2}q_D + \tfrac{1}{4} = 0\\
\frac{\partial \pi_M}{\partial p_B} &= -\tfrac{1}{2}\beta q_A^2 - \tfrac{1}{2}q_A + \beta q_B^2 + q_B + p_A - 2p_B + \tfrac{1}{2}p_C - \tfrac{1}{2}q_C + \tfrac{1}{4} = 0\\
\frac{\partial \pi_C}{\partial p_C} &= \beta q_C^2 + q_C + \tfrac{1}{2}p_B - 2p_C - \tfrac{1}{2}q_B + \tfrac{1}{2}p_D - \tfrac{1}{2}q_D + \tfrac{1}{4} = 0\\
\frac{\partial \pi_D}{\partial p_D} &= \beta q_D^2 + q_D + \tfrac{1}{2}p_A + \tfrac{1}{2}p_C - \tfrac{1}{2}q_A - \tfrac{1}{2}q_C - 2p_D + \tfrac{1}{4} = 0
\end{aligned}$$

These conditions imply prices of:

$$\begin{aligned}
p_A &= \tfrac{81}{145}\beta q_A^2 + \tfrac{64}{145}q_A + \tfrac{6}{145}\beta q_B^2 - \tfrac{6}{145}q_B + \tfrac{24}{145}\beta q_C^2 - \tfrac{24}{145}q_C + \tfrac{34}{145}\beta q_D^2 - \tfrac{34}{145}q_D + \tfrac{2}{5}\\
p_B &= \tfrac{6}{145}\beta q_A^2 - \tfrac{6}{145}q_A + \tfrac{81}{145}\beta q_B^2 + \tfrac{64}{145}q_B + \tfrac{34}{145}\beta q_C^2 - \tfrac{34}{145}q_C + \tfrac{24}{145}\beta q_D^2 - \tfrac{24}{145}q_D + \tfrac{2}{5}\\
p_C &= \tfrac{7}{145}\beta q_A^2 - \tfrac{7}{145}q_A + \tfrac{22}{145}\beta q_B^2 - \tfrac{22}{145}q_B + \tfrac{88}{145}\beta q_C^2 + \tfrac{57}{145}q_C + \tfrac{28}{145}\beta q_D^2 - \tfrac{28}{145}q_D + \tfrac{3}{10}\\
p_D &= \tfrac{22}{145}\beta q_A^2 - \tfrac{22}{145}q_A + \tfrac{7}{145}\beta q_B^2 - \tfrac{7}{145}q_B + \tfrac{28}{145}\beta q_C^2 - \tfrac{28}{145}q_C + \tfrac{88}{145}\beta q_D^2 + \tfrac{57}{145}q_D + \tfrac{3}{10}
\end{aligned}$$

I substitute into each firm's objective function to obtain first order conditions for the first stage of the game.

$$\begin{aligned}
\frac{\partial \pi_M}{\partial q_A} &= -\frac{4(2\beta q_A - 1)(-2258\beta q_A^2 + 2258q_A + 1417\beta q_B^2 - 1417q_B + 158\beta q_C^2 - 158q_C + 683\beta q_D^2 - 683q_D + 841)}{21025} = 0\\
\frac{\partial \pi_M}{\partial q_B} &= -\frac{4(2\beta q_B - 1)(1417\beta q_A^2 - 1417q_A - 2258\beta q_B^2 + 2258q_B + 683\beta q_C^2 - 683q_C + 158\beta q_D^2 - 158q_D + 841)}{21025} = 0\\
\frac{\partial \pi_C}{\partial q_C} &= -\frac{57(2\beta q_C - 1)(-14q_A - 44q_B + 114q_C - 56q_D + 14\beta q_A^2 + 44\beta q_B^2 - 114\beta q_C^2 + 56\beta q_D^2 + 87)}{21025} = 0\\
\frac{\partial \pi_D}{\partial q_D} &= -\frac{57(2\beta q_D - 1)(-44q_A - 14q_B - 56q_C + 114q_D + 44\beta q_A^2 + 14\beta q_B^2 + 56\beta q_C^2 - 114\beta q_D^2 + 87)}{21025} = 0
\end{aligned}$$

There are five symmetric solutions to this problem.

1. $\left[q_{A,B} = \frac{1}{2\beta}, q_{C,D} = \frac{1}{2}\sqrt{\frac{1}{\beta}}\left(\sqrt{\frac{1}{\beta}} + \sqrt{6}\right)\right]$
2. $\left[q_{A,B} = -\frac{1}{2\beta}\left(2\beta^2\sqrt{\frac{1}{\beta^3}} - 1\right), q_{C,D} = \frac{1}{2\beta}\right]$
3. $\left[q_{A,B} = \frac{1}{2\beta}\left(2\beta^2\sqrt{\frac{1}{\beta^3}} + 1\right), q_{C,D} = \frac{1}{2\beta}\right]$.
4. $\left[q_{A,B} = \frac{1}{2\beta}, q_{C,D} = \frac{1}{2\beta}\right]$
5. $\left[q_{A,B} = \frac{1}{2\beta}, q_{C,D} = \frac{1}{2}\left(\sqrt{\frac{1}{\beta}} - \sqrt{6}\right)\sqrt{\frac{1}{\beta}}\right]$

The only solution which satisfies the second order conditions and the feasibility conditions is $\left[q_{A,B} = \frac{1}{2\beta}, q_{C,D} = \frac{1}{2\beta}\right]$. The other roots are infeasible. They involve the merged firm capturing the entire market or the competitors capturing the entire market (Roots 1 and 3). They are "local maxima" due to the nature of profit functions but are not best responses. Roots 2 and 5 entail choices of negative quality. The equilibrium prices are $p_{A,B} = \frac{1}{20\beta}(8\beta + 5)$ and $p_{C,D} = \frac{1}{20\beta}(6\beta + 5)$. The merged firm earns profit of $\frac{4}{25}$ and the competitors earn $\frac{9}{100}$. The merged firm realizes a 28% increase in profit compared to the pre-acquisition profit of the two firms. The competitors realize a 44% increase in profit due to acquisition.

Proof of Lemma 6 Using the demand functions provided in the main text, the following first order conditions for the final stage of the game must be satisfied.

$$\begin{aligned}
\frac{\partial \pi_M}{\partial p_A} &= \beta q_A^2 + q_A - 2p_A + \tfrac{1}{2}p_B - \tfrac{1}{2}q_B + \tfrac{1}{2}p_D - \tfrac{1}{2}q_D + \tfrac{1}{4} = 0\\
\frac{\partial \pi_B}{\partial p_B} &= \beta q_B^2 + q_B + \tfrac{1}{2}p_A - 2p_B + \tfrac{1}{2}p_C - \tfrac{1}{2}q_A - \tfrac{1}{2}q_C + \tfrac{1}{4} = 0\\
\frac{\partial \pi_M}{\partial p_C} &= \beta q_C^2 + q_C + \tfrac{1}{2}p_B - 2p_C - \tfrac{1}{2}q_B + \tfrac{1}{2}p_D - \tfrac{1}{2}q_D + \tfrac{1}{4} = 0\\
\frac{\partial \pi_D}{\partial p_D} &= \beta q_D^2 + q_D + \tfrac{1}{2}p_A + \tfrac{1}{2}p_C - \tfrac{1}{2}q_A - \tfrac{1}{2}q_C - 2p_D + \tfrac{1}{4} = 0
\end{aligned}$$

These conditions imply prices of:

$$p_A = \frac{7}{12}\beta q_A^2 + \frac{5}{12}q_A + \frac{1}{6}\beta q_B^2 - \frac{1}{6}q_B + \frac{1}{12}\beta q_C^2 - \frac{1}{12}q_C + \frac{1}{6}\beta q_D^2 - \frac{1}{6}q_D + \frac{1}{4}$$
$$p_B = \frac{1}{6}\beta q_A^2 - \frac{1}{6}q_A + \frac{7}{12}\beta q_B^2 + \frac{5}{12}q_B + \frac{1}{6}\beta q_C^2 - \frac{1}{6}q_C + \frac{1}{12}\beta q_D^2 - \frac{1}{12}q_D + \frac{1}{4}$$
$$p_C = \frac{1}{12}\beta q_A^2 - \frac{1}{12}q_A + \frac{1}{6}\beta q_B^2 - \frac{1}{6}q_B + \frac{7}{12}\beta q_C^2 + \frac{5}{12}q_C + \frac{1}{6}\beta q_D^2 - \frac{1}{6}q_D + \frac{1}{4}$$
$$p_D = \frac{1}{6}\beta q_A^2 - \frac{1}{6}q_A + \frac{1}{12}\beta q_B^2 - \frac{1}{12}q_B + \frac{1}{6}\beta q_C^2 - \frac{1}{6}q_C + \frac{7}{12}\beta q_D^2 + \frac{5}{12}q_D + \frac{1}{4}$$

I substitute into each firm's objective function to obtain first order conditions for the first stage of the game.

$$\frac{\partial \pi_M}{\partial q_A} = -\frac{(2\beta q_A - 1)\left(-13\beta q_A^2 + 13q_A + 4\beta q_B^2 - 4q_B + 5\beta q_C^2 - 5q_C + 4\beta q_D^2 - 4q_D + 6\right)}{36} = 0$$
$$\frac{\partial \pi_M}{\partial q_C} = -\frac{(2\beta q_C - 1)\left(5\beta q_A^2 - 5q_A + 4\beta q_B^2 - 4q_B - 13\beta q_C^2 + 13q_C + 4\beta q_D^2 - 4q_D + 6\right)}{36} = 0$$
$$\frac{\partial \pi_B}{\partial q_B} = -2\left(\frac{5}{6}\beta q_B - \frac{5}{12}\right)\left(\frac{1}{6}\beta q_A^2 - \frac{1}{6}q_A - \frac{5}{12}\beta q_B^2 + \frac{5}{12}q_B + \frac{1}{6}\beta q_C^2 - \frac{1}{6}q_C + \frac{1}{12}\beta q_D^2 - \frac{1}{12}q_D + \frac{1}{4}\right) = 0$$
$$\frac{\partial \pi_D}{\partial q_D} = -2\left(\frac{5}{6}\beta q_D - \frac{5}{12}\right)\left(\frac{1}{6}\beta q_A^2 - \frac{1}{6}q_A + \frac{1}{12}\beta q_B^2 - \frac{1}{12}q_B + \frac{1}{6}\beta q_C^2 - \frac{1}{6}q_C - \frac{5}{12}\beta q_D^2 + \frac{5}{12}q_D + \frac{1}{4}\right) = 0$$

There are five symmetric solutions to this problem.

1. $\left[q_{A,C} = \frac{1}{2\beta}, q_{B,D} = \frac{1}{2}\sqrt{\frac{1}{\beta}}\left(\sqrt{\frac{1}{\beta}} + \sqrt{3}\right)\right]$
2. $\left[q_{A,C} = \frac{1}{2\beta}, q_{B,D} = \frac{1}{2\beta}\right]$
3. $\left[q_{A,C} = \frac{1}{2\beta}, q_{B,D} = \frac{1}{2}\left(\sqrt{\frac{1}{\beta}} - \sqrt{3}\right)\sqrt{\frac{1}{\beta}}\right]$
4. $\left[q_{A,C} = -\frac{1}{2\beta}\left(\sqrt{3}\beta^2\sqrt{\frac{1}{\beta^3}} - 1\right), q_{B,D} = \frac{1}{2\beta}\right]$
5. $\left[q_{A,C} = \frac{1}{2\beta}\left(\sqrt{3}\beta^2\sqrt{\frac{1}{\beta^3}} + 1\right), q_{B,D} = \frac{1}{2\beta}\right]$

The only solution which satisfies the second order conditions and the feasibility conditions is $\left[q_{A,C} = \frac{1}{2\beta}, q_{B,D} = \frac{1}{2\beta}\right]$. The other roots are infeasible. They involve the merged firm capturing the entire market or the competitors capturing the entire market (Roots 1 and 5). They are "local maxima" due to the nature of profit functions but are not best responses. Roots 3 and 4 entail choices of negative quality. The equilibrium prices are $\frac{1}{4\beta}(\beta + 1)$ and the profits of the merged firm is $\frac{1}{8}$ and of the competitors is $\frac{1}{16}$.

Proof of Proposition 2 Because $\frac{4}{25} > \frac{1}{8}$, a direct acquisition when variable costs are used to increase quality is strictly preferred to an indirect acquisition. The benefit of a direct acquisition is unrelated to β.

Proof of Corollary 1 The ordering of qualities post and pre-acquisition when $\beta > \frac{\sqrt{2372329}+503}{5220}$ is $q_{direct\ acq}^{competitor} > q_{indirect\ acq}^{competitor} > q_{pre\ acq} > q_{indirect\ acq}^{merged\ firm} > q_{direct\ acq}^{merged\ firm}$. First, I let $\Delta_1 = q_{direct\ acq}^{competitor} - q_{indirect\ acq}^{competitor} = \frac{1}{2700\beta(145\beta-23)(4\beta-1)}8669\beta + 21555\beta^2 + 406$. The first term is positive in the allowable range for β and the second term is positive for all $\beta > \frac{\sqrt{40146241}-8669}{43110} \approx -0.54$. Second, I let $\Delta_2 = q_{indirect\ acq}^{competitor} - q_{pre\ acq} = \frac{5}{432\beta(4\beta-1)} > 0$. Third, I let $\Delta_3 = q_{pre\ acq} - q_{indirect\ acq}^{merged\ firm} = \frac{1}{432}\frac{36\beta-5}{\beta(4\beta-1)} > 0$. Fourth, I let $\Delta_4 = q_{indirect\ acq}^{merged\ firm} - q_{direct\ acq}^{merged\ firm} = \frac{1}{1350\beta(145\beta-23)(4\beta-1)}(-503\beta + 2610\beta^2 - 203)$. The first term is positive in the allowable range and the second term is a parabola which is positive for $\beta > \frac{\sqrt{2372329}+503}{5220} \approx 0.39142$ and $\beta <, \frac{503-\sqrt{2372329}}{5220} \approx -0.1987$.

Proof of Corollary 2 The gap between the merged firm and its competitors for a direct acquisition is $Gap_1 = \frac{11}{2(145\beta-23)}$. The gap between the merged firm and its competitors for an indirect acquisition is $Gap_2 = \frac{1}{12(4\beta-1)}$. $Gap_1 - Gap_2 = \frac{1}{12}\frac{119\beta-43}{(145\beta-23)(4\beta-1)} > 0$ for all $\beta > \frac{43}{119} \approx 0.36134$ and the allowable zone is $\beta > .36471$.

Proof of Corollary 3 The average quality pre-acquisition is $\overline{q}_{pre\ acq} = \frac{5}{48\beta}$, the average quality after a direct acquisition near firm is $\overline{q}_{direct} = \frac{1}{100}\frac{1435\beta-228}{\beta(145\beta-23)}$ and the average quality after an indirect acquisition is $\overline{q}_{indirect} = \frac{1}{216}\frac{81\beta-20}{\beta(4\beta-1)}$. $\overline{q}_{pre\ acq} - \overline{q}_{direct} = \frac{1}{1200}\frac{905\beta-139}{\beta(145\beta-23)} > 0$ for all $\beta \gtrapprox 0.15862$. $2\overline{q}_{direct} - \overline{q}_{indirect} = \frac{1}{5400}\frac{-7663\beta+16335\beta^2+812}{\beta(145\beta-23)(4\beta-1)} > 0$ for all $\beta > \frac{\sqrt{5665489}+7663}{32670} \approx 0.30741$ and the allowable zone is $\beta > .36471$.

Proof of Lemma 7 With fixed investments in quality, the expression for total welfare is as follows:

$$W = benefit\ from\ consumption - transportation\ \cos t - investments\ in\ quality \tag{i}$$

The benefit from consumption when all 4 firms operate is:

$$B = \left((1 - x_4) + x_1\right)(v + q_A) + (x_2 - x_1)(v + q_q) + (x_3 - x_2)(v + q_C) \tag{ii}$$

$$+\left(x_4 - x_3\right)(v + q_D) \tag{iii}$$

The transportation costs are as follows:

$$\begin{aligned} T = {} & \int_0^{x_1} y dy + \int_{x_1}^{\frac{1}{4}}\left(\frac{1}{4} - y\right)dy + \int_{\frac{1}{4}}^{x_2}\left(y - \frac{1}{4}\right)dy + \int_{x_2}^{\frac{1}{2}}\left(\frac{1}{2} - y\right)dy \\ & + \int_{\frac{1}{2}}^{x_3}\left(y - \frac{1}{2}\right)dy + \int_{x_3}^{\frac{3}{4}}\left(\frac{3}{4} - y\right)dy + \int_{\frac{3}{4}}^{x_4}\left(y - \frac{3}{4}\right)dy + \int_{x_4}^{1}(1 - y)dy \end{aligned} \tag{iv}$$

The investments in quality are:

$$I = \beta\left(q_A^2 + q_B^2 + q_C^2 + q_D^2\right) \tag{v}$$

As long as prices are equal, consumers will buy from the firm that offers the combination of quality and transportation cost that maximizes utility. Using the expressions from Table 1, I construct the function for W and optimize for the qualities. The first order conditions are:

$$\frac{\partial W}{\partial q_A} = q_A - \frac{1}{2}q_B - \frac{1}{2}q_D - 2\beta q_A + \frac{1}{4} = 0$$
$$\frac{\partial W}{\partial q_B} = q_B - \frac{1}{2}q_A - \frac{1}{2}q_C - 2\beta q_B + \frac{1}{4} = 0$$
$$\frac{\partial W}{\partial q_C} = q_C - \frac{1}{2}q_B - \frac{1}{2}q_D - 2\beta q_C + \frac{1}{4} = 0$$
$$\frac{\partial W}{\partial q_D} = q_D - \frac{1}{2}q_C - \frac{1}{2}q_A - 2\beta q_D + \frac{1}{4} = 0$$

The solution to these equations is $q_A = q_B = q_C = q_D = \frac{1}{8\beta}$ and $W_{all\ firms} = \frac{1}{16}\frac{-\beta+16v\beta+1}{\beta}$. Here, the central planner serves customers at all four locations, the optimal quality for the firms is $\frac{1}{8\beta}$ and transportation costs are minimized. However, there are two discontinuous possibilities that need to be considered. First, the central planner could set high quality at one firm and 0 quality at the other firms: all consumers would travel to one firm to realize the benefit of high quality. Second, the central planner could set a high qualities at two firms located on opposite sides of the circular market, 0 quality at the other firms and have consumers travel to the firm that is closest.

1. Option 1: Here one firm serves all consumers. I assume the firm that invests in quality is Firm A and consumers travel at most $\frac{1}{2}$ to patronize Firm A. This implies that $W_{one\ firm} = v - 2\int_0^{\frac{1}{2}} x dx + q_A - \beta q_A^2$. Optimizing with respect to q_A, I obtain $q_A = \frac{1}{2\beta}$. This leads to total welfare of $\frac{1}{4}\frac{-\beta+4v\beta+1}{\beta}$.
2. Option 2: Here, two firms serve all consumers and prices are fixed at 0. I assume the firms that invest in quality are Firms A and C. The indifference point between Firm A and C is given by $x^* = \frac{1}{2}q_A - \frac{1}{2}q_C + \frac{1}{4}$. This implies that $W_{two\ firms} = v + 2x^* q_A + 2\left(\frac{1}{2} - x^*\right)q_C - 2\int_0^{x^*} x dx - 2\int_{x^*}^{\frac{1}{2}}\left(\frac{1}{2} - x\right)dx - \beta q_A^2 - \beta q_C^2$. Substituting I have $W = v + \frac{1}{2}q_A + \frac{1}{2}q_C + \frac{1}{2}q_A^2 + \frac{1}{2}q_C^2 - q_A q_C - \beta q_A^2 - \beta q_C^2 - \frac{1}{8}$. Optimizing with respect to q_A and q_C, I obtain $q_A = q_C = \frac{1}{4\beta}$. This leads to total welfare of $\frac{1}{8}\frac{-\beta+8v\beta+1}{\beta}$.

Straightforward calculations show that when $\beta < 1$, $W_{one\ firm} > W_{two\ firms} > W_{all\ firms}$ and when $\beta > 1$, $W_{all\ firms} > W_{two\ firms} > W_{one\ firm}$.

Proof of Proposition 3 In order to determine total welfare pre-acquisition and for direct and indirect acquisitions, note that all firms realize positive demand. Hence, the indifferent points of Table 1 determine the transportation costs and benefit associated with consumption at each firm. Let W be total welfare in the market.

$$W = Basic\ benefit + benefit_{quality} - transportation\ cost - investments\ in\ quality \quad \text{(vi)}$$

The basic benefit is v. The benefit created by quality when all 4 firms operate is:

$$\left(\left(1 - x_4\right) + x_1\right)q_A + \left(x_2 - x_1\right)q_B + \left(x_3 - x_2\right)q_C + \left(x_4 - x_3\right)q_D \quad \text{(vii)}$$

The transportation costs are as follows:

$$\int_0^{x_1} y dy + \int_{x_1}^{\frac{1}{4}}\left(\frac{1}{4} - y\right)dy + \int_{\frac{1}{4}}^{x_2}\left(y - \frac{1}{4}\right)dy + \int_{x_2}^{\frac{1}{2}}\left(\frac{1}{2} - y\right)dy + \int_{\frac{1}{2}}^{x_3}\left(y - \frac{1}{2}\right)dy + \int_{x_3}^{\frac{3}{4}}\left(\frac{3}{4} - y\right)dy + \int_{\frac{3}{4}}^{x_4}\left(y - \frac{3}{4}\right)dy + \int_{x_4}^{1}(1 - y)dy \quad \text{(viii)}$$

The investments in quality are:

$$\beta\left(q_A^2 + q_B^2 + q_C^2 + q_D^2\right) \quad \text{(ix)}$$

Simple substitution of the equilibrium values from Lemmas 3 and 4 yield the following expressions for Total Welfare.

$$W_{pre-acquisition} = \frac{1}{576}\frac{-36\beta + 576v\beta + 35}{\beta}$$

$$W_{direct} = \frac{-4325235\beta + 5290000v\beta + 17248850\beta^2 - 14191875\beta^3 - 66700000v\beta^2 + 210250000v\beta^3 + 316464}{10000\beta(145\beta - 23)^2}$$

$$W_{indirect} = \frac{-3069\beta + 5832v\beta + 8343\beta^2 - 5832\beta^3 - 46656v\beta^2 + 93312v\beta^3 + 340}{5832\beta(4\beta - 1)^2}$$

We know from Lemma 3 that the allowable range is $\beta > \frac{3\sqrt{43849}+2439}{8410} \approx .36471$.

Step 1 I let $\Delta_1 = W_{pre-acquisition} - W_{direct}$. This is positive when $-2100290\beta - 10961725\beta^2 + 37845000\beta^3 + 179171 > 0$. The roots are too long for presentation but are approximately $\left(-0.17656, 6.7205 \times 10^{-2}, 0.399\right)$. Only the third root lies in the feasible range, hence $\Delta_1 > 0$ when $\beta \gtrapprox 0.399$.

Step 2 I let $\Delta_2 = W_{direct} - W_{indirect}$. This is positive when $134588113\beta - 1450197016\beta^2 + 7317382365\beta^3 - 16068832650\beta^4 + 12261780000\beta^5 - 5877256 > 0$. Solving the expression numerically, I find 5 roots: 2 are complex numbers $7.2044 \times 10^{-2} + 7.2629 \times 10^{-2}i$ and $7.2044 \times 10^{-2} - 7.2629 \times 10^{-2}i$ and three are real 0.21250, 0.36766 and 0.58624. Hence $\Delta_2 > 0$ when $\beta \in (\approx 0.36766, \approx 0.58624)$.

Step 3 I let $\Delta_3 = W_{pre-acquisition} - W_{indirect}$. This is positive when $-1044\beta + 1944\beta^2 + 115 > 0$. The roots are $\frac{29+\sqrt{151}}{108} \approx 0.38230$ and $\frac{29-\sqrt{151}}{108} \approx 0.15474$. Only the

first root lies in the feasible range, hence $\Delta_3 > 0$ when $\beta \gtrapprox 0.38230$.

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s40547-022-00131-6.

Declarations

Conflict of Interest The author declares no competing interests.

References

1. Armstrong M, Chen Y (2009) Inattentive consumers and product quality. J Eur Econ Assoc 7(2–3):411–422
2. Atmar H, Hudson S, Koshy A, Rickert S, Slelatt R (2020) The next normal in consumer: Implications for Consumer Goods M&A. McKinsey Consumer Packaged Goods, June 24. https://www.mckinsey.com/industries/consumer-packaged-goods/our-insights/the-next-normal-in-consumer-implications-for-consumer-goods-m-and-a. Accessed 21 Oct 2022
3. Brander J, Eaton J (1984) Product line rivalry. Am Econ Rev 74:323–334
4. Brekke KR, Siciliani L, Straume OR (2017) Horizontal mergers and product quality. Canadian J Econ 50(4):1063–1103
5. Bulow J, Geanakoplos J, Klemperer P (1985) Multimarket oligopoly: strategic substitutes and strategic complements. J Polit Econ 93:488–511
6. Chambers C, Kouvelis P, Semple J (2006) Quality-Based Competition, profitability, and variable costs. Manag Sci 52(12):1884–1895
7. Chioveanu I (2012) Price and quality competition. J Econ 107(1):p23-44
8. Davidson C, Deneckere R (1984) Horizontal mergers and collusive behavior. Int J Ind Organ 2(2):117–132
9. Davidson C, Deneckere R (1985) Incentives to form coalitions with Bertrand competition. Rand J Econ 16(4):473–486
10. Esteban L, Hernández J (2014) Endogenous direct advertising and price competition. J Econ 112(3):p225-251
11. Fan Y (2013) Ownership consolidation and product characteristics: a study of the US daily newspaper market. Am Econ Rev 103(5):1598–1628
12. Farrell J, Shapiro C (1990) Horizontal mergers: an equilibrium analysis. Am Econ Rev 80(1):107–126
13. Ghose A, Huang Ke-Wei (2009) Personalized pricing and quality customization. J Econ Manag Strategy 18(4):p1095-1135
14. Ghosh B, Balachander S (2007) Competitive bundling and counterbundling with generalist and specialist firms. Manag Sci 53(1):159–168
15. Giraud-Heraud E, Hammoudi H, Mokrane M (1999) Industrial structure and strategic takeover in a differentiated market working paper 99-34, Paris X - Nanterre, U.F.R. de Sc. Ec. Gest. Maths Infor
16. Giraud-Heraud E, Hammoudi H, Mokrane M (2003) Multiproduct firm behavior in a differentiated market. Can J Econ 36(1):41–61
17. Hotelling H (1929) Stability in competition. Econ J 39:41–57
18. Iyer G (1998) Coordinating channels under price and nonprice competition. Mark Sci 17(4):338–355
19. Kirby J (2020) Maple glazed Tim Hortons. Macleans 133(3):36–37
20. Levy DT, Reitzes JD (1992) Anticompetitive effects of mergers in markets with localized competition, Journal of Law. Econ Organ 8(2):427–440
21. Lilien GL, Kotler P, Moorthy S (1992) Marketing models. Prentice-Hall, Inc, Englewood Cliffs, New Jersey, pp 223–236
22. Maskin E, Riley J (1984) Monopoly with incomplete information. Rand J Econ 15(2):171–196
23. McAfee R, Simons J, Williams M (1992) Preston horizontal mergers in spatially differentiated noncooperative markets. J Ind Econ 40(4):349–358
24. Mendelson H, Parlaktürk AK (2008) Product-line competition: Customization vs Proliferation. Manage Sci 54(12):2039–2053
25. Moorthy KS (1988) Product and price competition in a duopoly. Mark Sci 7(2):141–168
26. Motta M (1993) Endogenous quality Choice:Price versus quantity competition. J Ind Econ 41(2):113–131
27. Mussa M, Rosen S (1978) Monopoly and product quality. J Econ Theory 18(2):301–317
28. Norman G, Pepall L (2000) Profitable mergers in a cournot model of spatial competition. South Econ J 66(3):667–681
29. Panzar JC, Willig RD (1981) Economies of scope. Am Econ Rev 71(2):268–272
30. Perry MK, Porter RH (1985) Oligopoly and the incentive for horizontal merger. Am Econ Rev 75(1):219–227
31. Rhee B-D (1996) Consumer heterogeneity and strategic quality decisions. Manag Sci 42(2):157–172
32. Rothschild R (2000) Mergers under spatial competition, urban studies, special issue: location theory. Anal Appl 37(3):443–449
33. Salop S (1979) Monopolistic competition with outside goods. Bell J Econ 10(1):141–156
34. Shaked A, Sutton J (1982) Relaxing price competition through product differentiation. Rev Econ Stud 49(155):3–13
35. Stigler GJ (1950) Monopoly and oligopoly by merger. Am Econ Rev (Papers and Proceedings) 40:23–34
36. Tenn S, Froeb L, Tschantz S (2010) Mergers when firms compete by choosing both price and promotion. Int J Ind Organ 28(6):695–707
37. Tirole J (1988) The theory of industrial organization. MITPress, Cambridge Massacchusetts
38. Wernerfelt B (1986) Product line rivalry. Am Econ Rev 76(4):842–844
39. Wolinsky A (1983) Prices as signals of product quality. Rev Econ Stud 50(4):647–658
40. Williamson O (1968) Economies as an Antitrust defense: The welfare tradeoffs. Am Econ Rev 58(1):18–36
41. Willig R (2011) Unilateral competitive effects of mergers: upward pricing pressure, product quality, and other extensions. Rev Ind Organ 39(1/2):19–38